

Problem 1

(10 points)

Compute the derivative of the following functions directly from the definition

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- a) $f(x) = x^3$. (4 points)
- b) $f(x) = \sqrt{x}$. (4 points)
- c) $f(x) = x$. (1 points)
- d) $f(x) = c$ with some constant x . (1 points)

Problem 2

(14 points)

Compute the derivatives of the following functions

- a) $f(x) = \frac{x^2}{b-3x^2}$ where b is a constant (2 points)
- b) $g(t) = \cos(\omega t + \phi) + \sin(\omega t + \phi)$ where ω and ϕ are constants (2 points)
- c) $h(s) = \cos(s^2 + s) + \sin(s/2)$ (2 points)
- d) $j(x) = \ln(x^{a^2} + x^{-a^2})$ where a is a constant
Note: You can use $(\ln x)' = 1/x$ from the lecture (2 points)
- e) $k(x) = \ln(x^a + b^x)$ where a and b are constants (2 points)
- f) $l(x) = x^2 \exp(-x^2)$ (2 points)
- g) $m(x) = x^{x^2}$ (2 points)
Note for e) and g): You cannot directly work with something of the form a^x (with some a) but only with something of the form e^{cx} (with some c). Transform the function accordingly before differentiation.

Problem 3

(6 points)

Use the definition of the derivative, $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$, to show that the function $f(x) = |x|$ is not differentiable at $x = 0$.