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A generalized goal programming model is used to resolve a real-world human resource allocation problem involving allocating teachers to 22 private schools in St. Louis, Missouri. The model provides a solution that balances cost minimization with preference goals of the teachers, administrators, and schools.

A major role of the Blue Hills Home Corporation, St. Louis Division, is providing remedial educational services to private school children in the eastern half of Missouri. Teachers' job assignments are planned on a regional basis. Within the St. Louis metropolitan region, the children at 22 private schools use their services eight hours a day, over nine months. The Blue Hills Home Corporation (BHHC) employed 22 teachers to cover the private school childrens' needs.

Under the terms of the remedial education services agreement, the teachers were assigned to two different schools, four hours at each per day. The teachers would attend one school in the morning and another school in the afternoon. Once the teachers were assigned to two schools, they would go to both schools each day until a new assignment was made for the next school year. The distance the teachers travelled between schools was the basis for reimbursing them. For each school year, BHHC would assign the existing teachers to the schools where children required their services and were eligible to receive them.

Minimizing the traveling expense is only one of the criteria used in assigning jobs. Also considered were the preferences expressed by three groups of

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people: the teachers' supervisors, the teachers, and the individual school principals or administrators. During the year prior to the assignment, these three groups expressed their pleasure or displeasure with particular job assignments, and supervisors made recommendations based on observed productivity, and personal assessments of job attitude. Occasionally, teachers would ask in writing to be removed from or assigned to a specific school. Their preferences were usually based on the working environment, both physical and managerial, at the private schools where they were teachers. The school administrators, who had the authority to discontinue the remedial educational service, occasionally expressed preferences for particular teachers. They communicated with the administration of BHHC by phone or letter praising or criticizing specific educational teachers.

Considering all of the personal preferences of the line supervisors, the teachers, and the school administrators, as well as trying to minimize traveling costs, required the BHHC administration to spend a great deal of time making decisions. Decisions were made even more difficult by conflicting preferences among the supervisors, teachers, and school administrators.

Related Research

Assignment problem formulations date back to the mid-1950s [Dwyer 1954; Flood 1953; Kuhn 1955] and have continued to appear in recent literature [Norula and Oghu 1980; Seshan 1981]. The noncomputer solution procedure usually used to solve assignment problems is the Hungarian method [Lee, Moore, and Taylor 1981, pp. 313-317]. Real-world, large scale assignment problems, are often formulated as cost minimizing linear programming problems and solved with a computer as zero-one problems [Brown and Graves 1981].

Unfortunately, real-world problems do not always have single criterion goals such as cost minimization. When a problem has multiple goals, a multicriteria approach provides a means by which the problem situation can more accurately be incorporated into an optimization model. One of the most commonly used multicriteria optimization techniques is goal programming (GP).

Unfortunately, real-world problems do not always have single criterion goals . . .

The GP approach allows additional relevant criteria to be considered. In addition to the usual goal of cost minimization, judgmental preferences can be incorporated as separate goals in an assignment problem. The advantage of a goal programming model is that these goals can be weighted in accordance with opinion or in accordance with a derived mathematical weighting system. Including such humanistic information in the decision model helps to improve the resulting solution in real terms [Libby 1976] and to improve its psychological acceptance for implementation purposes [Langer 1975].

Goal programming has been used in assignment-related problems, such as manpower planning; Bres et al. [1980]

used goal programming to determine future manpower needs for job assignments. Most applications of goal programming to assignment problems are expressed as transportation type problems [Armstrong and Cook 1979; Kwak and Schniederjans 1979] or as location-allocation problems [Lee and Franz 1979; Lee, Green, and Kim 1981], and not as classic people-to-job assignment problems.

Model Formulation

For the assignment problem facing BHHC, we used a goal programming model with three priorities:

1. In formulating the goal constraints for the first priority level (P_1) the actual assignments had to be incorporated into the model [see Equation (2) and (3) in the Appendix]. Since each of the 22 teachers had to be assigned to two different schools each day, the problem had to be reduced by 22 to eliminate assigning teachers to the same school for both the morning and the afternoon. The model has a total of 462 [that is, $(22 \times 22) - 22$] decision variables, representing alternative assignments of two schools for each of the 22 teachers. A total of 44 goal constraints are required to express this assignment problem.
2. The structuring of the goal constraints at the second priority level (P_2) included substantial judgmental information. Three groups of constraints with appropriate weighting were structured to model the preferences of the supervisors, teachers, and school administrators. All of these preferential goal constraints were similarly structured [see Equation (4) in the Appendix]. However, mathematical weight attached to each deviational variable in the objective function was uniquely determined for the group it modeled.

Three supervisors jointly managed the 22 teachers. The three each submitted several recommendations on what they believe to be appropriate teacher assignments. Some of their recommendations conflicted. To give appropriate judgmental importance to these recommendations, the number of years of managerial experience each supervisor had accumulated with BHHC was used as a numerical weight. This scheme is consistent with modern decision-making research [Einhorn 1972, 1980]. Several teachers expressed preferences for assignment to the same school. To weight these requests, each teacher's seniority and productivity were combined in the following expression:

$$\text{Years of Service to BHHC} \times \text{Current Year's Productivity Index} = w_i.$$

Several teachers expressed preferences for assignment to the same school.

The years of service were taken from personnel records. The productivity index (a scale of 0.00 to 1.00 or low to high) was derived from a yearly evaluation made by supervisors and included in the employee's records. A few school administrators asked that the teachers assigned to their school be changed. An inverse ranking of the number of students to be served at each school gave us an appropriate weighting system for the individual schools. Collectively, the judgmental preferences of the supervisors, teachers, and school administrators added 17 goal constraints to the model with weights ranging from 0.7 to 6.0.

3. The third priority goal (P_3), minimization of the traveling costs, involves only one constraint [see Equation (5) in the Appendix]. The cost was obtained using traveling distance between the two schools, and the standard cost to BHHC for each mile of $0.185.

Once the problem was formulated it was solved using a modified simplex program for goal programming problems. The resulting solution required 276 seconds of execution time on an IBM 370-158.

Model Results and Analysis

On the initial solution of the problem the first goal of an integer solution was fully satisfied, but the second (personal preferences) and the third (travel cost minimization) were not fully satisfied. Examination of the personal preference goals uncovered dissatisfaction in each of the three groups. To see if one or more of these groups might have their goals fully satisfied (at the cost of increased dissatisfaction to the others), each of the three groups of constraints were given an alternative priority ranking. Thus six permutations of a model possessing five priority levels were run (Table 1).

In this sensitivity analysis of the personal preference goals only four of the six alternative solutions provide satisfaction to one or more of the supervisors, teachers, and school administrator preference goals. One solution (alternative 4, Table 1) minimized underachievement of all goals, which is the primary objective of goal programming. If, on the other hand, the objective is to satisfy fully as many groups of goals as possible, another alternative (number 3 on Table 1) is the best solution.

Implementation of the Model Solution

The model has been developed and implemented in part by the BHHC, St. Louis Division. It was used to make the yearly assignment of remedial teachers to private schools in the St. Louis region.

Prior to using the goal programming model, the BHHC administrations had used a tedious trial and error approach that consumed an enormous amount of time. The present goal programming method requires only a yearly update of the mathematical weighting and adjustments for any changes in the schools and personnel to be used in planning the next year's assignments. Not only were hours of administrators' labor saved, but all the parties affected by the solution (the supervisors, teachers, and the school administrators) were allowed some input. This fact was publicized and it significantly reduced the resistance to assignments when they were announced at the beginning of the year.

The costs of developing and running the computer solution to this problem were more than paid for just by the reduction in traveling expenses. During the next year, BHHC was to experience an increase in the standard mileage costs from

Alterate Solution		1		2		3		4		5		6	
		GOAL ORDER	GOAL ACHIEVEMENT	GOAL ORDER	GOAL ACHIEVEMENT	GOAL ORDER	GOAL ACHIEVEMENT	GOAL ORDER	GOAL ACHIEVEMENT	GOAL ORDER	GOAL ACHIEVEMENT	GOAL ORDER	GOAL ACHIEVEMENT
Priority	P_1	A	0	A	0	A	0	A	0	A	0	A	0
Level	P_2	B	0	B	0	C	0	C	0	D	2.1	D	2.1
	P_3	C	8.3	D	8.5	B	0	D	2.2	B	4.3	C	6.4
	P_4	D	8.4	C	8.6	D	15.7	B	8.4	C	4.4	B	6.7
	P_5	E	42.6	E	57.5	E	44.9	E	56.3	E	50.1	E	43.2

Goals: A = Assignment problem formulation (a positive value represents the number of teachers not assigned to a school)
B = Supervisor preferences (a positive value represents the weighted number of preferences not satisfied)
C = Teacher preferences (a positive value represents the weighted number of preferences not satisfied)
D = School administrator preferences (a positive value represents the weighted number of preferences not satisfied)
E = Traveling cost minimization (a positive value represents the total miles traveled daily)

Goal Achievement: 0 Represents a complete achievement

Table 1. The level of goal achievement obtained by using alternative goal ordering sequences.

the $0.170 per mile to $0.185 per mile. BHHC also increased their service from 20 schools to 22 schools. Based on the past year's assignments, the impact of the increased standard mileage costs and additional service to the new schools should have resulted in an increase in the total transportation budget of about 20% over the prior year's costs. The actual total transportation budget during the first year the assignments were made using the GP model only increased by about 5%. BHHC judged the average improvement in traveling distance to be about 10 to 15%.

Potential applications of the goal programming model are limited only by its assignment problem nature. Because the generalized model allows for a multi-criteria solution, this model might broaden the base of assignment problem applications. The application presented demonstrated how even the generalized goal programming model could be modified with alternative arrangements of priorities for use in a sensitivity analysis of the preemptive priority level goals.

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APPENDIX

$$\text{Minimize } Z = P_1 \sum_{j=1}^{n} (d_j^+ + d_j^-) + P_1 \sum_{i=1}^{n} (d_i^+ + d_i^-)$$

$$+ P_2 \sum_{j=1}^{n^*} \sum_{i=1}^{n^*} w_i(d_{ij}^- + d_{ij}^+) + P_3 d^+ \quad (\text{for } n^* \subseteq n) \tag{1}$$

subject to:

$$\sum_{j=1}^{n} x_{ij} + d_j^- - d_j^+ = 1 \quad (\text{for } i = 1, 2, \ldots, n) \tag{2}$$

$$\sum_{i=1}^{n} x_{ij} + d_i^- - d_i^+ = 1 \quad (\text{for } j = 1, 2, \ldots, n) \tag{3}$$

$$x_{ij} + d_{ij}^- - d_{ij}^+ = a_{ij} \quad (a_{ij} = 1 \text{ or } 0) \tag{4}$$

$$\sum_{j=1}^{n} \sum_{i=1}^{n} c_{ij} x_{ij} - d^+ = 0 \tag{5}$$

$$\text{and } x_{ij} = 0 \text{ or } 1 \quad (\text{for } i, j = 1, 2, \ldots, n)$$

Variable and Parameter Definition

x_{ij} {if 1, then person i is assigned to job j; if 0, then no assignment

d_j^- = negative deviation from an assignment goal constraint for the jth job (being assigned less than a whole job)

d_j^+ = positive deviation from an assignment goal constraint for the jth job (being assigned more than a whole job)

d_i^- = negative deviation from an assignment goal constraint for the ith person (assigning less than a whole person)

d_i^+ = positive deviation from an assignment goal constraint for the ith person (assigning more than a whole person)

d^+ = positive deviation in dollars from zero assignment cost goal

d_{ij}^- = negative goal deviation from preferred assignment between the ith person and jth job

d_{ij}^+ = positive goal deviation from preferred assignment between the ith person and jth job

c_{ij} = cost of assigning person i *to job* j

a_{ij} {if 1, then person i is preferred for assignment to the jth job; if 0, then person i is not preferred for assignment to the jth job

w_i = weight given to the ith person's preference in job assignments.

Priority Definitions

P_1 = satisfy basic assignment problem requirements of assigning all people to all jobs

P_2 = minimize dissatisfaction in assigning personnel by recognizing judgment preferences

P_3 = minimize cost of the assignment of personnel.

A letter from John P. Cole, Executive Director of Blue Hills Homes Corporation confirms "the use of the goal programming model by Marc Schniederjans to resolve a personnel assignment problem our corporation faced a few years back . . . it appears the G.P. model reduced transportation cost by 10% to 15% and saved a great deal of administrative planning time."