Automata, Computability and Complexity

Spring Semester 2023
Prof. Dr. Peter Zaspel

Assignment Sheet 1. Submit on **12:00 (noon), Monday, Feb. 13, 2023**.

Excercise 1. (Formal description of finite automata)

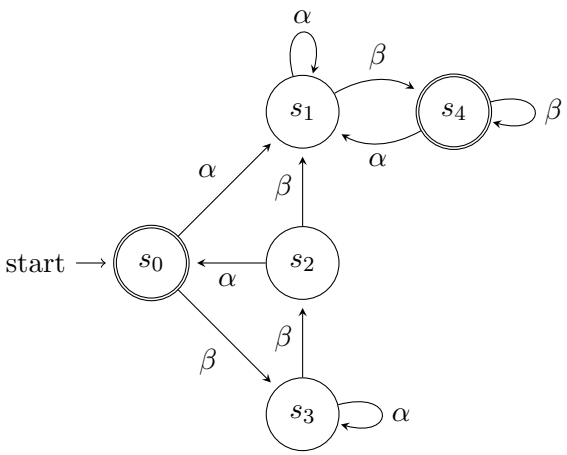
a) Draw the state transition diagram for the finite automaton

$$M_1 = (\{s_0, s_1, s_2, s_3, s_4\},\ \{\alpha, \beta\},\ \delta,\ s_0,\ \{s_0, s_4\})$$

where the transition function δ is:

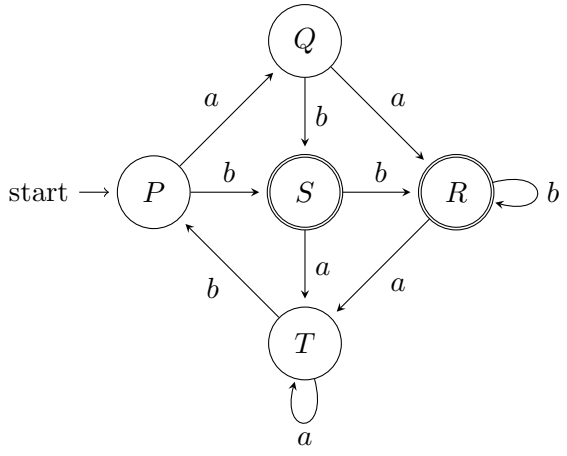
	α	β
s_0	s_1	s_3
s_1	s_1	s_4
s_2	s_0	s_1
s_3	s_3	s_2
s_4	s_1	s_4

Solution.



□

b) The state transition diagram of the automaton M_2 is given below. Using the usual conventions, describe M_2 as a 5-tuple.



Solution. $M_2 = (\{P, Q, R, S, T\}, \{a, b\}, \delta, P, \{S, R\})$ where δ is:

	a	b
P	Q	S
Q	R	S
R	T	R
S	T	R
T	T	P

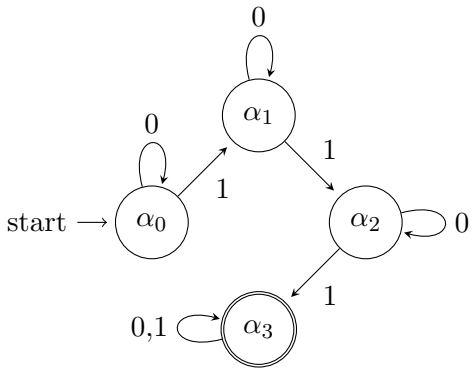
□

(4 Points)

Excercise 2. (Languages accepted by automata)

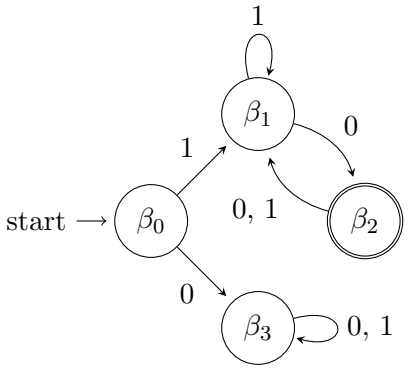
Describe the language accepted by the deterministic finite automata given below.

a)



Solution. $L_a = \{w : w \text{ contains at least 3 occurrences of 1}\}$. □

b)



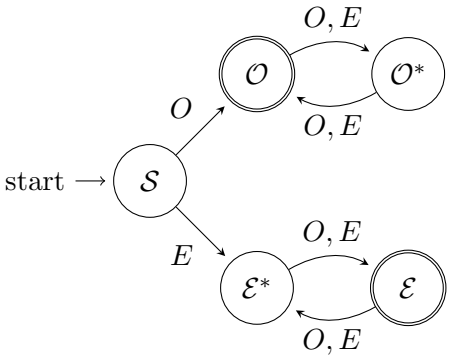
Solution. $L_b = \{w : w \text{ begins with 1, ends with an odd number of 0}\}$. □

(4 Points)

Excercise 3. (Regular languages)

Consider the language L over $\Sigma = \{O, E\}$ that contains all words that begin with O and have odd length or begin with E and have even length. Design a finite automaton that accepts L.

Solution.

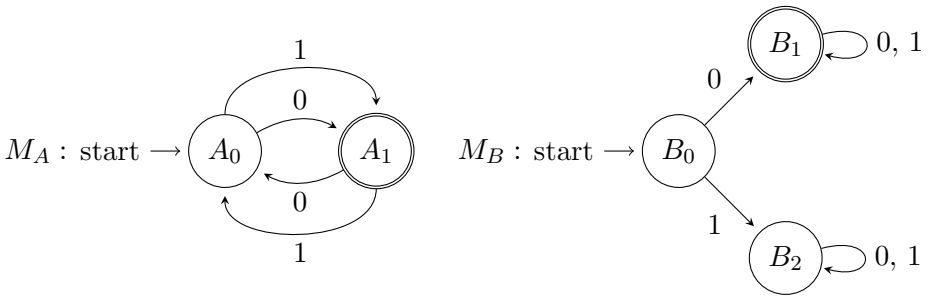


□

(4 Points)

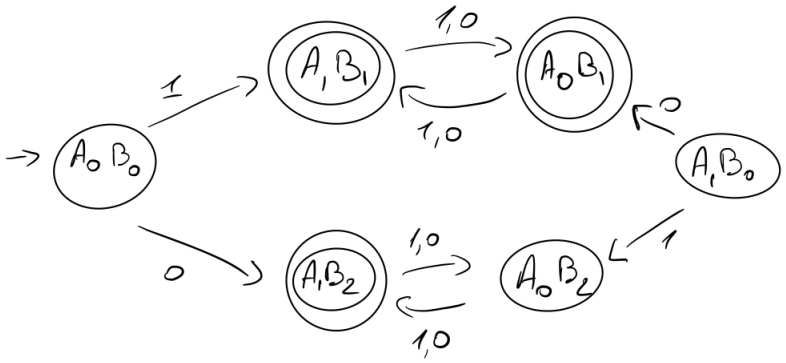
Excercise 4. (Closure property)

Consider the state transition diagrams of two automata M_A and M_B over a common alphabet $\Sigma = \{0, 1\}$ that accept the languages A and B respectively.

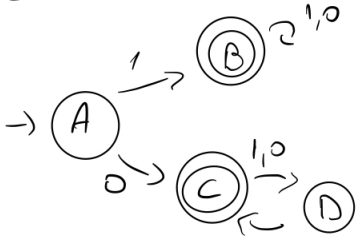


Describe (as a state transition diagram) an automaton M that accepts the language $A \cup B$.

Solution.



Note that A_1B_0 does not influence the language of the automaton at all and can therefore be excluded. Further simplifications result in:



□

(4 Points)