Problem 1

(10 points)

Use substitution to evaluate the following integrals.

a) $\int \frac{\sin(\pi/x^2)}{x^3}\mathrm{d}x$ **(2 points)**

b) $\int \frac{2\ln x}{x}\mathrm{d}x$ **(2 points)**

Use integration by parts to evaluate the following integrals.

c) $\int \cos(x)\ln(\sin x)\mathrm{d}x$ **(3 points)**

d) $\int_0^{\pi/2} x\cos(x)\sin(x)\mathrm{d}x$ **(3 points)**

Problem 2

(10 points)

a) Prove the reduction formula

$$\int \cos^n(x)\,\mathrm{d}x = \frac{1}{n}\,\cos^{n-1}(x)\,\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)\,\mathrm{d}x\,.$$

(5 points)

Hint: Use integration by parts and the fact that $\cos^2(x)+\sin^2(x)=1$.

b) Suppose that $f\colon \mathbb{R}\to\mathbb{R}$ continuous and *odd*, i.e., satisfies $-f(x)=f(-x)$. Show that

$$\int_{-a}^{a} f(x)\,\mathrm{d}x = 0\,.$$

(5 points)

Problem 3

(10 points + 5 bonus points)

Using 2a)

Homework 7

a) Evaluate $\int \cos^2(x)\,\mathrm{d}x$. **(4 points)**

b) Evaluate $\int \cos^3(x)\,\mathrm{d}x$. **(3 points)**

c) Evaluate $\int_0^{2\pi} \cos^5(x)\,\mathrm{d}x$. **(3 points)**

Bonus:
We will cover this on Tuesday, 26.11:
Find the area between the curves $x = 1 - y^2$ and $y = -2x - 1$. **(5 bonus points)**
Hint: At the end, integrate with respect to y not x.