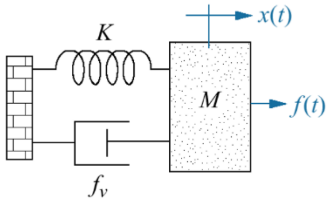
Practice Problems II - 02

Practice problems are supposed to help you digest the content of the lecture. It is important that you manage to solve them on your own. Before you write your solutions, you may of course ask questions, and discuss things. In order to prepare for the exam, already now, try to explicitly write down your solutions – clearly and easy to read. Apply definitions properly, and give explanations for what you are doing. That will help you to understand them later when you prepare for the final exam.

I. Model & Solve in Time Domain



Consider the second order ode

$$M\ddot{x} + f_v\dot{x} + Kx = 0,$$

where $\mathrm{M} = 4\ \mathrm{kg}, f_v = 0,$ and $K = 100\,\frac{N}{m}$.
In order to find the solution set, start by finding the characteristic exponents, i.e., use an exponential ansatz $x(t) = e^{\lambda t}$.

a) What are the relevant values of λ … including their units?

b) In order to specify a concrete solution for such a second order ode, we need two initial values, usually x_0 and $\dot{x}_0$. Here, we start the system at $x_0 = 10\ m$ with an initial speed $\dot{x}_0 = 0$. Write and sketch the solution.

II. Laplace Transforms & Solve in Frequency Domain

a) Find the Laplace transform of the equation

$$M\ddot{x} + f_v\dot{x} + Kx = 0$$

for the parameters and initial values mentioned in problem 2.

b) Solve for X(s).

c) Based on your solution, also find the solution in time domain, and compare.

$\boldsymbol{f(t)}$	$\boldsymbol{F(s) = \mathcal{L}\{f(t)\}(s)}$
$f(t)$	$\int_0^\infty f(t)\, e^{-st} dt$
e^{-at}	$\frac{1}{s+a}$
$\delta(t)$	1
$\frac{d}{dt} f(t)$	$sF(s) - f_0$
$\frac{d^2}{dt^2} f(t)$	$-sf_0 - \dot{f}_0 + s^2F(s)$

Hint: For complex values of λ, the rules for exponentials remain intact: In particular, $e^{\lambda t} = e^{(\sigma + j\omega)t} = e^{\sigma t}e^{j\omega t}$, also Euler's formula says: $e^{j\omega t} = \cos\omega t + j\sin\omega t$.