15 sept 2021 Calculus and linear Algebra. Wednesday.

Homework 2.

Problem 1

a) $-3x + 2y = 2$

$2y = 2 + 3x.$

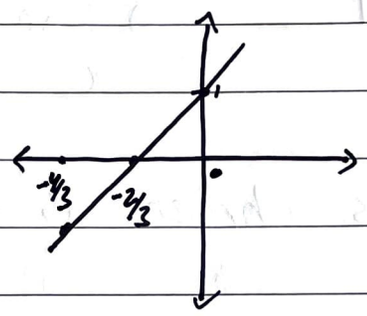
$y = \frac{3}{2}x + \frac{2}{2}$

$y = \frac{3}{2}x + 1$

y	-1	0	1
x	$-\frac{4}{3}$	$-\frac{2}{3}$	0

$-1 = \frac{3}{2}x + 1.$

$-4 = 3x. \quad x = -\frac{4}{3}.$

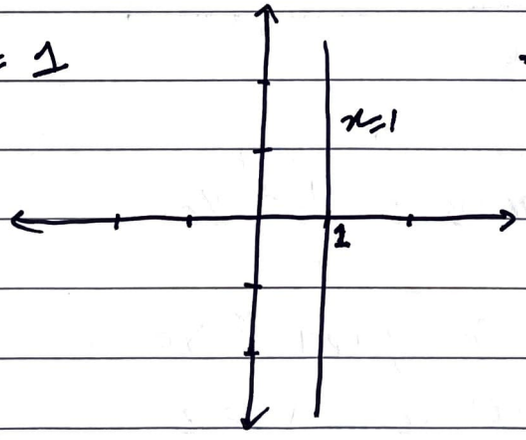


* It is ag graph of a function $y = f(x)$

domain $\Rightarrow \in \mathbb{R}$

Range $\Rightarrow \in \mathbb{R}$

b) $x = 1$



$\Rightarrow$ it is not a graph of a function $y = f(x)$.

c) $(x+2)^2 = 2(2y+1)$.

$x^2 + 4x + 4 = 4y + 2$.

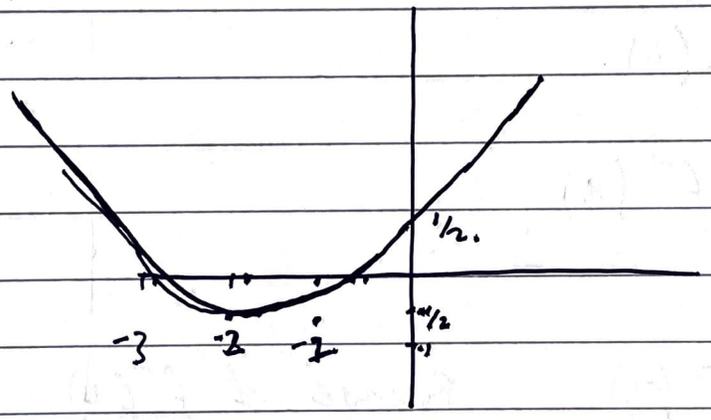
$y = \dfrac{x^2 + 4x + 4 - 2}{4}$.

$= \frac{1}{4}x^2 + x + \frac{1}{2}$.

$\frac{dy}{dx} = \frac{1}{2}x + 1 = 0 \rightarrow x = -2$. (Turning point).

$y = -\frac{1}{2}$.

y ~~$-3 \quad -2 \quad -1 \quad 0 \quad 1$~~



$\Rightarrow$ This ~~graph~~ is a graph of a function $y = f(x)$.

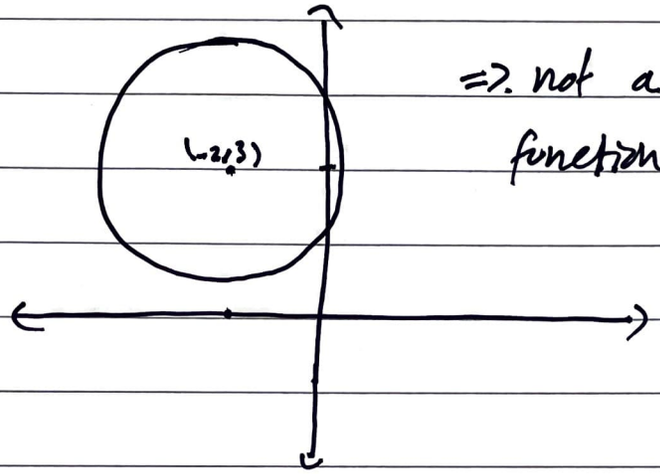
Domain $x \in \mathbb{R}$

Range $y \in [-\frac{1}{2}, +\infty)$

Vertex $(-2, -\frac{1}{2})$

d) $(x+2)^2 + (y-3)^2 = 6$.

It is a circle with radius $r = \sqrt{6}$
and centre $(-2, 3)$



$\Rightarrow$ not a graph of a function $y = f(x)$.

e) part a and part c have graphs of function $y = f(x)$ with domains $x \in \mathbb{R}$ and Range $y \in \mathbb{R}$ and $y \in [-\frac{1}{2}, +\infty)$ respectively.

f) Part a and part b have graph of function $x = g(y)$

part a:

$x = \frac{2}{3}y - \frac{2}{3}$

domain $x \in \mathbb{R}$

Rang $y \in \mathbb{R}$

part b:

$x = 1$

domain $g(y) = 1$

Range $-\infty < x < \infty$

Problem 2

$f(x) = 2^{x/2}$ $\quad f^{-1}(x) = ?$

$f(x) = y.$

$y = 2^{x/2}$ Switching variables.

$x = 2^{y/2}$

$\ln(x) = \frac{y}{2}\ln(2).$

$y = \frac{2\ln(x)}{\ln(2)} = f^{-1}(x)$

domain of $f(x)$
$= -\infty < x < \infty$

Range of $f(x)$

$0 < f(x) < \infty$

domain of $f^{-1}(x)$ $\quad$ Range of $f^{-1}(x)$

$\Rightarrow \quad x > 0$ $\quad\quad$ $\Rightarrow \quad -\infty < f(x) < \infty$

Sorry for not writing $\lim_{\square \to \square}$ with every step.

Problem 3

a) $\lim_{x \to 1} \; x^2 + 2x - 2$

$= 1 + 2 - 2$

$= 1$

b) $\lim_{s \to 0} \; \frac{s^3}{s^2}$

$= s^2$

$= 0$

c) $\lim_{t \to 4} \; \frac{t^2 - 16}{t - 4}$

$\frac{(t)^2 - (4)^2}{t-4}$ using $a^2 - b^2 = (a+b)(a-b)$

$= \frac{(t-4)(t+4)}{(t-4)}$

$\lim_{t \to 4} \; t + 4$

$= 4 + 4 = 8$

d) $\lim_{v \to 2} \; \frac{2 - v}{\frac{1}{2} - \frac{1}{v}} = \frac{2-v}{\frac{v-2}{2v}} = \frac{(2-v)}{\frac{-(2-v)}{2v}} = -2v.$

$\lim_{v \to 2} \; -2v$

$= -4$

e) $\lim_{y \to 0} \dfrac{\sqrt{2-y} - \sqrt{2+y}}{-4y}$

$$\frac{+\left(\sqrt{2+y} - \sqrt{2-y}\right)}{+4y}$$

$$= \frac{\sqrt{2+y} - \sqrt{2-y}}{4y} \times \frac{\sqrt{2+y} + \sqrt{2-y}}{\sqrt{2+y} + \sqrt{2-y}}$$

$$= \frac{\left(\sqrt{2+y} - \sqrt{2-y}\right)\left(\sqrt{2+y} + \sqrt{2-y}\right)}{4y\left(\sqrt{2+y} + \sqrt{2-y}\right)}$$

$$= \frac{2+y-(2-y)}{4y\left(\sqrt{2+y} + \sqrt{2-y}\right)}$$

$$= \frac{2y}{2 \cdot 4y\left(\sqrt{2+y} + \sqrt{2-y}\right)}$$

$$\lim_{y \to 0} \frac{1}{2} \frac{1}{\sqrt{2+y} + \sqrt{2-y}}$$

$$= \frac{1}{2} \frac{1}{\sqrt{2} + \sqrt{2}}$$

$$= \frac{1}{2} \frac{1}{2\sqrt{2}} = \frac{1}{4\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{8}$$