Wednesday

H.W #3

Introduction to Comp. Science

29th Sep 20

Problem 3.1

a) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

let (a,b) be an arbitrary element of $\cancel{(A \cap B) \times (C \cap D)}$ $(A \times B) \cap (C \times D)$

$(a,b) \in (A \times C) \cap (B \times D) \rightarrow$

$\rightarrow (a,b) \in (A \times C)$ & $(B \times D)$

$\rightarrow (a \in A, b \in C)$ & $(a \in B, b \in D)$

$(a \in A, \overset{a}{\cancel{b}} \in B)$ & $(b \in C, b \in D)$

$a \in b \in (A \cap B) \times (C \cap D)$ expression 1 ~~equation~~

let $(p,q) \in (A \cap B) \times (C \cap D)$

$(p,q) \in (A \cap B)$ & $(p,q) \in (C,D)$

$(p \in A, q \in B)$ & $(p \in C, q \in D)$

$(p \in A, p \in B)$ & $(q \in C, q \in D)$

$(A \times C) \cap (B \times D)$ expression 2.

$(A \cap B) \times (C \cap D) \subseteq (A \times C) \cap (B \times D)$

from expression 1 and 2

$(A \times C) \cap (B \times D) = (A \cap B) \times (C \cap D)$

b) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$

let $A = \{1\}$, $B = \{2\}$, $C = \{3\}$, $D = \{4\}$.

$A \cup B = \{1, 2\}$

$C \cup D = \{3, 4\}$.

→ $(A \cup B) \times (C \cup D) = \{(1,3), (1,4), (2,3), (2,4)\}$.

$A \times C = \{1, 3\}$, $B \times D = (2, 4)$

→ ~~AB~~ $(A \times C) \cup (B \times D) = \{(1,3), (2,4)\}$.

$(A \cup B) \times (C \cup D)$ is not equal to $(A \times C) \cup (B \times D)$.

Hence this equality is wrong.

Problem 3.2

a) $R = \{(a,b) \mid a,b \in \mathbb{Z} \land |a-b| \leq 3\}$

Reflective

let "a" be any integer number then
$|a-a| \leq 3$ shows that $(a,a) \in R \ \forall a \in \mathbb{Z}$

Symmitric

let (a,b) be any integer then

$(10-7) \leq 3$
$3 \leq 3 \rightarrow$ ✓

$|10-2| \leq 3$
$8 \leq 3 \rightarrow$ X So it's not symmitric

Transitive

let (a,b,c) be any intege then

$a=9, \ b=6, \ c=3$
$|9-6| \leq 3$
$3 \leq 3$

$|6-3| \leq 3$
$3 \leq 3$

$|9-3| \leq 3$
$6 \leq 3$

not transitive.

Reflective

b) $R = \{(a,b) \mid a,b \in \mathbb{Z} \wedge (a \bmod 10) = (b \bmod 10)\}$

let a be any integer.

$a \bmod 10 = a \bmod 10$

hence $(a,a) \in R \ \forall a\ \mathbb{Z}$

reflective ✓

Symmitric

let a, b be any integer.

$a \bmod 10 = b \bmod 10$

$b \bmod 10 = a \bmod 10$

Symmitric ✓

Transitive

let a, b, c be any integer.

$a \bmod 10 = b \bmod 10$

$b \bmod 10 = c \bmod 10$

$c \bmod 10 = a \bmod 10$

transitive ✓

Problem 3.3

a) $f: \mathbb{N} \longrightarrow \mathbb{N}$ with $f(x) = 2x^2$

$$f(x) = f(y)$$
$$2x^2 = 2y^2$$
$$x^2 = y^2$$
$$|x| = |y| \quad \text{for } x, y \in \mathbb{N}$$

~~Since~~ $x = y$, x, y arbitrary $\forall_{x,y} \in \mathbb{N}$

this show's it is injective.

b) $f: \mathbb{R} \rightarrow \mathbb{R}$ with $f(2) = x^2 + 6$

Problem 3.4

a) $f \cdot g$

$f(x) = x+1$; $2x+1$

$g(x) = 2x$ $\quad f \cdot g = 2x+1$

b) $f \cdot h$

$(x)^2 + 1$

$= x^2 + 1$

c) ~~g·h~~ $g \cdot f$

$= 2(x+1)$

$= 2x+2$

d $g \cdot h$

$2(x^2)$

$= 2x^2$

e $h \cdot f$

$(x+1)^2$

$= x^2 + 2x + 2$

f) $h \cdot g$

$(2x)^2$

$= 4x^2$

g) $f(g \cdot h)$

$2(x^2) + 1$

$= 2x^2 + 1$

h) $h(g \cdot f)$

$(2x+2)^2$

$= 4x^2 + 8x + 2.$

Problem 3.5

a) $\{x \mid x \in \{1 \dots a\}, a \bmod x == 0\}$ $a = 210$