Exercise 1 - K-means clustering

i	x_i	$C^{(0)}(i)$
1	$(6,6)^T$	0
2	$(-11,8)^T$	1
3	$(-6,7)^T$	1
4	$(4,5)^T$	2
5	$(0,-5)^T$	2
6	$(-1,-4)^T$	1

1st iteration

1) Compute centroids

$$m_0^{(1)} = \frac{1}{2}\left[\begin{pmatrix}6\\6\end{pmatrix} + \begin{pmatrix}-6\\7\end{pmatrix}\right] = \begin{pmatrix}0\\6.5\end{pmatrix}$$

$$m_1^{(1)} = \frac{1}{2}\left[\begin{pmatrix}-11\\8\end{pmatrix} + \begin{pmatrix}-1\\-4\end{pmatrix}\right] = \begin{pmatrix}-6\\2\end{pmatrix}$$

$$m_2^{(1)} = \frac{1}{2}\left[\begin{pmatrix}4\\5\end{pmatrix} + \begin{pmatrix}0\\-5\end{pmatrix}\right] = \begin{pmatrix}2\\0\end{pmatrix}$$

2) Compute the distances of the points from each centroid and assign to the closest

i	$m_0^{(1)}$	$m_1^{(1)}$	$m_2^{(1)}$
1	(36.25)	160	52
2	123.25	(61)	233
3	36.25	(25)	113
4	(18.25)	109	29
5	132.25	85	(29)
6	111.25	61	(25)

→

i	$C^{(1)}(i)$
1	0
2	1
3	1
4	0
5	2
6	2

↳ Distances are squared

2nd iteration

1) compute centroids

$$m_0^{(2)} = \frac{1}{2}\left[\begin{pmatrix}6\\6\end{pmatrix} + \begin{pmatrix}4\\5\end{pmatrix}\right] = \begin{pmatrix}5\\5.5\end{pmatrix}$$

$$m_1^{(2)} = \frac{1}{2}\left[\begin{pmatrix}-11\\8\end{pmatrix} + \begin{pmatrix}-6\\7\end{pmatrix}\right] = \begin{pmatrix}-8.5\\7.5\end{pmatrix}$$

$$m_2^{(2)} = \frac{1}{2}\left[\begin{pmatrix}0\\-5\end{pmatrix} + \begin{pmatrix}-1\\-4\end{pmatrix}\right] = \begin{pmatrix}-0.5\\-4.5\end{pmatrix}$$

2) Compute the distances and assign

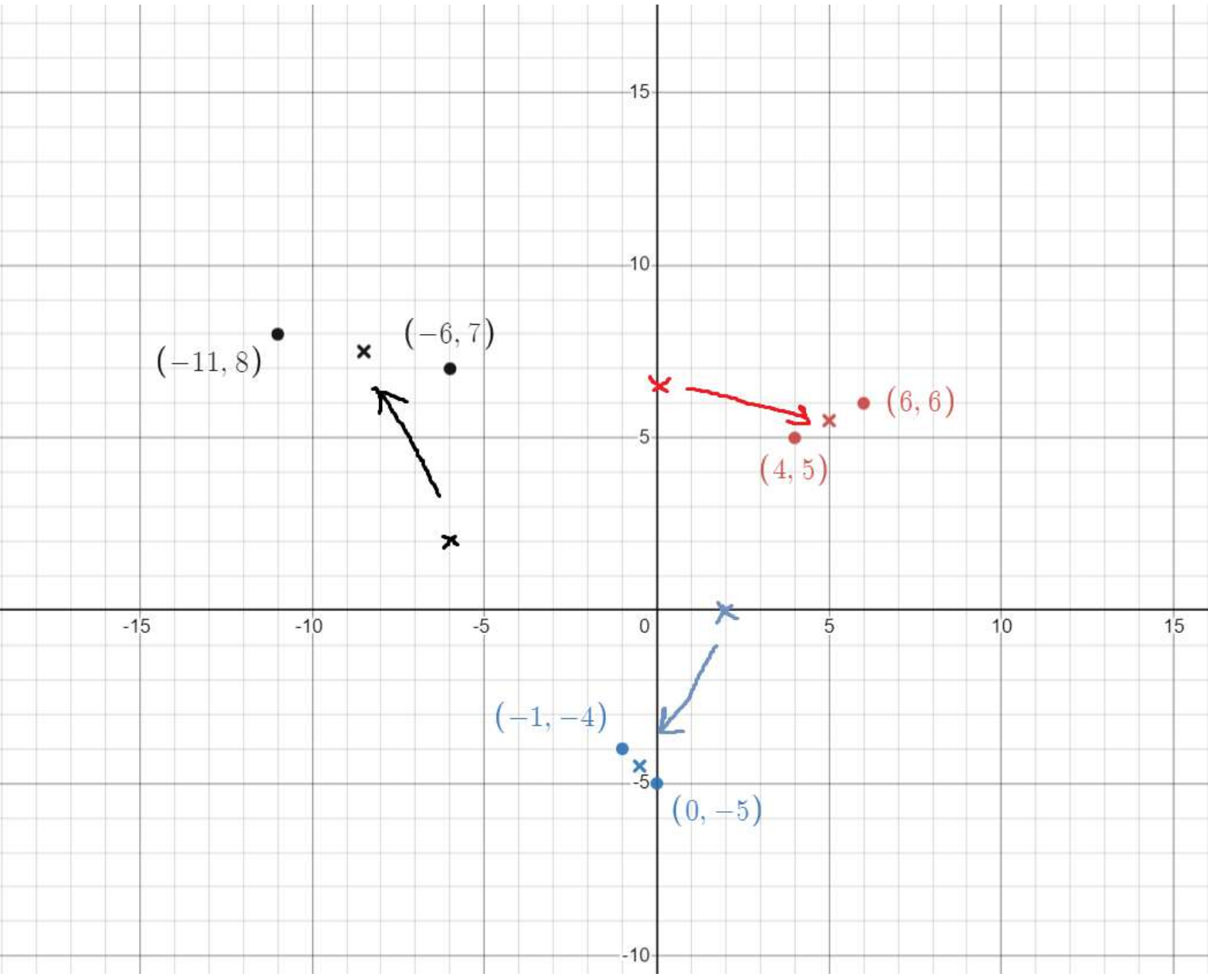
i	$m_0^{(2)}$	$m_1^{(2)}$	$m_2^{(2)}$
1	(1.25)	212.5	152.5
2	262.25	(6.5)	266.5
3	123.25	(6.5)	162.5
4	(1.25)	162.5	110.5
5	135.25	228.5	(0.5)
6	126.25	188.5	(0.5)

→

i	$C^{(2)}(i)$
1	0
2	1
3	1
4	0
5	2
6	2

$C^{(2)} = C^{(1)}$ → no change occurred since the previous iteration

=> K-means clustering algorithm is complete


15
10
5
-5
-10
-15
-10
-5
0
5
10
15
$(-11, 8)$
$(-6, 7)$
$(6, 6)$
$(4, 5)$
$(-1, -4)$
$(0, -5)$