Practice Problems - 02

Practice problems are supposed to help you digest the content of the lecture. It is important that you manage to solve them on your own. Before you write your solutions, you may of course ask questions, and discuss things. In order to prepare for the exam, already now, try to explicitly write down your solutions – clearly and easy to read. Apply definitions properly, and give explanations for what you are doing. That will help you to understand them later when you prepare for the final exam.

I. Rotation Matrices

1) Using rotation matrices, prove that 2D rotation is commutative but 3D rotation is not.

2) Consider a 2D point P, expressed in polar coordinates as [ρ, θ].
Define the rotation matrix M (in Cartesian coordinates) for a rotation by an angle φ about the origin. Use your rotation matrix M to find the new position P' after the rotation. Also express that new position in polar coordinates.

3) Referring to the previous problem, use geometrical considerations to calculate the point P', instead of a rotation matrix.

4) Write a matrix M in 3D that mirrors points about the yz-plane.

5) Construct a 3D matrix to rotate by -30° about the y-axis.

6) Calculate determinant and trace of the following matrix:

$$A = \begin{bmatrix} 3 & -1 \\ 2 & -3 \end{bmatrix}$$

Is this a rotation matrix?

II. Robot Motion – Homogeneous Coordinates

Let's consider a mobile robot that moves in the 2D space.
It starts at (0,0). It will move along the x-axis for 5 units (think of meters). Then it will rotate about the origin by 45°. Finally, it moves by a vector $(-2.5\sqrt{2}, \quad -2.5\sqrt{2})^T$.

Use homogeneous coordinates to calculate the final transformation matrix as a combination of the three individual transformations, and show the robot path in a graph. Did you get a simple result? How come? Interpret the second move!