Machine learning homework: 1

Q-1 a) Exactly 2 of A; B; C occur at same time.

$$P(A \cap B) + P(B \cap C) + P(A \cap C) - P(A \cup B \cup C)$$

$$\left[P(A) + P(B) - P(A \cup B)\right] + \left[P(B) + P(C) - P(B \cup C)\right] + \left[P(A) + P(C) - P(A \cup C)\right]$$

$$2P(A) + 2P(B) + 2P(C) - P(A \cup B) - P(A \cup C) - P(B \cup C) - P(A \cup B \cup C)$$

b) $P(A) + P(B) + P(C) - 2\left(P(A \cap B) + P(A \cap C) + P(B \cap C)\right)$

$P(A) + P(B) + P(C) - 2($ ~~$P(A) + P(B) - P(A \cup B) + P(A) + P(B)$~~
$2P(A) + 2P(B) + 2P(C) - P(A \cup B)$
$- P(A \cup C) - P(B \cup C))$.

Q-2

a) $\Omega = \{HHHH, HHHT, HHTT, HTTT, HHTH, HTHH, HTHT, HTTH, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT,\}$

b) discrete random variable $X = \{0, 1, 2, 3, 4\}$

PMF

$P(X=0) = 1/16$

$P(X=1) = 4/16$

$P(X=2) = 6/16$

$P(X=3) = 4/16$

$P(X=4) = 1/16$

CDF

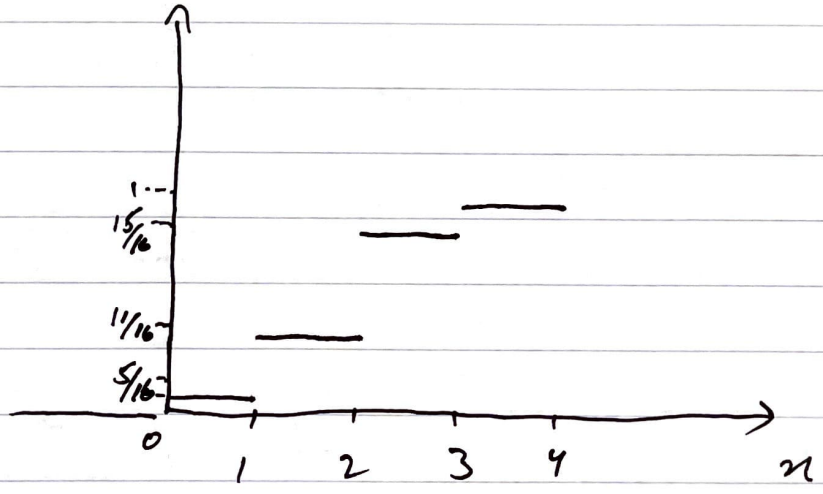
$P(x \leq 0) = 1/16$

$P(x \leq 1) = 5/16$

$P(x \leq 2) = 11/16$

$P(x \leq 3) = 15/16$

$P(x \leq 4) = 16/16 = 1$



c) $P(X>1) = 1 - P(X<=1) = 1 - \frac{1}{16} - \frac{4}{16} = \boxed{\frac{11}{16}}$

d) $E(x) = \sum x\, p(x)$

$= \left(0 \cdot \frac{1}{16}\right) + \left(1 \cdot \frac{4}{16}\right) + \left(2 \cdot \frac{6}{16}\right) + 3\left(\frac{4}{16}\right) + 4\left(\frac{1}{16}\right)$

$= 0 + \frac{4}{16} + \frac{12}{16} + \frac{12}{16} + \frac{4}{16} = \frac{8}{4} = \boxed{2.}$