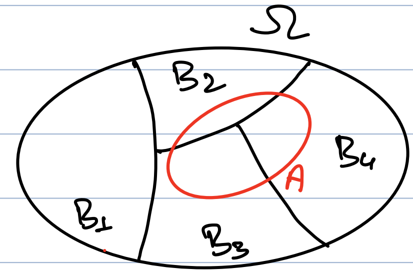
Tutorial 4 notes:

Oct 06

Law of total probability:



If $\Omega = \bigcup_{i=1}^{n} B_i$, for any $A \subseteq \Omega$:

$$\mathbb{P}[A] = \sum_{K=1}^{n} \mathbb{P}[A|B_i]\,\mathbb{P}[B_i]$$

Baye's Theorem:

If $\Omega = \bigcup_{i=1}^{n} B_i$, for any $A \subseteq \Omega$:

$$\mathbb{P}[B_i|A] = \frac{\mathbb{P}[A|B_i]\,\mathbb{P}[B_i]}{\underbrace{\sum_{K=1}^{n} \mathbb{P}[A|B_K]\,\mathbb{P}[B_K]}_{=\mathbb{P}[A]}}$$

⇝ Total probability of A.

Problems relating Baye's Theorem:

Bits are sent through two independent communication channels. If 1 is sent, it is received with the probability 0.8 in the first channel, and 0.9 in the second. Probabilities of correct transmission are the same if 0 is sent.

- Find the probability that at least one 1 is received if 1 are sent through both channels.
- 1 are sent with probability p through the first channel and q through the second one. Find the probability of getting 1 from both channels.
- If 1 is received in the first channel, what is the probability that 1 has been sent through this channel?

Say $\mathbb{P}_1, \mathbb{P}_2$ = First, Second channel.

$\mathbb{P}_1(1R|1S) = 0.8 \Rightarrow \mathbb{P}_1(0R|1S) = 0.2$

$\mathbb{P}_2(1R|1S) = 0.9 \Rightarrow \mathbb{P}_2(0R|1S) = 0.1$

1) $\mathbb{P}(\text{At least one } 1R \,|\, \text{Both } 1 \text{ sent}) = \mathbb{P}_1(1|1)\,\mathbb{P}_2(1|1) + \mathbb{P}_1(1|1)\,\mathbb{P}_2(0|1)$

$+ \mathbb{P}_1(0|1)\,\mathbb{P}_2(1|1) = 0.8 \cdot 0.9 + 0.8 \cdot 0.1 + 0.9 \cdot 0.2 = 0.98$

OR Can look at Complement event:

$\mathbb{P}[\text{At least one } 1R \,|\, \text{Both } 1 \text{ sent}] = 1 - \mathbb{P}[\text{no } 1R \,|\, \text{Both } 1 \text{ sent}]$

$= 1 - 0.2 \cdot 0.1 = 0.98$

2) $\mathbb{P}_1(1) = p\,\mathbb{P}_1(1|1) + (1-p)\,\mathbb{P}_1(1|0) = 0.8p + 0.2(1-p)$

$\mathbb{P}_2(1) = q\,\mathbb{P}_2(1|1) + (1-q)\,\mathbb{P}_2(1|0) = 0.9q + 0.1(1-q)$

$\mathbb{P}(\text{Receiving both } 1) = \mathbb{P}_1(1) \cdot \mathbb{P}_2(1)$

3) $$\mathbb{P}_1(1s|1r) = \frac{\mathbb{P}_1(1r|1s)\,\mathbb{P}(1s)}{\mathbb{P}_1(1r)} = \frac{0.8p}{0.8p + 0.2(1-p)}$$