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Homework 11

Calculus and Linear Algebra
Due: December 1, 2021

Problem 1

(5+5 points)

a) Solve the following system of linear equations using the method taught in class.

$$\begin{aligned} -x_1 + 2\,x_2 - x_3 &= 8 \\ 2\,x_1 + 3\,x_2 + 9\,x_3 &= 5 \\ -4\,x_1 - 5\,x_2 - 17\,x_3 &= -7 \end{aligned}$$

b) Let $\boldsymbol{v} = (3, 1, 3)^T$ be a vector expressed in coordinates with respect to the standard basis of $\mathbb{R}^3$. Find the coordinates of this vector with respect to the basis

$$\boldsymbol{b}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \boldsymbol{b}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \boldsymbol{b}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 2

(10 points)

Find conditions on α such that following system of linear equations has (a) exactly one solution, (b) no solutions, or (c) an infinite number of solutions.

$$\begin{aligned} 2\,x_1 - 2\,x_2 + \alpha\,x_3 &= -2 \\ 4\,x_1 - 4\,x_2 + 12\,x_3 &= -4 \\ 2\,x_1 + \alpha\,x_2 &= 2 \end{aligned}$$

Problem 3

(5+5 points)

a) Determine whether the following vectors form a basis of $\mathbb{R}^4$. If not, obtain a basis by adding and/or removing vectors from the set.

$$\boldsymbol{v}_1 = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad \boldsymbol{v}_2 = \begin{pmatrix} -1 \\ 2 \\ 2 \\ 1 \end{pmatrix}, \quad \boldsymbol{v}_3 = \begin{pmatrix} 0 \\ -1 \\ 2 \\ -1 \end{pmatrix}, \quad \boldsymbol{v}_4 = \begin{pmatrix} -2 \\ 2 \\ 1 \\ 0 \end{pmatrix}.$$

b) A matrix is called *singular* if the homogeneous linear system $A\boldsymbol{v} = \mathbf{0}$ has a "non-trivial" solution $\boldsymbol{v} \neq 0$.

Prove that $AB = 0$ implies that at least one of the matrices is singular.