Automata, Computability and Complexity

Spring Semester 2023
Prof. Dr. Peter Zaspel

Assignment Sheet 4.

Submit on **12:00, March 06, 2022**.

Excercise 1. (Regular and Non-regular Languages)

Fix an alphabet $\Sigma = \{1, -1\}$ and consider the languages

$$L_1 = \{w \in \Sigma^* : w = w_1 \dots w_n \text{ and } \prod_{i=1}^{n} w_i = 1\},$$
$$L_2 = \{w \in \Sigma^* : w = w_1 \dots w_n \text{ and } \sum_{i=1}^{n} w_i = 0\}.$$

Check by describing an NFA that L_1 is a regular language and prove that L_2 is not a regular language.

(4 Points)

Excercise 2. (Non-regular languages)

a) Proof that Dyck's language is not regular:

$$L_1 = \{u \in \Sigma^* |\text{ all prefixes of } u \text{ contain no more]'s than ['s and the number of ['s in } u \text{ equals the number of]'s; }\}$$

b) Proof that language of prime numbers in unary notation is not regular:

$$L_2 : \{aa, aaa, aaaaa, \dots\}.$$

(4 Points)

Excercise 3. (Describing context free languages)

a) Describe the language of the context-free grammar $G_1 = (\{A, B\}, \{0, 1, 2\}, R, A)$ with rules:

$$A \rightarrow 00A1 \,|\, B$$
$$B \rightarrow \varepsilon \,|\, 22B$$

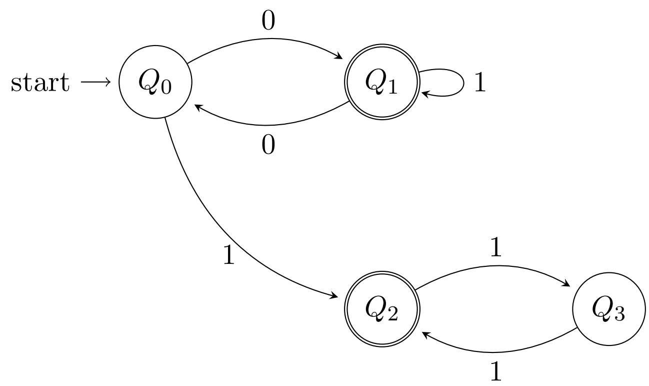
b) Describe the language of the context-free grammar $G_2 = (\{S, A, B\}, \{a, b\}, R, S)$ with rules:

$$S \rightarrow AB$$
$$A \rightarrow aAb \,|\, \varepsilon$$
$$B \rightarrow bBa \,|\, \varepsilon$$

(4 Points)

Excercise 4. (CFG conversions)

a) Convert the following finite automaton into a CFG.



b) Convert the context-free grammar $G_3 = (\{S, X, Y\}, \{a, b, c\}, R, S)$ with rules

$$\begin{aligned} S &\to aXbX \\ X &\to aY \mid bY \mid \varepsilon \\ Y &\to X \mid c \end{aligned}$$

into Chomsky normal form.

(4 Points)