I. Modelling in time domain

a) Mass 1 = $m_1\ddot{x}_1 + (K_1+K_2)x_1 - K_2x_2 = 0$

Mass 2 = $m_2\ddot{x}_2 + (K_2+K_3)x_2 - K_2x_1 - K_3x_3 = 0$

Mass 3 = $m_3\ddot{x}_3 + K_3x_3 - K_3x_2 = f(t)$

b) There is no x_3 term in equation related to body 1 because no component of m_3 is joined with body 1 and hence has no direct effect on it.

II. Solving a model

$$f_v\dot{x} + Kx = 0$$

a) $5\dot{x} + 12x = 0$ where $x = ae^{\lambda t}$

$$(5\lambda + 12)ae^{\lambda t} = 0$$

$$5\lambda + 12 = 0$$

$$\lambda = -\frac{12}{5}$$

$5\lambda + 12 = 0$

$\lambda = -\frac{12}{5}$ s

Hence $x(t) = ae^{-12/5\,t}$

b) $x_0 = 10$ m $\qquad a = x_0$

$x(t) = 10e^{-12/5\,t}$

$0.1 = 10e^{-12/5\,t}$

$\ln(0.1) = -\frac{12}{5}t \ \ln(10e)$

$\frac{5\ln(0.1)}{-12\ln(10e)} = t$

$t =$

III. Solving other models

a) $5\dot{x} = 100$

$\dot{x} = 20$

$\lambda ae^{\lambda t} = 0$ — For homogenous

$\lambda = 0$

$y = \dot{x} \implies y = 20$

$x = \int y \; dt$

$x = \int 20 \; dt$

$x = 20t$

b) $M\ddot{x} + f_v \dot{x} = 0$

$50\ddot{x} + 5\dot{x} = 0$ where $x = ae^{\lambda t}$

$(50\lambda^2 + 5\lambda)\, ae^{\lambda t} = 0$

$50\lambda^2 + 5\lambda = 0$

$\lambda_1 = 0 \qquad \lambda_2 = -0.1$

Hence $a_1 e^{0(t)}$ and $a_2 e^{-0.1t}$

$x(t) = a_1 e^0 + a_2 e^{-0.2t}$

c) $50\ddot{x} + 5\dot{x} = 100$

$50\dot{y} + 5y = 100 \rightarrow y = \dot{x} \rightarrow dx = y\,dt$

$$x = \int y \, dt$$

$$\dot{y} + 0.1y = 2$$

$$y = \int_0^t e^{-\lambda(t-\tau)} f(\tau) \, d\tau$$

$$\lambda = 0.1 \qquad f(\tau) = 2$$

$$y = \int_0^t e^{-0.1(t-\tau)} 2 \, d\tau = 2e^{-0.1} \int_0^t e^{0.1\tau} \, d\tau$$

$$= 2e^{-0.1t} \left(10e^{0.1\tau} \Big|_0^t \right) = 2e^{-0.1t} \left(10e^{0.1t} - 10 \right)$$

$$= 20 - 20e^{-0.1t}$$

$$\dot{y} = \frac{d}{dt} \left(20 - 20e^{-0.1t} \right) = -20 \cdot (0.1) e^{-0.1t} = 2e^{-0.1t}$$

$$x = \int 20 - 20e^{-0.1t} \, dt = 20t + \frac{20}{0.1} e^{-0.1t}$$

$$= 20t + 200 e^{-0.1t}$$