Practice exam for Probability and Random Processes
23.11.22

Choose all the correct answers in each problem. For every correct choice you will receive 1 **and for any wrong answer** −1 **points. For instance, if (a) and (c) are the correct answers, then choosing (a),(b),(c) results in 1 point, choosing (a) and (c) in 0 points, and choosing only (a) in 1 point.**

1. A dice is thrown twice. Denote the outcomes by X and Y. Choose the correct statement(s):

    A. $\mathbf{P}[X = Y] = 1/6$.

    B. X has a binomial distribution.

    C. X and Y are negatively correlated.

    D. $\mathbf{E}[X] = 3$.

2. A pair of fair dice are tossed. Let X denote the sum and Y be the maximum of the outcomes. Choose all the correct statement(s).

    A. X and Y are positively correlated.

    B. $\mathbf{E}[X] = 7$.

    C. $\mathbf{E}[Y] \leq 3$

    D. $\mathbf{P}[Y = 1] = 1/36$

3. Let X be a continuous random variable with the density function defined by $f(x) = 2x$ for $0 \leq x \leq 1$ and $f(x) = 0$ for other values of x. Choose all the correct statement(s)

    A. X has a uniform distribution.

    B. $\mathbf{P}[X \leq 1] = 1$.

    C. $F_X(t) = t^2$ for all values of t.

    D. The random variable $Y = X^2$ has a uniform distribution.

4. The length of sides of a rectangle R are independent random variables X and Y with uniform distribution in $(0, 1)$. Let A and P denote, respectively, the area and perimeter of this rectangle. Choose all the correct statement(s).

    A. $\mathbf{E}[A] = 1/4$.

    B. A has uniform distribution over $(0, 1)$.

    C. $\mathbf{E}[P] = 2$.

    D. The probability that R is a square is $1/2$.

5. Choose all the statement(s) that hold for all pairs of random variables X and Y:

    A. $\mathbf{E}[X + Y] = \mathbf{E}[X] + \mathbf{E}[Y]$

    B. $\mathbf{E}[XY] = \mathbf{E}[X] \cdot \mathbf{E}[Y]$

    C. $\mathbf{E}[X^2] \geq \mathbf{E}[X]^2$.

    D. If X and Y are independent and $\mathbf{E}[X] = 0$ then $\mathbf{E}[XY] = 0$.

6. Let X be a continuous random variable with the density function $f_X(x) = \frac{3}{16}x^2$ for $-2 < x < 2$ and $f_X(x) = 0$ otherwise. What is $\mathbf{P}[X > 1]$?

    A. $7/8$

    B. $7/16$

    C. $1/8$

    D. None of the above.

7. Suppose $\mathbf{E}[X] = 1$ and $\mathbf{Var}[X] = 2$. Choose all the correct statement(s).

    A. $\mathbf{E}[2X - 1] = 1$.

    B. X is discrete.

    C. X has a uniform distribution.

    D. $\mathbf{E}[X^2] \geq 2$.

8. The joint probability mass function of X and Y is given by the following table:

	$Y = 0$	$Y = 1$	$Y = 2$
$X = 0$	1/8	0	0
$X = 1$	1/8	1/8	0
$X = 2$	1/8	1/4	0
$X = 3$	0	0	1/8

Choose all the correct statement(s).

    A. $p_{X|Y}(3|2) = 1$.

    B. $\mathbf{P}[X = 1] < \mathbf{P}[Y = 1]$.

    C. X and Y are independent.

    D. $\mathbf{P}[X \geq Y] = 1$.

9. Let A be a 2×2 matrix whose entries are independent and each is uniformly chosen from the set $\{1, 2, 3\}$. Set $D = \det A$. Choose all the correct statement(s).

    A. $\mathbf{P}[D = 9] = 1/2$

    B. $\mathbf{E}[D] = 0$.

    C. D is a discrete random variable.

    D. $\mathbf{P}[D = 8] < 1/9$.

10. An integer N randomly from the set $\{1, 2, 3\}$. Once N is chosen, we throw N fair dice and denote by X the sum of scores obtained. For instance, if $N = 3$, three dice will be thrown and if the outcomes are $2, 3, 3$ then we set $X = 8$. The value of $\mathbf{E}[X]$ is given by

    A. 6

    B. 7

    C. 8

    D. None of the above.

11. Let X be a random variable with $\mathbf{E}[X] = 2$. Choose all the correct statement(s).

    A. $\mathbf{P}[X \geq 100] < 1/100$.

    B. If $X \geq 0$ then $\mathbf{P}[X > 4] \leq \frac{1}{2}$.

    C. $\mathbf{E}[X^2] = 4$.

    D. X has a Poisson distribution with parameter $\lambda = 2$.

12. If X and Y are independent random variables, which one of the following statement(s) is necessarily true:

    A. $\mathbf{Var}[XY] = \mathbf{Var}[X]\mathbf{Var}[Y]$

    B. $\mathbf{E}[XY] = \mathbf{E}[X]\mathbf{E}[Y]$

    C. $\mathbf{Var}[X + Y] = \mathbf{Var}[X] + \mathbf{Var}[Y]$.

    D. $\mathbf{P}[X = Y] = 0$.