**1.** Sea $p_k = 1 - \sum_{i=1}^{k-1} p_i$, $x_k = n - \sum_{i=1}^{k-1} x_i$, y

$$h(\boldsymbol{x}) = \binom{n}{x_1, \dots, x_k}.$$

De este modo, la función de probabilidad de la distribución multinomial puede ser escrita como

$$\begin{aligned}
f(x_1, \dots, x_{k-1}; \boldsymbol{p}) &= h(\boldsymbol{x}) \exp(\log(p_1^{x_1} \cdots p_k^{x_k})) = h(\boldsymbol{x}) \exp\Big(\sum_{i=1}^{k} x_i \log p_i\Big) \\
&= h(\boldsymbol{x}) \exp\Big(\sum_{i=1}^{k-1} x_i \log p_i + x_k \log p_k\Big) \\
&= h(\boldsymbol{x}) \exp\Big(\sum_{i=1}^{k-1} x_i \log p_i + \Big(n - \sum_{i=1}^{k-1} x_i\Big) \log\Big(1 - \sum_{i=1}^{k-1} p_i\Big)\Big) \\
&= h(\boldsymbol{x}) \exp\Big(\sum_{i=1}^{k-1} x_i \Big(\log p_i - \log\Big(1 - \sum_{i=1}^{k-1} p_i\Big)\Big) + n \log\Big(1 - \sum_{i=1}^{k-1} p_i\Big)\Big) \\
&= h(\boldsymbol{x}) \exp\Big(\sum_{i=1}^{k-1} x_i \log\Big(\frac{p_i}{1 - \sum_{j=1}^{k-1} p_j}\Big) + n \log\Big(1 - \sum_{i=1}^{k-1} p_i\Big)\Big).
\end{aligned}$$

Es decir, la distribución multinomial pertenece a la $\mathsf{FE}$ $(k-1)$-paramétrica con

$$b(\theta) = -n\Big(1 - \sum_{i=1}^{k-1} p_i\Big),$$

y

$$\eta_i(\boldsymbol{p}) = \log\Big(\frac{p_i}{1 - \sum_{j=1}^{k-1} p_j}\Big), \qquad T_i(\boldsymbol{x}) = x_i,$$

para $i = 1, \dots, k-1$.