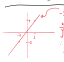
Homework 2 Solutions

① a) 

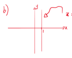
$-3x+2y=2$

x	2	-2
y	4	-2

$\therefore$ coordinates: $(2,4)$ and $(-2,-2)$

$2y = 3x+2$

$y = \underbrace{\tfrac{3}{2}}_{\text{gradient}}x + \underbrace{1}_{y\text{-intercept}}$

b) 

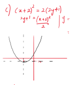
$x=1$

For every single coordinate of y, the value of x is 1. (x does not depend on y)

c) $(x+2)^2 = 2(2y+1)$

$2y+1 = \dfrac{(x+2)^2}{2}$ | $y = \left[\dfrac{(x+2)^2}{2} - 1\right]/2$

$= \dfrac{(x+2)^2-2}{4} = \dfrac{(x+2)^2}{4} - \dfrac{1}{2}$



when $x=0$, $y = \frac{2^2}{4} - \frac{1}{2} = \frac{1}{2}$

when $y=0$, $\frac{(x+2)^2}{4} = \frac{1}{2}$

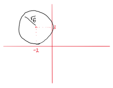
$x+2 = \pm\sqrt{2}$

$x = -2 \pm \sqrt{2}$

d) $(x+2)^2 + (y-3)^2 = 6$

$x+2=0$, $x=-2$ ← x-coordinate | $y-3=0$, $y=3$ ← y-coordinate | $r=\sqrt{6}$ ← radius

→ $(-2,3)$ is the midpoint of the circle



e) a) and c) are graphs of the function $y=f(x)$.
b) and d) are not graphs of $y=f(x)$.

a) → $f(x) = \frac{3}{2}x + 1$ | Domain : $\mathbb{R}$, Range : $\mathbb{R}$ } because they are linear, they can take all values

c) → $f(x) = \dfrac{(x+2)^2-2}{4}$ | Domain : $\mathbb{R}$, Range : $f(x) \geq -\frac{1}{2}$

f) a) and b) are graphs of the functions
c) and d) are not graphs of $x = g(y)$

a) → $3x = 2y-2$

$x = \frac{2y}{3} - \frac{2}{3}$

$g(y) = \frac{2y}{3} - \frac{2}{3}$ | Domain : $\mathbb{R}$, Range : $\mathbb{R}$

b) $x=1$

$g(y) = 1$ | Domain : $\mathbb{R}$, Range : 1

② $f(x) = 2^{x/2} \Rightarrow y = 2^{x/2} \Rightarrow \log_2 y = \frac{x}{2}$

$\therefore x = 2\log_2 y$

$\Rightarrow f^{-1}(x) = 2\log_2 y$

Domain $(f(x)) = \mathbb{R}$, every value of $x \in \mathbb{R}$, can be used as an input for $2^{x/2}$

Range $(f(x)) = \mathbb{R}^+$, only positive values can be reached by $2^{x/2}$ and $\lim_{x\to-\infty} 2^{x/2} = 0$

Domain $(f^{-1}(x)) = \mathbb{R}^+$, $2\log_2 x$ is only defined for positive x

Range $(f^{-1}(x)) = \mathbb{R}$, all real numbers can be an output for $2\log_2 x$

③ (a) $\lim_{x\to1} x^2 + 2x - 2 = 1^2 + (2\times1) - 2 = 1$

(b) $\lim_{s\to0} \dfrac{s^3}{s} = \lim_{s\to0} s^2 = 0$

(c) $\lim_{t\to4} \dfrac{t^2-16}{t-4} = \lim_{t\to4} \dfrac{\cancel{(t-4)}(t+4)}{\cancel{(t-4)}} = \lim_{t\to4} (t+4)$

$= 4+4 = 8$

(d) $\lim_{v\to2} \dfrac{2-v}{\frac{1}{2}-\frac{1}{v}} = \lim_{v\to2} \dfrac{2-v}{\frac{v-2}{2v}} = \lim_{v\to2} (-2v)\dfrac{\cancel{(2-v)}}{\cancel{(2-v)}}$

$= \lim_{v\to2} (-2v) = -2\times2 = -4$

(e) $\lim_{y\to0} \dfrac{\sqrt{2-y}-\sqrt{2+y}}{-4y} = -\dfrac{1}{4}\lim_{y\to0} \dfrac{\sqrt{2-y}-\sqrt{2+y}}{y}$

$= -\dfrac{1}{4}\lim_{y\to0} \dfrac{2-y-2-y}{y\left(\sqrt{2-y}+\sqrt{2+y}\right)} = -\dfrac{1}{4}\lim_{y\to0} \dfrac{-2y}{y\left(\sqrt{2-y}+\sqrt{2+y}\right)}$

$= \dfrac{2}{4}\lim_{y\to0} \dfrac{1}{\sqrt{2-y}+\sqrt{2+y}} = \dfrac{1}{2}\left(\dfrac{1}{\sqrt{2}+\sqrt{2}}\right) = \dfrac{1}{4\sqrt{2}}$