# Assignment Sheet 4, Problem 1

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## 1 Solution

Start by finding expected (squared) prediction error for $f$ (use the fact that we know the exact relation between $X$ and $Y$, i.e. $\rho(x,y) = \rho_X(x)\delta(y - e^x)$ with uniform distribution $\rho_x(x) = 1/2 \cdot \mathbb{1}_{[-1,1]}$):

$$\mathrm{EPE}(f) = \mathbb{E}[L_2(Y, f(X))] = \int_{-1}^{1} 1/2 \cdot (e^x - (1+x))^2 \, \mathrm{d}x \approx 0.061$$

As we can see, the error is not too large, and we can further improve our results by using the best quadratic approximation for $e^x$, given by $f^{(2)}(x) = 1 + x + \frac{x^2}{2}$ (see Taylor's theorem), drastically improving the results:

$$\mathrm{EPE}(f^{(2)}) \approx 0.005$$

As for the second part, due to our initial strong assumption $\rho(x,y) = \rho_X(x)\delta(y - e^x)$ the regressor can be directly deduced (in line with our expectation):

$$\mathbb{E}[Y|X = x] = e^x$$