Machine Learning

Spring Semester 2023
Prof. Dr. Peter Zaspel

Assignment Sheet 11.

Submit on **Tuesday, May 2, 2023, 10:00**.

Excercise 1. (Polynomial models)

In this paper & pencil task, you compare the quadratic model based on the basis functions $\{1, X, X^2\}$ to the linear model for the following training set.

$$\mathcal{T} = \{(0, -5), (1, -2), (2, 5)\}.$$

a) Compute the linear model using least squares for the given training data.

b) Compute the quadratic model using least squares for the given training data.

c) Draw (by hand) the linear and the quadratic model in a plot, together with the training samples.

(4 Points)

Excercise 2. (Kernel-based model)

You are given the training data

$$\mathcal{T} = \{(-2, 3)^\top, (0, -1)^\top, (3, 4)^\top\}.$$

a) Use the Gaussian kernel with kernel width $\sigma = 1$ and compute by hand the kernel-based model using least squares for the given training data.

b) Repeat the previous task, but this time you compute by hand the kernel-based model using ridge regression with regularization parameter $\lambda = 1$.

c) Draw both above models including the training data in one plot.

(4 Points)

Excercise 3. (Kernel matrix)

Prove Lemma 10.2 from the lecture notes.

(4 Points)

Programming Exercise 1. In this task, we implement kernel ridge regression. To this end, we start from Example 10.6 from the lecture notes, which is available as Jupyter notebook. However, instead of using skit-learn to compute the ridge regression model, you replace that part of the code by your own implementation of kernel ridge regression, which follows Algorithm 17 from the lecture notes. Due to the existing implementation, it will be easier to evaluate, whether your implementation is correct.
Reference solutions will only be provided in Python+Matplotlib. The submission format for Python is a Jupyter notebook. The submission format for C/C++ is standard source files. Choose an appropriate format for the Gnuplot-related submission.

(4 Points)