Practice exam for Probability and Random Processes

12.10.22

This exam consists of two parts. The first part consists of 12 multiple choice problems, each 5 points. In this part only choose the correct answer(s). The second part consist of two problems each 20 points. For this part of the exam, you need to show all your work.

Part I: Multiple choice problems

Each question has 5 points. There may be more than one correct answer for each problem. Choose **all** of them.

1. For which value of $0 \leq n \leq 10$ takes the binomial coefficient $\binom{10}{n}$ the largest possible value?
   A. 4
   B. 5
   C. 6
   D. 7

2. A fair die is thrown three times. What is the probability that the sum of the numbers shown is 4?
   A. $2/6^3$
   B. $3/6^3$
   C. $4/6^3$
   D. 0.

3. Suppose A and B are events with $\mathbf{P}[A] = 0.6$ and $\mathbf{P}[B] = 0.7$. Choose the statement(s) that are logically follow from these.
   A. $\mathbf{P}[A \cup B] = 1$.
   B. $\mathbf{P}[A \cap B] \geq 0.3$.
   C. $\mathbf{P}[A \cup B] \geq 0.6$.
   D. $\mathbf{P}[A|B] < \mathbf{P}[B|A]$.

4. Three horses A, B, C are in a race. A is twice as likely to win as B, and B is twice as likely to win as C. What is the probability that A wins?
   A. 1/5
   B. 1/6
   C. 1/7
   D. 1/8

5. Let A, B, C denote three events in a sample space. Which one of the following describe the event that A and B occur, but C does not occur?
   A. $A \cap B \cap C^c$
   B. $A \cup B \cup C^c$
   C. $(A \cap B \cap C)^c$
   D. None of the above

6. A box contains 2 white gloves and 2 blue gloves. Two gloves are drawn at random. Find the probability that they are a match (same color).
    A. 1/2
    B. 1/3
    C. 2/3
    D. none of the above.

7. Suppose 4 marbles are placed in 4 boxes at random. Find the probability that none of the boxes is empty.
    A. 1/64
    B. 3/16
    C. 3/64
    D. 3/32

8. Suppose A and B are events with $\mathbf{P}[A \cup B] = 0.8$, $\mathbf{P}[A] = 0.4$ and $\mathbf{P}[B] = 0.5$. What is $\mathbf{P}[A|B]$?
    A. 0.4
    B. 0.3
    C. 0.2
    D. none of the above.

9. A real number x is randomly chosen from the interval $[1, 2]$. What is the probability that $x^2 \geq 3$?
    A. $\sqrt{3} - 1$
    B. 1
    C. $2 - \sqrt{3}$
    D. $\sqrt{3}/2$

10. A bias coin has the probability 2/3 of turning up heads. The coin is thrown 4 times. What is the probability that the total number of heads shown is 3?
    A. 16/40
    B. 8/27
    C. 32/81
    D. 1/2

11. A box contains 7 red marbles and 3 white marbles. Two marbles are drawn from the box one after the other. Find the probability that the first is red and the second is white.
    A. 21/100
    B. 7/30
    C. 7/10
    D. 7/27

12. Let X be a discrete random variable taking values $-1, 0, 1$, each with probability 1/3. Find the probability of the event that $X^2 \leq 1$
    A. 1/3
    B. 2/3
    C. 1
    D. None of the above.

1 Part II: Written problems

1. Three machines A, B, and C produce, respectively, 40 percent, 10 percent, and 50 percent of the items in a factory. The percentage of defective items produced by the machines is, respectively, 2 percent, 3 percent, and 4 percent. An item from the factory is selected at random.

    1. Find the probability that the item is defective.
    2. If the item is defective, find the probability that the item was produced by machine A.

2. Suppose M is an integer randomly chosen from the set $\{1, 2 \ldots, 8\}$. Once M is chosen, the integer N is chosen from the set of integers between (and including) 1 and M. For instance if it turns out that $M = 3$, then N can take one of the values $1, 2, 3$, each with probability $1/3$.

    1. Find the probability of the event that $M = N$.
    2. If $M = N$, what is the probability that $M = 3$.