№1

$u = \begin{bmatrix} 1 \\ -3 \end{bmatrix} \quad \vartheta = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$

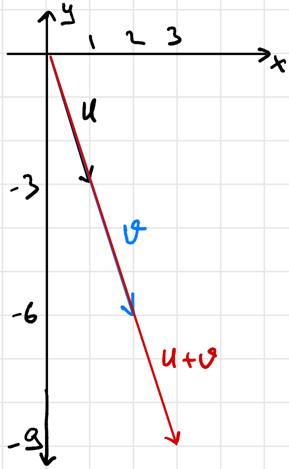
Euclidean Vector Norm $\|x\|_2 = \sqrt{\sum x_i^2}$

p-norm: $\|x\|_p = \sqrt[p]{\sum |x_i|^p}$

a) $\|u\|_2 = \sqrt{1^2 + (-3)^2} = \sqrt{10}$ $\quad \|\vartheta\|_2 = \sqrt{2^2 + (-6)^2} = \sqrt{40}$

b) $u + \vartheta = \begin{bmatrix} 1 \\ -3 \end{bmatrix} + \begin{bmatrix} 2 \\ -6 \end{bmatrix} = \begin{bmatrix} 3 \\ -9 \end{bmatrix}$

c)



d) $\vartheta - u = \begin{bmatrix} 2 \\ -6 \end{bmatrix} - \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

e) $7\vartheta - 5u = 7\begin{bmatrix} 2 \\ -6 \end{bmatrix} - 5\begin{bmatrix} 1 \\ -3 \end{bmatrix} =$

$= \begin{bmatrix} 14 \\ -42 \end{bmatrix} - \begin{bmatrix} 5 \\ -15 \end{bmatrix} = \begin{bmatrix} 9 \\ -27 \end{bmatrix}$

f) $\vartheta \cdot u = |\vartheta| \cdot |u| \cdot \cos 0^\circ =$

$= \sqrt{10} \cdot \sqrt{40} = \sqrt{400} = 20$

g) $\vartheta \times u = |\vartheta| \cdot |u| \cdot \sin 0^\circ = 0$

Note that usually cross product is a vector.

h) independent if

$\alpha\vartheta + \beta u = 0$ has only one solution $\alpha = \beta = 0$.

$\alpha \begin{bmatrix} 2 \\ -6 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ -3 \end{bmatrix} = 0$

$2\alpha + \beta = 0$

$-6\alpha + (-3)\beta = 0 \Rightarrow 2\alpha + \beta = 0$

$\Rightarrow$ it has infinite solutions.

№2.

u in the first quadrant of xy plane $\|u\| = 3$

$\|v\| = 5$ and v lies along positive z-axis. $\Rightarrow$

$$\Rightarrow v = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \qquad u = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \quad \text{where } x, y \geq 0 \text{ and } \sqrt{x^2+y^2} = 3$$

a) $u \times v = (0 \cdot \hat{i} + 0 \cdot \hat{j} + 5\hat{k}) \times$

$\times (x \cdot \hat{i} + y\hat{j} + 0 \cdot \hat{k}) = 5\hat{k} \times (x\hat{i} + y\hat{j}) =$

$$= 5x\hat{j} - 5y\hat{i} = \begin{bmatrix} -5y \\ 5x \\ 0 \end{bmatrix}$$

$\hat{i} \times \hat{j} = \hat{k}$ and $\hat{j} \times \hat{i} = -\hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$ and $\hat{k} \times \hat{j} = -\hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$ and $\hat{i} \times \hat{k} = -\hat{j}$

$\hat{i} \times \hat{i} = 0 \quad \hat{j} \times \hat{j} = 0 \quad \hat{k} \times \hat{k} = 0$

$\|u \times v\| = \sqrt{(-5y)^2 + 5x^2} = 5\sqrt{x^2+y^2} =$

$= 5 \cdot 3 = 15$

b) $-5y < 0$ c) $5x > 0$ d) $0 = 0$

№3

$\|u\| = 2\sqrt{2} \qquad \|v\| = 2\sqrt{2} \qquad \|u - v\| = 2\sqrt{2}$

a) $\|u+v\|^2 = (u+v) \cdot (u+v) = \|u\|^2 + 2(u \cdot v) + \|v\|^2$

$\|u-v\|^2 = (u-v) \cdot (u-v) = \|u\|^2 - 2(u \cdot v) + \|v\|^2$

$\|u+v\|^2 + \|u-v\|^2 = 2(\|u\|^2 + \|v\|^2) \Rightarrow$

$\Rightarrow \|u+v\|^2 = 2(8+8) - 8 = 24 \Rightarrow \|u+v\| = \sqrt{24}$

b) $\|u+v\|^2 - \|u-v\|^2 = 4(u \cdot v) = 24 - 8 = 16 \Rightarrow$

$\Rightarrow u \cdot v = 4 \Rightarrow u \cdot v = \|u\| \, \|v\| \cdot \cos\theta$

$$\cos\theta = \frac{u \cdot v}{\|u\| \, \|v\|} = \frac{4}{2\sqrt{2} \cdot 2\sqrt{2}} = \frac{1}{2} \qquad \theta = 60^\circ, 300$$

Matrices

N1

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 2 & -3 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 4 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 4 & -1 \\ 1 & -2 \end{bmatrix}$$

a) $A + B = \begin{bmatrix} 3 & -1 & 0 \\ 2 & -3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ -2 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

b) $A + C =$ Not possible

c) $2C + \frac{3}{2} I_2 = 2\begin{bmatrix} 4 & -1 \\ 1 & -2 \end{bmatrix} + \frac{3}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9.5 & -2 \\ 2 & -2.5 \end{bmatrix}$

N2.

$$A = \begin{bmatrix} 1 & 3 \\ 2 & -2 \\ 3 & 1 \\ -1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 2 & -2 & 2 & 1 \end{bmatrix}$$

a) $A^T = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 3 & -2 & 1 & 0 \end{bmatrix}$

b) $A + B$ not possible

c) $A^T + B = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 3 & -2 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 3 & 0 \\ 2 & -2 & 2 & 1 \end{bmatrix} =$

$= \begin{bmatrix} 2 & 2 & 6 & -1 \\ 5 & -4 & 3 & 1 \end{bmatrix}$

d)

$A \cdot B = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 3 & 0 \\ 2 & -2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+6 & -6 & 3+6 & 3 \\ 2+4 & -4 & 6+4 & 2 \\ 3+2 & -2 & 9+2 & 1 \\ -1+0 & 0 & -3 & 0 \end{bmatrix}$

$= \begin{bmatrix} 7 & -6 & 9 & 3 \\ 6 & -4 & 10 & 2 \\ 5 & -2 & 11 & 1 \\ -1 & 0 & -3 & 0 \end{bmatrix}$

e) $B \cdot A = \begin{bmatrix} 1 & 0 & 3 & 0 \\ 2 & -2 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 6 \\ 3 & 4 \end{bmatrix}$

N3.

$A = \begin{bmatrix} 1 & -1 & 3 \end{bmatrix}$

a) $A^2 = A \cdot A \quad \varnothing$

b) $A \cdot A^T = \begin{bmatrix} 1 & -1 & 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} = 1 + 1 + 9 = 11$

c) $A^T A = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} \times \begin{bmatrix} 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 3 \\ -1 & 1 & -3 \\ 3 & -3 & 9 \end{bmatrix}$