TRANSLATED PAPER

Predictive model of sound propagation in porous materials: Extension of applicability in Kato model

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Abstract: In a previous paper, it was shown that the Kato model, with the material density and fiber diameter, can be used as a predictive model that enables the acoustic design (optimization of the fiber mixing rate) of fibrous materials. However, it does not have sufficient analytical precision for a material with high-bulk-density, so the model has limited applicability. Also, since the material density and fiber diameter are required as parameters, the Kato model cannot be applied to fibrous materials or foamed materials, the parameters of which are unknown. To solve these problems, in this paper, a technique of applying the Kato model to fibrous materials with high-bulk-density, fibrous materials with unknown material density and fiber diameter, and foamed materials is shown to extend its application.

Keywords: Effective density, Bulk modulus, Cylindrical tube, Porous material

1. INTRODUCTION

In this study, the scope of application of the Kato model [1] is expanded for predicting sound propagation in fiber nonwoven fabric. Currently, popular predictive models include the Delany-Bazley model [2], which uses the flow resistivity of fiber nonwoven fabric as a parameter, and the Johnson-Allard model [3], which can be applied to foamed materials. Thus, most predictive models utilize flow resistivity as a parameter. Flow resistivity is relatively easy to measure and is recognized as an important physical property for predicting sound propagation in porous materials. However, predictive models using flow resistivity as a parameter cannot optimize the fiber mixing rate from the view point of the acoustic design of nonwoven fabric. To achieve this, the author developed the Kato model in which the parameters are the material density and fiber diameter of fiber nonwoven fabric. This technology can enable the acoustic design (optimization of the fiber mixing rate) of fiber nonwoven fabric.

However, in fiber nonwoven fabric with high-bulk-density, the Kato model does not provide predictive accuracy, limiting its applicability [1]. In addition, the Kato model is not applicable to the prediction of sound propagation in foamed materials because it uses fiber diameter as a parameter. Furthermore, the Kato model assumes a cylindrical fiber blend, and the material density and fiber diameter must be known, so acoustic design (optimization of the fiber mixing rate) with fibers with irregular cross sections, natural fibers such as cotton and wool, and recycled fibers of unknown composition is not possible. In other words, the Kato model is limited to predicting the blending of man-made cylindrical chemical fibers such as polyester and polypropylene fibers. Therefore, the Kato model is not very effective in terms of practicality. To solve these problems, in this study the applicability of the Kato model is expended to fiber nonwoven fabric with high-bulk-density, fiber nonwoven fabric with unknown material density and fiber diameter, and foamed materials.

2. KATO MODEL [1]

The Kato model is based on the cylindrical tube model [4,5] and uses the density of air ρ_0, specific heat ratio γ, and atmospheric pressure P_0 to obtain the effective density ρ_{eff} and bulk modulus K_{f} of air in fiber nonwoven fabric using the following equations.

$$\rho_{\mathrm{eff}} \simeq \rho_0\left[1 - j\frac{8}{s^2}\sqrt{1 + j\frac{s^2}{12}} - s_{\mathrm{v}}\right] \qquad (1)$$

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$$K_f \simeq \frac{\gamma P_0}{\gamma - (\gamma - 1)\left[1 - j\frac{8}{s'^2}\sqrt{1 + j\frac{s'^2}{12}} - s_t\right]^{-1}} \tag{2}$$

s and s' are defined by the equations below using the viscous characteristic length Λ, thermal characteristic length Λ', angular frequency ω, density of air ρ_0, viscosity coefficient μ, and Prandtl number B^2.

$$s = \sqrt{\frac{\rho_0 \omega \Lambda^2}{\mu}} \tag{3}$$

$$s' = \sqrt{\frac{\rho_0 \omega B^2 \Lambda'^2}{\mu}} \tag{4}$$

The thermal characteristic length Λ' is defined by the following equation from the volume V of the voids in the fiber nonwoven fabric and the surface area A of the fibers in contact with the voids.

$$\Lambda' = 2\frac{V}{A} \tag{5}$$

The thermal characteristic length Λ' of the fiber nonwoven fabric can be theoretically obtained from the equivalent material density ρ_s, equivalent fiber diameter D, and bulk density ρ using Eq. (5) rearranged to the following equation.

$$\Lambda' = \frac{D(\rho_s - \rho)}{2\rho} \tag{6}$$

The equivalent material density ρ_s and the equivalent fiber diameter D are theoretically obtained physical quantities that replace one fiber material without changing the value of the thermal characteristic length Λ' when the fiber material is arbitrarily mixed. The equivalent material density ρ_s is defined as below, where the weight ratio of each fiber material to be blended is Y_x and the individual material density is ρ_{sx}.

$$\rho_s = \frac{1}{\displaystyle\sum_{x=1}^{n} \frac{Y_x}{\rho_{sx}}} \tag{7}$$

The equivalent fiber diameter D is defined by the following equation, where D_x is the diameter of each fiber to be blended.

$$D = \frac{1}{\rho_s \displaystyle\sum_{x=1}^{n} \frac{Y_x}{D_x \rho_{sx}}} \tag{8}$$

The viscosity characteristic length Λ in general fiber nonwoven fabric is defined by the following theoretical equation by Allard [3].

$$\Lambda = \frac{\Lambda'}{2} \tag{9}$$

s_v and s_t are experimentally determined through acoustic tube measurements of polyester fiber nonwoven fabric, and are defined as below. They are correction terms that compensate for the lack of attenuation for the characteristics of the air inside the cylindrical tube.

$$s_v = j\frac{C_v}{\omega}\left(\frac{1}{\Lambda} - \frac{1}{\Lambda'}\right) \tag{10}$$

$$s_t = j\left[\frac{C_t}{\omega}\left(\frac{1}{\Lambda} - \frac{1}{\Lambda'}\right)\right]^{3/4} \tag{11}$$

C_v and C_t are determined using the fiber diameter D and are expressed by the following equations.

$$C_v = \frac{2\mu}{\rho_0 D} \tag{12}$$

$$C_t = \frac{4\mu}{3\rho_0 D} \tag{13}$$

The above are the equations of the Kato model for predicting the sound propagation in a fiber nonwoven fabric at an arbitrary bulk density using the material density of the fiber material to be blended, the fiber diameter, and its blending ratio as parameters. Thus, the Kato model does not use flow resistivity.

The prediction accuracy of the Kato model is higher than that of the Johnson-Allard model. Both models use an approximation of the Bessel function based on the cylindrical tube model, but the Kato model improves on the approximation by the Johnson-Allard model. The real and imaginary parts of the complex quantities calculated using the Johnson-Allard model are mismatched because the air in the cylindrical tubes and the air in the fiber nonwoven fabric exhibit different characteristics. The Kato model corrects this by using the correction terms s_v and s_t. For example, the bulk modulus of the Kato model becomes that of the Johnson-Allard model if the improved approximation and correction term s_t are not used. Thus, the Kato model can be described as a predictive model that is an improvement of the Johnson-Allard model.

3. APPLICATION METHODS FOR HIGH-BULK-DENSITY FIBER NONWOVEN FABRIC

To apply the Kato model to a fiber nonwoven fabric with high-bulk-density, it is necessary to take into account the tortuosity and the contact area between fibers [1]. The tortuosity α_∞ is related to the linearity of sound waves in the porous material and is expressed by the following equation using the sound velocity c_0 in air at infinite frequency and the apparent sound velocity c in the porous material.

$$\alpha_\infty = \left(\frac{c_0}{c}\right)^2 \tag{14}$$

Since the apparent sound velocity c in the porous material is slower than the sound velocity c_0 in air, the tortuosity α_∞ is represented by a value greater than 1. The actual value of α_∞ is usually obtained by measuring the apparent sound velocity c in the porous material in the ultrasonic band [6].

Sound waves in fiber nonwoven fabric with low bulk density can be assumed to travel straight, so there is no problem as $\alpha_\infty = 1$. However, the high-bulk-density inhibits the straightness of sound waves, so the change in the tortuosity α_∞ cannot be ignored. The effective density ρ_{eff} of air in the fiber nonwoven fabric is affected by the tortuosity α_∞, so Eq. (1) should be redefined as follows [3].

$$\rho_{\mathrm{eff}} \simeq \alpha_\infty \rho_0 \left[1 - j\frac{8}{s^2}\sqrt{1 + j\frac{s^2}{12}} - s_{\mathrm{v}}\right] \tag{15}$$

If the contact area ratio between fibers (contact area between fibers/total surface area of fibers) is χ, the theoretical equation (6) for the thermal characteristic length Λ' should be redefined as the following effective value.

$$\Lambda'_{\mathrm{eff}} = \frac{D(\rho_{\mathrm{s}} - \rho)}{2\rho(1-\chi)} \tag{16}$$

Since this equation considers the contact area between fibers, the surface area of fibers in contact with the actual voids is taken into account. By quantifying the tortuosity α_∞ and contact area ratio χ, we can apply the Kato model to fiber nonwoven fabric with high-bulk-density.

3.1. Relationship between Tortuosity and Contact Area Ratio in Normal-Incidence Sound Absorption Coefficient

In this study, how the tortuosity α_∞ and contact area ratio χ affect the calculation results of the normal-incidence sound absorption coefficient is investigated. In Eqs. (1) to (13) of the Kato model, we change Eq. (1) for effective density to Eq. (15), which takes into account the tortuosity α_∞, and change Eq. (6) for the theoretical thermal characteristic length Λ' to Eq. (16), which takes into account the contact area ratio χ between fibers. In other words, the thermal characteristic length Λ' in Eq. (4) and Eqs. (9) through (11) is replaced by the value of Λ'_{eff} calculated using Eq. (16). Then, from the obtained effective density ρ_{eff} and bulk modulus K_{f}, the characteristic impedance Z_{c} and propagation constant Γ are obtained by calculating the following equations.

$$Z_{\mathrm{c}} = \sqrt{\rho_{\mathrm{eff}} K_{\mathrm{f}}} \tag{17}$$

$$\Gamma = j\omega\sqrt{\frac{\rho_{\mathrm{eff}}}{K_{\mathrm{f}}}} \tag{18}$$

These equations are used to calculate the normal-incidence sound absorption coefficient under the condition of no back air space.

Assume the following for polyester fiber nonwoven fabric.

Equivalent material density $\rho_{\mathrm{s}} = 1{,}380\,\mathrm{kg/m^3}$
Equivalent fiber diameter $D = 15.7\,\mu\mathrm{m}$
Bulk density $\rho = 100\,\mathrm{kg/m^3}$
Thickness $d = 20\,\mathrm{mm}$

Also, $\alpha_\infty = 1.00$, 1.25, 1.50, 2.00, and $\chi = 0.00$, 0.25, 0.50, and 0.75 are assumed. The calculation results of the normal-incidence sound absorption coefficient are shown in Figs. 1 and 2. In the study of α_∞, $\chi = 0.00$ is assumed for all conditions. In the study of χ, $\alpha_\infty = 1.00$ is assumed for all conditions. From the calculation results, we can see that the tortuosity α_∞ affects the frequency response and the contact area ratio χ affects the damping response. Thus, it was found that α_∞ and χ show different characteristics for the normal-incidence sound absorption coefficient.

3.2. Experimental Equation for Tortuosity and Contact Area Ratio

The tortuosity α_∞ and contact area ratio χ are difficult

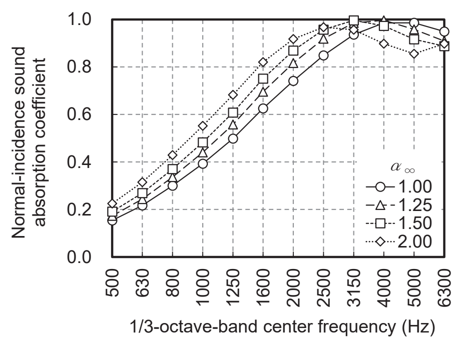

Fig. 1 Effect of tortuosity α_∞ on normal-incidence sound absorption coefficient.

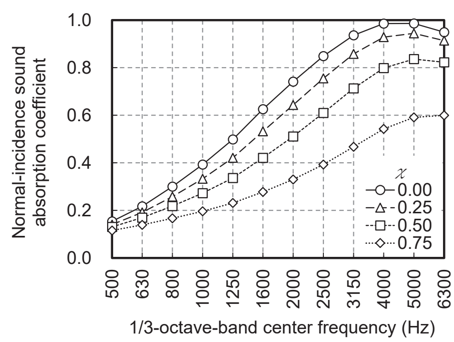

Fig. 2 Effect of contact area ratio χ on normal-incidence sound absorption coefficient.

Table 1 Specifications of polyester materials used in the experiments.

Main fiber diameter μm	Binder fiber diameter μm	Equivalent fiber diameter D μm
57.9	20.1	37.1
41.5	20.1	31.5
34.8	20.1	28.6
24.9	20.1	23.3
14.3	20.1	15.7

Weight ratio (main fiber 70%, binder fiber 30%)

to obtain theoretically. There are several existing methods for measuring α_∞, such as using the relationship of Eq. (14). However, it has been reported that it is difficult to perform experiments on samples with high attenuation [6]. There is also no direct means of measuring χ. Therefore, in this study, α_∞ and χ that are consistent with the experimental values of the normal-incidence sound absorption coefficient of polyester fiber nonwoven fabric are estimated. The polyester fiber nonwoven fabric utilized in this study comprised the five materials shown in Table 1, which were used in the development of the Kato model [1]. The equivalent material density was $\rho_s = 1{,}380\,\mathrm{kg/m^3}$, since 100% polyester was used.

The estimation method for α_∞ and χ was applied to mathematical programming as a minimum problem of the sum of the squares of the differences between the experimental and calculated values of the normal-incidence sound absorption coefficient (1/3-octave-band center frequency from 500 to 6,300 Hz), and the optimal solutions for α_∞ and χ were obtained simultaneously. α_∞ and χ each show different responses to the normal-incidence sound absorption coefficient, so this method of estimating them simultaneously is effective. The normal-incidence sound absorption coefficient experiments were conducted using the Brüel & Kjær 4206-type acoustic tube for high-frequency applications (inner diameter ϕ of 29 mm), with five types of equivalent fiber diameter D, as shown in Table 1, and a total of 60 samples with bulk densities ρ ranging from 15 to 200 $\mathrm{kg/m^3}$. The estimated results are shown in Figs. 3 and 4.

As a result, it was confirmed that both α_∞ and χ could be shown on a curve with the theoretical value of the thermal characteristic length Λ' given by theoretical equation (6) on the horizontal axis. Figures 3 and 4 show the predicted curves. The experimental equations are shown below.

$$\alpha_\infty = 1 + \left(\frac{5.45 \times 10^{-5}}{\Lambda'}\right)^{3/2} \tag{19}$$

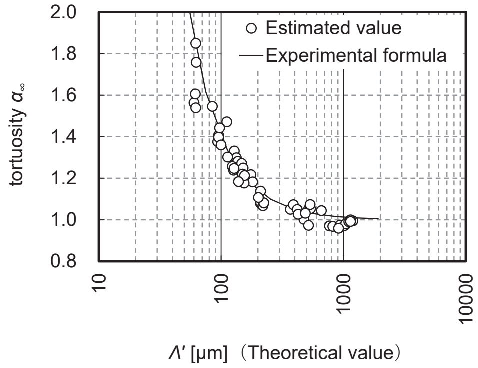

Fig. 3 Polyester fiber nonwoven fabric tortuosity.

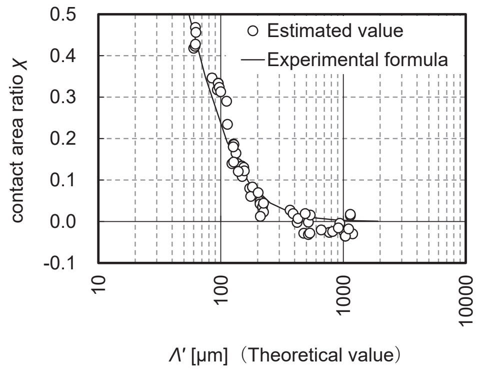

Fig. 4 Polyester fiber nonwoven fabric contact area ratio.

$$\chi = 1 - \frac{1}{1 + \left(\dfrac{5.45 \times 10^{-5}}{\Lambda'}\right)^2} \tag{20}$$

The thermal characteristic length Λ' in these equations means the value determined from the theoretical equation (6), and its unit is m. Since the tortuosity α_∞ is greater than or equal to 1 and the contact area ratio χ is between 0 and 1, the experimental equation was obtained by considering these factors.

As the bulk density increases, the volume of voids per unit volume decreases and the surface area of the fiber increases, so the thermal characteristic length Λ' becomes smaller, as can be seen from the definition, Eq. (5). In other words, the bulk density is higher when the Λ' is smaller. Therefore, the results in Figs. 3 and 4 show that the higher the bulk density, the larger the tortuosity α_∞ and the contact area ratio χ.

Note that α_∞ and χ are both dimensionless parameters. This requires that the unit of 5.45×10^{-5} in the equation be m, but the physical meaning of this value is unclear.

3.3. Validation Results of Experimental Equation Based on Normal-Incidence Sound Absorption Coefficient

The experimental equations (19) and (20) for the

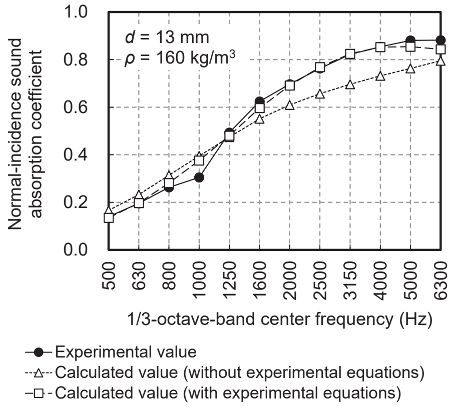

Fig. 5 Example calculations with and without application of experimental equations for α_∞ and χ (polyester fiber nonwoven fabric).

tortuosity α_∞ and contact area ratio χ were verified using the normal-incidence sound absorption coefficient of the polyester fiber nonwoven fabric. Experimental values of the normal-incidence sound absorption coefficient and values calculated with and without the application of the experimental equation are shown in Fig. 5. Assume the following for fiber nonwoven fabric.

Equivalent material density $\rho_s = 1{,}380\,\mathrm{kg/m^3}$
Equivalent fiber diameter $D = 15.7\,\mu\mathrm{m}$
Bulk density $\rho = 160\,\mathrm{kg/m^3}$
Thickness $d = 13\,\mathrm{mm}$

Under the above conditions, $\alpha_\infty = 1.87$ and $\chi = 0.453$. It can be seen that the tortuosity α_∞ and contact area ratio χ must be taken into account for fiber nonwoven fabric with high-bulk-density.

Next, the validity of the experimental equations (19) and (20) for the tortuosity α_∞ and contact area ratio χ was confirmed for polypropylene fiber nonwoven fabric. The experimental and calculated values of normal-incidence sound absorption coefficients are shown in Fig. 6. Assume the following for fiber nonwoven fabric.

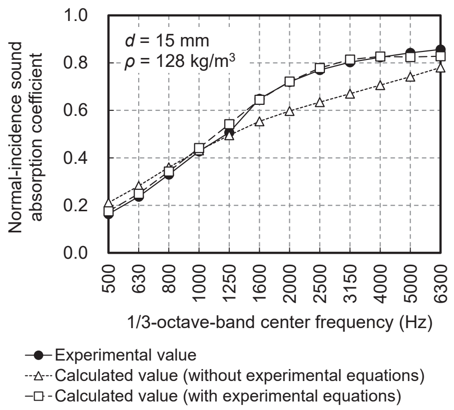

Fig. 6 Example calculations with and without application of experimental equations for α_∞ and χ (polypropylene fiber nonwoven fabric).

Equivalent material density $\rho_s = 910\,\mathrm{kg/m^3}$
Equivalent fiber diameter $D = 19.5\,\mu\mathrm{m}$
Bulk density $\rho = 128\,\mathrm{kg/m^3}$
Thickness $d = 15\,\mathrm{mm}$

Under the above conditions, $\alpha_\infty = 1.88$ and $\chi = 0.456$. The validation results show that the experimental equations (19) and (20) for the tortuosity α_∞ and contact area ratio χ obtained from the polyester fiber nonwoven fabric experiments are valid for different materials.

3.4. Consideration

The tortuosity α_∞ and the contact area ratio χ were confirmed to be correlated with the theoretical value of the thermal characteristic length Λ'. Then, the experimental equations for α_∞ and χ with Λ' as the parameter were presented to confirm the validity of these experimental equations. These experimental equations are based on experiments with polyester fiber nonwoven fabric using an acoustic tube, but the validity of these formulas was also confirmed with polypropylene fiber nonwoven fabric. Therefore, it is suggested that α_∞ and χ are independent of the material to be blended and are determined by the theoretical value of Λ'. In fact, the validity of these experimental equations was separately confirmed for several other types of fiber nonwoven fabric.

However, for materials with extremely high-bulk-density and Λ' smaller than $60\,\mu\mathrm{m}$, the validity of these experimental equations has not been confirmed. Generally, such extremely high-bulk-density fiber nonwoven fabric is used as board-shaped hard fiber nonwoven fabric without the purpose of sound absorption. Acoustic tube measurements of rigid fiber materials are difficult because of measurement errors due to bending vibration. Therefore, data for rigid fiber materials were not included in the experimental equation. The experimental values in Figs. 5 and 6 also show a drop in the sound absorption coefficient that may be attributed to bending vibration, and it was determined that these samples were the limit of the data for constructing the experimental equation. Also, as can be seen from Figs. 3 and 4, when Λ' is small, α_∞ and χ are significantly high. In fact, the estimated values of α_∞ shown in Fig. 3 are found to be widely scattered for samples with low theoretical values of Λ'. These experimental equations certainly improve the prediction accuracy compared with the case where α_∞ and χ are not taken into account. However, it should be noted that these experimental equations may not provide the necessary prediction accuracy for hard fiber nonwoven fabric with extremely high-bulk-density.

Also, no comparison was made between the experimental values of tortuosity and the values calculated by the experimental equations shown here. This study was focused on practicality, and the experimental equation was constructed with the aim of eliminating the tortuosity experiments. Therefore, the validity of this experimental equation may be limited to the Kato model. In other words, it is not clear whether the database from tortuosity experiments can be applied to the Kato model. Comparison of the experimental tortuosity value with the value calculated by the experimental equation is a subject for future work.

4. APPLICATION METHOD FOR FIBER NONWOVEN FABRIC WITH UNKNOWN MATERIAL DENSITY AND FIBER DIAMETER

Coarse felt is a recycled product that is made by breaking apart the cut scraps of clothing and other materials and turning them into batting, which is then used as the raw material for fiber nonwoven fabric. It is a highly versatile material, and the prediction of sound propagation in this material is essential. However, since the material density and fiber diameter of coarse felt are unknown, it is necessary to obtain these properties in order to apply the Kato model. Therefore, a method to estimate the material density and fiber diameter of coarse felt is presented and applied to the Kato model. Some of the materials used in coarse felt are cotton, polyester, and polypropylene. The material density of these individual materials ranges from 900 to 1,500 kg/m^3. Thus, the equivalent material density ρ_s of coarse felt is in the range 900 to 1,500 kg/m^3.

4.1. Estimation of Equivalent Fiber Diameter by Mathematical Programming

In the Kato model, once the equivalent material density ρ_s and the equivalent fiber diameter D are determined, the propagating sound properties at any bulk density can be obtained with these properties as constants. Sound absorption coefficient is obtained by using these properties plus thickness. In other words, once ρ_s and D of the coarse felt are determined, it is expected that propagating sound properties can be obtained for any bulk density, provided that there is no change in the material composition. Therefore, an attempt is made to estimate ρ_s and D, which minimize the difference between the experimental and calculated values of the normal-incidence sound absorption coefficient, by mathematical programming. The coarse felt of the study sample had a material thickness $d = 30$ mm and area density $m = 1.0$ kg/m^2 (bulk density $\rho = 33$ kg/m^3). However, a problem arose where an impossible estimate was calculated for ρ_s. That is, ρ_s was estimated to be outside the range of 900 to 1,500 kg/m^3. The only unknown parameters for coarse felt in the Kato model are ρ_s and D. The ρ_s of this coarse felt ranges from 900 to 1,500 kg/m^3. Therefore, the method was changed to one where ρ_s is varied between 900 and 1,500 kg/m^3 at intervals of 100 kg/m^3 and the corresponding D is estimated for each ρ_s, to be consistent with the experimental values of the normal-incidence sound absorption coefficient.

D was estimated by using mathematical programming to obtain the optimal solution for D as a minimum problem of the sum of squares of the difference between the experimental and calculated values of the normal-incidence sound absorption coefficient (1/3-octave-band center frequency from 500 to 6,300 Hz). The results are shown in Table 2. A physical property that is close to the properties of the actual coarse felt is considered to be any of these combinations.

4.2. Verification Results Based on Normal-Incidence Sound Absorption Coefficient

Calculated and experimental values of the normal-incidence sound absorption coefficient for all seven combinations of ρ_s and D shown in Table 2 are shown in Fig. 7. The calculations took into account tortuosity and the fiber-to-fiber contact area ratio, and used experimental equations (19) and (20) obtained from experiments on polyester fiber nonwoven fabric. For coarse felt, the application of these experimental equations improves the prediction accuracy. Therefore, Eq. (15), which includes

Table 2 Optimal solution for equivalent fiber diameter D estimated using experimental values of coarse felt normal-incidence sound absorption coefficient.

Equivalent material density ρ_s kg/m^3	Equivalent fiber diameter D μm
900	27
1,000	25
1,100	23
1,200	21
1,300	20
1,400	19
1,500	18

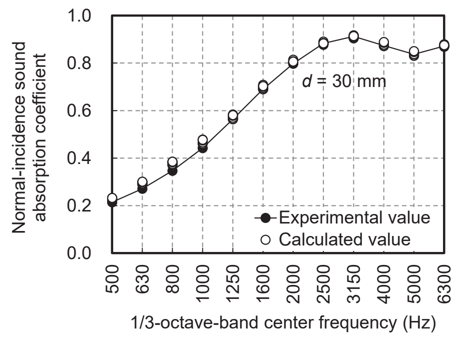

Fig. 7 Calculated normal-incidence sound absorption coefficient of coarse felt.

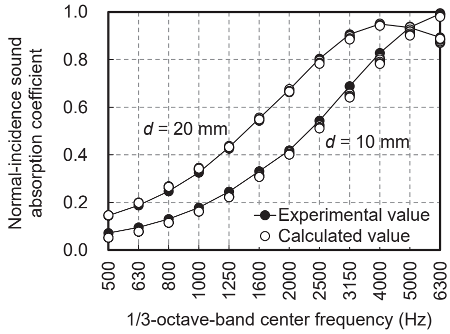

Fig. 8 Calculated normal-incidence sound absorption coefficient of compressed coarse felt.

the tortuosity α_∞ in the effective density ρ_{eff}, and Eq. (16) for the effective thermal characteristic length Λ'_{eff} were used. Figure 7 shows plots of seven different calculated values, which overlap, confirming that there is no significant difference between any of the combinations used.

Coarse felt of material thickness $d = 30\,\mathrm{mm}$ was pressed to thicknesses of $d = 20\,\mathrm{mm}$ and $d = 10\,\mathrm{mm}$. Figure 8 shows the calculated and experimental values of the normal-incidence sound absorption coefficient for all seven combinations of ρ_{s} and D shown in Table 2. The calculation results show that the Kato model can be applied to materials whose material density and fiber diameter are unknown by estimating these properties. However, it was also found that it is not possible to determine which of the combinations of ρ_{s} and D shown in Table 2 is closest to the actual characteristics.

4.3. Consideration

In the Kato model, coarse felt with unknown material density and fiber diameter was used, where the material density ρ_{s} was set to an arbitrary value, and the corresponding fiber diameter D was estimated by mathematical programming using the normal-incidence sound absorption coefficient data. Using this estimation method, it was found that the prediction of the normal-incidence sound absorption coefficient can be achieved even when the thickness changes due to the compaction process. It is found that the normal-incidence sound absorption coefficient can be obtained with good accuracy for any of the seven combinations of ρ_{s} and D shown in Table 2. Therefore, it can be said that there is no practical problem in estimating the corresponding D from the approximate ρ_{s} and using them as parameters, even if the actual ρ_{s} and D are not theoretically obtained. In other words, the actual ρ_{s} and D are not required.

Sound absorption in porous materials is mainly due to the viscous resistance of the air in the pores. The condition for increased viscous resistance in fiber nonwoven fabric is an increase in the surface area of the fibers in contact with the voids. Under the conditions of unchanged fiber diameter and bulk density, the lower the material density, the greater the amount of fibers per unit volume, and thus the greater the surface area and the greater the viscous resistance. Under the conditions of unchanged material density and bulk density, the smaller the fiber diameter, the greater the viscous resistance because the surface area increases. In other words, the lower the equivalent material density ρ_{s}, the greater the viscous resistance, but the thicker the fiber diameter, the smaller the viscous resistance, so the relationship between ρ_{s} and D shown in Table 2 is a combination that does not change the viscous resistance. This means that there are an infinite number of combinations of ρ_{s} and D corresponding to a certain normal-incidence sound absorption coefficient. Thus, the method of estimating ρ_{s} and D simultaneously by mathematical programming cannot identify these values, so it can be understood that an impossible value for ρ_{s} was estimated. If one wants to obtain a theoretically realistic equivalent material density ρ_{s}, one can calculate it from porosity. The following equation holds for the relationship among the porosity ϕ, the equivalent material density ρ_{s}, and the bulk density ρ.

$$\phi = 1 - \frac{\rho}{\rho_{\mathrm{s}}} \tag{21}$$

The equivalent material density ρ_{s} is a physical property newly defined in the previous paper [1] and derived under the condition that Eq. (21) holds. From this relationship, the equivalent material density ρ_{s} can be obtained. This method is also useful if a porosity database is available.

Since most predictive models use physical quantities such as flow resistivity, which vary with bulk density, as parameters, it is necessary to obtain the physical quantities for each bulk density. However, the Kato model does not require this, and as shown in the previous section, using the normal-incidence sound absorption coefficient, thickness, and area density of a single fiber nonwoven fabric, the sound propagation characteristics for any bulk density of that material blend can be obtained. In other words, with these databases or catalog values, there is no need to prepare actual fiber nonwoven fabric. Even if it is necessary to predict sound propagation in fiber nonwoven fabric with various thicknesses, the Kato model is more effective than other predictive models in practical use, because it can obtain characteristics for all thicknesses.

5. APPLICATION METHODS FOR FOAMED MATERIALS

Foamed materials are used as porous materials for soundproofing purposes, as are fiber nonwoven fabric. The Kato model was developed as a predictive model for fiber nonwoven fabric. Thus, it is currently not applicable to foamed materials. Therefore, in this study, the application of the Kato model to foamed materials was examined.

5.1. Unified Prediction Equation for Fibrous and Foamed Materials

The Kato model uses fiber diameter, material density, and bulk density as parameters. Fiber diameter is a property unique to fiber nonwoven fabric, and to apply the Kato model to foamed materials, it is necessary to rewrite the equation to eliminate fiber diameter. The fiber diameter in the Kato model is treated as a parameter for the theoretical determination of the thermal characteristic length Λ' shown in Eq. (6). The thermal characteristic length is treated as a parameter in the Johnson-Allard model [3] for both fibrous and foamed materials. Thus, this implies that the thermal characteristic length is a physical property applicable to all materials. Therefore, the Kato model is expected to be applicable as a predictive model for foamed materials by setting the unknown variables of foamed materials to thermal characteristic lengths. By substituting the equivalent fiber diameter D in Eqs. (6) into Eqs. (12) and (13), we can rewrite C_v and C_t as the following equations with the thermal characteristic length Λ' as a parameter.

$$C_v = \frac{\mu(\rho_s - \rho)}{\rho_0 \rho \Lambda'} \tag{22}$$

$$C_t = \frac{2\mu(\rho_s - \rho)}{3\rho_0 \rho \Lambda'} \tag{23}$$

Substitute Eqs. (22) and (23) into Eqs. (10) and (11) for s_v and s_t as below.

$$s_v = j\frac{\mu(\rho_s - \rho)}{\omega\rho_0 \rho \Lambda'}\left(\frac{1}{\Lambda} - \frac{1}{\Lambda'}\right) \tag{24}$$

$$s_t = j\left[\frac{2\mu(\rho_s - \rho)}{3\omega\rho_0 \rho \Lambda'}\left(\frac{1}{\Lambda} - \frac{1}{\Lambda'}\right)\right]^{3/4} \tag{25}$$

s_v and s_t are correction terms introduced to compensate for the sound attenuation deficiency of the sound propagating in the fiber nonwoven fabric, compared with the cylindrical tube [1]. If the attenuation of sound propagation in the fiber nonwoven fabric and in the foamed materials is by the same mechanism, Eqs. (24) and (25) should also hold for the foamed materials. These two equations are also applicable to fiber nonwoven fabric, and they are the unified prediction equations for fiber nonwoven fabric and foamed materials.

5.2. Validation Results of the Kato Model Using the Unified Forecasting Equation

The foamed material used for verification was continuous flexible polyurethane foam synthesized by the polyaddition reaction of polyol and isocyanate. The sample was of thickness $d = 20\,\text{mm}$, bulk density $\rho = 57\,\text{kg/m}^3$, and material density $\rho_s = 1{,}200\,\text{kg/m}^3$. This sample is made of a material whose tortuosity α_∞ is not negligible, and the effective density ρ_{eff} of Eq. (15), which takes the tortuosity α_∞ into account, is applied. Thus, the parameters of the Kato model for foamed materials are five characteristic properties: material density, bulk density, tortuosity, viscous characteristic length, and thermal characteristic length.

The physical properties of polyurethane foam tortuosity α_∞, viscous characteristic length Λ, and thermal characteristic length Λ' are difficult to obtain theoretically. Although these physical properties are also parameters that aid the Johnson-Allard model, it is difficult to obtain them directly from experiments, and currently, they are generally obtained by fitting to the experimental values of the normal-incidence sound absorption coefficient using an acoustic tube [7]. Thus, each property value is estimated in this study as well. These properties an estimated by applying mathematical programming as a minimum problem of the sum of the squares of the differences between the experimental and calculated values of the normal-incidence sound absorption coefficient (1/3-octave-band center frequency from 500 to 6,300 Hz) to obtain the optimal solution for each property value. However, as in the case of fiber nonwoven fabric, the relationship between Λ and Λ' is assumed to be $\Lambda = \Lambda'/2$ as shown in Eq. (9). The specifications and estimated properties of this sample are shown in Table 3, and the experimental and calculated values of the normal-incidence sound absorption coefficient are shown in Fig. 9. These results indicate that the Kato model can be applied to foamed materials.

Table 3 Specifications and estimated physical properties, tortuosity α_∞, viscous characteristic length Λ, and thermal characteristic length Λ' for foamed materials.

d mm	ρ kg/m^3	ρ_s kg/m^3	α_∞	Λ µm	Λ' µm
20	57	1,200	2.66	150	300

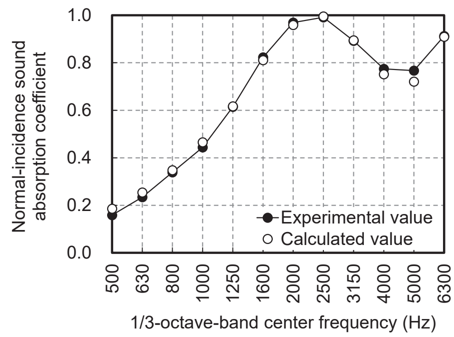

Fig. 9 Calculation results for normal-incidence sound absorption coefficient of polyurethane foam.

However, the assumption that $\Lambda = \Lambda'/2$ is confirmed separately for several other foamed materials, so that agreement with experimental results can be obtained, although it is unclear whether this assumption is theoretically correct. Depending on the structure of voids, this assumption may not hold true and is open to further investigation.

5.3. Simple Modeling of Cells in Foamed Materials

In the case of general foamed materials, the voids are structured as cells, and these cells form intricately interconnected voids. Consider a simple model in which the cell shape is assumed to be a sphere and that all cell diameters are the same. Neglecting the area of the interconnecting pores between cells, the cell volume V and surface area A are defined by the following equations, where D_c is the cell diameter.

$$V = \frac{1}{6}\pi {D_c}^3, \quad A = \pi {D_c}^2 \tag{26}$$

Therefore, the next thermal characteristic length Λ' follows from Eq. (5).

$$\Lambda' = 2\frac{V}{A} = \frac{D_c}{3} \tag{27}$$

Thus, the thermal characteristic length can be replaced by the simply modeled cell diameter, which can be used as a parameter. I considered that the cell diameter, rather than the thermal characteristic length, would give a better physical picture. In the case of the calculation example in Fig. 9, the thermal characteristic length Λ' is 300 µm, so the cell diameter D_c is 0.9 mm.

However, this cell diameter D_c does not represent the actual cell diameter of foamed materials. These are only properties assuming a simple model of foamed materials. The area of the interconnecting holes between cells is neglected and the cells are assumed to be spheres, which is different from the actual cells. This should be noted.

5.4. Semiclosed-Cell Foamed Materials

Some foamed materials, such as styrofoam, are formed with closed cells. The viscous resistance of the air in the pores is the main factor for sound absorption in porous materials, but sound absorption due to viscous resistance cannot be expected in foamed materials formed only with closed cells. For this reason, most foamed materials for sound absorption have open cells. However, there are semiclosed-cell foamed materials in which open cells and closed cells are mixed. Let us consider the case of applying the Kato model to such foamed materials.

The volume and surface area of a closed cell do not affect the airborne sound. Therefore, the closed cell in the predictive model should be considered equivalent to a solid-filled state. In other words, only open cells should be considered for porosity in the predictive model for semiclosed-cell foamed materials. Therefore, the relationship between the equivalent material density ρ_s and the bulk density ρ in the theoretical equation (21) of porosity does not hold. The aforementioned unified prediction equations (24) and (25) for fibrous and foamed materials are conditional on the theoretical equation (21) for porosity being valid. Therefore, the unified prediction equation for fibrous and foamed materials presented here cannot be applied to semiclosed-cell foamed materials. The reason why material density and bulk density are parameters in the Kato model for foamed materials is that the unified prediction equation includes the theoretical equation for porosity. Therefore, with the effective porosity ϕ_{eff}, which considers only open cells, as an unknown parameter, the unified prediction equations (24) and (25) for fibrous and foamed materials are rewritten as follows.

$$s_v = j\frac{\mu\phi_{\mathrm{eff}}}{\omega\rho_0\Lambda'(1-\phi_{\mathrm{eff}})}\left(\frac{1}{\Lambda} - \frac{1}{\Lambda'}\right) \tag{28}$$

$$s_t = j\left[\frac{2\mu\phi_{\mathrm{eff}}}{3\omega\rho_0\Lambda'(1-\phi_{\mathrm{eff}})}\left(\frac{1}{\Lambda} - \frac{1}{\Lambda'}\right)\right]^{3/4} \tag{29}$$

These equations allow porosity, tortuosity, and cell diameter (or characteristic length) to be parameters of the Kato model for semiclosed-cell foamed materials. However, as a practical matter, there is no experimental method of obtaining the effective porosity ϕ_{eff} (except for the independent bubble property). sucha Thus, the prediction of airborne sound in semiclosed-cells is an issue to be addressed in the future.

5.5. Consideration

s_v and s_t in the unified prediction equation in the Kato model are correction terms introduced to compensate for the lack of damping of the fiber nonwoven fabric relative to cylindrical tubes. Since the unified prediction equations (24) and (25) could be applied to foamed materials, it can be said that the mechanism of airborne sound attenuation in foamed materials is not different from that in fiber nonwoven fabric.

The difference between fibrous and foamed materials can be seen in the tortuosity value. The tortuosity of the foamed materials shown here is estimated to be 2.66, which is larger than the tortuosity of the fiber nonwoven fabric in this study. Since tortuosity is related to the straightness of sound waves, it can be inferred that the open cells in cellular foamed materials from a labyrinth, producing such a result. As tortuosity increases, the frequency response of the normal-incidence sound absorption coefficient changes, as shown in Fig. 1, as if the material were thicker. Tortuosity α_∞ in a cylindrical tube is defined as follows, where θ is the inclination of the cylindrical tube [3].

$$\alpha_\infty = \frac{1}{\cos^2 \theta} \tag{30}$$

This means that for $\alpha_\infty = 1$, the path length of the propagating sound is $\sqrt{\alpha_\infty}$ times longer. The $\alpha_\infty = 2.66$ corresponds to a path length about 1.6 times longer than $\alpha_\infty = 1$. As tortuosity increases, the path of propagating sound within the porous material becomes longer, which is thought to result in properties similar to those of thicker materials. Thus, it can be understood that foamed materials generally have a larger tortuosity and behave as if they are thicker than fiber nonwoven fabric.

Equations (28) and (29) for s_v and s_t in semiclosed-cell foamed materials are also unified prediction equations that hold for general foamed and fiber nonwoven fabric. Substituting the theoretical equation (21) for porosity into the effective porosity ϕ_eff in these equations, we obtain agreement with the unified prediction equations (24) and (25). Since the Kato model can be applied to both fibrous and foamed materials, it can be said that the sound propagation in a porous material is determined by the physical properties porosity, tortuosity, and thermal characteristic length if $\Lambda = \Lambda'/2$ in Eq. (9) holds. For example, in a semiclosed-cell foamed material, the diameter of a spherical cell (void) would be represented by its thermal characteristic length, the tortuosity would represent the slope (length) of the cell from the top to the bottom, and the porosity would represent the number of open cells. In the Kato model, bulk density, mass density, and fiber diameter are used as parameters to theoretically obtain porosity and thermal characteristic length. In other words, the Kato model can be considered an equivalent model in which the voids in the porous material are replaced by simple shape information, from which the sound propagation characteristics are derived. Flow resistivity is a property that has pressure as a unit and is not directly related to geometry. In the Kato model, only characteristics related to shape information (volume, area, and length) are used, making this a distinctive feature of the predictive model. In this paper, the experimental equations for the tortuosity of the fiber nonwoven fabric and the contact area ratio between fibers have been presented they can be said to yield the real shape information (volume, area, and length) experimentally.

6. CONCLUSION

The scope of application of the Kato model was expanded. For fiber nonwoven fabric with high-bulk-density, the experimental equations for tortuosity and the fiber-to-fiber contact area ratio were shown to improve the prediction accuracy of the normal-incidence sound absorption coefficient. For coarse felt with unknown material density and fiber diameter, it was shown that the normal-incidence sound absorption coefficient at a given thickness can be predicted by estimating the equivalent fiber diameter. For foamed materials, the equation was changed to one that does not use fiber diameter, and a unified prediction equation applicable to both fibers and foamed materials was presented, showing that the prediction of the normal-incidence sound absorption coefficient is possible.

Although this paper was focused on the practical aspects and methods to expand the applicability of the Kato model were presented, it is impossible to confirm the validity of the equations and methods presented here for all materials, and many additional experiments should be conducted.

The Kato model is a predictive model for airborne sound in porous materials. In the future, using Biot theory [8], I plan to expand the Kato model to a predictive model that can also handle solid propagating sound in porous materials and to investigate the effectiveness of such a model.

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