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Variable Neighborhood Search Based Algorithms for High School Timetabling

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Abstract

This work presents the application of Variable Neighborhood Search (VNS) based algorithms to the High School Timetabling Problem. The addressed model of the problem was proposed by the Third International Timetabling Competition (ITC 2011), which released many instances from educational institutions around the world and attracted seventeen competitors. Some of the VNS algorithm variants were able to outperform the winner of Third ITC solver, which proposed a Simulated Annealing - Iterated local Search approach. This result, coupled with another reports in literature points that VNS based algorithms are a practical solution method for providing high quality solutions for some hard timetabling problems. Moreover they are easy to implement with few parameters to adjust.

Keywords: Variable Neighborhood Search, High School Timetabling Problem, Third International Timetabling Competition

1. Introduction

The High School Timetabling Problem is faced by many educational institutions around the world. The basic search version consists in assigning teacher×class activities to timeslots and rooms, in such a way that no teacher, class or room is involved with more than one event at time. Generally, this

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assignment is repeated weekly until the end of the semester. Many other constraints are considered in real problems, like availability of teachers, to avoid idle times and to limit the number of lessons of the same subject taught to a class in a day.

Beyond it is practical importance, this problem was proven to be $\mathcal{NP}$-Hard [1, 2]. Progresses in heuristic and exact approaches for tackling these problems in a major goal of current research in Operations Research and Artificial Intelligence.

Three international competitions (ITCs) were made to bring the attention of scientists and practitioners for this problem, with the objective of performing comparisons of different methods in a controlled computational environment: the first one happened in 2003 [3] and was won by Kostuch [4] with a 3-phase Simulated Annealing (SA) [5] based approach. In 2007, the second one [6] started and was composed by three separated tracks, which were mostly won by Müller [7] also with a Simulated Annealing based approach. The last one [8] happened in 2012 and was won by a Simulated Annealing - Iterated Local Search [9] approach.

As the results of the competitions show, local search methods are defining the state-of-art heuristic solvers for educational timetabling problems. In special, the Simulated Annealing metaheuristic, which composed the solver of all winners. The role of exact methods which employ Integer Programming, such as the proposed in [10, 11, 12], appears to be still very limited for tackling the problems and instances which appeared in these competitions, considering the absence of these techniques in submissions. This scenario contrasts with the first International Nurse Rostering Competition [13], for instance, where two of the first places used Integer Programming in some form.

This paper presents a computational study of Variable Neighborhood Search and its variants applied to the Third ITC problem. The results indicate that the proposed method outperforms the state-of-art method.

The remaining of this work is organized as follows: Section 2 presents the problem considered in this paper: the Third ITC problem, Section 3 presents our solution approach, Section 4 presents computational experiments and finally, Section 5, concludes our paper and discusses future works.

2. High School Timetabling Problem Model

The roots of the School Timetabling model considered in this paper: the model of the Third ITC, are in the Benchmarking project for (High) School Timetabling[1]. The project, which involved a group of researchers in this area, started with the ambitious goal of developing a XML format capable of modeling different school timetabling problems arising in diverse institutions around the world. Initial versions of this project appeared in the PATAT 2008 conference [14], with an improved version named XHSTT published later [15]. Nowadays, the project site holds approximately 50 datasets from 11 countries. The project site also includes an evaluator to validate solutions and the best known solutions are kept, so that the results of newly proposed methods can be immediately confronted with previously obtained results. Some of the previous models which are now in XHSTT are [16, 17, 18, 19, 20, 10, 21]. The model is split in three main entities: Time and Resources, Events and Constraints. A solution consists of a set of assignments of times and resources to the events.

2.1. Times and Resources

The time entity consists of a timeslot, which is an indivisible interval of time. Timeslots do not overlap and can be grouped in timegroups. Resources are entities which attend events. Typical resources are students, teachers and rooms [21]:

students: a group of students attends to events (lessons); important constraints associated with students are the control of their idle times and the number of lessons taken by day;

teachers: teachers perform their academic tasks in events; the allocation of teachers for specific teaching activities can be preassigned or not; when teachers are not preassigned, they should be assigned according to their qualifications and workload limits;

rooms: the usage of rooms for hosting events must be observed: some events require rooms with a given capacity and/or a set of special features.

[1] `http://www.utwente.nl/ctit/hstt/`

2.2. Events

An event (*instance event*) is a meeting between resources, usually representing a simple *lesson* or a set of lessons (event group). Each instance event needs to be scheduled into one or more *solution events.* Timeslot assignments to events are called *meets* and the assignment of resources to events are *tasks.* The term *course* is used to designate a group of students who attend to the same events. Other kinds of events, like meetings, are allowed by the model [21]. The following attributes can be specified for events, the first one is the only obligatory:

duration: represents the number of timeslots which have to be assigned to the event;

course : a course is a grouping of events: events declared in the same course constitute a course of study in one subject for one group of students;

pre-assigned resources: to attend the event ;

workload: that will be added to the total workload of resources assigned to the event ;

pre-assigned timeslot: some events have only one timeslot in which they can be assigned .

2.3. Constraints

Post *et al.* [21] groups the constraints in three categories: basic constraints of scheduling, constraints of events and constraints of resources. The objective function $f(.)$ is computed considering the summation of penalties for deviations in different constraints and events/resources which they refer. The flexibility of XHSTT allows the inclusion of non-linear terms in the cost function used to compute the penalties[15]. The constraints are also divided in hard constraints, whose satisfaction is mandatory; and soft constraints, whose satisfaction is desirable but not obligatory. Costs for violations in these two types of constraints are summed in two separated costs: the infeasibility cost and the quality cost, defining an hierarchical objective function. Each instance can define whether a constraint is hard or soft, its weight and the type of cost function used (eg.: linear or quadratic). For more details, see [15].

2.3.1. Basic Constraints of Scheduling

1. Assign Time: assign timeslots to each event;
2. Assign Resource: assign the resources to each event;
3. Prefer Times: indicates that some event have preference for a particular timeslot(s);
4. Prefer Resources: Indicates that some event have preference for a particular resource(s).

2.3.2. Constraints to Events

1. Link Events: to schedule a set of events to the same starting time;
2. Spread Events: specify the allowed number of occurrences for event groups in time groups between a minimum a maximum number of times; this constraint can be used, for example, to define a daily limit of lessons;
3. Avoid Split Assignments: for each event, assign a particular resource to all of its meets;
4. Distribute Split Events: for each event, assign between a minimum and a maximum meets of a given duration;
5. Split Events: limits the number of non-consecutive meets that an event should be scheduled and its duration.

2.3.3. Constraints to Resources

1. Avoid Clashes: assign the resources without clashes (i.e. without assign the same resource to more than one event at a timeslot);
2. Avoid Unavailable Times: avoid assigning resources on the times that they are not available;
3. Limit Workload Constraint: schedule the workload of the resources between a minimum and a maximum bound;
4. Limit Idle Times: the number of idle times in each time group should lie between a minimum and a maximum bound for each resource; typically, a time group consists of all timeslots of a given week day;
5. Limit Busy Times: the number of busy times in each time group should lie between a minimum and a maximum bound for each resource;
6. Cluster Busy Times: the number of time groups with a timeslot assigned to a resource should lie between a minimum and a maximum limit; this can be used, for example, to concentrate teacher's activities in as few days as possible.

3. Solution Approach

Our approach uses the Kingston High School Timetabling Engine (KHE) [22] to generate initial solutions. Afterwards, we implemented the Variable Neighborhood Search metaheuristic and some of its variants to perform local search around this solution. These elements will be explained in the following subsections.

3.1. Build Method

The KHE is a platform for handling instances of the addressed problem. It also provides a solver, used to build initial solutions in the presented approach. This solver was chosen to generate the initial solutions since it is able to find reasonably good initial solutions in short amounts of time.

The incorporated solver is based on the concept of Hierarchical Timetabling [23], where smaller allocations are joint to generate bigger blocks of allocation until a full representation of the solution is developed. Hierarchical Timetabling is supported by the Layer Tree data structure [23], consisting of nodes that represent the required meet and task allocation. An allocation may appear in at most one node. A Layer is a subset of nodes having the propriety that none of them can be overlapped in time. Commonly, nodes are grouped in a Layer when share resources.

The hard constraints of the problem are modeled to this data structure and then a Matching problem is solved to find the times/resources allocation. The Matching is done by connecting each node to a timeslot or resource respecting the property of Layer. For full details, see [23, 22].

3.2. Neighborhood Structure

Six neighborhood structures were used:

1. Event Swap (ES). Two events e_1 and e_2 have their timeslots t_1 and t_2 swapped;
2. Event Move (EM). An event e_1 is moved from timeslot t_1 to another timeslot t_2;
3. Event Block Move (EBM). Works like ES, but when moving events with different durations in contiguous timeslots, keeps these events adjacent;
4. Resource Swap (RS). Two events e_1 and e_2 have their assigned resources r_1 and r_2 swapped. Resources r_1 and r_2 should play the same role to allow the swap (e.g. both have to be teachers);

5. Resource Move (RM). An event e_1 has its assigned resource r_1 replaced by a new resource r_2;
6. Kempe Move (KM). Two times t_1 and t_2 are fixed and one seeks the best path at the bipartite conflict graph containing all events in t_1 and t_2; arcs are built from conflicting events which are in different timeslots and their cost is the cost of swapping the timeslots of these two events.

The sef of neighborhoods is quite similar to the one used in Fonseca [9].

3.3. Variable Neighborhood Search

The Variable Neighborhood Search Method was proposed by Mladenovic and Hansen [24] and consists in a local search method that explores the search space by making systematic changes in the neighborhood structures.

In each iteration, a neighborhood structure k is selected according to the order presented in Section 3.2. A random neighbor $s^{'}$ is generated in this neighborhood. Afterwards, a descent method is applied to $s^{'}$. If the best solution found by descent method, $s^{''}$, is better than the best known solution, it is updated and the neighborhood structure is set to the first one. Otherwise, the search continues in the next neighborhood structure. When we explore the last neighborhood structure $k_{max} = 6$, the search goes back to the first neighborhood. This process continues until a stop condition is reach.

A key component of VNS algorithms is the descent phase (Algorithm 1, line 5). The ability to quickly reach good local optima is critical to the success of the method. Our implementation aims at the fast generation of high quality solutions which tend to be local optima with respect to many neighborhoods. Thus, at each iteration of the descent phase, a different neighborhood can be considered, with the following probabilities of selection: if the instance requires the assignment of resources (i.e. there exists at least one ASSIGN RESOURCE constraint), the neighborhood is chosen based on the following probabilities: ES $= 0.20$, EM $= 0.38$, EBM $= 0.10$, RS $= 0.20$, RM $= 0.10$ and KM $= 0.02$. Otherwise, the neighborhoods RS and RM are not used and the odds become: ES $= 0.40$, EM $= 0.38$, EBS $= 0.20$ and KM $= 0.02$. Since the union of all these neighborhoods is usually a very large search space, composed with many flat landscapes, we employed Random Non-Ascendent (RNA) movements in the descent phase, with the stopping criterion of 1,000,000 non improvement iterations. These values were empirically adjusted.

Algorithm 1 presents the basic implementation of VNS, denoted here as BVNS. Note that the adopted stop condition is a timeout, to be discussed in Section 4. Some variations of VNS implemented are present in the following subsections. Some successful examples of application of VNS can also be found at [25, 26, 27].

Algorithm 1: Basic VNS (BVNS) algorithm

```
Input: Initial solution $s$.
Output: Best solution $s$ found.
1  while $elapsedTime < timeout$ do
2      $k \leftarrow 1$;
3      while $k \leq k_{max}$ do
4          Generate a random neighbor $s^{'} \in N_k(s)$;
5          $s^{''} \leftarrow descentMethod(s^{'})$;
6          if $f(s^{''}) \leq f(s)$ then
7              $s \leftarrow s^{''}$;
8              $k \leftarrow 1$;
9          else
10             $k \leftarrow k + 1$;
11 return $s$;
```

3.3.1. Reduced Variable Neighborhood Search

A reduction to the original Variable Neighborhood Search Method was also proposed by Mladenovic and Hansen [24], in which we do not have a descent phase (Algorithm 1, line 5) to improve the generated solution $s^{'}$ at each iteration. This may improve the VNS performance in cases in which the complete exploration of the defined neighborhoods is too computationally expensive. This reduction was called Reduced Variable Neighborhood Search (RVNS).

3.3.2. Sequential Variable Neighborhood Descent

Another variation of the original VNS method is the Sequential Variable Neighborhood Descent (SVND) [28]. The main difference between the basic VNS and SVND method is instead of allowing all neighborhood structures to be explored in the descent phase, we allow only a subset of the available neighborhood structures at each iteration. In our implementation, we made the local search at each iteration considering only one neighborhood structure k ($s'' \leftarrow descentMethod_k(s')$).

3.3.3. Skewed Variable Neighborhood Search

Taking larger and larger neighborhoods, the information related to the best local optimum dissolves and VNS degenerates into multistart [29]. To deal with this cases a new variant of VNS, the Skewed Variable Neighborhood Search (SVNS), was proposed. In this variant, we have a relaxed rule to accept the candidate solution s''. The relaxed rule uses an evaluation function linear in the distance from the incumbent: i.e. $f(s'')$ is replaced by $f(s'') - \alpha \times \rho(s, s'')$, where $\rho(s, s'')$ is the distance from s to s'' and α a parameter. To compute the distance between two solutions we used the following metric: for each solution we compute a string with n positions, where n is the number of events. In each position there is an ordered pair indicating the meeting and tasks which are associated with this event. Then, $\rho(s, s'')$ is the hamming distance of these two strings. After some experiments, we set $\alpha = 1.0$.

In our implementation, we made the local search at each iteration considering only one neighborhood structure k ($s'' \leftarrow descentMethod_k(s')$).

4. Computational Experiments

All experiments ran on an Intel ® Core i5 2.4 Ghz computer with 4Gb of RAM under the Ubuntu 11.10 operating system. The programming language used was C++ compiled with the GNU Compiler Collection version 4.6.1. All generated solutions were validated by the HSEval validator (`http://sydney.edu.au/engineering/it/~jeff/hseval.cgi`). We considered the timeout of the competition in all experiments, which was 1000 seconds[2].

Results are expressed by the pairs x/y, where x contains the feasibility measure and y the quality measure. Our solver along with our solutions and reports can be found at `https://sites.google.com/site/georgehgfonseca/producaoacademica`
We invite the interested reader to validate our results.

[2]The CPU time was adjusted in our computer using the ITC benchmark software, which suggested 1500 seconds.

4.1. Dataset Characterization

The set of instances available from Third ITC `http://www.utwente.nl/ctit/hstt/archives/X` was originated from many countries and ranges from small instances to huge challenging ones. Table 1 presents the main features of these problems.

Table 1: Features of considered instances from Third ITC

Instance	**Times**	**Teachers**	**Rooms**	**Classes**	**Lessons**
BrazilInstance2	25	14		6	150
BrazilInstance3	25	16		8	200
BrazilInstance4	25	23		12	300
BrazilInstance6	25	30		14	350
FinlandElementarySchool	35	22	21	291	445
FinlandSecondarySchool2	40	22	21	469	566
Aigio1stHighSchool10-11	35	37		208	532
Italy_Instance4	36	61		38	1101
KosovaInstance1	62	101		63	1912
Kottenpark2003	38	75	41	18	1203
Kottenpark2005A	37	78	42	26	1272
Kottenpark2008	40	81	11	34	1118
Kottenpark2009	38	93	53	48	1301
Woodlands2009	42	40			1353
Spanishschool	35	66	4	21	439
WesternGreeceUniversity3	35	19		6	210
WesternGreeceUniversity4	35	19		12	262
WesternGreeceUniversity5	35	18		6	184

4.2. Obtained Results

In the first set of experiments we evaluated the proposed methods using the same metric employed in the Third ITC : average results produced in a restricted time limit, as discussed in the beginning of this section. Table 2 presents the obtained average results of the basic VNS method (BVNS) and its variations: RVNS, SVND and SVNS. We also included in this table the results of the KHE engine [30] initial solutions as well as the results of the Third ITC winner, a Simulated Annealing-Iterated Local Search approach [9]. These results are presented, respectively, in columns KHE and SA-ILS. Each cell includes the average result of five independent executions[3] of one method on a given instance.

[3]Random seeds from 1 to 5.

Table 2: Average results produced with VNS variants and other approaches in the restricted time limit of the Third ITC

Instance	**KHE[30]**	**SA-ILS[9]**	**BVNS**	**RVNS**	**SVND**	**SVNS**
BrazilInstance2	4 / 90	1.0 / 63.9	0.0 / 40.6	2.2 / 71.4	0.6 / 63.8	0.0 / 39.6
BrazilInstance3	3 / 240	0.0 / 127.8	0.0 / 113.0	2.4 / 151.4	1.6 / 136.8	0.0 / 119.0
BrazilInstance4	39 / 144	17.2 / 99.6	4.8 / 108.2	21.0 / 112.8	13.6 / 103.4	3.8 / 123.4
BrazilInstance6	11 / 291	4.0 / 223.5	0.0 / 157.4	6.0 / 271.0	2.2 / 231.2	0.0 / 151.4
FinlandElementarySchool	9 / 30	0.0 / 4.0	0.0 / 3.4	2.6 / 7.4	0.0 / 4.0	0.0 / 3.8
FinlandSecondarySchool2	2 / 1821	0.0 / 0.4	0.0 / 0.4	0.6 / 86.8	0.0 / 1.0	0.0 / 0.4
Aigio1stHighSchool10-11	14 / 757	0.0 / 15.3	0.4 / 10.2	11.2 / 200.0	4.8 / 259.0	0.2 / 8.2
ItalyInstance4	39 / 21238	0.0 / 658.4	0.0 / 409.0	0.4 / 2666.6	0.0 / 1271.0	0.0 / 324.8
KosovaInstance1	1333 / 566	14.0 / 6934.4	1.2 / 20.4	31.6 / 278.8	2.0 / 75.6	1.2 / 17.4
Kottenpark2003	3 / 78440	0.6 / 90195.8	2.0 / 10217.2	2.4 / 34766.0	2.8 / 7937.8	2.0 / 9694.4
Kottenpark2005A	35 / 23677	33.9 / 27480.4	33.8 / 19059.2	35.0 / 22914.0	27.0 / 10118.0	33.8 / 18547.6
Kottenpark2008	63 / 140083	25.7 / 31403.7	15.6 / 23962.0	36.8 / 38936.6	16.8 / 33443.6	15.8 / 24024.2
Kottenpark2009	55 / 211095	36.6 / 154998.5	35.0 / 8543.0	45.4 / 148601.0	31.2 / 8563.0	33.2 / 9667.0
Woodlands2009	19 / 0	2.0 / 15.8	2.0 / 8.2	10.8 / 16.4	2.0 / 14.4	2.0 / 6.2
Spanish school	1 / 4103	0.0 / 865.2	0.0 / 907.8	0.0 / 3068.0	0.0 / 1126.0	0.0 / 724.2
WesternGreeceUniversity3	0 / 30	0.0 / 5.6	0.0 / 5.4	0.0 / 20.4	0.0 / 15.2	0.0 / 5.0
WesternGreeceUniversity4	0 / 41	0.0 / 7.4	0.0 / 6.4	0.0 / 30.0	0.0 / 23.6	0.0 / 5.6
WesternGreeceUniversity5	17 / 44	0.0 / 0.0	0.0 / 0.0	2.8 / 16.2	1.2 / 3.0	0.0 / 0.0
Average	**91.5 / 26816.11**	**7.5 / 17394.4**	**5.3 / 3531.8**	**11.7 / 14011.9**	**5.9 / 3521.7**	**5.1 / 3525.7**

Table 3 presents the ordering of the presented methods according to the Third ITC rules. Each solver receives a rank in each instance ranging from 1 (best) to 4 (worst) according to the average costs of solutions obtained. The solver with the smaller average rank is considered the best.

Table 3: Solvers ranking

Instance	**SA-ILS[9]**	**BVNS**	**RVNS**	**SVND**	**SVNS**
BrazilInstance2	4.0	2.0	5.0	3.0	1.0
BrazilInstance3	3.0	1.0	5.0	4.0	2.0
BrazilInstance4	4.0	2.0	5.0	3.0	1.0
BrazilInstance6	4.0	2.0	5.0	3.0	1.0
FinlandElementarySchool	3.5	1.0	5.0	3.5	2.0
FinlandSecondarySchool2	2.0	2.0	5.0	4.0	2.0
Aigio 1st High School 2010-2011	1.0	3.0	5.0	4.0	2.0
Italy_Instance4	3.0	2.0	5.0	4.0	1.0
KosovaInstance1	4.0	2.0	5.0	3.0	1.0
Kottenpark2003	1.0	3.0	4.0	5.0	2.0
Kottenpark2005A	4.0	3.0	5.0	1.0	2.0
Kottenpark2008	4.0	1.0	5.0	3.0	2.0
Kottenpark2009	4.0	3.0	5.0	1.0	2.0
Woodlands2009	4.0	2.0	5.0	3.0	1.0
Spanish school	2.0	3.0	5.0	4.0	1.0
WesternGreeceUniversityInstance3	3.0	2.0	5.0	4.0	1.0
WesternGreeceUniversityInstance4	3.0	2.0	5.0	4.0	1.0
WesternGreeceUniversityInstance5	2.0	5.0	4.0	2.0	2.0
Average	3.08	2.28	4.89	3.25	1.50

Brito *et al.* [31] presented another VNS based approach to this problem. In their work, they used the Simulated Annealing algorithm to perform the local search at each iteration of VNS. They used the public set of instances from ITC to evaluate their approach since the hidden set was not released yet.

Table 4 presents the comparison between the SVNS method and the results presented by Brito *et al.* [31]. The best result on each instance is highlighted in bold.

4.3. Discussion of Results

For some instances, even the production of feasible solutions configures a hard task. These instances commonly define most of constraints as hard constraints. The VNS approach and its variations were able to find 12 out

Table 4: Comparative between SA-VNS approach [31] and SVNS approach

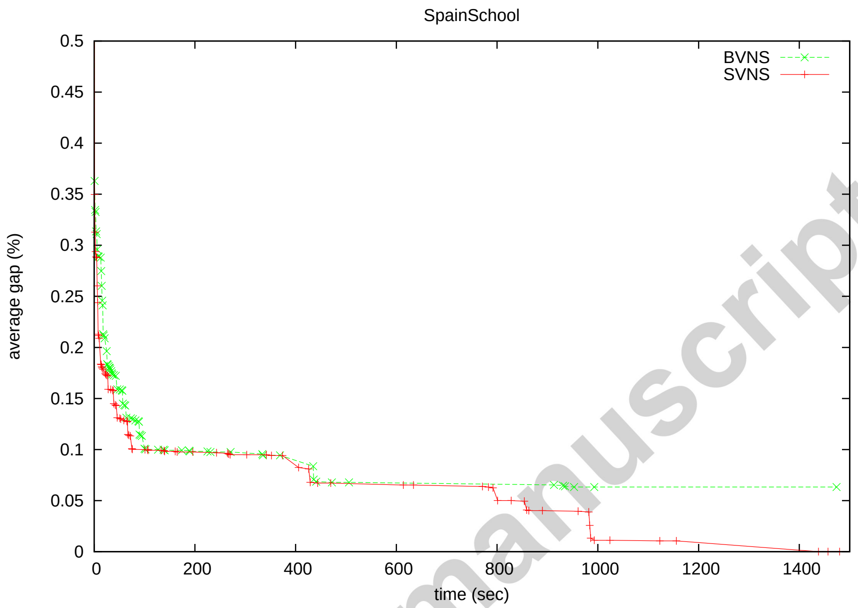
Instance	SA-VNS[31]	SA-RVNS[31]	SVNS
AustraliaBGHS98	11 / 475	11 / 475	**9 / 411**
AustraliaSAHS96	**18 / 52**	18 / 52	19 / 30
AustraliaTES99	9 / 187	9 / 187	**9 / 177**
BrazilInstance1	0 / 21	0 / 44	**0 / 17**
BrazilInstance4	12 / 123	12 / 153	**1 / 90**
BrazilInstance5	4 / 148	4 / 184	**0 / 78**
BrazilInstance6	4 / 213	4 / 213	**0 / 151**
BrazilInstance7	11 / 267	11 / 318	**0 / 242**
EnglandStPaul	2 / 48758	2 / 48450	**1 / 26258**
FinlandArtificialSchool	19 / 12	19 / 12	**6 / 5**
FinlandCollege	**1 / 49**	1 / 77	2 / 32
FinlandHighSchool	**0 / 16**	0 / 73	0 / 29
FinlandSecondarySchool	**0 / 114**	0 / 129	1 / 94
GreecePatras3rdHS2010	0 / 12	0 / 20	**0 / 0**
GreecePreveza3rdHS2008	0 / 37	0 / 33	**0 / 4**
ItalyInstance1	**0 / 20**	0 / 31	0 / 24
Kottenpark2003	1 / 72413	0 / 85372	**0 / 9365**
Kottenpark2005	20 / 28710	20 / 28482	**18 / 10052**
SouthAfricaLewitt2009	0 / 78	0 / 74	**0 /8**

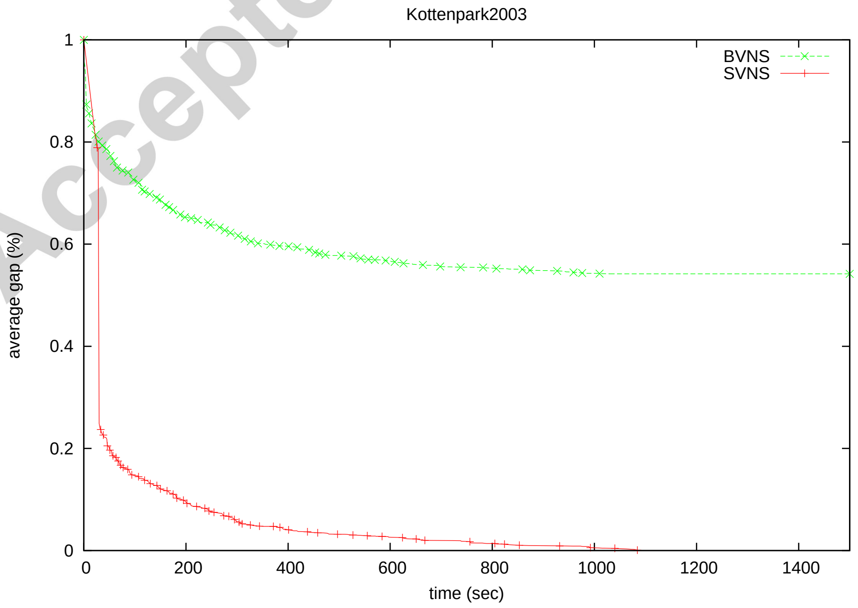
of 18 feasible solutions to the instance set, one more than the Third ITC winner.

As it can be seen in Table 2, the VNS based approach was able to outperform the Third ITC winner. More specifically, the SVNS algorithm presented better results in most of the instances. One explanation to this result may be the fact that SVNS has an improved mechanism to escape from large valleys which does not relies only on randomness.

To understand the positive effect of the controlled diversification in SVNS we plotted in Figure 1 the evolution in time of the relative distance (gap) of the cost of the incumbent solution to the best known solution in BVNS and SVNS for two hard instances. Considering the incumbent solution cost c and the best known solution cost c^* the gap is computed at each time instant as $\frac{c-c^*}{c^*}$. The cost is a fixed point value where the integer part corresponds to the feasibility cost and the fractional part to the quality cost. As it can be seen in Figure 1, while in the beginning of the search both methods are comparable, SVNS improves solutions much more often as the search process advances.

Figure 1: Evolution of the distance to the best known solution in the search process of BVNS and SVNS





The algorithm RVNS presented a poor performance. We believe that the fact that it does not systematically reaches different local optima contributed to these poor results. Moreover, an excessive exploration in larger, more expensive neighborhoods, may also slow down the search in cases where improvement movements could be found in smaller neighborhoods.

We compared the method who found the best results, SVNS, to the SA-VNS approach presented by Brito *et al.* [31]. SVNS was able to outperform the SA-VNS results in 14 out of 19 instances. This result points that a descent method may be more effective than the Simulated Annealing method to perform local search at each iteration of VNS for this problem. The RNA descent method which we implemented has a smaller computational cost and has only one parameter to tune. Note that SA-VNS performed better than SVNS in the easy instances, since the computational cost of Simulated Annealing is not a problem in these cases.

5. Concluding Remarks

The VNS algorithm showed strong results when applied to the High School Timetabling Problem, outperforming the Third ITC winner approach. This result, coupled with another recent reports in literature [25, 26, 27, 32] points that VNS and its variations are very good alternatives for the heuristic solution of timetabling and scheduling problems.

Contrary to the tradition established by the last three timetabling competitions, where the best timetabling solvers incorporated Simulated Annealing in some form, we demonstrated that a proper implementation of the Skewed VNS method provides an excellent heuristic for the High School Timetabling problem. We consider that the VNS approach has two important advantages when compared to SA based approaches: VNS usually has less parameters to tune and these parameters are not sensible to scales.

Some possible future works are (1) to implement and to evaluate another metaheuristics to this problem, like evolutionary algorithms; (2) implement other neighborhood movements; and (3) develop a graphical user interface and allow schools and universities from all around the world to produce their instances and solve them with our solver.

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