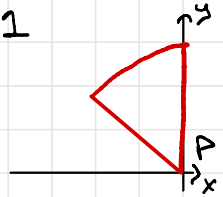
Homework 5

N1

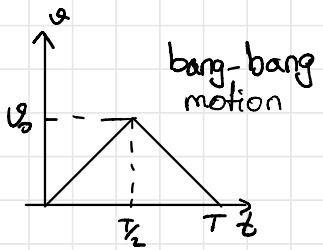


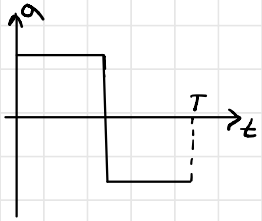
$$P = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & 0 & 3\sqrt{2} \\ 0 & 1 & -3\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

a) $P' = T_2(R(T_1 P)) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

b)





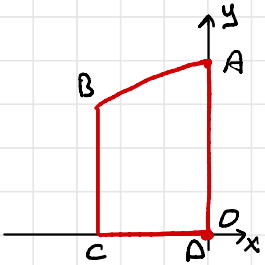
$d = 6$

$$d = \int_0^T v(t)\,dt = \frac{v_0 \cdot T}{2} \Rightarrow v_0 = \frac{2d}{T} = \frac{2 \cdot 6}{8} = \frac{6}{4} = 1.5 \ \frac{\text{unit}}{s}$$

$$a = \frac{v_0 - 0}{T/2} = \frac{2(1.5-0)}{8} = \frac{3}{8} \ \frac{\text{unit}}{s^2}$$

Problems #2 and #3 have similar solutions with different numbers

N4



$$A=\begin{bmatrix}0\\12\end{bmatrix} \quad B=\begin{bmatrix}-6\sqrt{2}\\6\sqrt{2}\end{bmatrix} \quad C=\begin{bmatrix}-6\sqrt{2}\\0\end{bmatrix}$$

$$D=\begin{bmatrix}0\\0\end{bmatrix}$$

a) $\vec{BC}\cdot\vec{CD} = |\vec{BC}|\cdot|\vec{CD}|\cdot\cos 90^\circ = 6\sqrt{2}\cdot 6\sqrt{2}\cdot 0 = 0$

$\vec{BC}\times\vec{CD} = |\vec{BC}|\cdot|\vec{CD}|\cdot\sin 90^\circ = 6\sqrt{2}\cdot 6\sqrt{2}\cdot 1 = 72$

b) $\vec{OA}+\vec{CB} = \begin{pmatrix}0\\12\end{pmatrix}+\begin{pmatrix}6\sqrt{2}\\0\end{pmatrix}=\begin{pmatrix}6\sqrt{2}\\12\end{pmatrix}$

$\vec{OA}-\vec{CB} = \begin{pmatrix}0\\12\end{pmatrix}-\begin{pmatrix}6\sqrt{2}\\0\end{pmatrix}=\begin{pmatrix}-6\sqrt{2}\\12\end{pmatrix}$

c) Dot product is commutative

Cross product is not commutative (because of direction)

Sum of two vectors is commutative

Difference of two vectors is not commutative.

d) $T_1=\begin{bmatrix}1&0&0\\0&1&12\\0&0&1\end{bmatrix}$ $R=\begin{bmatrix}\cos 45^\circ & -\sin 45^\circ & 0\\ \sin 45^\circ & \cos 45^\circ & 0\\ 0 & 0 & 1\end{bmatrix}$

$T_2=\begin{bmatrix}1&0&0\\0&1&-6\sqrt{2}\\0&0&1\end{bmatrix}$ $T_3=\begin{bmatrix}1&0&6\sqrt{2}\\0&1&0\\0&0&1\end{bmatrix}$ $O=\begin{bmatrix}0\\0\\1\end{bmatrix}$

$$D = T_3\left(T_2\left(R\left(T_1 O\right)\right)\right)=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

e) $a = 1\ \frac{m}{s^2}$ $\quad |\vec{OA}| = 12\ m.$

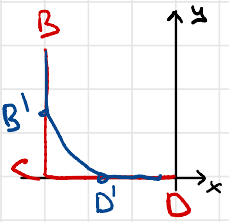
$$d = v_0 t + \frac{at^2}{2} = \frac{at^2}{2} \Rightarrow t = \sqrt{\frac{2d}{a}} = \sqrt{\frac{2 \cdot 12}{1}} = 2\sqrt{6}\ s$$

$$a = \frac{v_f - v_i}{t} \Rightarrow v_f = at = 2\sqrt{6}\ \frac{m}{s}$$

f) $$v = r \cdot \omega \rightarrow \omega = \frac{v}{r} = \frac{2\sqrt{6}}{12} = \frac{\sqrt{6}}{6}\ \frac{rad}{s}$$

$$a_c = \frac{v^2}{r} = \frac{4 \cdot 6}{12} = 2\ \frac{m}{s^2}$$

g) $v = 1\ \frac{m}{s}$ $\quad \Delta T = 3s$



$$d_1 = \frac{1}{2} \Delta T \cdot v_1 = \frac{1}{2} \cdot 3 \cdot 1 = 1.5\ m$$

$$\frac{d_2}{v_2} = \frac{d_1}{v_1} \Rightarrow d_2 = \frac{v_2}{v_1} \cdot d_1 = d_1 = 1.5m$$

$$\Rightarrow CB' = CD' = d_1 = d_2 = 1.5m$$

Problem 5 has the same solution.