Introduction to Computer Science
Jacobs University Bremen
Dr. Jürgen Schönwälder

Course: CH-232-A
Date: 2021-09-24
Due: 2021-10-01

ICS 2021 Problem Sheet #3

Problem 3.1: *cartesian products* (1+1 = 2 points)

Prove or disprove the following two propositions:

a) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

b) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$

Problem 3.2: *reflexive, symmetric, transitive* (3 points)

For each of the following relations, determine whether they are reflexive, symmetric, or transitive. Provide a reasoning.

a) The absolute difference of the integer numbers a and b is less than or equal to 3.

$$R = \{(a,b) | a,b \in \mathbb{Z} \wedge |a-b| \leq 3\}$$

b) The last digit of the decimal representation of the integer numbers a and b is the same.

$$R = \{(a,b) | a,b \in \mathbb{Z} \wedge (a \bmod 10) = (b \bmod 10)\}$$

Problem 3.3: *total, injective, surjective, bijective functions* (1+1 = 2 points)

Are the following functions total, injective, surjective, or bijective? Expain why or why not.

a) $f : \mathbb{N} \mapsto \mathbb{N}$ with $f(x) = 2x^2$

b) $f : \mathbb{R} \mapsto \mathbb{R}$ with $f(x) = x^2 + 6$

Problem 3.4: *function composition* (1 point)

Given the functions $f(x) = x + 1$. $g(x) = 2x$, and $h(x) = x^2$, determine an expression for the following function compositions:

a) $f \circ g$

b) $f \circ h$

c) $g \circ f$

d) $g \circ h$

e) $h \circ f$

f) $h \circ g$

g) $f \circ (g \circ h)$

h) $h \circ (g \circ f)$

Problem 3.5: *list comprehensions (haskell)* (1+1 = 2 points)

Your list comprehensions should be correct, they do not have to be efficient. You are not getting points for a list comprehension simply returning a hard coded solution list. In other words, your list comprehensions should continue to function correctly if parameters are changed.

a) Write a list comprehension that returns all positive factors of the number 210. Try to write the list comprehension in such a way that 210 can easily be replaced by a different number.

b) Write a list comprehension that returns a list of Pythagorean triads (a, b, c), where a, b, c are positive integers in the range $1..100$ and the Pythagorean triad is defined as $a^2 + b^2 = c^2$. The list should not contain any "duplicates" where a and b are swapped. If the list contains $(3, 4, 5)$ (since $3^2 + 4^2 = 25 = 5^2$), then is should not also include $(4, 3, 5)$.