Robotics
PS06 Solution

Problem 1

A differential drive robot has two drive units, each with

- a left respectively right motor with a variable speed s_L, respectively s_R measured in rounds per minute (rpm).
- a planetary gear box with a 1:100 reduction, i.e., the wheel axis turns 100 times slower than the motor axis (but it has 100 times the torque)
- a wheel with a radius $r = 10$cm

The distance D between the two wheels is 30cm. The speeds of the two motors are measured by quadrature encoders at a frequency of 100 Hz, i.e., 100 times per second.

The coordinate frame of the robot follows the standards, i.e., it is as follows. The x-axis points from the center of motion of the robot to its front and it is co-aligned with zero degrees; angles are measured counterclockwise.

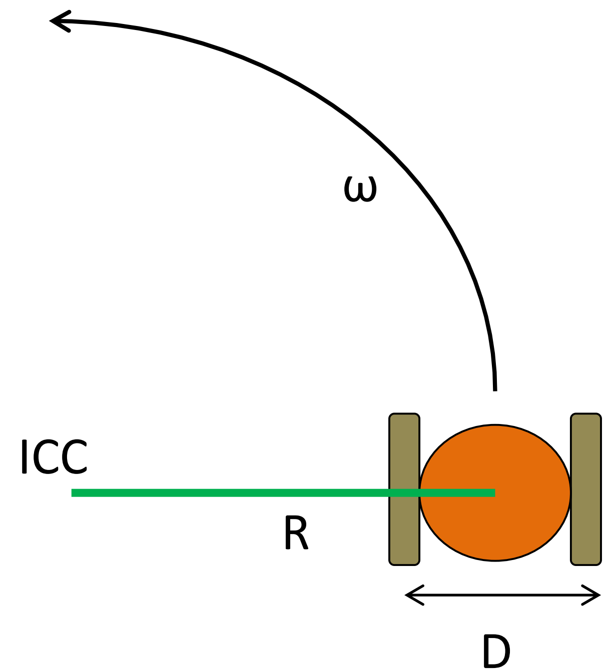
Suppose the robot drives with constant (motor-)speeds $N_L = 18849$ rpm, $N_R = 15708$ rpm over 40 msec. Suppose its initial pose is $(0, 0, 0)^T$. Derive its pose after 40 msec.

Problem 1: arc model

rotation ω
around Instantaneous Center of Curvature (ICC)
with radius R

$$R = \frac{D}{2}\frac{v_r + v_l}{v_r - v_l}$$

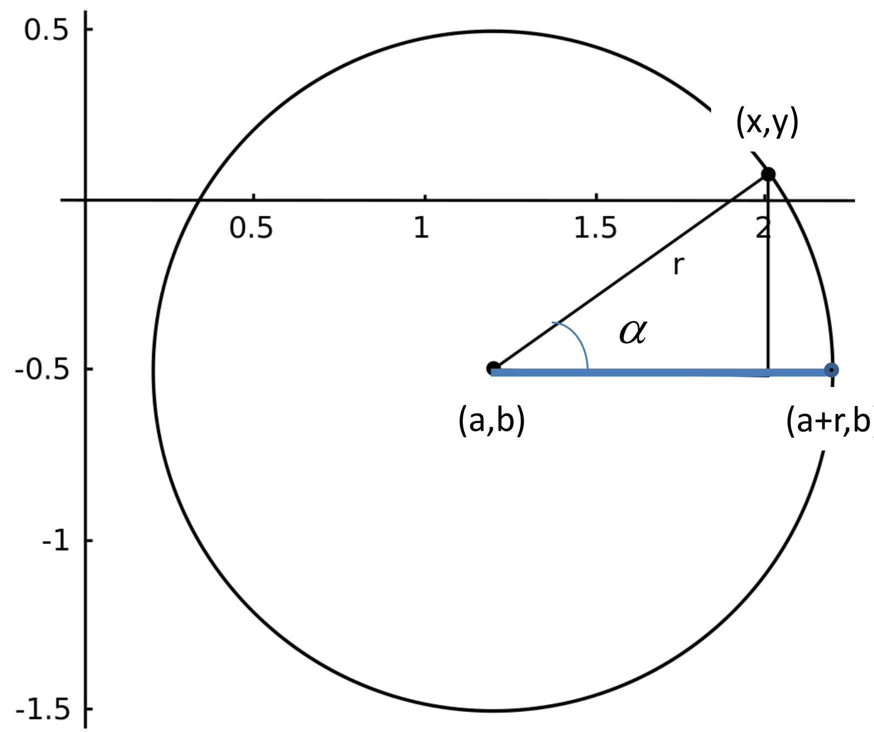
$$\omega = \frac{v_r - v_l}{D}$$



Problem 1: arc model

note arc: circle around (a,b), radius r

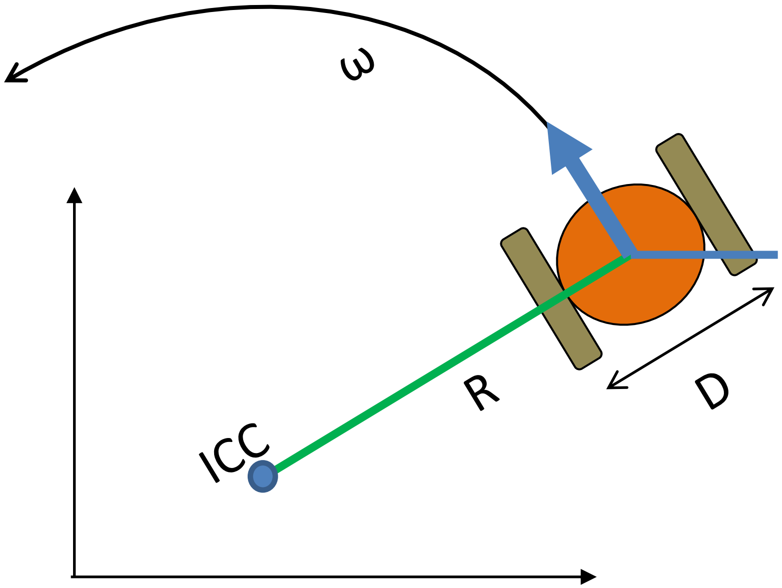
$$x = a + r \cdot \cos(\alpha)$$
$$y = b + r \cdot \sin(\alpha)$$



Problem 1: arc model

note: ICC is perpendicular
to robot forward orientation

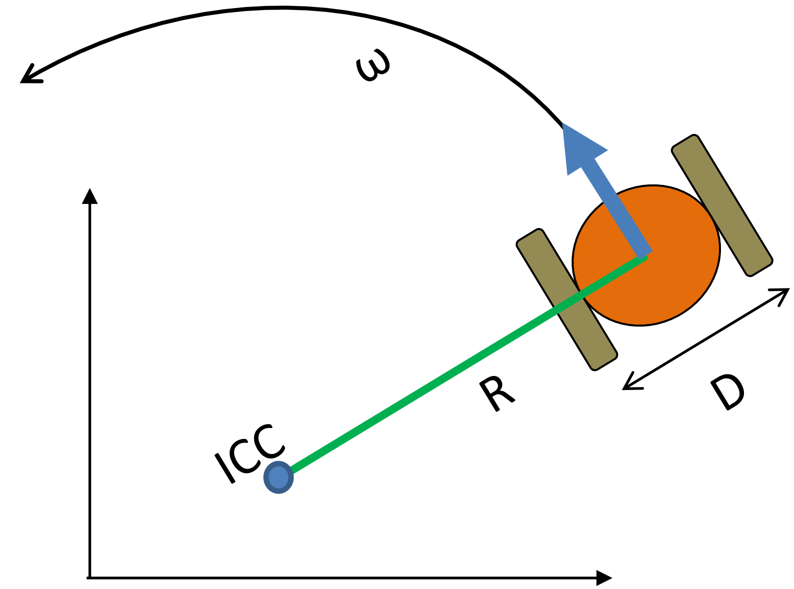
$\alpha = \theta + \pi/2$



$$\cos(\alpha) = \cos(\theta + \pi/2) = -\sin(\theta)$$
$$\sin(\alpha) = \sin(\theta + \pi/2) = \cos(\theta)$$

$$\begin{aligned} x &= a + r \cdot \cos(\alpha) \\ y &= b + r \cdot \sin(\alpha) \end{aligned} \quad \Rightarrow \quad \begin{aligned} x &= a - r \cdot \sin(\theta) \\ y &= b + r \cdot \cos(\theta) \end{aligned}$$

Problem 1: arc model



$$\omega = \frac{v_r - v_l}{D} \qquad R = \frac{D}{2}\frac{v_r + v_l}{v_r - v_l}$$

$$p_{ICC} = \left(x_{ICC}, y_{ICC}\right)^T$$
$$= \left(x - R\sin(\theta), y + R\cos(\theta)\right)^T$$

$$\begin{pmatrix} x_{t+\Delta t} \\ y_{t+\Delta t} \\ \theta_{t+\Delta t} \end{pmatrix} = \begin{pmatrix} \cos(\omega\Delta t) & -\sin(\omega\Delta t) & 0 \\ \sin(\omega\Delta t) & \cos(\omega\Delta t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_t - x_{ICC} \\ y_t - y_{ICC} \\ \theta_t \end{pmatrix} + \begin{pmatrix} x_{ICC} \\ y_{ICC} \\ \omega\Delta t \end{pmatrix}$$

Problem 1

$$v_{wheel} = r \cdot \omega_{wheel-axis}$$
$$= r \cdot GR \cdot \omega_{motor-axis}$$

- proper velocities in SI units
- $1\ RPM = 2\pi \frac{1}{60} \frac{rad}{sec}$
- $1\ rad = \frac{1\ m}{1\ m}$ („virtual“ unit)

$$v_{\mathrm{l}} = 0.1m \cdot \frac{1}{100} \cdot 18849RPM$$
$$= 0.1m \cdot \frac{1}{100} \cdot 18849/60 \cdot 2\pi \frac{rad}{sec}$$
$$= 1.9739 \frac{m}{sec}$$

$$v_{\mathrm{r}} = 0.1m \cdot \frac{1}{100} \cdot 15708RPM$$
$$= 0.1m \cdot \frac{1}{100} \cdot 15708/60 \cdot 2\pi \frac{rad}{sec}$$
$$= 1.6449 \frac{m}{sec}$$

Problem 1

$$\omega = \frac{v\mathrm{r} - v\mathrm{l}}{D} = \frac{(1.6449\ - 1.9739)\frac{m}{s}}{0.3m} = -1.0966\frac{rad}{s}$$

v_l (m/s)	1.9739
v_r (m/s)	1.6449
D (m)	0.3

$$R = \frac{D}{2}\frac{v\mathrm{r} + v\mathrm{l}}{v\mathrm{r} - v\mathrm{l}} = \frac{0.3m}{2}\frac{(1.6449 + 1.9739)\frac{m}{s}}{(1.6449\ - 1.9739)\frac{m}{s}} = -1.65m$$

start pose:

$(x,y,\theta)^T=(0,0,0)^T$

$$\begin{aligned} p_{ICC} &= \left(x_{ICC}, y_{ICC}\right)^T \\ &= \left(x - R\sin(\theta), y + R\cos(\theta)\right)^T \\ &= \left(0 + 1.65\sin(0), 0 - 1.65\cos(0)\right)^T = \left(0, -1.65\right)^T \end{aligned}$$

Problem 1

$$\omega = -1.0966\frac{rad}{s}$$
$$R = -1.65m$$
$$p_{ICC} = (0, -1.65)^T$$

$$\omega\Delta t = -1.0966\frac{rad}{s} \cdot 0.04\text{s} = -0.0439\text{ rad}$$

$$\begin{pmatrix} x_{t+\Delta t} \\ y_{t+\Delta t} \\ \theta_{t+\Delta t} \end{pmatrix} = \begin{pmatrix} \cos(\omega\Delta t) & -\sin(\omega\Delta t) & 0 \\ \sin(\omega\Delta t) & \cos(\omega\Delta t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_t - x_{ICC} \\ y_t - y_{ICC} \\ \theta_t \end{pmatrix} + \begin{pmatrix} x_{ICC} \\ y_{ICC} \\ \omega\Delta t \end{pmatrix}$$

$$= \begin{pmatrix} \cos(-0.0439) & -\sin(-0.0439) & 0 \\ \sin(-0.0439) & \cos(-0.0439) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 - 0 \\ 0 + 1.65 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -1.65 \\ -0.0439 \end{pmatrix}$$

$$= \begin{pmatrix} 0.9990 & -0.0439 & 0 \\ -0.0439 & 0.9990 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1.65 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -1.65 \\ -0.0439 \end{pmatrix}$$

$$= \begin{pmatrix} 0.0724 \\ -0.0016 \\ -0.0439 \end{pmatrix}$$

Problem 2

Given an omni-drive robot with 4 motors with omni-wheels W_i that are evenly spaced apart at 90^o starting with 0^o, i.e., W_1 is at 0^o, W_2 is at 90^o, and so on. The distance from the center of motion to each wheel is R, the wheel radius is r and the angular velocity of each wheel is ω_i.

Derive the inverse Kinematics of this robot, i.e., derive the matrix M with

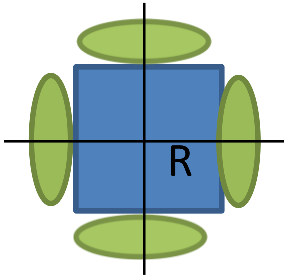
$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{pmatrix} = M \cdot \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}$$

for the translational velocity $V_t = (V_x, V_y)^T = (\dot{x}, \dot{y})^T$ and angular velocity $\omega = \dot{\theta}$ of the robot.

Problem 2

just extend the multiple omni-wheels example from the lecture

- 3 wheels in the lecture, here 4
- but simply all the same distance R to center of motion



$$\omega_i = \frac{1}{r}\left(-\sin(\alpha_i)\,\dot{x} + \cos(\alpha_i)\,\dot{y} + R\dot{\theta}\right)$$

here with $\alpha_1 = 0^o, \alpha_2 = 90^o, \alpha_3 = 180^o, \alpha_4 = 270^o$

Problem 2

$$\omega_i = \frac{1}{r}\left(-\sin(\alpha_i)\,\dot{x} + \cos(\alpha_i)\,\dot{y} + R\dot{\theta}\right)$$

here with $\alpha_1 = 0^o, \alpha_2 = 90^o, \alpha_3 = 180^o, \alpha_4 = 270^o$

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{pmatrix} = \frac{1}{r}\begin{pmatrix} -\sin(\alpha_1) & \cos(\alpha_1) & R \\ -\sin(\alpha_2) & \cos(\alpha_2) & R \\ -\sin(\alpha_3) & \cos(\alpha_3) & R \\ -\sin(\alpha_4) & \cos(\alpha_4) & R \end{pmatrix}\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \frac{1}{r}\begin{pmatrix} 0 & 1 & R \\ -1 & 0 & R \\ 0 & -1 & R \\ 1 & 0 & R \end{pmatrix}\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}$$

Multiple Omni-Wheels

Inverse Kinematics, general n≥3

$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \vdots \\ \omega_n \end{pmatrix} = \frac{1}{r} \begin{pmatrix} -\sin(\alpha_1) & \cos(\alpha_1) & R \\ -\sin(\alpha_2) & \cos(\alpha_2) & R \\ -\sin(\alpha_3) & \cos(\alpha_3) & R \\ & \vdots & \\ -\sin(\alpha_n) & \cos(\alpha_n) & R \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}$$

(in local robot frame)

Multiple Omni-Wheels

same for Forward Kinematics

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} -\sin(\alpha_1) & \cos(\alpha_1) & R \\ -\sin(\alpha_2) & \cos(\alpha_2) & R \\ -\sin(\alpha_3) & \cos(\alpha_3) & R \\ & \vdots & \\ -\sin(\alpha_n) & \cos(\alpha_n) & R \end{pmatrix}^{-1} r \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \vdots \\ \omega_n \end{pmatrix}$$

(in local robot frame)

Multiple Omni-Wheels

same for Forward Kinematics

matrix not square!!!

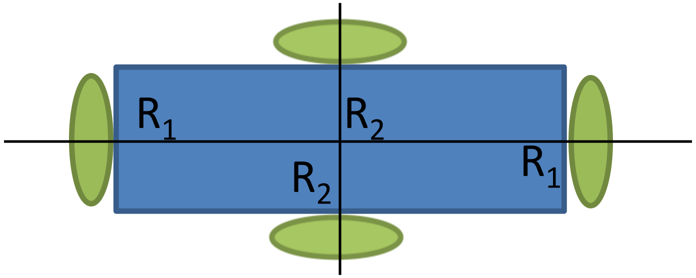
pseudo-inverse!!!

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} -\sin(\alpha_1) & \cos(\alpha_1) & R \\ -\sin(\alpha_2) & \cos(\alpha_2) & R \\ -\sin(\alpha_3) & \cos(\alpha_3) & R \\ & \vdots & \\ -\sin(\alpha_n) & \cos(\alpha_n) & R \end{pmatrix}^{+} r \begin{pmatrix} \omega_1 \\ \\ \omega_3 \\ \vdots \\ \omega_n \end{pmatrix}$$

(in local robot frame)

Multiple Omni-Wheels

different distances R_i



$$\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{pmatrix} = \frac{1}{r} \begin{pmatrix} -\sin(\alpha_1) & \cos(\alpha_1) & R_1 \\ -\sin(\alpha_2) & \cos(\alpha_2) & R_2 \\ -\sin(\alpha_3) & \cos(\alpha_3) & R_1 \\ -\sin(\alpha_4) & \cos(\alpha_4) & R_2 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix}$$