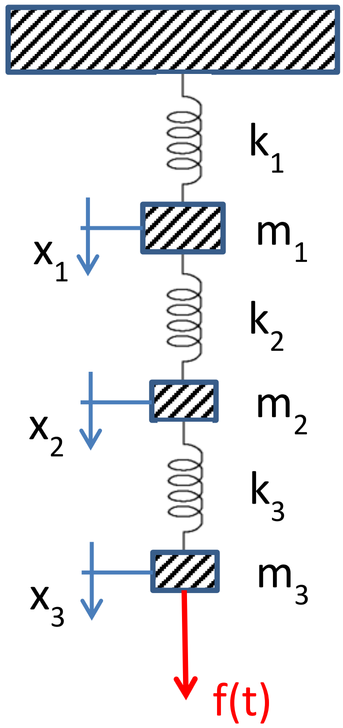
Introduction to RIS
Spring term 2022
Date: Mar 25, 2022

Course CH-220-A
M. Bode

Practice Problems II - 01

Practice problems are supposed to help you digest the content of the lecture. It is important that you manage to solve them on your own. Before you write your solutions, you may of course ask questions, and discuss things. In order to prepare for the exam, already now, try to explicitly write down your solutions – clearly and easy to read. Apply definitions properly, and give explanations for what you are doing. That will help you to understand them later when you prepare for the final exam.

I. Modeling in Time Domain



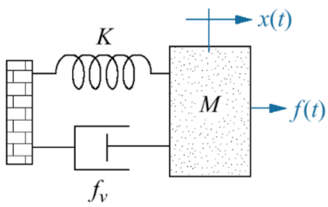
a) Write the time domain model of the system shown to the right.

In order to do so, write one equation per body (mass) involved, that is three in this case:
For each body/equation, think of its inertial term,
"mass · acceleration" , and equate that to the total of all the forces acting upon the respective body, that is, in the case of body 1, combine the forces exerted by the springs as they result from positions x_1, x_2, x_3 of the three bodies.

b) Explain: Why is there no x_3 – term in the equation related to body 1?

Mind: The coordinates x_1, x_2, x_3 are chosen in a way such that for $x_1 = x_2 = x_3 = 0$, the springs are relaxed, that is, do not exert any forces at all.

II. Solving a Model



Consider the first order ode

$$f_v \dot{x} + Kx = 0,$$

where $f_v = 5\frac{Ns}{m}, and \;\; K = \frac{12N}{m}$. In order to find the solution set, use an exponential ansatz $x(t) = x_0 e^{\lambda t}$.

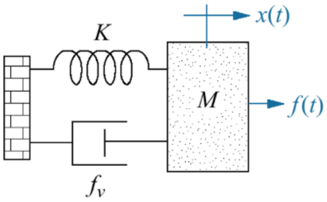
a) What is the value of λ … including its units?

b) For such first order ode, we have one initial value. If we start the system at $x_0 = 10\ m$, the initial value is specified. How long does it take until $x(t)$ reaches $10\ cm$?

III. Solving other Models

An exponential ansatz $x(t) = x_0 e^{\lambda t}$ works in many contexts but you still need to remain flexible … in particular when considering inhomogeneous odes.

a) Consider the first order ode



$$f_v \dot{x} = f(t),$$

where $f_v = 5\frac{Ns}{m}$, and $f(t) = 100\ N$.

Write the set of solutions $x(t)$.

b) Consider the second order ode

$$M\ddot{x} + f_v \dot{x} = 0,$$

where M=50 kg, $f_v = 5\frac{Ns}{m}$.

Write the set of solutions $x(t)$.

c) Consider the second order ode

$$M\ddot{x} + f_v \dot{x} = f(t),$$

where $\mathrm{M} = 50$ kg, $f_v = 5\frac{Ns}{m}$, and $f(t) = 100\ N$.

Write the set of solutions $x(t)$.