21 sept 2021

ICS HW #2

Problem 2.1

$y^3 + yx^2 \leq x^3 + xy^2 \qquad y \leq x.$

$p \longrightarrow q$ prooving it by contrapositive.

$\neg q \longrightarrow \neg p.$

if $y \leq x$ is false then
$y^3 + yx^2 \leq xy^2 + x^3$ is also false.

$y > x \longrightarrow y^3 + yx^2 > x^3 + xy^2$

$\Rightarrow$ Multiplying both sides by $(y^2 + x^2)$
in $y > x$.

$y(x^2 + y^2) > x(x^2 + y^2).$

$y^3 + x^2y > x^3 + xy^2$

hence proved that $y \leq x$ and $y^3 + yx^2 \leq x^3 + xy^2$
are contrapositive.

Problem 2.2

$$1^2 + 3^2 + 5^2 + \dots (2n-1)^2 =$$

$$\sum_{k=1}^{n} (2k-1)^2 = \frac{2n(2n-1)(2n+1)}{6}$$

Proof by Induction

Let $n=1$

$$\sum_{k=1}^{1} (2-1)^2 = \frac{2(2-1)(2+1)}{6}.$$

$$1 = \frac{2 \times 3}{6}$$

$1 = 1$ RHS = LHS.

$$n = k+1 \rightarrow \frac{2n(2n+1)(2n-1)}{6} \cancel{+(2k+1)^2}$$

$$= \frac{2(k+1)(2(k+1)+1)(2(k+1)-1)}{6}$$

$$= \frac{(2k+1)(2k+2)(2k+3)}{6}$$

$$\sum_{k=1}^{k+1} (2k-1)^2 = \sum_{k=1}^{k} (2k-1)^2 + (2(k+1)-1)^2$$

$$= \frac{2k(2k-1)(2k+1)}{\cancel{6}\,3!} + \frac{(2k+1)^2 \times 6}{6}$$

This equation holds for k+1

It follows by Induction ~~tat~~ that $1^2 + 3^2 + 5^2 \cdots (2n-1)^2$

$= \sum_{k=1}^{n} (2k-1)^2$ holds for arbitary nonnegative integers n.

Problem 2.3

a) The operator "==" is a non associative operator means it is neither left nor right associative. If it appears for eg:

> 1 == 1 == 1

it gives precedence parsing error. because the interpreter ~~does~~ or could not determine whether to start from left or right.

b) \$ operator is right associative and it's precedence is 0.

> (^) 2 \$ $\underbrace{(*)\ 5\ \$\ (+)\ 2\ 3}$.
> 33554432

←

> (*) 5 5 => 25
> (^) 2 25 = 2^25
> 33554432.

\$ infix notation = $2^{\wedge}(5*(2+3))$

without \$ = (^) 2 ((*) 5 ((+) 2 3))