MAT-266: Análisis de Regresión **Nombre:** ________________________
Certamen 2. Mayo 10, 2023 **Profesor:** Felipe Osorio

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1. (50 pts) Sea

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon},$$

donde $\boldsymbol{\epsilon} \sim \mathsf{N}_n(\boldsymbol{0}, \sigma^2\boldsymbol{I})$, $\boldsymbol{X} \in \mathbb{R}^{n\times p}$ con $\operatorname{rg}(\boldsymbol{X}) = p$ y $\boldsymbol{\beta} \in \mathbb{R}^p$. Considere la partición $\boldsymbol{X} = (\boldsymbol{X}_1, \boldsymbol{X}_2)$ y $\boldsymbol{\beta} = (\boldsymbol{\beta}_1^\top, \boldsymbol{\beta}_2^\top)^\top$, donde $\boldsymbol{\beta}_1 \in \mathbb{R}^{p_1}$ $(p = p_1 + p_2)$. Suponga además que $\boldsymbol{\beta}_2 = \boldsymbol{0}$. En este caso tenemos,

$$\boldsymbol{Y} = \boldsymbol{X}_1\boldsymbol{\beta}_1 + \boldsymbol{\epsilon}.$$

Defina:

$$\begin{aligned} Q_1 &= (\boldsymbol{Y} - \boldsymbol{X}_1\boldsymbol{b}_1)^\top(\boldsymbol{Y} - \boldsymbol{X}_1\boldsymbol{b}_1) = \boldsymbol{Y}^\top(\boldsymbol{I} - \boldsymbol{X}_1(\boldsymbol{X}_1^\top\boldsymbol{X}_1)^{-1}\boldsymbol{X}_1^\top)\boldsymbol{Y} \\ Q_2 &= (\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{b})^\top(\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{b}) = \boldsymbol{Y}^\top(\boldsymbol{I} - \boldsymbol{X}(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\boldsymbol{X}^\top)\boldsymbol{Y}. \end{aligned}$$

Usando que $\boldsymbol{X}(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\boldsymbol{X}^\top\boldsymbol{X}_1 = \boldsymbol{X}_1$, determine la distribución de $Q = (Q_1 - Q_2)/\sigma^2$. ¿Son Q y Q_2/σ^2 independientes?

2. (50 pts) Suponga el modelo de regresión lineal $Y_i \sim \mathsf{N}(\beta_1 + \beta_2 x_i, \sigma^2)$, donde

$$x_i = \begin{cases} -1, & i = 1, \dots, k, \\ 0, & i = k+1, \dots, k+l, \\ k, & i = k+l+1, \end{cases},$$

con $n = k + l + 1$.

a) Obtenga la matriz de diseño $\boldsymbol{X}$.

b) Calcular $\widehat{\beta}_1$ y $\widehat{\beta}_2$.

c) ¿Son $\widehat{\beta}_1$ y $\widehat{\beta}_2$ independientes?