CH-231-A

Algorithms and Data Structures

ADS

Lecture 39

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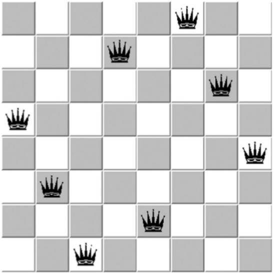
The Eight-Queens Problem

- The eight queens problem is a classical puzzle of positioning eight queens on an 8×8 chessboard such that no two queens threaten each other.
- This a classical textbook backtracking problem.



Eight Queens: Representation

- What is concise, efficient representation for an n-queens solution, and how big must it be?
- Since no two queens can occupy the same column, we know that the n columns of a complete solution must form a permutation of n.
- By avoiding repetitive elements, we reduce our search space to just $8! = 40,320$ quick for any reasonably fast machine.
- The critical routine is the candidate constructor.
- We repeatedly check whether the k^{th} square on the given row is threatened by any previously positioned queen.
- If so, we move on, but if not we include it as a possible candidate.
- Algorithm can find the $365,596$ solutions for $n = 14$ in minutes.

String Matching

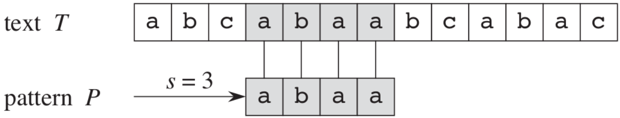
- Text-editing programs often need to find all occurrences of a pattern in the text.
- Among their many other applications, string-matching algorithms search for particular patterns in DNA sequences.
- Internet search engines also use them to find web pages relevant to queries.

String Matching Problem (1)

- We assume that the text is an array $T[1..n]$ of length n and that the pattern is an array $P[1..m]$ of length $m \leq n$.
- We further assume that the elements of P and T are characters drawn from a finite alphabet Σ.
- For example, we may have $\Sigma = \{0, 1\}$ or $\Sigma = \{a, b, ..., z\}$.
- The character arrays P and T are often called strings of characters.

String Matching Problem (2)

- The pattern P occurs with shift s in text T (or, P occurs beginning at position $s+1$ in text T)
- If P occurs with shift s in T, then we call s a valid shift; otherwise, we call s an invalid shift.
- The string-matching problem is the problem of finding all valid shifts with which a given pattern P occurs in a given text T.



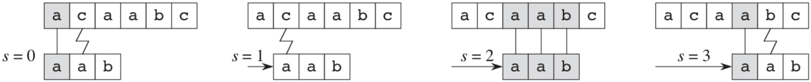
Notation and Terminology

- Σ^* the set of all finite-length strings formed using characters from the alphabet Σ
- ϵ is the zero-length empty string
- $|x|$ is the length of a string x
- xy is the concatenation of two strings x and y
- $w \sqsubset x$ means w is a prefix of a string x, i.e., $x = wy$
- $w \sqsupset x$ means w is a suffix of a string x, i.e., $x = yw$

Naive String-Matching Algorithm

The naive algorithm finds all valid shifts using a loop that checks the condition $P[1..m] = T[s+1..s+m]$ for each of the $n - m + 1$ possible values of s.

```
NAIVE-STRING-MATCHER(T, P)
n = T.length
m = P.length
for s = 0 to n - m
    if P[1..m] == T[s+1..s+m]
        print "Pattern occurs with shift" s
```



(a) (b) (c) (d)

Naive String-Matching Time Complexity

- For example, consider the text string a^n (a string of n a's) and the pattern a^m.
- For each of the $n - m + 1$ possible values of the shift s, the implicit loop on line 4 to compare corresponding characters must execute m times to validate the shift.
- The worst-case running time is thus $\Theta((n - m + 1)m)$, which is $\Theta(n^2)$ if $m = \lfloor n/2 \rfloor$.
- The naive string-matcher is inefficient because it entirely ignores information gained about the text for one value of s when it considers other values of s.

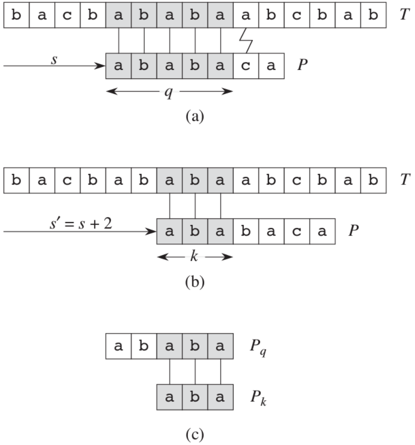
String Matching with Finite Automata

- Many string-matching algorithms build a finite automaton that scans the text string T for all occurrences of the pattern P.
- The matching time used after preprocessing the pattern to build the automaton is $\Theta(n)$.
- The time to build the automaton, however, can be large if Σ is large.
- The Knuth-Morris-Pratt algorithm has a clever way around this problem.

Knuth-Morris-Pratt Algorithm

- It is a linear-time string-matching algorithm due to Knuth, Morris, and Pratt.
- Its matching time is $\Theta(n)$ using just an auxiliary function π, which we precompute from the pattern in time $\Theta(m)$ and store in an array $\pi[1..m]$.

Prefix Function for a Pattern (1)



Prefix Function for a Pattern (2)

- Subfigure (a) shows a particular shift s of a template containing the pattern $P = ababaca$ against a text T.
- For this example, $q = 5$ of the characters have matched successfully, but the 6$^{\text{th}}$ pattern character fails to match the corresponding text character.
- The information that q characters have matched successfully determines the corresponding text characters.
- Knowing these q text characters allows us to determine immediately that certain shifts are invalid.
- The shift $s' = s + 2$ shown in subfigure (b) of the figure, however, aligns the first three pattern characters with three text characters that must necessarily match.