CH-231-A

Algorithms and Data Structures

ADS

Lecture 22

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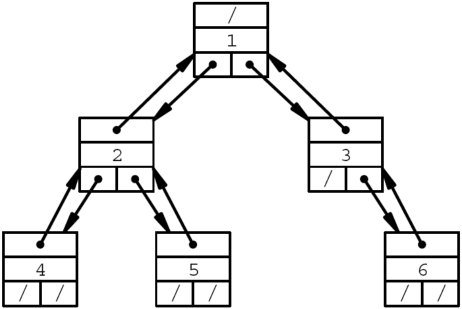
Representing Rooted Trees

- Traversing a rooted tree requires us to know about the hierarchical relationships of their nodes.
- Similar to linked list implementations, such relationships can be stored by using pointers.

Binary Tree

- Binary trees T have an attribute $T.root$.
- They consist of nodes x with attributes $x.parent$ (short $x.p$), $x.left$, and $x.right$ in addition to $x.key$.



d-ary Trees

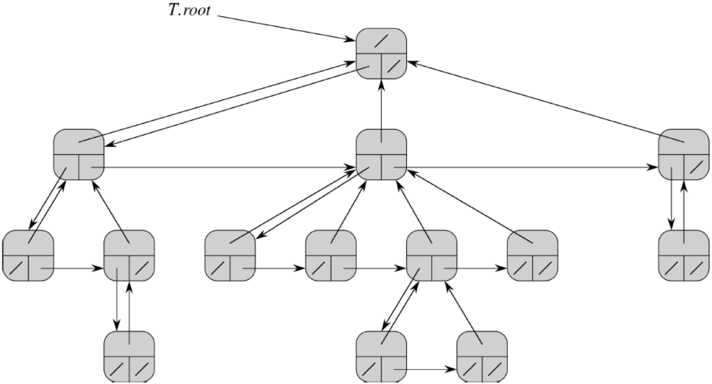
- d-ary trees are rooted trees with at most d children per node.
- They can be handled analogously to binary trees.

```
struct node {
    int val;
    node* parent;
    node* child[d];
};

typedef node* tree;
```

Rooted Trees with Arbitrary Branching

Rooted trees T with arbitrary branching consist of nodes x with attributes $x.p$, $x.leftmost\text{-}child$, and $x.right\text{-}sibling$ in addition to $x.key$.



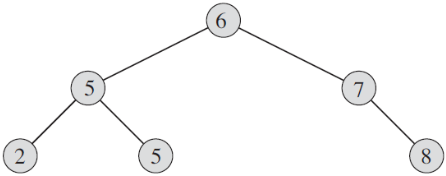
Discussion

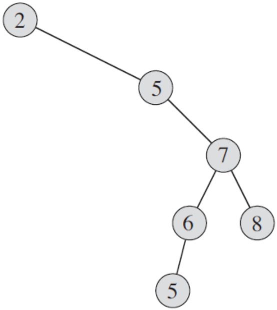
- Representing trees with pointers allows for a simple and intuitive representation.
- It also allows for a dynamic data management.
- Modifying operations can be implemented efficiently.
- However, extra memory requirements exist for storing the pointers.

Binary Search Tree: Definition

- A binary search tree (BST) is a binary tree with the following property:
  - Let x be a node of the BST.
  - If y is a node in the left subtree of x, then $y.key \leq x.key$.
  - If y is a node in the right subtree of x, then $x.key \leq y.key$.
- The idea of a BST data structure is to support efficient dynamic set operations, many in $O(h)$, where h is the tree's height.

Binary Search Tree: Examples





Query: In Order Visit

- Visit all nodes in order and execute an operation:

Function DFS-Inorder-Visit(Node n)

if $n = NIL$ **then return**;
DFS-Inorder-Visit($n.left$) ;
n.Operation() ;
DFS-Inorder-Visit($n.right$) ;

- The operation could, e.g., be printing the key.
- This tree traversal is also referred to as in-order tree walk.
- Time complexity ($n =$ number of nodes):
  $O(nk)$ when assuming that the operation is in $O(k)$.

Query: Searching

- Recursive tree search:

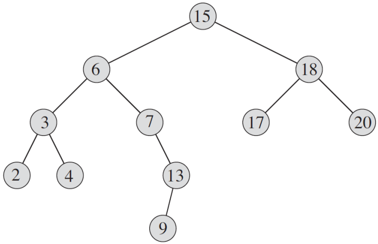
TREE-SEARCH(x, k)

```
if x == NIL or k == x.key
    return x
if k < x.key
    return TREE-SEARCH(x.left, k)
else return TREE-SEARCH(x.right, k)
```

- Iterative tree search:

ITERATIVE-TREE-SEARCH(x, k)

```
while x ≠ NIL and k ≠ x.key
    if k < x.key
        x = x.left
    else x = x.right
return x
```



Time complexity: $O(h)$