HW #6

Problem 1

1) $f(x) = \frac{x^2}{4-x^2}$ domain $x \in R \setminus \{-2, 2\}$

2) y-intercept $f(0) = 0$

x-intercept $\frac{0^2}{4-0^2} = 0$

3) Horizontal Asymptotes:

$$\lim_{x \to +\infty} \left(\frac{x^2}{4-x^2} \right) \div x^2$$

$$\lim_{x \to +\infty} \frac{1}{\frac{4}{x^2} - x^2} = \frac{1}{4 \times 0 - 1} = -1$$

$$\lim_{x \to -\infty} \frac{1}{\frac{4}{x^2} - x^2} = -1$$

$y = -1$

4) Vertical Asymptotes:

$\cancel{\lim}\ f(x) = \dfrac{x^2}{4-x^2}$, $x \neq -2$, $x \neq 2$.

$\lim\limits_{x \to 2} \dfrac{x^2}{4-x^2}$ does not exist

$\lim\limits_{x \to -2} \dfrac{x^2}{4-x^2}$ does not exist

~~Vert~~ so $x = -2$ and $x = 2$

5) $\dfrac{2x(4-x^2) - x^2(4-x^2)\,-2x}{(4-x^2)^2}$

$f'(x) = \dfrac{8x}{(4-x^2)^2} = 0$

$x = 0$

$(0,0)$ is local Minimum

6) $f'(x) = \dfrac{8x}{(4-x^2)^2}$

$$f''(x) = \frac{8(4-x^2)^2 - 8x(2\cdot(4-x^2)\cdot -2x)}{(4-x^2)^4}$$

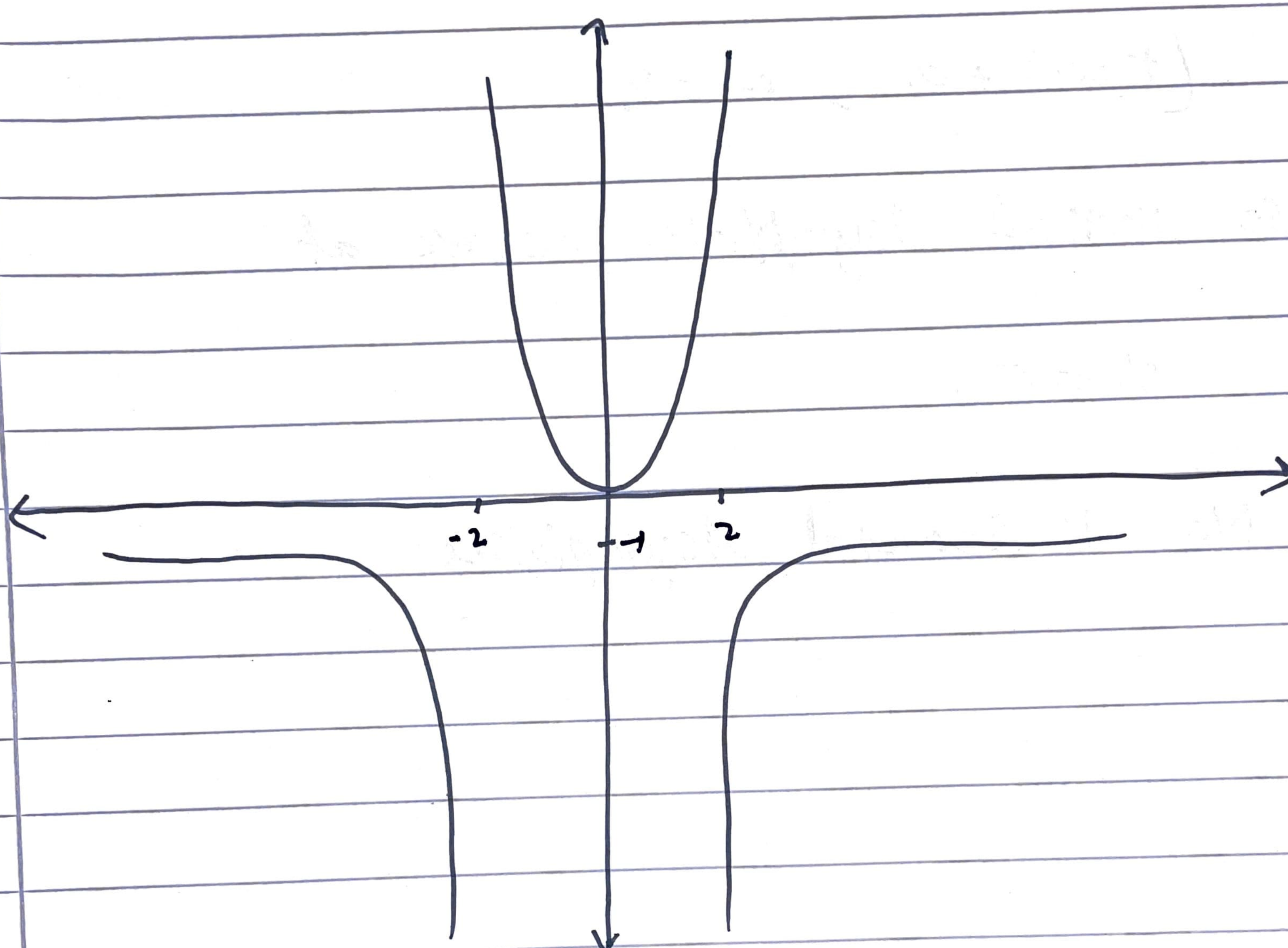
$$= \frac{32+24x^2}{(4-x^2)^3} = 0$$

$24x^2 = -32$ so $x \notin \mathbb{R}$

$x^2 = -32/24$

No inflection point at $x = -2$ or 2

$\boxed{\text{No inflection points}}$



-2
-1
2

<u>Problem 2</u> $f(x) = -\ln(x) + \sqrt{x}$

1) Domain $x \in (0, +\infty)$

2) y-intercept

$f(0) = -\ln(0) + \sqrt{0}$ undefined

no y-intercept. or x-intercept

~~3)~~ 4)
3) $\lim_{x \to 0} -\ln(x) + \sqrt{x} = +\infty$

$(+\infty) + a \; , \; a \in \mathbb{R}$

So vertical Asymptote occurs at

$x = 0$

~~4)~~ 3) No Horizontal Asymptotes

5) $f(x) = -\ln(x) + \sqrt{x}$

$$f'(x) = \frac{-1}{x} + \frac{1}{2\sqrt{x}} = 0$$

$$\frac{1}{2\sqrt{x}} = \frac{1}{x}$$

$$2\sqrt{x} = x$$
$$4x = x^2$$
$$x = 4$$

$f(1) = -\frac{1}{2}$
$f(5) = 0.0236$

min at 4

$$f(4) = -\ln(\cancel{4}) \cancel{+} 2$$

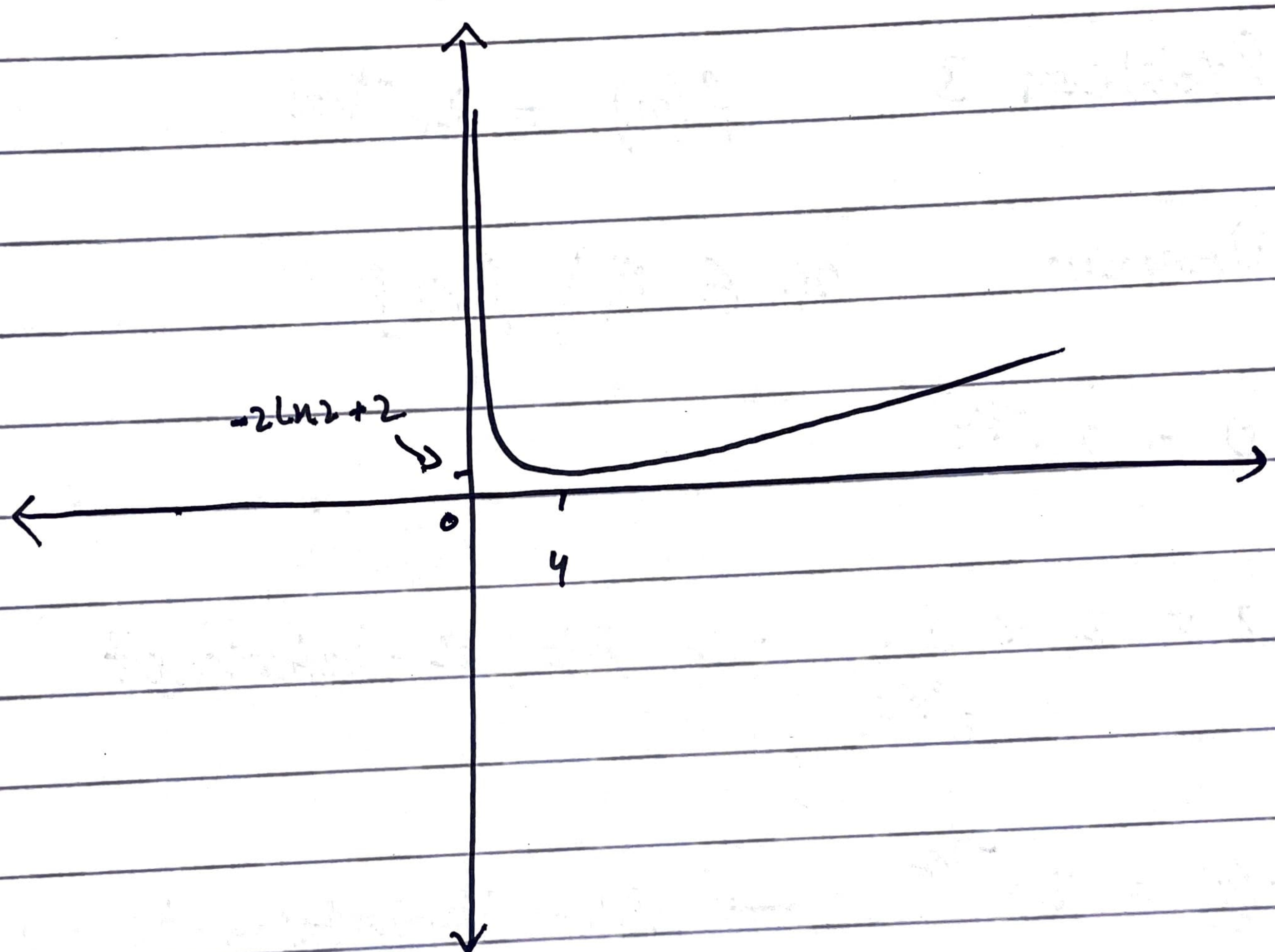
minima is at $-2\ln(2) + 2$

$$(4, -2\ln(2) + 2)$$

6) $f''(x) = \frac{1}{x^2} - \frac{1}{2} \frac{1}{2 \times 1.5} = 0$

$$= x = 16$$

concave down at $x = 16$



$-2\ln 2+2$
0
4

Problem 3 $\quad f(x) = 2e^{-4/x}$

1) Domain $\quad x \in \mathbb{R} \setminus \{0\}$

2) $0 = 2e^{-4/x}$

$0 = 2 \times \frac{1}{e^{4/x}}$ $\rightarrow$ No x-intercept

$f(0) = 2e^{-4/0}$ $\rightarrow$ No y-intercept

3) Horizontal Asymtotes

$\lim_{x \to +\infty} 2e^{-4/x}$ $\qquad$ $\lim_{x \to -\infty} 2e^{-4/x}$

$2e^{0}$

$= 2$ $\qquad$ $= 2$

$\boxed{y = 2}$

4) Vertical Asymtotes

$\lim_{x \to 0^{+-}} 2e^{-4/e}$

$2 \quad +\infty$

So Vertical Asymptote occur at $x = 0$

5) $f(x) = 2e^{-4/x}$

$f'(x) = 2 \cdot e^{-4/x} \cdot 4 \times \frac{1}{x^2}$

$f'(x) = \frac{8}{e^{4/x} \cdot x^2} = 0$

$8 = 0$ ~~[illegible]~~

No relative extrema points.

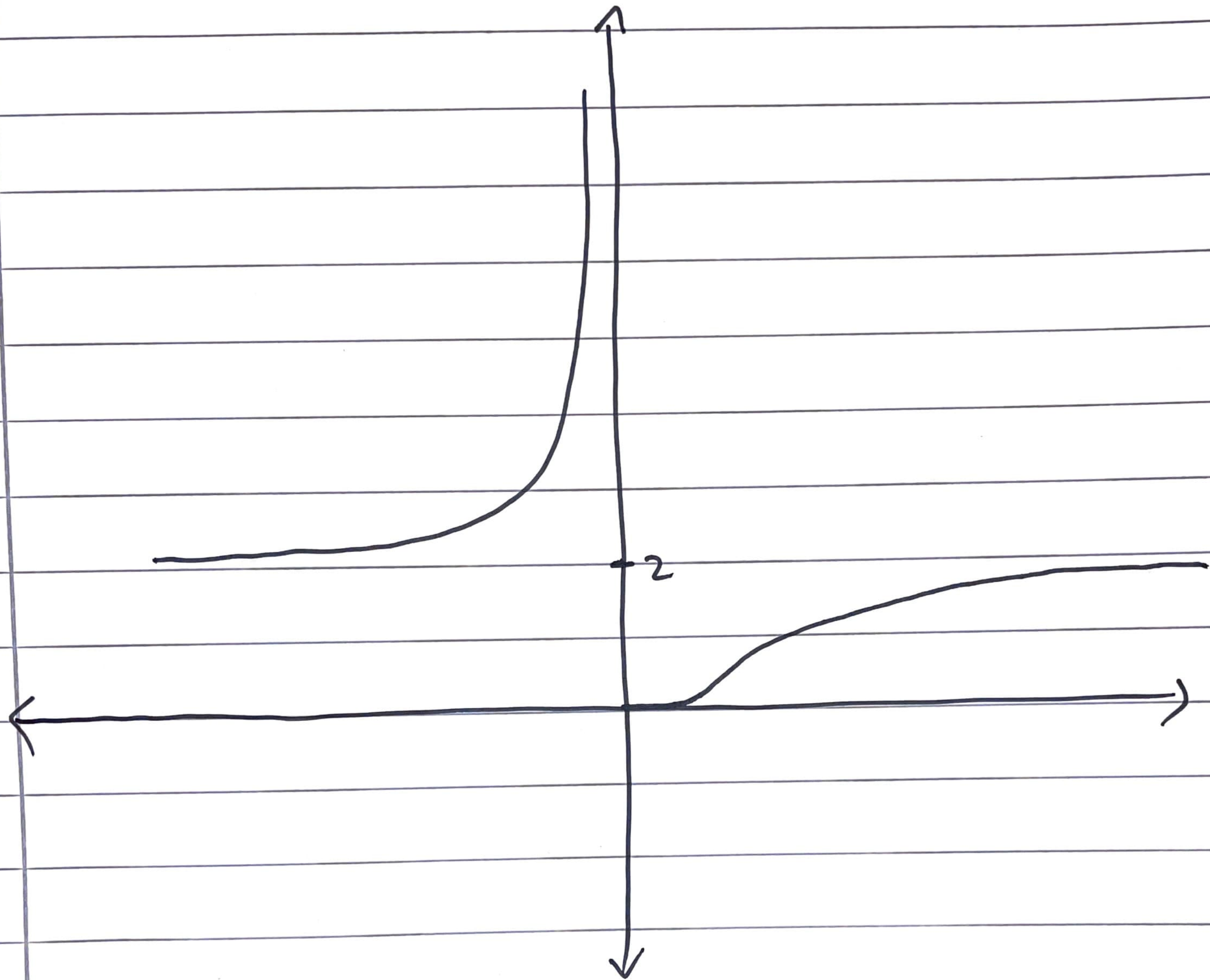
6) $f'(x) = \frac{8}{e^{4/x} \cdot x^2}$

$f''(x) = -8 \frac{\frac{d}{dx}\left(e^{4/x} \cdot x^2\right)}{\left(e^{4/x} \cdot x^2\right)^2}$

$= \frac{-8 \cdot \left(e^{4/x} \cdot -4 \cdot \frac{1}{\cancel{x^2}} \cancel{x^2} + e^{4/x} \times 2x\right)}{\left(e^{4/x} \cdot x^2\right)^2}$

$f''(x) = \frac{32e^{4/x} - 16xe^{4/x}}{\left(e^{4/x} x^2\right)^2}$

No concavity.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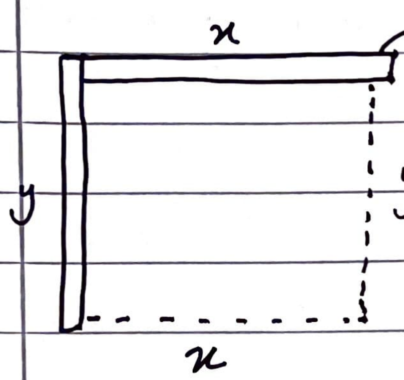
Problem 4



→ Fixed (hedged).

$x + y = 50.$

$$\text{Area} = x \cdot y.$$
$$= x(50 - x).$$
$$= 50x - x^2.$$

$$\frac{dA}{dx} = 50 - 2x. = 0$$

$$= x = \frac{-50}{-2} = \boxed{25}$$

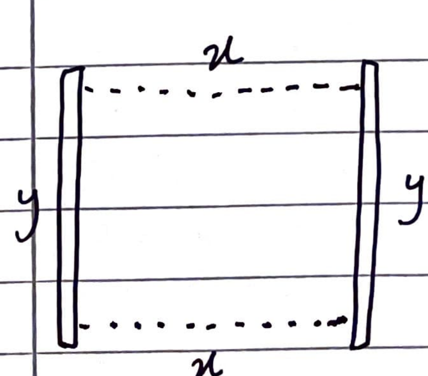
$$\frac{d^2A}{dx^2} = -2x + 50 = -2 \quad \text{Maxima.}$$

$x = 25$ $\qquad$ $25 + y = 50$

$y = 25$

$\text{Area} = 25 \times 25 = 625\,m^2$

case 2:



$2x = 50$

$x = 25$

So $y \in \mathbb{R}$. No bounds for y so area cannot be determined. y can be any value.

Q-5 $(x-h)^2 + (y-k)^2 = r^2$ centre at orign (0,0)

So

$x^2 + y^2 = r^2$

$d_1 = \sqrt{1^2 + (-1, -0)^2} - r$

$= \sqrt{2} - r$

$d_1^2 = (\sqrt{2} - r)^2$

$d_2 = \sqrt{1^2 + 0^2} - r$

$= 1 - r$

$d_2^2 = (1-r)^2$

eq of circle $f(r) = d_1^2 + d^2$

$= 1 + r^2 - 2r + 2 + r^2 - 2\sqrt{2}r$

$= 2r^2 - 2r - 2\sqrt{2}r + 3$

taking derivative.

$= 4r - 2 - 2\sqrt{2} = 0$

$4r = 2 + 2\sqrt{2}$

$r = \frac{2 + 2\sqrt{2}}{4} = 1.207$

$\boxed{r = 1.207}$