Two Dimensional Geometric Transformation

2D Geometric Transformation :-

1) Translation
2) Rotation
3) Scaling
4) Reflecting
5) shearing

1) Translation

Translation vector: (t_x, t_y)

$$\boxed{\begin{aligned} &\bullet\ x' = x + t_x \\ &\bullet\ y' = y + t_y \end{aligned}}$$ ← مقدار الحركة

Ex : $p(\overset{x}{2}, \overset{y}{3})$
$(\overset{t_x}{5}, \overset{t_y}{-3})$

$p'(7, 0)$

2) Rotation

Anticlock wise
clockwise

$p(x, y)$

• θ → rotation origin

• $p'(x', y')$

$$\boxed{\begin{aligned} &\bullet\ x' = x\cos\theta - y\sin\theta \\ &\bullet\ y' = x\sin\theta + y\cos\theta \end{aligned}}$$

→ CCW θ +ve
→ CW θ -ve

depends on 2 factors
1) Rotation Angle θ
• clockwise (-ve) Anticlockwise (+ve)
2) Rotation axis (x_p, y_p)

Ex : $p(2, 3)$
rotation ccw by $\theta = 90°$ find p'

$x' = x\cos\theta - y\sin\theta = 2\cos(90) - 3\sin(90) = -3$

$y' = x\sin\theta + y\cos\theta = 2\sin(90) + 3\cos(90) = 2$

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3) Scaling

$p(x,y) \xrightarrow[\text{origin}]{\text{scaling}} p'(x',y')$

(sx, sy) scaling factor

$$\boxed{\begin{array}{l} \bullet\ x' = x\,sx \\ \bullet\ y' = y\,sy \end{array}}$$

depends on 2 factors
1) scaling factor (sx, sy)
- If $sx, sy > 1$ (Amplification)
- If $sx, sy < 1$ (Attenuation)
- If $s_x = s_y$, uniform scaling

2) Fixed point (x_f, y_f)

Ex $p(2,3)$
Scaling by $(\underset{sx}{0.5}, \underset{sy}{4})$

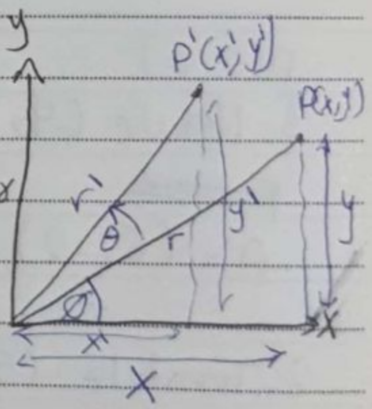
$p' = (1, 12)$

* Final Exam :- proof of hypotense

$x = r.\cos\cancel{\theta}$
$y = r\sin\cancel{\theta}$

$$x' = r\cos(\theta+\emptyset) = r\cos\theta\cos\emptyset - r\sin\theta\sin\emptyset = x\cos\cancel{\theta} - y\sin\cancel{\theta}$$

$$y' = r\sin(\theta+\emptyset) = r\cos\emptyset\sin\theta + r\sin\emptyset\cos\theta$$
$$= x\sin\cancel{\theta} + y\cos\theta$$



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Sequence of Geometric Transformation : Sequence is important
Ex: 1: Scale (0.3, 0.3)   2. Rotate (-90°)   3. Translate (5, 1)

Ex : $p = (2, 6)$ $p' = ??$
$\boxed{p' = (6.8, 0.4)}$

1. Scale (0.3, 0.3) = $x' = x s_x$, $y' = y s_y$
$p' = (0.6, 1.8)$

2. Rotate (-90°) $x' = x\cos\theta - y\sin\theta$, $y' = x\sin\theta + y\cos\theta$
$p = (0.6, 1.8)$ $= 0 - (-1.8)$ $y' = -0.6 + 0$
$p'' = (1.8, -0.6)$

3) Translate (5, 1) $x' = x + t_x$, $y' = y + t_x$
$p'' = (1.8, -0.6)$
$p''' = (6.8, 0.4)$

$p(2, 6)$
1. Rotate (-90) $x' = x\cos\theta - y\sin\theta$, $y' = x\sin\theta + y\cos\theta$
~~p = (cos~~ $= 2\cos(-90) - 6\sin(-90)$ $= 2\sin(-90) + 6\cos(-90)$
$p' = (6, -2)$ $= 6$ $=$

2. Translate (5, 1)
$p'' = (11, -1)$

3) Scale (0.3, 0.3)
$p''' = (3.3, -0.3)$
$\boxed{\therefore \text{Final } p\ (3.3, -0.3)}$

translation :- (t_x, t_y)

$x' = x + t_x$

$y = y + t_y$

Scaling (S_x, S_y)

$x' = x\,s_x$

$y' = y\,s_x$

Rotation θ ↗ ccw +ve ↘ cw −ve

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

Ex Assume $p(2,3)$

sequence
① Rotation by $\theta = 90°$
② Scaling $(3,5)$
③ translation $(-1,4)$

① Rotation by $\theta = 90°$

$x' = x\cos\theta - y\sin\theta$

$= 2\cos 90° - 3\sin 90 = -3$

$y' = x\sin\theta + y\cos\theta$

$= 2\sin(90°) + 3\cos(90°) = 2$

$(-3, 2)$

② Scaling $(3,5)$

$x' = x\,S_x = -3 * 3 = -9$

$y' = y\,s_y = 2 * 5 = 10$

$(-9, 10)$

③ translation $(-1,4)$

$x' = x + t_x = -9 - 1 = -10$

$y' = y + t_y = 10 + 4 = 14$

final point $\boxed{\text{final point} = (-10, 14)}$

Matrix Revision

properties of Matrix

1) Non commutative : $AB \neq BA$
2) Associative ; $(AB)C = A(BC)$
3) Distributive : $A(B+C) = AB + AC$
4) Scalar Multiplication : $(KA)B = A(KB) = K(AB)$ $\boxed{K = \text{scalar Value}}$

Ex : $A_{2\times3} \times B_{3\times4} = C_{2\times4}$

$$\begin{bmatrix} -1 & 4 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} 9 & -3 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} (-1*9)+(4*6) & (-1*3)+(4*1) \\ (2*9)+(3*6) & (2*(-3))+(3*1) \end{bmatrix}$$

$$= \begin{bmatrix} 15 & 7 \\ 36 & -3 \end{bmatrix}$$

Transformation Matrix for Translation

$x' = x + tx$

$y' = y + ty$

$$\underbrace{\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix}}_{\text{transformation matrix for translation}} \underbrace{\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}}_{\text{Input}} = \underbrace{\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}}_{\text{Output}}$$

To be able to Multiply them as no. of columns = no. of rows

Example : The point (2,1) was translated by the vector (-3,2) What is the value of (x', y') ?

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$$

$\boxed{x' = -1 \ , \ y' = 3}$

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Transformation Matrix for Rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} x & y & \text{Free} \end{array} \\ \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

Example: A point (2,2) was rotated 90 degrees counter clockwise about the origin. What is the value of (x', y')?

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$x' = -2$

$y' = 2$

Transformation Matrix for Scaling

$x' = x\,Sx$

$y' = y\,sy$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Example: Scaling by the factor (2, 0.5) to the point (1, 4) find (x', y')?

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$x' = 2$, $y' = 2$

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Sequence of Transformation by Matrix:-

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \end{bmatrix} \begin{bmatrix} 2 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Output Transformation Input

Ex:- What is the Value of (x', y') after applying the sequence on the point $(4,4)$?

1) Scaling $(0.5, 0.5)$
2) Clock wise rotation 90 degrees
3) Translation With the Vector $(1,1)$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-90) & -\sin(-90) & 0 \\ \sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$$

Successive Transformation

1) Two or more successive translations are additive

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Sigma t_x \\ 0 & 1 & \Sigma t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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Example: Two successive Translation are applied to the point (1,5) with translation Vectors (3,2) and (-2,1) find (x', y')?

$t_{x_1}\ t_{y_1}$ $\quad t_{x_2}\ t_{y_2}$

3-2=1

2+1=3

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 1 \end{bmatrix}$$

$x' = 2 \ , \ y' = 8$

Example Derive the geometric transformation matrix for 2 successive translations with vectors $T_1(t_{x_1}, t_{y_1})$ $T_2(t_{x_2}, t_{y_2})$

proof:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_{x_2} \\ 0 & 1 & t_{y_2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & t_{x_1} \\ 0 & 1 & t_{y_1} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_{x_1}+t_{x_2} \\ 0 & 1 & t_{y_1}+t_{y_2} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2) Two or more successive rotations are additive

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\sum^{n}_{i}\theta) & -\sin(\Sigma\theta) & 0 \\ \sin(\Sigma\theta) & \cos(\Sigma\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Example Given the point (2,2) then after applying 90 degrees rotation clockwise, followed by 45 degrees rotation counterclockwise, what's the value of (x', y')?

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(-90+45) & -\sin(-90+45) & 0 \\ \sin(90+45) & \cos(-90+45) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.70 & 0.70 & 0 \\ -0.70 & 0.70 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

Successive Transformation

3) Two or more successive scaling are multiplicative:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx_1 * Sx_2 * ... * Sx_n & 0 & 0 \\ 0 & Sy_1 * Sy_2 * ... * Sy_n & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Ex:- Given the point (3,4), after applying scaling with factor (1,2) followed by scaling with factor (0.5, 0.6) What is the Value of (x', y')?

p(3,4)
→ (1,2)
→ (0.5, 0.6)
(0.5, 1.2)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 4.8 \\ 1 \end{bmatrix}$$

3×3 3×1

$x' = 1.5$
$y' = 4.8$

Example prove that three successive scaling with factors $S_1(sx_1, sy_1)$, $S_2(sx_2, sy_2)$, $S_3(sx_3, sy_3)$ are multiplicative.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx_3 & 0 & 0 \\ 0 & Sy_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Sx_2 & 0 & 0 \\ 0 & Sy_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Sx_1 & 0 & 0 \\ 0 & Sy_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} Sx_1 Sx_2 Sx_3 & 0 & 0 \\ 0 & Sy_1 Sy_2 Sy_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Inverse Transformation

1) Inverse Translation:

$T^{-1}(tx, ty) = T(-tx, -ty)$

Ex. p(2,3)

$T(1,4) \hookrightarrow p'(3,7)$

$T^{-1}(1,4) \hookrightarrow (2,3)$

2) Inverse Rotation

$R^{-1}(\theta) = R(-\theta)$

3) Inverse Scaling

$S^{-1}(sx, sy) = S(\frac{1}{sx}, \frac{1}{sy})$

Ex: $S(2,4) = S(\frac{1}{2}, \frac{1}{4})$

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Rotation around a fixed (pivot) point (x_p, y_p):

a) original position of object & pivot point

b) Translation of object so that pivot point (x_r, y_r) is at origin

c) Rotation about origin

d) Translation of object so that the pivot point is Returned to position (x_r, y_r)

Rotation Matrix about point (x_p, y_p):

$$R((x_p, y_p), \theta) = T(-x_p, -y_p) * R(\theta) * T^{-1}(x_f, y_f)$$

① translation $(-x_p, -y_p)$
② Rotation by θ
③ translation (x_p, y_p)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \overset{③}{\begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix}} \overset{②}{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}} \overset{①}{\begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix}} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & x_p(1-\cos\theta) + y_p \sin\theta \\ \sin\theta & \cos\theta & y_p(1-\cos\theta) - x_p \sin\theta \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Example: we want to rotate the point $(-4, -1)$ $90°$ counterclockwise about the point $(2, 1)$, find the value of (x', y')?

$p(-4, -1)$

$\theta = 90°$

$(2, 1)$ — x_f, y_f

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 90° & -\sin 90° & 2(1-\cos 90°) + \sin 90° \\ \sin 90° & \cos 90° & (1-\cos\theta) - 2\sin 90° \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 3 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -4 \\ -1 \\ 1 \end{bmatrix} \quad x' = 4, \; y' = -5$$

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Scaling around a fixed (pivot) point (x_p, y_p):-

① Translation $(-x_p, -y_p)$ to origin

② Scaling (S_x, S_y)

③ Translation (x_p, y_p) back

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_p \\ 0 & 1 & y_p \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_p \\ 0 & 1 & -y_p \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} S_x & 0 & x_p(1-S_x) \\ 0 & S_y & y_p(1-S_y) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Example:- Given the point (3,4), after applying scaling with factor (1,2) about the point (2,2) what is the Value $P(x', y')$?

$P(3,4)$

$S(\overset{s_x}{1}, \overset{s_y}{2})$

$(\underset{x_f}{2}, \underset{x_y}{2})$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & x_f(1-S_x) \\ 0 & S_y & y_f(1-S_y) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 2(1-1) \\ 0 & 2 & 2(1-2) \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$x' = 3, y' = 6$

Reflection

Cases:-

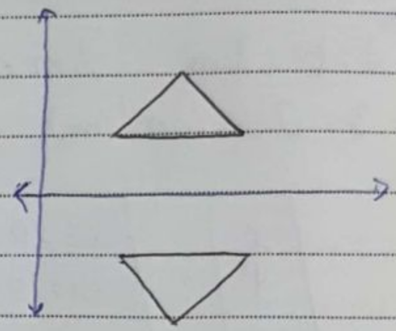
1. Reflection about x-axis
2. Reflection about y-axis
3. Reflection about x-y plane
4. Reflection about $y = mx + b$

1. Reflection about x-axis $y=0$

$x' = x$

$y' = -y$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



2. Reflection about y-axis $x=0$

$x' = -x$

$y' = y$

$$\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3. Reflection about xy plane (Reflection about origin)

$x' = -x$

$y' = -y$

Ex:-

Reflection axis	(2,1)	(3,5)	(4,1)
x-axis	(2,-1)	(3,-5)	(4,-1)
y-axis	(-2,1)	(-3,5)	(-4,1)
xy plane	(-2,-1)	(-3,-5)	(-4,-1)

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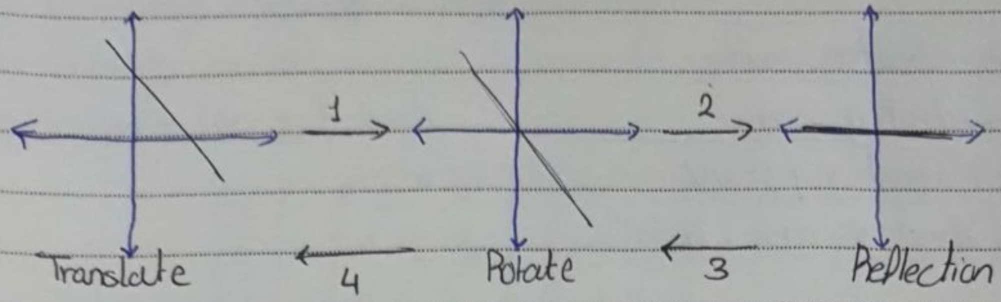
4. Reflection about $y = mx + b$



Steps:

1. $b = \cancel{2m}$ 2. $m =$ 4. 2θ 6. $\sin(2\theta)$

3. $\theta = \tan^{-1}(m)$ 5. $\cos(2\theta)$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 2\theta & -\sin 2\theta & b\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta & b(1+\cos 2\theta) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Example $p(3,4)$

$y = -2x + 6$

$b = 6$, $m = -2$

$\theta = \tan^{-1}(-2) = 63.4$

$2\theta = 2 \cdot (63.4) = 126.8$

$\cos 2\theta = \cos 126.8 = -0.6$

$\sin 2\theta = \sin 126.8 = 0.8$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -0.6 & -0.8 & 6 \cdot (0.8) \\ -0.8 & 0.6 & 6 \cdot (1 + (-0.6)) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} -0.6 & -0.8 & 4.8 \\ -0.8 & 0.6 & 2.4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$x' = -0.2$ $y' = 2.4$

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Example $p(2,3)$
1. Scaling $(5,4)$
2 Reflection about x-axis $y=0$ $x'=x, y'=-y$
3. Rotation ccw by $\theta=90°$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$x'=12$ $y'=10$

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Shearing

- Transformation that distorts the shape of an object.
  example: convert a square to a parallelogram

Shear

An x-direction shear relative to the X-axis (origin):

origin → $x' = x + sh_x \cdot y$

$y' = y$

$$\begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

y = y reference → $x' = x + sh_x \cdot (y - y_{ref})$

$y' = y$

$$\begin{bmatrix} 1 & sh_x & sh_x * y_{ref} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Ex: p(2,3)
x direction $sh_x = 0.5$
find $p'(x', y')$
$x' = x + sh_x \cdot y = 2 + (0.5 * 3) = 3.5$
$y' = y = 3$ $p'(3.5, 3)$

Ex: p(2,3)
Shearing (x) with $sh_x = 0.5$ relative to $y_{ref} = 2$
$x' = x + sh_x (y - y_{ref}) = 2 + (0.5 * 3) = 2.5$
$y' = y = 3$ $p'(2.5, 3)$

An y-direction shear relative to the y-axis (origin)

$x' = x$

origin → $y' = y + sh_y \cdot x$

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

x = x reference → $x' = x$

$y' = y + sh_y \cdot (x - x_{ref})$

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & -sh_y * \\ 0 & 0 & 1 \end{bmatrix}$$

Ex: p(2,3)
y direction $sh_y = 0.5$
find $p'(x', y')$
$x' = x = 2$
$y' = y + sh_y x = 3 + (0.5 * 2) = 4$
$p'(2, 4)$

Ex: p(2,3)
Shearing (y) with $sh_y = 0.5$ relative to x_{ref} =
$x' = x = 2$
$y' = y + sh_y (x - x_{ref}) = 3 + (0.5 * (2 - 1)) = 3.5$
$p'(2, 3.5)$

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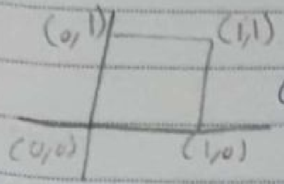
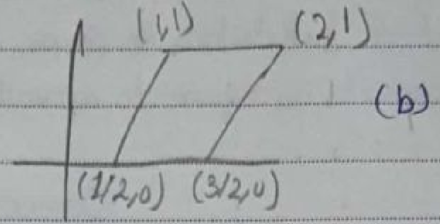
Example:-



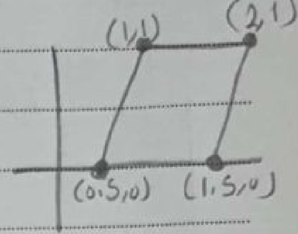


A unit square (a) is transformed to a shifted parallelogram (b) with $sh_x = 0.5$ and $y_{ref} = -1$

$$x' = x + sh_x\,(y - y_{ref}) = x + 0.5(y+1)$$
$$y' = y$$

(0,0)	$x' = 0+0.5(0+1) = 0.5$ $y' = 0$ (0.5,0)
(1,0)	(1.5,0) $x' = 1.5$ $y' = 0$
(0,1)	(1,1) $x' = 1$ $y' = 1$
(1,1)	(2,1) $x' = 2$ $y' = 1$

⟶ draw graph

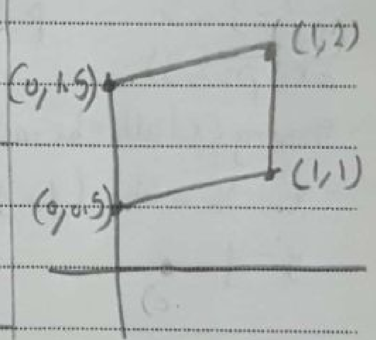


Example:-

A unit square (a) is turned into a shifted parallelogram (b) with parameter Values $sh_y = 0.5$ and $x_{ref} = -1$

$$x' = x$$
$$y' = y + sh_y\,(x - x_{ref}) = y + 0.5(x+1)$$

(0,0)	$x' = 0$ $y' = 0 + 0.5(0+1) = 0.5$ $P'(0, 0.5)$
(1,0)	$x' = 1$ $y' = 0 + 0.5(1+1) = 1$ $P'(1,1)$
(0,1)	$x' = 0$ $y' = 1 + 0.5(0+1) = 1.5$ $P'(0, 1.5)$
(1,1)	$x' = 1$ $y' = 1 + 0.5(1+1) = 2$ $P'(1,2)$



End of 12th Exam.