Robotics

PS04 Solution

1 Problem

Given the planar (2D) robot arm from figure 1 with a rotational joint in the origin of the world frame and a prismatic joint linked to it with the respective variables α (rotation) and l (translation), with $l \in [500, 1000]$.

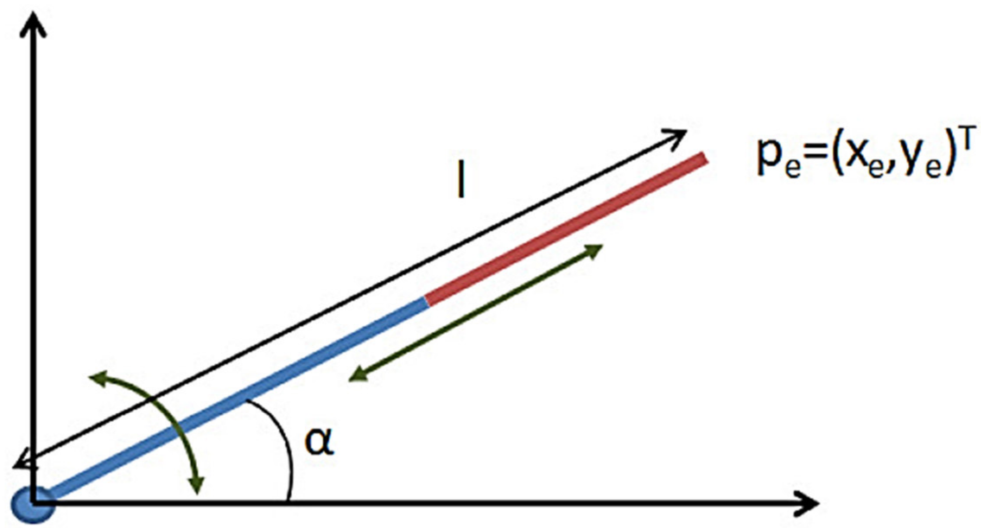


Figure 1: A planar robot arm with a rotational and a prismatic joint.

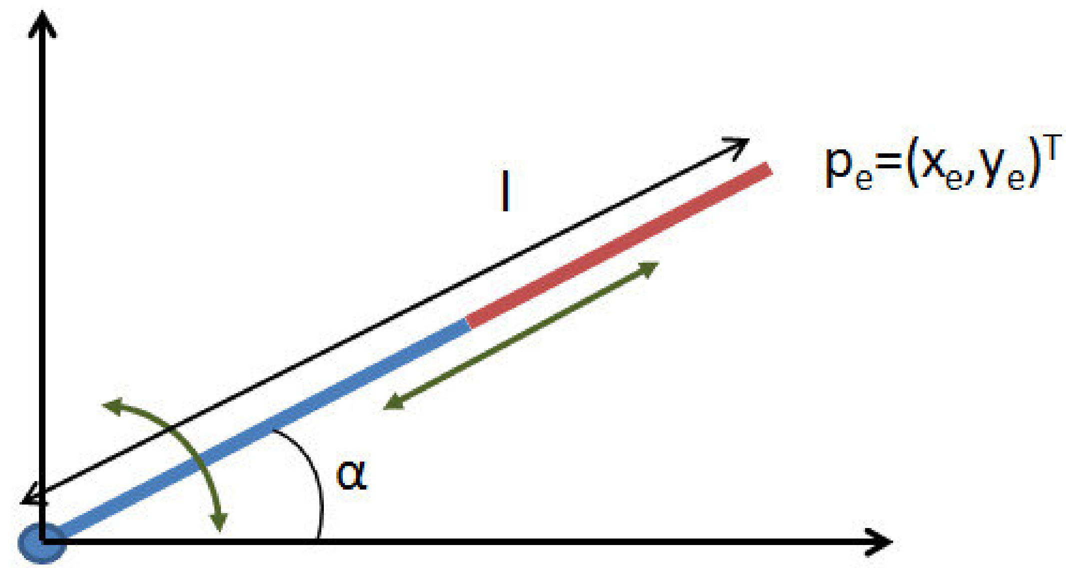
Provide the forward kinematics for the position $p_e = (x_e, y_e)$ of the end-effector of this robot.

Problem 1

$$ {}^{F_2}p_e = {}^{F_1}_{F_2}T(l)\,{}^{F_0}_{F_1}R(\alpha)\,{}^{F_0}o $$

$$ \Rightarrow $$

$$ {}^{F_0}p_e = {}^{F_0}R(\alpha)\,{}^{F_0}T(l)\,{}^{F_0}o $$



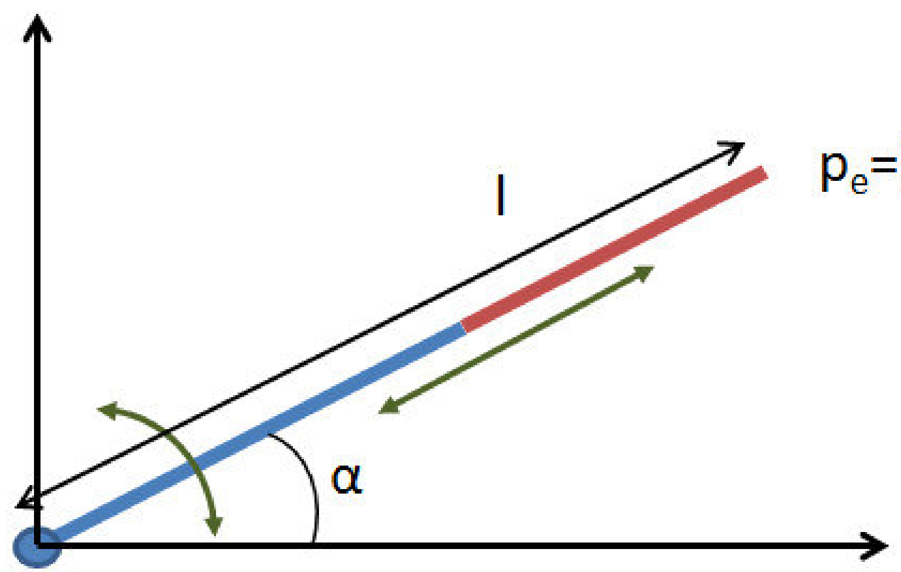
$$ p_e = \begin{pmatrix} x_e \\ y_e \\ 1 \end{pmatrix} = {}^{F_0}_{F_1}R(\alpha)\,{}^{F_1}_{F_2}T(l) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & l \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} $$

Problem 1

$$\begin{pmatrix} x_e \\ y_e \\ 1 \end{pmatrix} = \begin{pmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & l \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} c\alpha & -s\alpha & c\alpha \cdot l \\ s\alpha & c\alpha & s\alpha \cdot l \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} c\alpha \cdot l \\ s\alpha \cdot l \\ 1 \end{pmatrix}$$



$p_e=(x_e,y_e)^T$

2 Problem

Take the robot from the previous problem and find

- the proper Jacobian matrix
- the numerical approximation of the Jacobian at point (2,3) with $\delta = 0.1$

as basis for inverse kinematics.

Problem 2

$$f(\alpha,l)=\begin{pmatrix} x_e \\ y_e \end{pmatrix}=\begin{pmatrix} c\alpha\cdot l \\ s\alpha\cdot l \end{pmatrix}$$

$$Df(\alpha,l)=\begin{pmatrix} \dfrac{\partial c\alpha\cdot l}{\partial\alpha} & \dfrac{\partial c\alpha\cdot l}{\partial l} \\ \dfrac{\partial s\alpha\cdot l}{\partial\alpha} & \dfrac{\partial s\alpha\cdot l}{\partial l} \end{pmatrix}=\begin{pmatrix} -s\alpha\cdot l & c\alpha \\ c\alpha\cdot l & s\alpha \end{pmatrix}$$

Problem 2

Note:

$$\sin'(ax+b) = a\cos(ax+b)$$
$$\cos'(ax+b) = -a\sin(ax+b)$$

$$f(\alpha_1,\alpha_2) = \sin(\alpha_1+\alpha_2)$$
$$\Rightarrow \frac{\partial f(\alpha_1,\alpha_2)}{\partial \alpha_1} = \cos(\alpha_1+\alpha_2)$$
$$[a=1, x=\alpha_1, b=\alpha_2]$$

Problem 2

$$Df(\alpha, l) = \begin{pmatrix} -s\alpha \cdot l & c\alpha \\ c\alpha \cdot l & s\alpha \end{pmatrix} \qquad Df(1,2) = \begin{pmatrix} -1.683 & 0.540 \\ 1.081 & 0.841 \end{pmatrix}$$

note (1,2):
1 - in radians
2 - in *m*
[SI as default]

$$D_{\delta=0.1} f(1,2) = \begin{pmatrix} \dfrac{f_1(1+\delta,2) - f_1(1,2)}{\delta} & \dfrac{f_1(1,2+\delta) - f_1(1,2)}{\delta} \\ \dfrac{f_2(1+\delta,2) - f_2(1,2)}{\delta} & \dfrac{f_2(1,2+\delta) - f_2(1,2)}{\delta} \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{c(1.1)\cdot 2 - c(1)\cdot 2}{0.1} & \dfrac{c(1)\cdot 2.1 - c(1)\cdot 2}{0.1} \\ \dfrac{s(1.1)\cdot 2 - s(1)\cdot 2}{0.1} & \dfrac{s(1)\cdot 2.1 - s(1)\cdot 2}{0.1} \end{pmatrix} = \begin{pmatrix} -1.734 & 0.540 \\ 0.995 & 0.841 \end{pmatrix}$$