

Practice Problems - 01

Practice problems are supposed to help you digest the content of the lecture. It is important that you manage to solve them on your own. Before you write your solutions, you may of course ask questions, and discuss things. In order to prepare for the exam, already now, try to explicitly write down your solutions – clearly and easy to read. Apply definitions properly, and give explanations for what you are doing. That will help you to understand them later when you prepare for the final exam.

I. Vectors

1) Consider the following vectors:

$$u = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, v = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$$

- a) Calculate the (Euclidean) 2-norm of u and v .
- b) Calculate the sum of the two vectors
- c) Show the result geometrically
- d) Calculate the difference $v - u$
- e) Calculate $7v - 5u$
- f) Calculate the dot product
- g) Calculate the cross product $u \times v$
- h) Are u and v linearly independent? Why?

2) Suppose that $u \in \mathbb{R}^3$ is a vector which lies in the first quadrant of the xy -plane and has length 3 and that $v \in \mathbb{R}^3$ is a vector that lies along the positive z -axis and has length 5.

- a) Calculate $\|u \times v\|$
- b) The x -coordinate of $u \times v$ is ... 0 (choose $<$, $>$, or $=$, and motivate the answer)
- c) The y -coordinate of $u \times v$ is ... 0 (choose $<$, $>$, or $=$, and motivate the answer)
- d) The z -coordinate of $u \times v$ is ... 0 (choose $<$, $>$, or $=$, and motivate the answer)

3) Suppose that u and v are vectors in \mathbb{R}^3 , both of length $2\sqrt{2}$ and that the length of $u - v$ is also $2\sqrt{2}$.

- a) Calculate $\|u + v\|$
- b) Calculate the angle between u and v

II. Matrices

1) Consider the following matrices:

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & -3 & 2 \end{bmatrix}; B = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 4 & 1 \end{bmatrix}; C = \begin{bmatrix} 4 & -1 \\ 1 & -2 \end{bmatrix}$$

Calculate if possible

- a) $A + B$
- b) $A + C$
- c) $2C + \frac{3}{2}I_2$

2) Consider the following matrices:

$$A = \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \\ -1 & 0 \end{bmatrix}; B = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 2 & -2 & 2 & 1 \end{bmatrix}$$

Calculate if possible

- a) A^T
- b) $A + B$
- c) $A^T + B$
- d) AB
- e) BA

3) Consider the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 3 \end{bmatrix}$$

Calculate if possible

- a) A^2
- b) AA^T
- c) $A^T A$