1. Tenemos que

$$\boldsymbol{X} = \begin{pmatrix} \boldsymbol{X}_{(i)} \\ \boldsymbol{x}_i^\top \end{pmatrix}, \qquad \boldsymbol{Y} = \begin{pmatrix} \boldsymbol{Y}_{(i)} \\ Y_i \end{pmatrix}, \qquad \boldsymbol{W} = \begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \omega \end{pmatrix}.$$

De este modo,

$$\boldsymbol{X}^\top \boldsymbol{W}\boldsymbol{X} = (\boldsymbol{X}_{(i)}^\top, \boldsymbol{x}_i) \begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \omega \end{pmatrix} \begin{pmatrix} \boldsymbol{X}_{(i)} \\ \boldsymbol{x}_i^\top \end{pmatrix} = \boldsymbol{X}_{(i)}^\top \boldsymbol{X}_{(i)} + \omega \boldsymbol{x}_i \boldsymbol{x}_i^\top$$

$$\boldsymbol{X}^\top \boldsymbol{W}\boldsymbol{Y} = (\boldsymbol{X}_{(i)}^\top, \boldsymbol{x}_i) \begin{pmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \omega \end{pmatrix} \begin{pmatrix} \boldsymbol{Y}_{(i)} \\ Y_i \end{pmatrix} = \boldsymbol{X}_{(i)}^\top \boldsymbol{Y}_{(i)} + \omega \boldsymbol{x}_i Y_i$$

Sabemos que $\boldsymbol{X}_{(i)}^\top \boldsymbol{X}_{(i)} = \boldsymbol{X}^\top \boldsymbol{X} - \boldsymbol{x}_i \boldsymbol{x}_i^\top$ y $\boldsymbol{X}_{(i)}^\top \boldsymbol{Y}_{(i)} = \boldsymbol{X}^\top \boldsymbol{Y} - \boldsymbol{x}_i Y_i$. De este modo,

$$\boldsymbol{X}^\top \boldsymbol{W}\boldsymbol{X} = \boldsymbol{X}^\top \boldsymbol{X} - (1-\omega)\boldsymbol{x}_i \boldsymbol{x}_i^\top = (\boldsymbol{X}^\top \boldsymbol{X})\big(\boldsymbol{I} - (1-\omega)(\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{x}_i \boldsymbol{x}_i^\top\big),$$

cuya matriz inversa es dada por

$$(\boldsymbol{X}^\top \boldsymbol{W}\boldsymbol{X})^{-1} = \Big\{\boldsymbol{I} + \frac{(1-\omega)(\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{x}_i \boldsymbol{x}_i^\top}{1-(1-\omega)\boldsymbol{x}_i^\top (\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{x}_i}\Big\}(\boldsymbol{X}^\top \boldsymbol{X})^{-1}.$$

Además, $\boldsymbol{X}^\top \boldsymbol{W}\boldsymbol{Y} = \boldsymbol{X}^\top \boldsymbol{Y} - (1-\omega)\boldsymbol{x}_i Y_i$. Esto permite escribir

$$\begin{aligned}
\widehat{\boldsymbol{\beta}}(\omega) &= (\boldsymbol{X}^\top \boldsymbol{W}\boldsymbol{X})^{-1}\boldsymbol{X}^\top \boldsymbol{W}\boldsymbol{Y} \\
&= \Big\{\boldsymbol{I} + \frac{(1-\omega)(\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{x}_i \boldsymbol{x}_i^\top}{1-(1-\omega)\boldsymbol{x}_i^\top (\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{x}_i}\Big\}\{\widehat{\boldsymbol{\beta}} - (1-\omega)(\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{x}_i Y_i\} \\
&= \widehat{\boldsymbol{\beta}} - \frac{1}{1-(1-\omega)h_{ii}}\Big[\{1-(1-\omega)h_{ii}\}(1-\omega)(\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{x}_i Y_i \\
&\quad - (1-\omega)(\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{x}_i \widehat{Y}_i + (1-\omega)h_{ii}(\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{x}_i Y_i\Big] \\
&= \widehat{\boldsymbol{\beta}} - \frac{(1-\omega)(\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{x}_i e_i}{1-(1-\omega)h_{ii}}.
\end{aligned}$$

2. Note que

$$(\boldsymbol{X}^\top \boldsymbol{X} + k\boldsymbol{G}^\top \boldsymbol{G})^{-1} = (\boldsymbol{X}^\top \boldsymbol{X})^{-1} - (\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{G}^\top \Big\{\frac{1}{k}\boldsymbol{I} + \boldsymbol{G}(\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{G}^\top\Big\}^{-1}\boldsymbol{G}(\boldsymbol{X}^\top \boldsymbol{X})^{-1}.$$

De ahí que,

$$\lim_{k\to\infty}(\boldsymbol{X}^\top \boldsymbol{X} + k\boldsymbol{G}^\top \boldsymbol{G})^{-1} = (\boldsymbol{X}^\top \boldsymbol{X})^{-1} - (\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{G}^\top \{\boldsymbol{G}(\boldsymbol{X}^\top \boldsymbol{X})^{-1}\boldsymbol{G}^\top\}^{-1}\boldsymbol{G}(\boldsymbol{X}^\top \boldsymbol{X})^{-1}.$$

Por otro lado,

$$\begin{aligned}
(\boldsymbol{X}^\top \boldsymbol{X} + k\boldsymbol{G}^\top \boldsymbol{G})^{-1}k\boldsymbol{G}^\top \boldsymbol{g} &= (\boldsymbol{X}^\top \boldsymbol{X} + k\boldsymbol{G}^\top \boldsymbol{G})^{-1}k\boldsymbol{G}^\top \boldsymbol{G}\boldsymbol{G}^\top (\boldsymbol{G}\boldsymbol{G}^\top)^{-1}\boldsymbol{g} \\
&= (\boldsymbol{X}^\top \boldsymbol{X} + k\boldsymbol{G}^\top \boldsymbol{G})^{-1}(k\boldsymbol{G}^\top \boldsymbol{G} + \boldsymbol{X}^\top \boldsymbol{X} - \boldsymbol{X}^\top \boldsymbol{X})\boldsymbol{G}^\top (\boldsymbol{G}\boldsymbol{G}^\top)^{-1}\boldsymbol{g} \\
&= \{\boldsymbol{I} - (\boldsymbol{X}^\top \boldsymbol{X} + k\boldsymbol{G}^\top \boldsymbol{G})^{-1}\boldsymbol{X}^\top \boldsymbol{X}\}\boldsymbol{G}^\top (\boldsymbol{G}\boldsymbol{G}^\top)^{-1}\boldsymbol{g}.
\end{aligned}$$

Lo anterior permite notar,

$$\lim_{k\to\infty}(\boldsymbol{X}^\top\boldsymbol{X}+k\boldsymbol{G}^\top\boldsymbol{G})^{-1}k\boldsymbol{G}^\top\boldsymbol{g}=(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\boldsymbol{G}^\top\{\boldsymbol{G}(\boldsymbol{X}^\top\boldsymbol{X})^{-1}\boldsymbol{G}^\top\}^{-1}\boldsymbol{g}.$$

y esto muestra el resultado.

3. La información puede ser organizada en la siguiente tabla:

Variables	p	RSS_p	R_p^2	R^2_{adj}	s_p^2	C_p	AIC
–	1	2715.764	0.000	0.000	226.314	442.913	73.444
X_1	2	1265.687	0.534	0.492	115.063	202.547	65.519
X_2	2	906.336	0.666	0.636	82.394	142.485	61.178
X_3	2	1939.401	0.286	0.221	176.309	315.152	71.067
X_4	2	883.867	0.675	0.645	80.352	138.730	60.852
X_1, X_2	3	57.905	0.979	0.975	5.791	2.678	27.420
X_1, X_3	3	1227.072	0.548	0.458	122.707	198.093	67.117
X_1, X_4	3	74.762	0.972	0.966	7.476	5.496	30.742
X_2, X_3	3	415.442	0.847	0.816	41.544	62.437	53.037
X_2, X_4	3	868.880	0.680	0.616	86.888	138.225	62.629
X_3, X_4	3	175.738	0.935	0.922	17.574	22.373	41.853
X_1, X_2, X_3	4	48.111	0.982	0.976	5.346	3.041	27.011
X_1, X_2, X_4	4	47.973	0.982	0.976	5.330	3.018	26.974
X_2, X_3, X_4	4	50.836	0.981	0.975	5.648	3.497	27.728
X_1, X_3, X_4	4	73.815	0.973	0.964	8.202	7.338	32.576
X_1, X_2, X_3, X_4	5	47.864	0.982	0.973	5.983	5.000	28.944

De este modo, el mejor modelo corresponde a $Y=\beta_0+\beta_1x_1+\beta_2x_2+\epsilon$.