

Machine learning homework: 1

Q-1 a) Exactly 2 of A; B; C occur at same time.

$$P(A \cap B) + P(B \cap C) + P(A \cap C) - P(A \cup B \cup C)$$

$$\begin{aligned} & \downarrow \\ & [P(A) + P(B) - P(A \cup B)] + [P(B) + P(C) - P(B \cup C)] \\ & \quad + [P(A) + P(C) - P(A \cup C)] \end{aligned}$$

$$\begin{aligned} & 2P(A) + 2P(B) + 2P(C) - P(A \cup B) - P(A \cup C) - P(B \cup C) \\ & \quad - P(A \cup B \cup C) \end{aligned}$$

b) $P(A) + P(B) + P(C) - 2(P(A \cap B) + P(A \cap C) + P(B \cap C))$

$$\begin{aligned} & P(A) + P(B) + P(C) - 2(\cancel{P(A)} + \cancel{P(B)} - \cancel{P(A \cup B)} + \cancel{P(A \cup C)} + \cancel{P(B \cup C)}) \\ & \quad 2P(A) + 2P(B) + 2P(C) - P(A \cup B) - P(A \cup C) - P(B \cup C) \end{aligned}$$

Q-2

a) $\Omega = \{HHHH, HHHT, HHTT, HTTT, HHTH, HTHH, HTTH, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTT\}$

b) discrete random variable $X = \{0, 1, 2, 3, 4\}$

PMF $P(X=0) = 1/16$

$P(X=1) = 4/16$

$P(X=2) = 6/16$

$P(X=3) = 4/16$

$P(X=4) = 1/16$

CDF

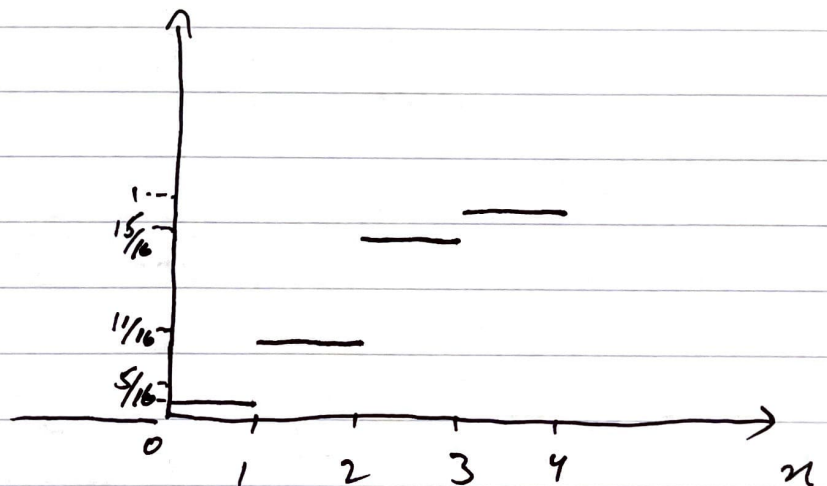
$P(X \leq 0) = 1/16$

$P(X \leq 1) = 5/16$

$P(X \leq 2) = 11/16$

$P(X \leq 3) = 15/16$

$P(X \leq 4) = 16/16 = 1$



$$c) P(X > 1) = 1 - P(X \leq 1) = 1 - \frac{1}{16} - \frac{4}{16} = \boxed{\frac{11}{16}}$$

$$d) E(X) = \sum x p(x)$$

$$= \left(0 \cdot \frac{1}{16}\right) + \left(1 \cdot \frac{4}{16}\right) + \left(2 \cdot \frac{6}{16}\right) + 3\left(\frac{4}{16}\right) + 4\left(\frac{1}{16}\right)$$

$$= 0 + \frac{4}{16} + \frac{12}{16} + \frac{12}{16} + \frac{4}{16} = \frac{8}{4} = \boxed{2}$$

Q-3
$$P(x) = \begin{cases} c |x| (1+x)(1-x) & \text{if } -1 \leq x \leq 1 \\ 0 & \text{else,} \end{cases}$$

a) normalization condition.

$$\int_{-1}^1 P(x) dx = c \int_{-1}^1 |x| (1-x)(1+x) dx.$$

$$= c \int_{-1}^1 (x-1)(x+1) |x| dx$$

$$\frac{x}{|x|} \int_{-1}^1 \frac{|x|}{x} (x-1)(x+1) |x| dx.$$

$$= \frac{x}{|x|} \int_{-1}^1 x(x-1)(x+1) dx.$$

$$\begin{aligned} u &= (x-1)(x+1). \\ du &= 2x dx. \\ dx &= \frac{du}{2x}. \end{aligned}$$

$$= \frac{x}{|x|} \int \frac{x du}{2x} \Rightarrow \frac{u^2}{2} \cdot \frac{1}{2}.$$

$$= \frac{x}{|x|} \frac{(x-1)^2 (x+1)^2}{4} \dots c$$

$$= \left| -c \frac{x(x-1)^2 (x+1)^2}{4|x|} \right|_{-1}^1$$

$$1 = \frac{c}{2} \cdot \frac{1}{2} = 1 \quad c = 2$$