Homework 1

Problem 1

(2+3+5 points)

a) Find the (complex) roots of the polynomial $p(x) = 2x^2 + 12x + 26$.

b) Find the values of parameter b for which the equation $bx^2 - bx + 2 = 0$ has no real roots.

c) Find all roots (real or complex) of the polynomial $p(x) = x^6 - x^5 - 3x^4 - 3x^3 - 22x^2 + 4x + 24$
.
Hint: $x = 3$ is a root. Divide out the associated linear factor and continue with more roots that are easy to guess.

Problem 2

(3+3+4 points and (2+3) bonus points)
Assuming that $z = a + ib$ is a complex number, compute real and imaginary parts of

a) $\frac{1}{(z^*)^2}$

b) $\frac{2+z}{2z+2}$

c) $(z^*)^2 z$

Bonus: $|x|$ is the absolute value function:
In the case of $x \in \mathbb{C}$: $|x| = \sqrt{xx^*}$
In the case of $x \in \mathbb{R}$: $|x| = \sqrt{x^2}$ or in other words $|x| = x$ if $x \geq 0$; $|x| = -x$ if $x < 0$.
In both cases $|x| \in \mathbb{R}$ and $|x| \geq 0$.

d) Compute $|\frac{1-i}{2+i}|$. Use the definition of the absolute value function for complex numbers.

d) Characterize the set of real numbers x that satisfy $|4x + 2| \leq |2x - 3|$.
Hint: You cannot directly work with $|\ |$. Use the definition of absolute value for real numbers to change the inequality into an equivalent problem <u>without</u> $|\ |$. For that, you can apply certain functions to both sides of the inequality without changing the inequality.

Problem 3

(4+3+3 points)
Proof the following for complex numbers z and w, i.e. $z, w \in \mathbb{C}$.

a) $\frac{z^*}{w^*} = (\frac{z}{w})^*$

b) $\mathrm{Re}(z) = \frac{z+z^*}{2}$

c) $\mathrm{Im}(z) = \frac{z-z^*}{2i}$

$\mathrm{Re}(z)$ and $\mathrm{Im}(z)$ are the real and complex part of z, respectively. I.e. if $z = a + bi$, then $\mathrm{Re}(z) = a$ and $\mathrm{Im}(z) = b$.