Automata, Computability and Complexity

Spring Semester 2023
Prof. Dr. Peter Zaspel

Assignment Sheet 2. Submit on **Monday, Feb. 20, 2023, 12:00 (noon)**.

Excercise 1. (Formal description of nondeterministic finite automata)

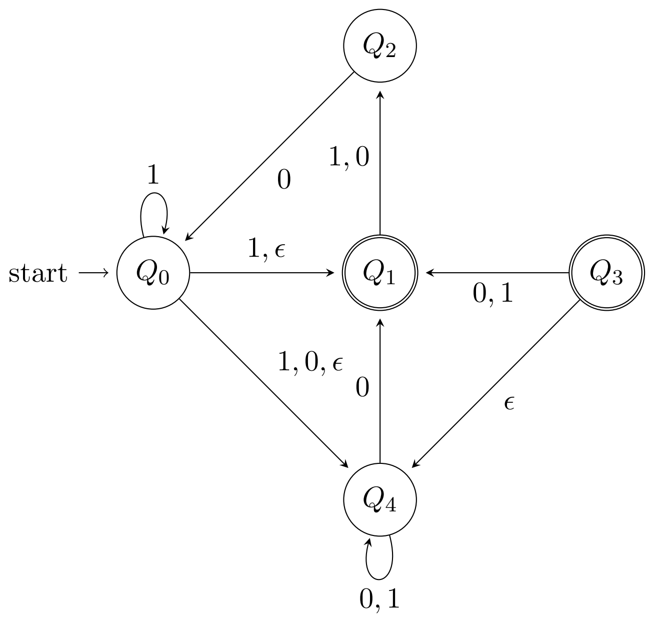
a) Draw the state transition diagram for the NFA

$$N_1 = (\{s_0, s_1, s_2, s_3\},\ \{0,1\},\ \delta,\ s_0,\ \{s_1\})$$

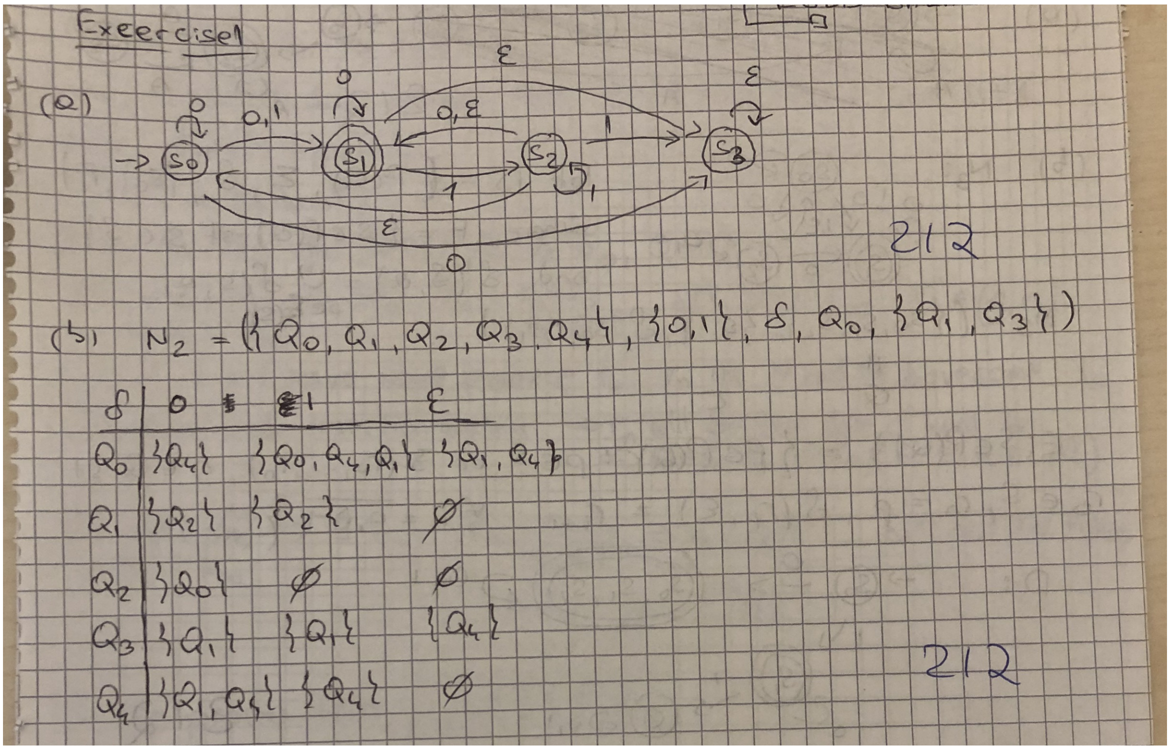
where the transition function δ is:

	0	1	ϵ
s_0	$\{s_0, s_1, s_3\}$	$\{s_1\}$	$\emptyset$
s_1	$\{s_1\}$	$\{s_2\}$	$\{s_3\}$
s_2	$\{s_1\}$	$\{s_2, s_3\}$	$\{s_0, s_1\}$
s_3	$\emptyset$	$\emptyset$	$\{s_3\}$

b) The state transition diagram of the automaton N_2 is given below. Using the usual conventions, describe N_2 as a 5-tuple.



Solution.



(b) $N_2 = (\{Q_0, Q_1, Q_2, Q_3, Q_4\}, \{0,1\}, \delta, Q_0, \{Q_1, Q_3\})$

δ	0	1	ε
Q_0	$\{Q_4\}$	$\{Q_0, Q_4, Q_1\}$	$\{Q_1, Q_4\}$
Q_1	$\{Q_2\}$	$\{Q_2\}$	$\emptyset$
Q_2	$\{Q_0\}$	$\emptyset$	$\emptyset$
Q_3	$\{Q_1\}$	$\{Q_1\}$	$\{Q_4\}$
Q_4	$\{Q_1, Q_4\}$	$\{Q_4\}$	$\emptyset$

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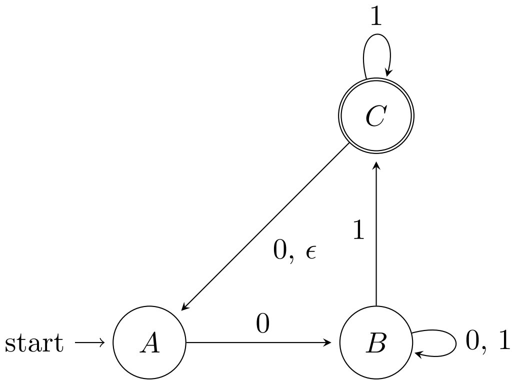
□

(4 Points)

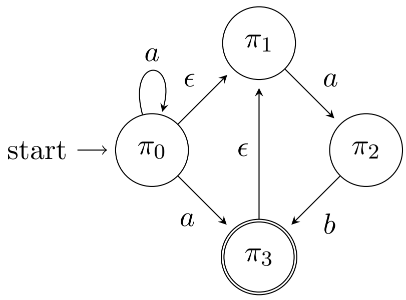
Excercise 2. (Languages accepted by nondeterministic automata)

Describe the language accepted by the nondeterministic finite automata given below.

a)



b)



Solution.

Excercise 2

(a) $L(NFA) = \{ w = \overline{w_1 \dots w_n} : w_i \in \{0,1\}, w_1 = 0, w_n = 1 \}$

2|2

(b) $L(NFA) = \{ w = \overline{w_1 \dots w_n} : w_i \in \{a,b\}, w_1 = a$, and if $w_i = b$ $(2 \leq i \leq n)$ then $i = n$ or $i \leq n-2$, $w_{i+1} = a$, $w_{i+2} = b \}$

= words starting with a and ending in 0 or more sequences of ab with nothing between them

2|2

$\square$

(4 Points)

Excercise 3. (Equivalence of NFAs and DFAs)

a) Consider the language L over $\sum = \{A, B\}$ that contains all words beginning with A and that satisfy at least one of these two conditions:

- Every B is immediately followed by an A
- The symbol B appears an even amount of times

Design a NFA that accepts L with at most 5 states.

b) Given the NFA

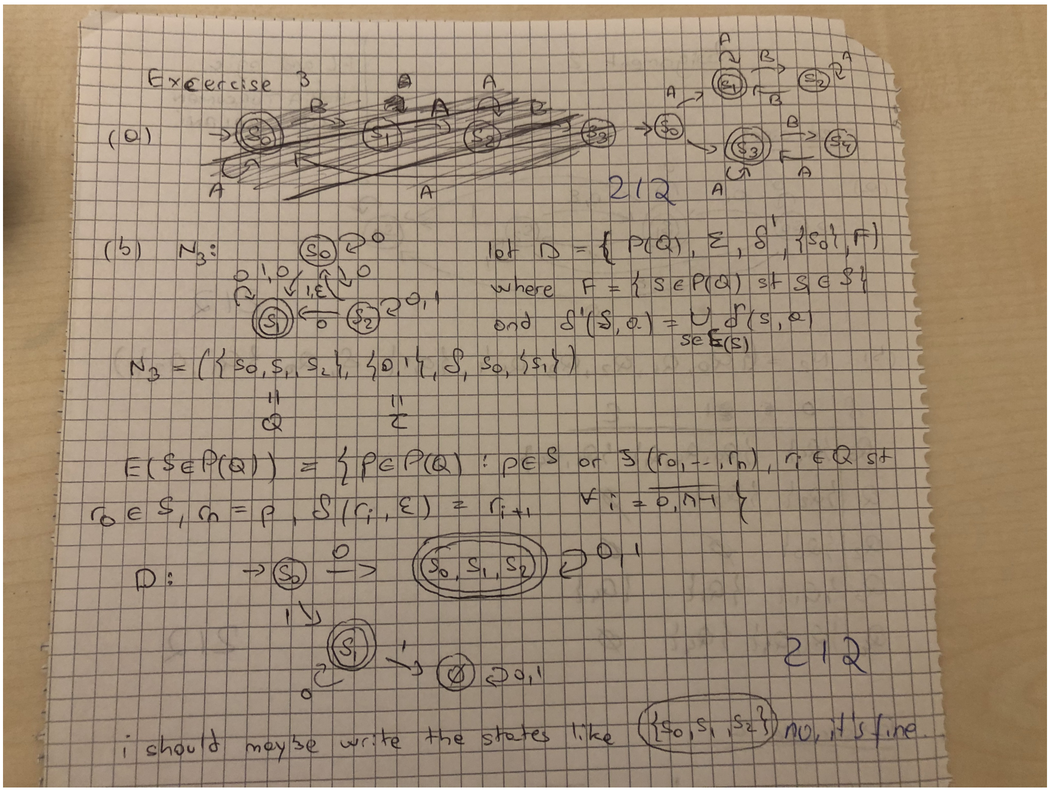
$$N_3 = (\{s_0, s_1, s_2\},\ \{0,1\},\ \delta,\ s_0,\ \{s_1\})$$

with the transition function

	0	1	ϵ
s_0	$\{s_0, s_1, s_2\}$	$\{s_1\}$	$\emptyset$
s_1	$\{s_1\}$	$\emptyset$	$\emptyset$
s_2	$\{s_1, s_2\}$	$\{s_0, s_2\}$	$\{s_0\}$

design a DFA that accepts the same language using the subset construction (see the proof of Theorem from the lecture).

Solution.



$\square$

(4 Points)

Excercise 4. (Variants of NFAs)

Given a language L over an alphabet $\sum$, define the complement language $L^C = \{w \in \sum^* | w \notin L\}$. Given an STD of an NFA accepting L, provide a general recipe how to construct a NFA that accepts L^C.

Solution.

Exercise 4 a

L over Σ. $L^c := \{ w \in \Sigma^* \mid w \notin L \}$ N - an NDA

N accepts L. $\Rightarrow L(N) = L$

From theorem 1.3. we have that $\exists \hat{M}$ s.th. $\hat{M}$ is an DFA and $L(\hat{M}) = L(N) = L$

$\hat{M} = (Q, \Sigma, \delta, s_0, F)$

Take M - DFA sth. $M = (Q, \Sigma, \delta, s_0, Q-F)$

If $\underset{w = w_0 \dots w_{n+1}}{w \in L(M)} \Rightarrow \exists r_0 \dots r_n : \begin{matrix} r_0 = s_0 \\ r_n \in Q-F \end{matrix} \quad \delta(r_i, w_i) = r_{i+1} \; \forall i \in (0, n)$

In $\hat{M}$ w will have same sequence $r_0 \dots r_n$ as M and $\hat{M}$ have same s_0, states and transition function $\Rightarrow r_n$ will be final state of word w $\Rightarrow r_n \in Q-F \Rightarrow r_n \notin F \Rightarrow w \notin L(\hat{M})$

If $\underset{w = w_0 \dots w_{n+1}}{w \notin L(\hat{M})} \Rightarrow \exists r_0, \dots, r_n : \begin{matrix} r_0 = s_0 \\ r_n \notin F \end{matrix} \Rightarrow r_n \in Q-F \quad \delta(r_i, w_i) = r_{i+1} \; \forall i \in (0,n)$

$\Rightarrow w \in L(M)$

$\Rightarrow$ We proved that $\boxed{L(M) = L^c}$

2/2

□

(4 Points)