HW #5

Calculus & linear Algebra

Problem 1

a) $xy^2 = 3x + \cancel{d}\, y$ at $(2,2)$

$$x \cdot 2y \frac{dy}{dx} + y^2 = 3 + \frac{dy}{dx}$$

$$2xy \frac{dy}{dx} - \frac{dy}{dx} = 3 - y^2$$

$$\frac{dy}{dx}(2xy - 1) = 3 - y^2$$

$$\frac{dy}{dx} = \frac{3 - y^2}{2xy - 1}$$

$$= \frac{3 - 4}{8 - 1} = -1/7$$

eq of tangent $= y - y_1 = m(x - x_1)$

$$y - 2 = -1/7\, x + 2/7$$

$$\boxed{y = -1/7\, x + 16/7}$$

b) ~~d~~ $x^{3/2}\sqrt{y} + x\sqrt[3]{y} = 12$

$$\frac{3}{2}x^{1/2}\sqrt{y} + \frac{1}{2}x^{3/2}y^{-1/2}\frac{dy}{dx} + \frac{1}{3}x\,y^{-2/3}\frac{dy}{dx} + \sqrt[3]{y} = 0.$$

$$\frac{dy}{dx}\left(3x^{3/2}\sqrt[3]{y^2} + 2x\sqrt{y}\right) = -3\left(3\sqrt{x}\,y^{5/3} + 2y^{3/2}\right)$$

$$\frac{dy}{dx} = \frac{-3\left(3\sqrt{x}\,y^{5/3} + 2y^{3/2}\right)}{3x^{3/2}y^{2/3} + 2x\sqrt{y}}$$ put 2, 8.

$$= \frac{-3\left(3\sqrt{2}\,8^{5/3} + 2\cdot 8^{3/2}\right)}{3\cdot 2^{3/2}\,8^{2/3} + 2\cdot 2\sqrt{8}}$$

$$= \frac{-24}{2}$$

$$= -12$$

eq of tangent.

$y - 8 = -12(x-2)$

$y = -12x + 24 + 8$

$\boxed{y = -12x + 32}$

c) $xy^2 = 3x + y.$

$\frac{d}{dy}(xy^2 = 3x + y)$?

$2xy + \frac{dx}{dy}y^2 = 3\frac{dx}{dy} + \cancel{y}\,1$

$\frac{dx}{dy}y^2 - 3\frac{dx}{dy} = 1 - 2xy$

$\frac{dx}{dy}(y^2 - 3) = 1 - 2xy$

$\frac{dx}{dy} = \frac{1-2xy}{y^2-3}$ $\quad 2,2.$

$= \frac{1-8}{4-3} = \frac{-7}{1}$ for $\frac{dx}{dy}$

$\cancel{\frac{dy}{dx} = \frac{-1}{7}}$ ~~reciprocal.~~

$x - x_1 = m(y - y_1)$

$x - 2 = -7(y-2)$

$x - 2 = -7y + 14$

$x - 16 = -7y$

$y = \frac{x-16}{-7}$ $\boxed{= -\tfrac{1}{7}x + \tfrac{16}{7}}$ same as a) part

Problem 2

a) $\cancel{\frac{d}{ds}}$ $\quad \cancel{\frac{ds}{d}}$ $\quad \frac{dV}{ds} = 0.001\pi \qquad \frac{dr}{ds} = ?$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dr} = \cancel{\frac{8}{3}}\pi r^2 \times \frac{4}{\cancel{3}} \times 3$$

$$= 4\pi r^2$$

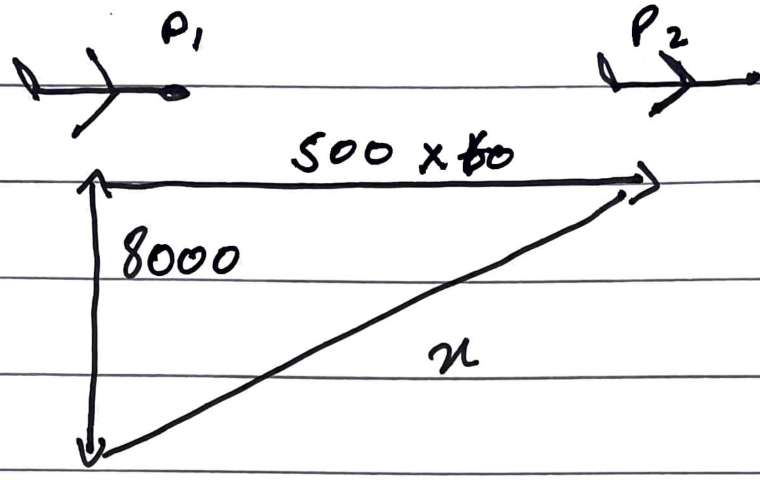
$$\frac{dV}{dr} = \frac{dv}{ds} \cdot \frac{ds}{dr}$$

$$4\pi r^2 = 0.001\pi \cdot \frac{ds}{dr} \qquad r = 20 \times 10^{-2}$$

$$\frac{4\cancel{\pi}(0.2)^2}{0.001\cancel{\pi}} = \frac{ds}{dr}$$

$$= 160$$

$$\frac{dr}{ds} = 1/160 = 0.00625$$



$$x = \sqrt{8000^2 + (30000)^2}$$

$$= 31048.35$$

$$\frac{dx}{dt} = \frac{30,000}{31048.36} \times 800 = 483.12 \text{ m/s.}$$

Problem 3

a) show $\frac{d}{dx}\left(\cos^{-1}(x)\right) = -\frac{1}{\sqrt{1-x^2}}$

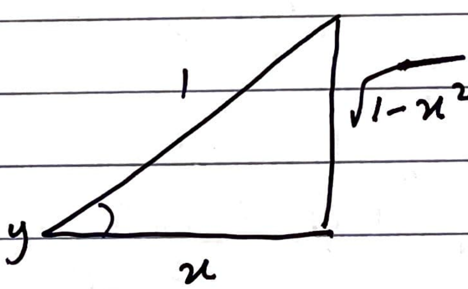
$$y = \cos^{-1} x$$

$$\cos y = \cancel{\cos}\left(\cos^{-1}(x)\right)$$

$$\cos y = x$$

$$\frac{d \cos y}{dy} = \frac{d(x)}{dy}$$

$$-\sin(y) \cdot \frac{dy}{dx} = 1$$



$$\frac{dy}{dx} = -\frac{1}{\sin(y)}$$

$$\sin y = \sqrt{1-x^2}$$

$$= -\frac{1}{\sqrt{1-x^2}}$$

b) $f(x) = 2x^3 - 6x + 9$

$f'(x) = 6x^2 - 6 = 0.$

$\cancel{6}x^2 = \cancel{6}$

$x^2 = 1$

$x = \pm\sqrt{1}$

Critical points

$y = 2(1)^3 - 6(1) + 9$	$y = 2(-1)^3 - \{6(-1)\} + 9$
$= 2 - 6 + 9$	$= -2 + 6 + 9$
$= 5$	$= 13$
$P_1\ (1, 5)$	$P_2\ (-1, 13)$

$f''(x) = 12x \qquad x = \pm 1$

$12(-1)$

-12 is less than 0 so it is a local Maximum

$12(1)$

$= 12$ is greater than 0 so local minima

c) $g(x) = 2x^3 + 6x + 9$

$g'(x) = 6x^2 + 6 = 0$

$6x^2 = -6$

$x^2 = -1$

$x = \pm i$

Since function has imaginary roots so local Maxima or Minima cannot be determined.

d) $h(t) = \sin(\omega t)$

$h'(t) = \cos(\omega t) \cdot \omega = 0$

$\cos(\omega t) = 0$

Since $\cos(x)$ is a periodic function it has ~~mini~~ infinite maxima and minima

$\omega t = \frac{\pi}{2}$ and $\omega t = \frac{3}{2}\pi$

critical point

$\sin(\pi/2) = 1$ $\sin\left(\frac{3\pi}{2}\right) = -1$

maxima minima