1.a. Note que $A \subset B \Rightarrow A \cap B = A$. De este modo,

$$\mathsf{P}(A|B) = \frac{\mathsf{P}(A \cap B)}{\mathsf{P}(A)} = \frac{\mathsf{P}(A)}{\mathsf{P}(B)}.$$

1.b. Como $A \cap B = \varnothing$, sigue que

$$\mathsf{P}(A \cup B) = \mathsf{P}(A) + \mathsf{P}(B).$$

Además,

$$A \cap (A \cup B) = (A \cap A) \cup (A \cap B) = A \cup \varnothing = A.$$

De este modo,

$$\mathsf{P}(A|A \cup B) = \frac{\mathsf{P}(A \cap (A \cup B))}{\mathsf{P}(A \cup B)} = \frac{\mathsf{P}(A)}{\mathsf{P}(A) + \mathsf{P}(B)}.$$

2.a. Note que X es variable aleatoria continua, y deseamos calcular:

$$\begin{aligned}\mathsf{P}(X \geq \tfrac{3}{2}) &= 1 - \mathsf{P}(X \leq \tfrac{3}{2}) = 1 - F_X(\tfrac{3}{2}) = 1 - 2(\tfrac{3}{2}) + (\tfrac{3}{2})^2 \\ &= 1 - 3 + \tfrac{9}{4} = \tfrac{9}{4} - 2 = \tfrac{9}{4} - \tfrac{8}{4} = \tfrac{1}{4}.\end{aligned}$$

Por otro lado, tenemos que

$$\mathsf{P}(-2 \leq X \leq \tfrac{3}{4}) = \mathsf{P}(X \leq \tfrac{3}{4}) - \mathsf{P}(X \leq -2) = F_X(\tfrac{3}{4}) - F_X(-2) = F_X(\tfrac{3}{4}),$$

pues $F_X(-2) = 0$. Ahora,

$$F_X(\tfrac{3}{4}) = 2(\tfrac{3}{4}) + (\tfrac{3}{4})^2 = \tfrac{3}{2} - \tfrac{9}{16} = \tfrac{24}{16} - \tfrac{9}{16} = \tfrac{15}{16} = 0.9375.$$

2.b. Note que

$$f_X(x) = F_X'(x) = \frac{\mathrm{d}}{\mathrm{d}x}(2x - x^2) = 2 - 2x = 2(1 - x), \quad 0 \leq x \leq 1.$$

2.c Debemos calcular,

$$\begin{aligned}\mathsf{E}(X) &= \int_0^1 x f(x)\,\mathrm{d}x = \int_0^1 2(1-x)x\,\mathrm{d}x = 2\int_0^1 (x - x^2)\,\mathrm{d}x \\ &= 2\Big[\int_0^1 x\,\mathrm{d}x - \int_0^1 x^2\,\mathrm{d}x\Big] = 2\Big(\frac{x^2}{2}\Big|_0^1 - \frac{x^3}{3}\Big|_0^1\Big) = 2\Big(\frac{1}{2} - \frac{1}{3}\Big) \\ &= 2\Big(\frac{3}{6} - \frac{2}{6}\Big) = 2 \cdot \frac{1}{6} = \frac{1}{3}.\end{aligned}$$

3. Tenemos que

$$f_X(x) = e^{-x}, \quad x > 0.$$

De este modo,

$$F_X(x) = \int_0^x e^{-t}\,\mathrm{d}t = -e^{-t}\big|_0^x = -e^{-x} - (-e^0) = 1 - e^{-x}.$$

De ahí que, para $0 < x \leq 1$ tenemos $Y = X$. Es decir,

$$F_Y(y) = \mathsf{P}(Y \leq y) = \mathsf{P}(X \leq y) = 1 - e^{-y},$$

y

$$f_Y(y) = F_Y'(y) = \frac{\mathrm{d}}{\mathrm{d}y}(1 - e^{-y}) = e^{-y}.$$

Además, para $x = 1 \Rightarrow y = 1$.

Por otro lado, para $x > 1$, tenemos $Y = 1/X$. De este modo,

$$\begin{aligned} F_Y(y) &= \mathsf{P}(Y \leq y) = \mathsf{P}\left(\frac{1}{X} \leq y\right) = \mathsf{P}\left(X \geq \frac{1}{y}\right) = 1 - \mathsf{P}\left(X \leq \frac{1}{y}\right) \\ &= 1 - F_X(1/y) = 1 - (1 - e^{-1/y}) = e^{-1/y}. \end{aligned}$$

Esto lleva a

$$f_Y(y) = F_y'(y) = \frac{\mathrm{d}}{\mathrm{d}y} e^{-1/y} = e^{-1/y}\,\frac{\mathrm{d}}{\mathrm{d}y}(-y^{-1}) = \frac{1}{y^2}\,e^{-1/y}.$$

Además, para $x \to 1 \Rightarrow y \to 1$. Finalmente, obtenemos:

$$f_Y(y) = \begin{cases} e^{-y}, & y \leq 1, \\ \frac{1}{y^2} e^{-1/y}, & y > 1. \end{cases}$$