CH-231-A

Algorithms and Data Structures

ADS

Lecture 33

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Spring 2022

Graph Representations: Directed and Undirected Graphs

Definition:

- A directed graph (digraph) $G = (V, E)$ is an ordered pair consisting of
    - a set V of vertices and
    - a set $E \subset V \times V$ of edges.
- In an undirected graph $G = (V, E)$, the edge set E consists of unordered pairs of vertices.

Number of Edges and Vertices

- In a graph, the number of edges is bound by $|E| = O(|V|^2)$.
- If G is connected, then $|E| \geq |V| - 1$.
- Hence, for a connected graph we get $\lg |E| = \Theta(\lg |V|)$.

Adjacency Matrices
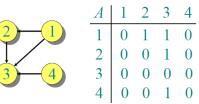
Definition:

The adjacency matrix of a graph $G = (V, E)$ with $V = \{1, ..., n\}$ is the $n \times n$ matrix A given by

$$A[i, j] = \left\{ \begin{array}{ll} 1 & \text{if } (i, j) \in \text{E}, \\ 0 & \text{if } (i, j) \notin \text{E}. \end{array} \right.$$

Dense representation: Storage requirements are $\Theta(|V|^2)$.

Example:

A	1	2	3	4
1	0	1	1	0
2	0	0	1	0
3	0	0	0	0
4	0	0	1	0

Adjacency List

Definition:

An adjacency list of a vertex $v \in V$ is the list $Adj[v]$ of vertices adjacent to v.

Example:

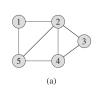$Adj[1] = \{2, 3\}$
$Adj[2] = \{3\}$
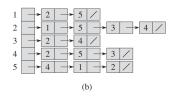$Adj[3] = \{\}$
$Adj[4] = \{3\}$
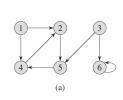
Sparse representation:

▶ Storage requirements for $Adj[v]$ is $\Theta(|\text{outgoing edges from } v|)$.

▶ Storage requirement for $Adj[v]$ for all $v \in V$ is $\Theta(|E|)$.
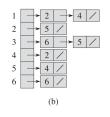
▶ Overall storage requirement is $\Theta(|V| + |E|)$.
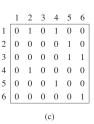
Examples for Undirected & Directed Graphs

	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

(a)　　　　　　(b)　　　　　　(c)

	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

(a)　　　　　　(b)　　　　　　(c)

Application Example: Neighboring States