Robotics
Problem Sheet 2

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Notes

The homework serves as preparation for the exams. It is strongly recommended that you solve them before the given deadline - but you do not need to hand them in. Feel free to work on the problems as a group - this is even recommended.

1 Problem

Proof that when turning in circles you end up where you started. Or more concretely: given the motion `move(`α`,` d`)` (in 2D is sufficient) that turns with angle α and then makes a translation by a distance d, proof that the sequence of motions `move(90,` d`)`, `move(90,` d`)`, `move(90,` d`)`, `move(90,` d`)` executed in pose p_{start} gets you into pose p_{end} with $p_{start} = p_{end}$.

2 Problem

Suppose an object, e.g., the earth, has the pose P_e and a 2nd object, e.g., the moon, with pose P_m is rotating around it with angle θ around the z-axis of P_e.

What is the new pose of P'_m for

$$\theta = 90^o, \quad p_e = \begin{pmatrix} 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & -4 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad p_m = \begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & -1 & 0 & 7 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3 Problem

Given a world-frame F_w as identity matrix and an object with pose P_o with

$$p_o = \begin{pmatrix} 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & -4 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Suppose the object rotates by 90^o around the z-axis of F_w. What is the new pose P'_o of the object?
- Suppose world frame is an observer/sensor, who/which rotates by 90^o around its z-axis. What is the new pose P'_o of the object?

4 Problem

Given a simple robot arm with

- a base frame F_0 as identity matrix
- a rotational joint j_1 that can rotate along the z-axis of F_0 with angle θ_1
- a link l_1 of length 5 along the z-axis of F_0
- a rotational joint j_2 at the end of l_1 that can rotate with angle θ_2 around the y-axis and where its frame F_2 is co-aligned with the base frame for $\theta_1 = \theta_2 = 0$
- a link l_2 of length 3 along the z-axis of F_2
- an end-effector, e.g., a gripper, with pose $P_g = F_3$ at the end of link l_2

How can we express the pose of the end-effector with homogeneous matrices? What is the exact pose P_g for $\theta_1 = 90^o$ and $\theta_2 = 180^o$?