# HW #7

## Problem 1

a) $\int \frac{\sin\left(\pi/x^2\right)}{x^3}\,dx.$  $u = \pi/x^2$

$$\frac{du}{dx} = \frac{-2\pi}{x^3}$$

$$\rightarrow \int \frac{\sin(\cancel{\pi} u)}{x^3} \cdot \frac{-x^3}{2\pi}\,du$$

$$dx = \frac{-x^3}{2\pi}\,du.$$

$$\rightarrow \frac{-1}{2\pi} \int \sin(u)\,du$$

$$\rightarrow \frac{+1}{2\pi} \times (+\cos(u) + c)$$

$$\rightarrow \frac{\cos\left(\pi/x^2\right)}{2\pi} + C$$

b) $\int \frac{2\ln(x)}{x}\,dx$

$u = \ln(x)$

$\frac{du}{dx} = \frac{1}{x}$

$\rightarrow \int \frac{2 \cdot u}{\cancel{x}} \cancel{x}\,du$

$dx = x\,du.$

$\rightarrow 2 \int u \, du$

$\rightarrow \cancel{2} \cdot \frac{u^2}{\cancel{2}}$ ~~du~~ $+ C$

$\rightarrow \ln^2(x) + C$

c) $\int \cos(x) \ln(\sin(x))\, dx.$

$u = \sin(x)$

$\frac{du}{dx} = \cos(x)$

$\rightarrow \int \cancel{\cos(x)} \ln(u) \frac{du}{\cancel{\cos(x)}}$

$\rightarrow \int 1 \cdot \ln(u)\, du$ * Integration by Parts.

$\rightarrow u \cdot \ln(u) - \int 1\, du$

$\rightarrow u \ln(u) - u$

$\rightarrow \sin(x) \ln(\sin(x)) - \sin(x) + C.$

$\sin(x)\left(\ln(\sin(x)) - 1\right) + C$

d) $\displaystyle\int_0^{\pi/2} x\cos(x)\sin(x)\,dx$

$f = x$

$g' = \sin x \cos x.$

$g = \int \cancel{\sin x} \cos x \cdot \frac{1}{\cancel{\cos x}}\, du.$

$= \frac{u^2}{2}$

$= \frac{\sin^2(x)}{2}.$

$\int f g' = fg - \int f' g$

$\rightarrow x\int \cos(x)\sin(x) - \int \frac{d}{dx}(x)\cos(x)\sin(x)$

$\rightarrow \frac{x \cdot \sin^2(x)}{2} - \int 1 \cdot \frac{\sin^2(x)}{2}$

$\rightarrow \frac{x\sin^2(x)}{2} - \left(\frac{\cos(x)\sin x}{2} \cdot \frac{-1}{2} + \frac{x}{4}\right)$

$\rightarrow \frac{x\sin^2(x)}{2} + \frac{\cos x \sin(x)}{4} - \frac{x}{4} + C$

$\Rightarrow \frac{\frac{\pi}{2}\sin^2(\frac{\pi}{2})}{2} + \frac{\cos(\frac{\pi}{2})\sin(\frac{\pi}{2})}{4} - \frac{\frac{\pi}{2}}{4} = \cancel{\frac{\pi}{4}} + 0\ \frac{\pi}{8} = \cancel{\frac{\pi}{8}}\ \frac{\pi}{8}$

$\Rightarrow \frac{0 \cdot \sin^2(0)}{2} + \frac{\cos(0)\sin(0)}{4} - \frac{0}{4} = 0$

$\frac{\pi}{8} - 0 = \boxed{\frac{\pi}{8}}$

Problem 2

a) $\int \cos^n(x)\,dx$ prove with reduction formula.

$= \int \cos^{n-1}(x) \cdot \cos(x)\,dx$

$\int u\,dv = uv - \int v\,du$

let $u = \cos^{n-1}(x)$
$du = (n-1)\cos^{n-2}(x) \cdot \sin(x)$.

$dv = \cos(x)\,dx$.

$\int \cos^{n-1}(x)\cos(x)\,dx = \cos^{n-1}(x) \cdot \sin(x) - \int \sin(x) \cdot (n-1)\cos^{n-2}(x)\,dx$.

$= \cos^{(n-1)}(x) \cdot \sin(x) + \int (n-1)\cos^{n-2}(x)\sin^2(x)\,dx$

$= \cos^{(n-1)}(x) \cdot \sin(x) + \int (n-1)\cos^{n-2}(x)(1-\cos^2 x)\,dx$.

$= \cos^{(n-1)}(x) \cdot \sin(x) + \int (n-1)\cos^{n-2}(x) - \int (n-1)\cos^{n-2}(x)\cos^2(x)\,dx$.

$$\int \cos^n(x)\,dx + (n-1)\int \cos^n(x)\,dx$$

$$n\int \cos^n(x)\,dx = \cos^{n-1}(x)\cdot\sin(x) + (n-1)\int \cos^{n-2}(x)\,dx$$

$$\rightarrow \left(\cos^{n-1}(x)\cdot\sin x + (n-1)\int \cos^{n-2}(x)\,dx\right)\frac{1}{n}.$$

$$\frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx$$

proved !

b) $I = \int_{-a}^{a} f(x)\,dx.$

$$= \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx.$$

$u = -x$ new limits $=$ $a, 0$

$du = -dx$

$$= -\int_{a}^{0} f(-u)\,du + \int_{0}^{a} f(x)\,dx$$

flip

$$= -\int_{0}^{a} f(u)\,du + \int_{0}^{a} f(x)\,dx$$

$$= \cancel{-\int_{0}^{a} f(x)\,dx} + \cancel{\int_{0}^{a} f(x)\,dx}$$

$= \underline{\underline{0}}$ hence proved that

$\int_{-a}^{a} f(x)\,dx$ where $f(x)$ is odd $= 0$

a) $\int \cos^{2}(x)\, dx$

$$\frac{1}{2}\int \cos^{2-2}(x)\, dx + \frac{\cos(x)\sin(x)}{2}$$

$$\frac{\cos(x)\sin(x)}{2} + \frac{1}{2}\int 1\, dx.$$

$$\frac{\cos(x)\sin(x)}{2} + \frac{1}{2}x . + C$$

$$= x + \frac{\cos(x)\sin(x)}{2} + C$$

b) $\int \cos^3(x)\, dx.$

$= \frac{1}{3}\cos^2(x)\sin(x) + \frac{2}{2}\int \cos x\, dx.$

$\frac{\cos^3(x)\sin(x)}{3} + \sin x + C$

c) $\int_0^{2\pi} \cos^5(x)\, dx$

$\frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{5}\int \cos^3(x)\, dx$

$\frac{4}{5}\left(\frac{1}{3}\cos^2(x)\sin(x) + \frac{2}{3}\int \cos(x)\, dx\right)$

$\rightarrow \frac{1}{5}\cos^4(x)\sin(x) + \frac{4}{15}\cos^2 \sin x + \frac{8}{15}\sin x + C$

$\frac{1}{5}\cos^4(2\pi)\sin 2\pi) + \frac{4}{15}\cos^2 \sin 2\pi + \frac{8}{15}\sin(2\pi) = 0$

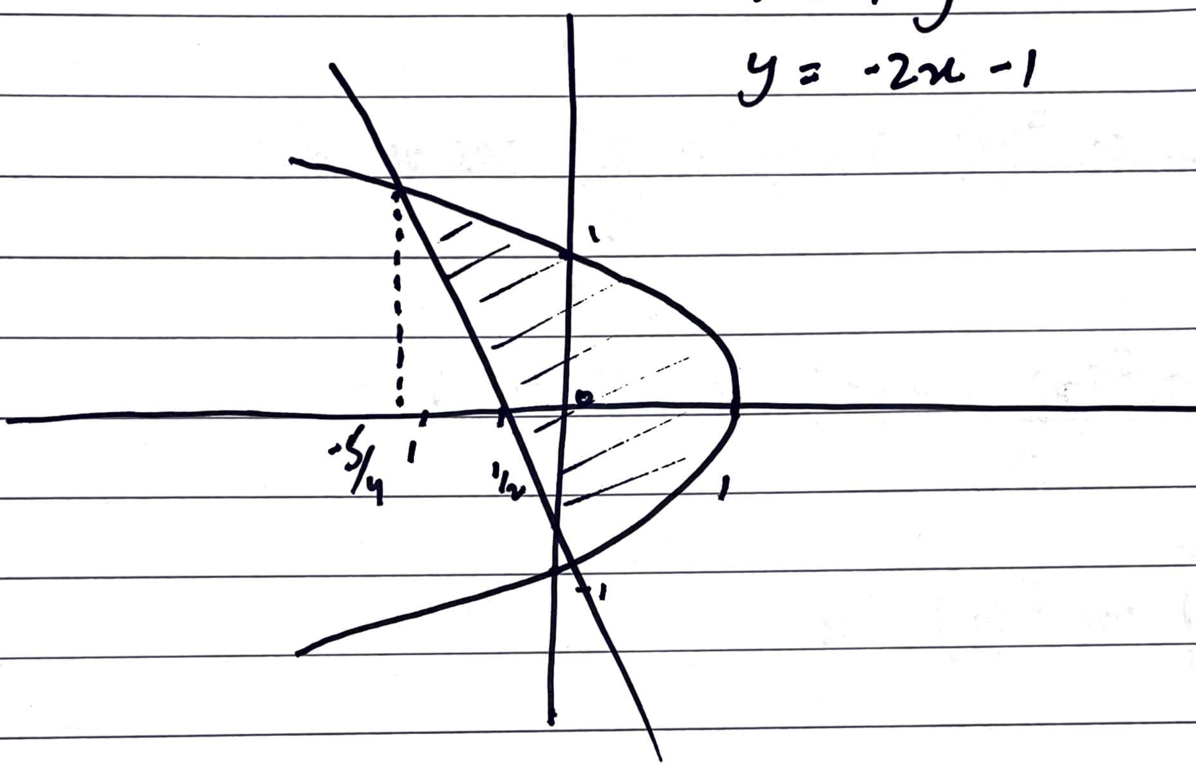
$\boxed{= 0}$

$\sin(0) = 0$
$\sin(2\pi) = 0.$

## Bonus Questions

$x = 1 - y^2$

$y = -2x - 1$




$y = -2(1-y^2) - 1$

$= -2 + 2y^2 - 1$

$0 = 2y^2 - y - 3$

$y = -1, \; y = 3/2$

$x = 1 - \left(\frac{3}{2}\right)^2$

$= -5/4.$

$y = \sqrt{1-x}$

$\int \sqrt{1-x} \times \frac{-1}{-1} \, dx.$

$= \frac{-(1-x)^{3/2}}{3} \cdot 2 + C$

$y = -2x - 1$

$\int -2x - 1 \, dx.$

$= \frac{-\cancel{2}x^2}{\cancel{2}} - x$

$= -x^2 - x.$

$\left| \frac{-2}{3}(1-x)^{3/2} - (-x^2 - x) \right|_{-5/4}^{0} + \left| \frac{-2}{3}(1-x)^{3/2} + x^2 + x \right|_{0}^{1}.$

$\frac{-2}{3}(1-0)^{3/2} - 0 - \left( \frac{-2}{3}\left(1-\frac{\cancel{5}}{4}\right)^{3/2} - \left(\frac{-5}{4}\right)^2 - \left(-\frac{5}{4}\right) \right)$

$= \frac{61}{48} + \left( \frac{-2}{3}(1-0)^{3/2} + 1 + 1 \right)$

$\frac{61}{48} + \frac{4}{3}$

$= \boxed{\frac{125}{48}}$