Introduction to Computer Science
Jacobs University Bremen
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Module: CH-232
Date: 2021-10-22
Due: 2021-10-29

ICS 2021 Problem Sheet #7

Problem 7.1: *quine-mccluskey algorithm* (2+4+3+1 = 10 points)

Consider integer numbers in the range 0...63 that can be represented using six bits. The boolean function $F(X_5, X_4, X_3, X_2, X_1, X_0)$ is true when the number $(X_5X_4X_3X_2X_1X_0)_2$ is a Fibonacci number and false otherwise.

a) Provide a boolean expression in DNF defining the function F. What is the cost of the DNF expression?

b) Calculate the prime implicants of F.

c) Construct the prime implicant chart and identify the essential prime implicants. What is a minimal set of prime implicants covering the function F?

d) Write out a minimal boolean expression defining F using mathematical logic notation. What is the cost of the minimal boolean expression?

For calculating the cost of a boolean expression, we only consider logical $\wedge$ and $\vee$ operations.

Solution:

a) The Fibonacci numbers in the range 0...63 are the numbers $0, 1, 2, 3, 5, 8, 13, 21, 34, 55$. Hence, the Boolean function F is defined by the following DNF expression:

$$F = m_0 + m_1 + m_2 + m_3 + m_5 + m_8 + m_{13} + m_{21} + m_{34} + m_{55}$$

The cost of the expression is $10 \cdot 5 + 9 = 59$ logical $\wedge$ and $\vee$ operations.

b) Prime implicants:

minterm	pattern	used	minterms	pattern	used	minterms	pattern	used
m_0	000000	✓	$m_{0,1}$	00000-	✓	$m_{0,1,2,3}$	0000–	
			$m_{0,2}$	0000-0	✓			
			$m_{0,8}$	00-000				
m_1	000001	✓	$m_{1,3}$	0000-1	✓			
			$m_{1,5}$	000-01				
m_2	000010	✓	$m_{2,3}$	00001-	✓			
			$m_{2,34}$	-00010				
m_8	001000	✓						
m_3	000011	✓						
m_5	000101	✓	$m_{5,13}$	00-101				
			$m_{5,21}$	0-0101				
m_{34}	100010	✓						
m_{13}	001101	✓						
m_{21}	010101	✓						
m_{55}	110111							

The set of prime implicants is: $m_{0,1,2,3}, m_{0,8}, m_{1,5}, m_{2,34}, m_{5,13}, m_{5,21}, m_{55}$

c) Prime implicant chart:

	m_0	m_1	m_2	m_3	m_5	m_8	m_{13}	m_{21}	m_{34}	m_{55}	
$m_{0,1,2,3}$	✓	✓	✓	✓							(essential for m_3)
$m_{0,8}$	✓					✓					(essential for m_8)
$m_{1,5}$		✓			✓						
$m_{2,34}$			✓						✓		(essential for m_{34})
$m_{5,13}$					✓		✓				(essential for m_{13})
$m_{5,21}$					✓			✓			(essential for m_{21})
m_{55}										✓	(essential for m_{55})
covered	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	

The set of essential prime implicants is: $m_{0,1,2,3}, m_{0,8}, m_{2,34}, m_{5,13}, m_{5,21}, m_{55}$ In this case, the essential prime implicants are sufficient to cover all minterms of the function F.

d) We obtain:

$$\begin{aligned} F(X_5, X_4, X_3, X_2, X_1, X_0) = &(\neg X_5 \wedge \neg X_4 \wedge \neg X_3 \wedge \neg X_2) \vee \\ &(\neg X_5 \wedge \neg X_4 \wedge \neg X_2 \wedge \neg X_1 \wedge \neg X_0) \vee \\ &(\neg X_4 \wedge \neg X_3 \wedge \neg X_2 \wedge X_1 \wedge \neg X_0) \vee \\ &(\neg X_5 \wedge \neg X_4 \wedge X_2 \wedge \neg X_1 \wedge X_0) \vee \\ &(\neg X_5 \wedge \neg X_3 \wedge X_2 \wedge \neg X_1 \wedge X_0) \vee \\ &(X_5 \wedge X_4 \wedge \neg X_3 \wedge X_2 \wedge X_1 \wedge X_0) \end{aligned}$$

The cost of the minimal expression is $(3+4+4+4+4+5)+5 = 29$ logical $\wedge$ and $\vee$ operations.

Marking:

a)
- *1pt for the correct DNF expression*
- *1pt for the correct cost*

b)
- *1pt for correctly sorting and classifying the minterms*
- *2pt for obtaining the combined minterms in the first iteration*
- *1pt for obtaining the combined minterm in the second iteration*

c)
- *0.5pt for each correctly derived essential prime implicant*
- *-0.5pt if the non-essential prime implicant is marked as essential*
- *-0.5pt if the obtained coverage is wrong*

d)
- *0.5pt for writing out the function correctly*
- *0.5pt for the correct cost*