



Name: _____

Matriculation ID: _____

INSTRUCTIONS

- Make sure to write your name and ID on the first page and every page thereafter.
- The question booklet consists of **6 pages**. Make sure you have all of them.
- Regarding question 5, you should only do two out of three options. You can pick the ones you prefer. Clearly state which ones they are.
- Keep quiet during the exam. For assistance, raise your hand and an invigilator will come to see you
- Answer the questions in the spaces provided after each question. If you run out of room for an answer, continue on the back of the page.
- The mark of each question is printed next to it.
- Use of mobile phones or other unauthorized electronic devices or material in the exam room is prohibited. Only simple calculators are allowed during the exam.
- Make sure you read and sign the **Declaration Of Academic Integrity** shown below.

Question	1	2	3	4	5	Total
Points	18	16	20	32	14	100
Score						

Declaration of Academic Integrity

By signing below, I pledge that the answers of this exam are my own work without the assistance of others or the usage of unauthorized material or information.

Signature:

Good luck!

1. (18 points)

Compute the following limits, if they exist. Else, argue why the limit does not exist.

$$\boxed{4} \quad (\text{a}) \quad \lim_{x \rightarrow -2} \frac{\frac{1}{x} - \frac{1}{2}}{x^3 - 8}$$

$$\boxed{7} \quad (\text{b}) \quad \lim_{y \rightarrow \infty} \frac{e^{-y} \sin(y) \cos(y)}{y}$$

$$\boxed{7} \quad (\text{c}) \quad \lim_{r \rightarrow 1} \frac{|r-1|}{2r-2}$$

2. (16 points)

- 6 (a) Show that the equation $x^6 - 5x - 5 = 0$ has at least one solution on the interval $[-1, 0]$.
- 10 (b) Compute the derivative of $f(x) = \frac{1}{x^2}$ directly from its definition as the limit of a difference quotient.

3. (20 points)

Consider the function $f(x) = \frac{x^2}{2 - x^2}$.

What is the domain of f ? Find the horizontal and vertical asymptotes, local minima, local maxima, and reflection points of f . Identify the regions where the graph of f is concave up or concave down. Finally, sketch the graph.

4. **(32 points)**

Solve the following:

10

(a) Integrate $\int \frac{x+1}{x^2(x-1)} dx$

6

(b) Integrate $\int x^{-3} e^{-1/x^2} dx$

8

(c) Integrate $\int_0^{2\pi} e^x \cos(x) dx$

8

(d) Differentiate $f(t) = t^3$

5. (14 points)

Choose only two of the below:

- 7 (a) Find the area between the curves $x = y^2$ and $0 = -x - y^2 + 2$ (in absolute terms).
- 7 (b) A farmer owns an 8 km long stretch of land between two parallel rivers that are 1500 m apart. What is the area of the largest rectangular enclosure he can fence off with (i) 1 km of fencing and (ii) 4 km of fencing, assuming that no fence is needed along the rivers?
- 7 (c) Use implicit differentiation to find an equation for the tangent line to the graph of $\sin(2x + y) = y^3 \sin(x)$ at the point $(0, 0)$.