Capacitated Lot Sizing Problem in Refrigerator Production System

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This paper presents a formulation for scheduling problem of door sub-assembly lines in a refrigerator factory ran by company H. (Company H is a disguised name of a worldwide home appliance maker.) The main topic is to determine lot sizes and production sequence for H company's mixed-models assembly line. A solution computed by Gurobi solver is updated in the github.

Background - Refrigerator Production Process

A refrigerator is basically made up of four parts: inner liner, U shell, door, and cooling system. The inner liner and U shell together form the main cabinet of a refrigerator. After filling insulation foam between the inner liner and U shell, the cabinet goes through a foaming (heating) process. Finally, after a cabinet is successfully foamed and cooled, other components, modules, and doors are added onto the cabinet. Below picture illustrates the basic production process of a refrigerator.

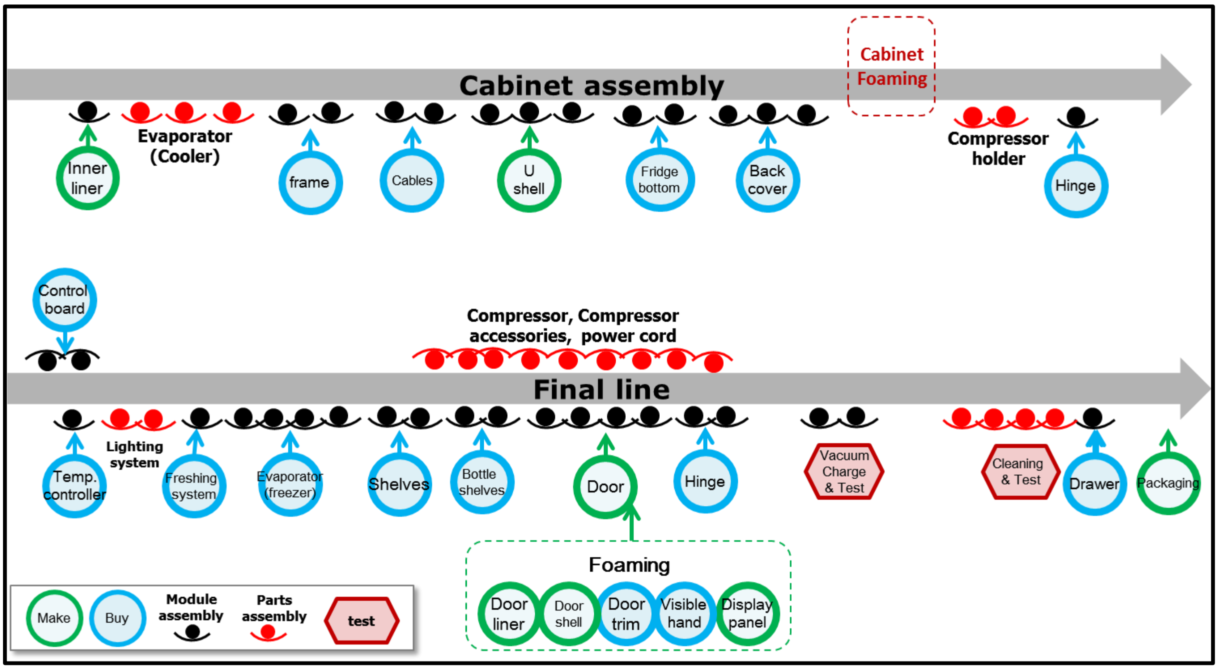

Figure 1

H company manufactured 5 types of refrigerators:

1) **two-door** low-end refrigerators
2) **two-door** high-end refrigerators
3) **three-door** low-end refrigerators
4) **three-door** high-end refrigerators, and
5) **six-door** refrigerators.

Each refrigerator type had approximately 10 – 15 different models. In order to meet various market needs and better line of balance (LOB), H company adopted mixed-product assembly line to produce various models. Each line switched to assemble different models hourly. Sometimes, it even assembled 2 different types (e.g. two-door low-end refrigerator and three-door low-end refrigerator) during the same period of time. This mixed-product assembly strategy made production scheduling and inventory controlling process much more complicated.

Problem Description

H company wanted to determine lot sizes and production sequence for its door assembly line. The capacity was finite. When switching between models, a setup was required. The setups were dependent of the sequence and treated as lost production time. There was a single facility but multiple items could be produced at the same period of time. The inventory storage cost was important as doors were bulky. The trade-off between lost capacity/set-up time due to frequent changeovers and inventory storage cost was the main concern.

Figure 2 illustrates the door sub-assembly process. After a door shell was pre-assembled, a worker placed the metal shell and door liner into a mold holder. (The six squares in the figure). The conveyer carrying the 6 holders would move at a predefined speed and go through a heating process to foam the insulation between the liner and shell. After the heating process, the foamed door would be taken out of the holder. Then, the next door shell and liner would be placed into the empty holder. The conveyer kept moving until setups were required to change the holders.

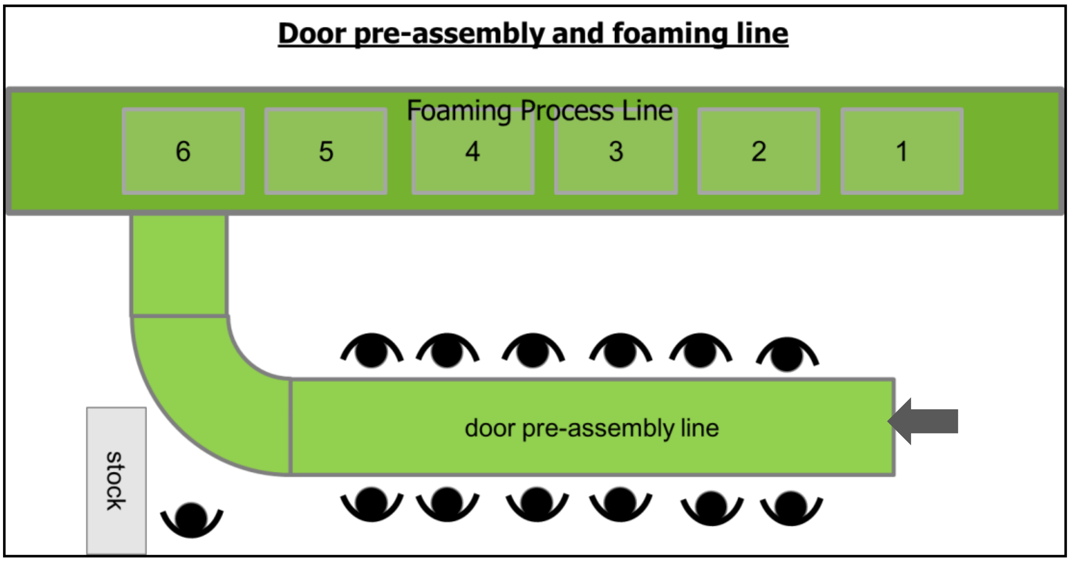

Figure 2

Recall that each type of refrigerator had 10-15 different models. Some models could share the same mold holders and some could not. Table 1 contains one week real daily demands for 15 different models, which were all assembled at one single production line. The commonality and availability of mold holder were listed at the last columns. The various product demands are treated as deterministic and time varying (dynamic) over a finite planning horizon, such as that generated from a material requirements planning (MRP) system.

#	Product ID	Type of fridge	Daily Demand (unit)							Number of available molds		
			Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Sun.	Cabinet	Door (for cooler)	Door (for freezer)
1	A1	2 door (low end)	200	220	220	80	0	0	(usually) Sunday off	14	12	12
2	A2		10	0	0	0	0	0				
3	A3		35	0	0	0	0	0			5	5
4	A4		0	100	110	0	0	0			8	8
5	A5		350	130	120	230	230	230				
6	A6		0	110	110	110	110	110				
7	A7		445	445	445	445	445	445			15	15
8	A8		720	720	720	720	720	720				
9	A9		300	320	320	320	320	320			4	4
10	A10		100	105	115	220	220	220			7	7
11	B1	3 door (low end)	405	780	900	850	725	700		9	14	14
12	B2		0	20	0	50	175	200				
13	B3		400	100	0	0	0	0				
14	B4		90	0	0	0	0	0				
15	B5		100	100	100	100	100	100			3	3
		sum	3155	3150	3160	3125	3045	3045				

Table 1

Our main goal is to determine lot sizes and production sequence for the 15 models. The conveyer of door assembly line could carry a maximum of 26 mold holders. Each holder could accommodate only one freezer or cooler door. (Note: each refrigerator requires two door sets – one freezer and one cooler). The door assembly line ran at a speed that it could produce 12 doors per holder per hour.

Setting up a mold holder took roughly 30 to 40 minutes. Multiple holders could be setup simultaneously. Each changeover caused more defects which could be treated as setup costs. During a changeover, the whole line had to stop. H company usually utilized the break time between two shifts to do setups. Often, an additional setup is required within the shift.

Transformation of the Demands Data

Since the main constraint is mold setup, we only care about the demand for each type of mold during each period of time. Thus, we can aggregate daily demands of multiple models by their mold types as new demand data for modeling.

In addition, we split daily demands into four periods. Each period has five hours for production. There is one hour break for possible setup between each period. The logic behind this split is because H company usually utilized the break time between two shifts or the middle point within a shift to do setups. It is also faster for a solver to determine scheduling for four periods instead of 24 hours a day. The following table shows two-day scheduling after our transformation.

#	Product ID	Type of fridge	Mold Type (i)	Begin. Inv (t=0)	Mon. #1	Mon. #2	Mon. #3	Mon. #4	Mon	Tue. #1	Tue. #2	Tue. #3	Tue. #4	Tue
					t=1	t=2	t=3	t=4	sum	t=5	t=6	t=7	t=8	sum
1	A1	2 door (low end)	**#0**	50	52	53	52	53	210	55	55	55	55	220
2	A2													
3	A3		**#1**	10	8	9	9	9	35	0	0	0	0	0
4	A4		**#2**	90	87	88	87	88	350	85	85	85	85	340
5	A5													
6	A6													
7	A7		**#3**	200	291	291	291	292	1165	291	291	291	292	1165
8	A8													
9	A9		**#4**	70	75	75	75	75	300	80	80	80	80	320
10	A10		**#5**	25	25	25	25	25	100	26	26	26	27	105
11	B1	3 door (low end)	**#6**	150	222	223	225	225	895	225	225	225	225	900
12	B2													
13	B3													
14	B4													
15	B5		**#7**	25	25	25	25	25	100	25	25	25	25	100
		sum							3155					3150

Table 2

Mathematical Formulation of the CLSP

We use the following notation in the mathematical formulation of the capacitated lot sizing problem:

- X_{it} : number of molds of type i in conveyer holders during period t
- Y_{it}^{+}, Y_{it}^{-} : variables to convert absolute values into a linear programming (see below)
- D_{it} : demand for door models corresponding to mold i during period t
- I_{it} : ending inventory of door models i at time t
- M_i : maximum number of molds i available
- SC : setup cost (\$/per changeover)
- IC : inventory holding cost (\$/per unit)

The problem is to find a solution that minimize total inventory holding costs and setup costs:

$$\text{Min.} \quad \text{SC} * (1/2) * \sum_i \sum_t |X_{i,t} - X_{i,t-1}| + \text{IC} * \sum_i \sum_t I_{it}$$

In order to convert absolute values into linear programming, we modify the objective function as below:

$$\textbf{Min.} \quad \textbf{SC} *(1/2) * \sum_i \sum_t (\boldsymbol{Y_{it}^{+}} + \boldsymbol{Y_{it}^{-}}) + \textbf{IC} * \sum_i \sum_t \boldsymbol{I_{it}}$$

subject to:

1) $X_{i,t} - X_{i,t-1} = Y_{it}^{+} - Y_{it}^{-}$, i = 0, ……, 7, t=1, ….., 24

2) $I_{i,t-1} + 5 * 12 * X_{it} - D_{it}$, i = 0, ……, 7, t=1, ….., 24

3) $Y_{it}^{+}, Y_{it}^{-}, I_{it}, X_{it}, D_{it}, \geq 0$

4) $\sum_{i=0}^{7} X_{it} = 13$, for all t

5) $X_{it} <= M_i$, for all t

Computation Software

1) Gurobi optimizer 6.5.1
2) Anaconda Python 2.7
3) OS: OS X El Capitan, 10.11,4

References

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3) Robert R. Love, R. R. Vemuganti, ***"The Single-Plant Mold Allocation Problem with Capacity and Changeover Restrictions"***, Operations Research, Volume 26 Issue 1, February 1978 , Pages 159-165