CH-231-A

Algorithms and Data Structures

ADS

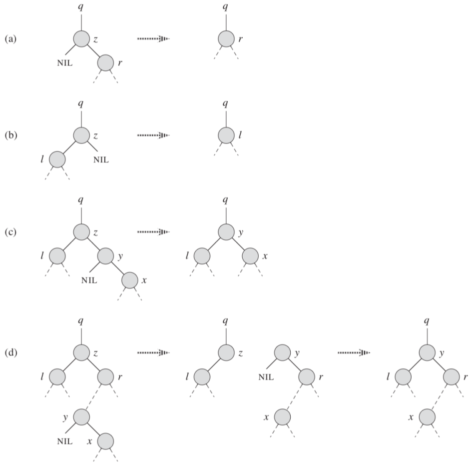
Lecture 26

Dr. Kinga Lipskoch

Spring 2022

Deletion (Remember BST)

```
TREE-DELETE(T, z)
if z.left == NIL
    TRANSPLANT(T, z, z.right)
elseif z.right == NIL
    TRANSPLANT(T, z, z.left)
else y = TREE-MINIMUM(z.right)
    if y.p ≠ z
        TRANSPLANT(T, y, y.right)
        y.right = z.right
        y.right.p = y
    TRANSPLANT(T, z, y)
    y.left = z.left
    y.left.p = y
```



Deletion (RB) (1)

```
TREE-DELETE(T, z)
if z.left == NIL
    TRANSPLANT(T, z, z.right)
elseif z.right == NIL
    TRANSPLANT(T, z, z.left)
else y = TREE-MINIMUM(z.right)
    if y.p ≠ z
        TRANSPLANT(T, y, y.right)
        y.right = z.right
        y.right.p = y
    TRANSPLANT(T, z, y)
    y.left = z.left
    y.left.p = y
```

```
RB-DELETE(T, z)
y = z
y-original-color = y.color
if z.left == T.nil
    x = z.right
    RB-TRANSPLANT(T, z, z.right)
elseif z.right == T.nil
    x = z.left
    RB-TRANSPLANT(T, z, z.left)
else y = TREE-MINIMUM(z.right)
    y-original-color = y.color
    x = y.right
    if y.p == z
        x.p = y
    else RB-TRANSPLANT(T, y, y.right)
        y.right = z.right
        y.right.p = y
    RB-TRANSPLANT(T, z, y)
    y.left = z.left
    y.left.p = y
    y.color = z.color
if y-original-color == BLACK
    RB-DELETE-FIXUP(T, x)
```

Deletion (RB) (2)

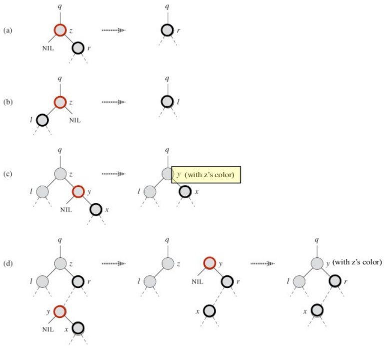
- node y
  - either removed (a/b)
  - or moved in the tree (c/d)
  - y-original-color
- node x
  - the node that moves into y's original position
  - x.p points to y's original parent (since it moves into y's position, note special case in 12/13)

RB-DELETE(*T*, *z*)

```
1  y = z
2  y-original-color = y.color
3  if z.left == T.nil
4      x = z.right
5      RB-TRANSPLANT(T, z, z.right)
6  elseif z.right == T.nil
7      x = z.left
8      RB-TRANSPLANT(T, z, z.left)
9  else y = TREE-MINIMUM(z.right)
10     y-original-color = y.color
11     x = y.right
12     if y.p == z
13         x.p = y
14     else RB-TRANSPLANT(T, y, y.right)
15         y.right = z.right
16         y.right.p = y
17     RB-TRANSPLANT(T, z, y)
18     y.left = z.left
19     y.left.p = y
20     y.color = z.color
21 if y-original-color == BLACK
22     RB-DELETE-FIXUP(T, x)
```

Deletion (RB) (3)

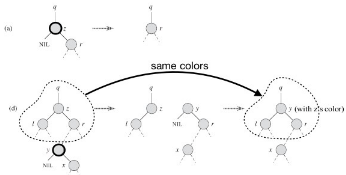
- y-original-color == red




```
RB-DELETE(T, z)
y = z
y-original-color = y.color
if z.left == T.nil
    x = z.right
    RB-TRANSPLANT(T, z, z.right)
elseif z.right == T.nil
    x = z.left
    RB-TRANSPLANT(T, z, z.left)
else y = TREE-MINIMUM(z.right)
    y-original-color = y.color
    x = y.right
    if y.p == z
        x.p = y
    else RB-TRANSPLANT(T, y, y.right)
        y.right = z.right
        y.right.p = y
    RB-TRANSPLANT(T, z, y)
    y.left = z.left
    y.left.p = y
    y.color = z.color
if y-original-color == BLACK
    RB-DELETE-FIXUP(T, x)
```

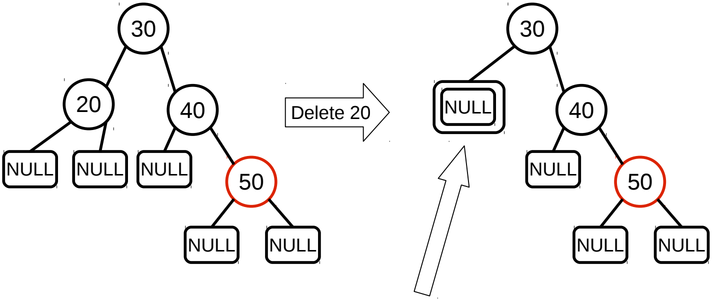
Deletion (RB) (4)

- y-original-color == red
  - no problem
- y-original-color == black
  - violations might occur (2,4,5)
  - main idea to fix
    - x gets an "**extra black**" & needs to get rid of it
  - 4 cases




```
RB-DELETE(T, z)
y = z
y-original-color = y.color
if z.left == T.nil
    x = z.right
    RB-TRANSPLANT(T, z, z.right)
elseif z.right == T.nil
    x = z.left
    RB-TRANSPLANT(T, z, z.left)
else y = TREE-MINIMUM(z.right)
    y-original-color = y.color
    x = y.right
    if y.p == z
        x.p = y
    else RB-TRANSPLANT(T, y, y.right)
        y.right = z.right
        y.right.p = y
    RB-TRANSPLANT(T, z, y)
    y.left = z.left
    y.left.p = y
    y.color = z.color
if y-original-color == BLACK
    RB-DELETE-FIXUP(T, x)
```

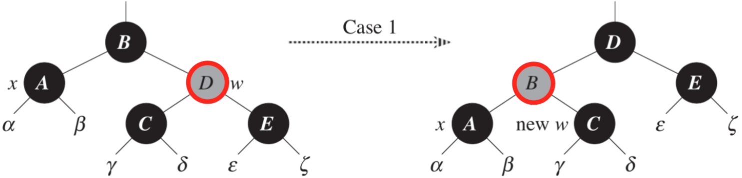
Extra Black or Double Black Node



Child caries „extra black“ information
also called „double black“ node

Fixing Red-Black Tree Properties (1)

Case 1: x's sibling w is red.
Transform to Case 2, 3, or 4 by left rotation and changing colors of nodes B and D.

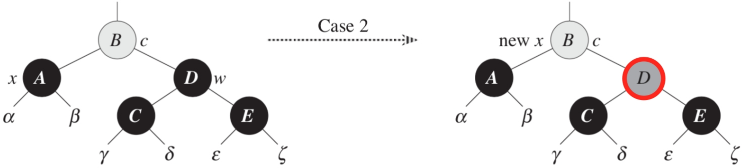


x = node with extra black
w = x's sibling

if $w.color$ == RED
    $w.color$ = BLACK
    $x.p.color$ = RED
    LEFT-ROTATE($T, x.p$)
    $w = x.p.right$

Fixing Red-Black Tree Properties (2)

Case 2: x's sibling w is black and the children of w are black.
Set color of w to red and propagate upwards.

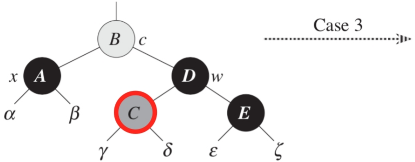


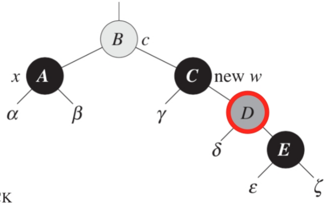
x = node with extra black
w = x's sibling
c = color of the node

if $w.left.color$ == BLACK and $w.right.color$ == BLACK
$\quad w.color = \text{RED}$
$\quad x = x.p$

Fixing Red-Black Tree Properties (3)

Case 3: x's sibling w is black and the left child of w is red, while the right child of w is black.
Transform to Case 4 by right rotation and changing colors of nodes C and D.

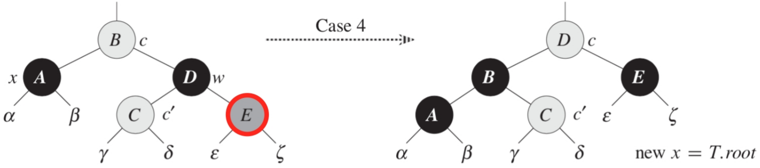




else if $w.right.color$ == BLACK
$w.left.color$ = BLACK
$w.color$ = RED
RIGHT-ROTATE(T, w)
$w = x.p.right$

Fixing Red-Black Tree Properties (4)

Case 4: x's sibling w is black and the right child of w is red. Perform a left-rotate and change colors of B, D, and E. Then, the loop terminates.



$w.color = x.p.color$
$x.p.color = \text{BLACK}$
$w.right.color = \text{BLACK}$
$\text{LEFT-ROTATE}(T, x.p)$

Fixing Red-Black Tree Properties (5)

```
RB-DELETE-FIXUP(T, x)
while x ≠ T.root and x.color == BLACK
    if x == x.p.left
        w = x.p.right
        if w.color == RED
            w.color = BLACK                                  // case 1
            x.p.color = RED                                  // case 1
            LEFT-ROTATE(T, x.p)                              // case 1
            w = x.p.right                                    // case 1
        if w.left.color == BLACK and w.right.color == BLACK
            w.color = RED                                    // case 2
            x = x.p                                          // case 2
        else if w.right.color == BLACK
                w.left.color = BLACK                         // case 3
                w.color = RED                                // case 3
                RIGHT-ROTATE(T, w)                           // case 3
                w = x.p.right                                // case 3
            w.color = x.p.color                              // case 4
            x.p.color = BLACK                                // case 4
            w.right.color = BLACK                            // case 4
            LEFT-ROTATE(T, x.p)                              // case 4
            x = T.root                                       // case 4
    else (same as then clause with "right" and "left" exchanged)
x.color = BLACK
```

Time complexity: $O(h) = O(\lg n)$

Conclusion

Modifying operations on red-black trees can be executed in $O(\lg n)$ time.