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Problem 1 [2 x 8 Points]: Consider the function

$$f(x) = \begin{cases} 4x^2 + 6x & \text{if } x \geq 1, \\ -x + k & \text{if } x < 1. \end{cases}$$

a) Find $k \in \mathbb{R}$ such that f is continuous on whole $\mathbb{R}$. Show that f is continuous on whole $\mathbb{R}$ for the selected value of k.

b) Using the value of k found in a) and using the definition of the derivative as the limit of the difference quotient prove or disprove that f is differentiable in $x = 1$.

*Hint: You will **not** get credit for just applying rules for differentiation. You **must use the definition of derivative as limit of difference quotient.***

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Problem 3 [2 x 8 Points]: For the following sets of vectors, find the condition on the parameter $b \in \mathbb{R}$ such that the vectors are linearly independent:

a) $\left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -b \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ b \end{pmatrix} \right\}$

b) $\left\{ 1 + x^2 + bx^3,\ x + x^2,\ 1 + bx - x^3 \right\}$

Problem 5 [2 x 8 Points]: Consider

$$A = \begin{pmatrix} 1 & 4 & 3 & 2 & 5 \\ 4 & 8 & 12 & 9 & 0 \\ 3 & 4 & 9 & 7 & -5 \\ 2 & 8 & 6 & 5 & 6 \end{pmatrix}, \qquad \vec{b} = \begin{pmatrix} 2 \\ 8 \\ 6 \\ 4 \end{pmatrix}.$$

a) Determine the rank, nullspace, and nullity of the matrix A.

b) Find a particular solution $\vec{p}$ for the non-homogeneous system $A\vec{x} = \vec{b}$. Use $\vec{p}$ and your result from a) to describe the general solution to this non-homogeneous system.

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Problem 6 [2 x 8 Points]: Compute the following integrals:

a) $\displaystyle\int_0^1 \frac{1}{\sqrt{x}(\sqrt{x}+1)}\, dx$

b) $\displaystyle\int_0^3 \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$

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Problem 1 [8 + 10 Points]: Given the following functions $f(x)$, find $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$.

a) $f(x) = x^2 - 4$,

b) $f(x) = 4x^3 + 3x^2 + x$.

*Hint: You will **not** get credit for applying rules for differentiation. You **must calculate the limit of the difference quotient.***

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Problem 2 [12 Points]: Compute the following integral.

$$\int x^7 \sqrt{5 + 3x^4}\, dx.$$

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Problem 3 [6 + 10 + 8 Points]: Consider the linear map $T : \mathbb{R}^3 \to \mathbb{R}^4$, where $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2x_1 + x_2 \\ x_2 - x_3 \\ 2x_2 + 4x_3 \\ x_1 + x_3 \end{pmatrix}$.

a) Find the standard matrix A (i.e. for the Euclidean bases) associated with T.

b) Determine nullity and rank of A. Give reasoning.

c) Determine the nullspace of A. Give reasoning.

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Problem 5 [2 x 8 Points]: Are the following sets of vectors linearly dependent? Prove or disprove.

a) $\left\{ \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 5 \end{pmatrix} \right\}$

b) $\left\{ 1 + t - t^3, -2 + 3t - t^2 + 2t^3, 1 + t^2 + 5t^3 \right\}$

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Problem 6 [8 + 10 Points]: Given $f(x) = \sqrt{25 - x^2}$ consider the Mean Value Theorem (MVT) for this function over the interval $[-3, 5]$.

a) Sketch the graph of this function in the coordinate system below. Label the points that determine the secant relevant to the application of the Mean Value Theorem.

b) Find the value $c \in [-3, 5]$ which is guaranteed to exist by the theorem. Place the point $(c, f(c))$ in the coordinate system below.