Constructor University
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Homework 1

Numerical Methods
Due: February 17, 2023

Problem 1

(5+5+3+5 points) Let $f(x) = e^{i\omega x}$ with some real number ω.

a) Compute the Taylor series for f around $c = \frac{\pi}{2}$.

b) Use the Taylor series truncated after the n-th term to compute $f(\pi)$ for $n = 1, \ldots, 5$ and a general ω.

c) Compare values calculated in b) with the actual value of $f(\pi)$ for $\omega = 1$ and create a plot for the errors of the real part and imaginary part as a function of n. (Hint: Use Euler's formula)

d) Show that the Taylor series for $f(x) = e^{i\omega x}$ around $c = \frac{\pi}{2}$ converges to f for $x \in [\frac{\pi}{2}, \pi]$.

Problem 2

(5+5+2 points)

a) Compute the Taylor series for $f(x) = \sin(3x^2)$ around $c = 0$. (Hint: compute for $\sin(x)$ then substitute).

b) The Taylor series for $f(x) = \frac{\sqrt{x+1}}{2}$ around $c = 0$ represents the function for $|x| \leq 1$. Show the Taylor expansion for $n = 1$ and the remainder term. Calculate the number of correct digits for $x = 0.0001$ and $x = -0.0001$.

c) Convert the following from one base to another and write down you calculations as an expansion:

i) $(530)_{10}$ to $(\ldots)_2$

ii) $(1.1011)_2$ to $(\ldots)_8$