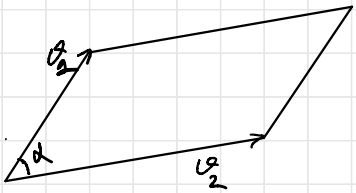
Linear Algebra



Area $= 40$ $\quad \|\vec{v}_1\| = 10$

$\alpha = \frac{\pi}{6}$

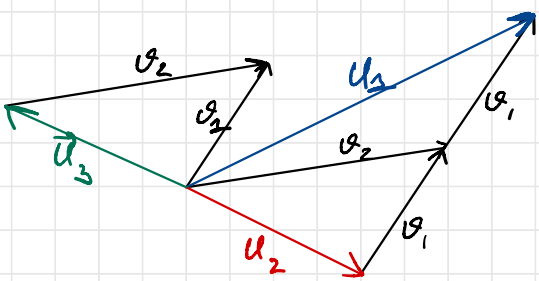
1) $\text{Area} = \|\vec{v}_1\| \cdot \|\vec{v}_2\| \cdot \sin\alpha = 10 \cdot \sin\frac{\pi}{6} \cdot \|\vec{v}_2\| \Rightarrow$

$\Rightarrow \|\vec{v}_2\| = \dfrac{40}{10 \cdot \frac{1}{2}} = 8$

2) $\vec{v}_1 \cdot \vec{v}_2 = \|\vec{v}_1\| \cdot \|\vec{v}_2\| \cdot \cos\alpha = 10 \cdot 8 \cdot \frac{\sqrt{3}}{2} = 40\sqrt{3}$

3) $\vec{v}_1 \times \vec{v}_2 = \|\vec{v}_1\| \cdot \|\vec{v}_2\| \cdot \sin\alpha = \text{Area} = 40$

4)



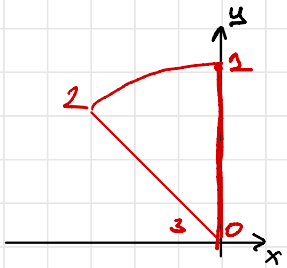
$\vec{v}_1 + \vec{v}_2 = \vec{u}_1$

$\vec{v}_2 - \vec{v}_1 = \vec{u}_2$

$\vec{v}_1 - \vec{v}_2 = \vec{u}_3$

Robot Motion - Homogeneous Coordinates

1)



$$P = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

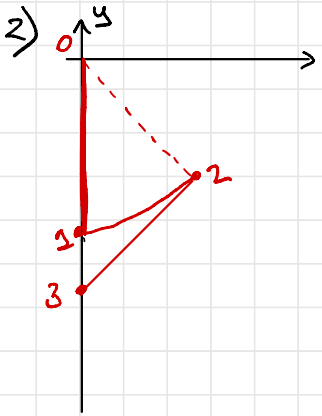
$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & 0 & 3\sqrt{2} \\ 0 & 1 & -3\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$P' = T_2\left(R\left(T_1 P\right)\right) = T_2\left(R\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right)\right) =$$

$$= T_2\left(\begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 3\sqrt{2} \\ 0 & 1 & -3\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -3\sqrt{2} \\ 3\sqrt{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

2)



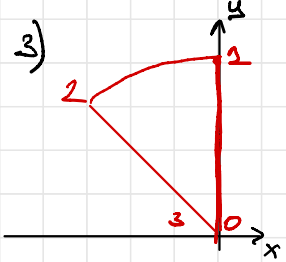
$$P = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & 0 & -3\sqrt{2} \\ 0 & 1 & -3\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix}$$

$$P' = T_2\left(R\left(T_1 P\right)\right) = \begin{bmatrix} 3\sqrt{2} \\ -6\sqrt{2} \\ 1 \end{bmatrix}$$

3)



$$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 1 & 0 & 4\sqrt{2} \\ 0 & 1 & -4\sqrt{2} \\ 0 & 0 & 1 \end{bmatrix} \qquad P = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$P' = T_2\left(R\left(T_1 P\right)\right) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$