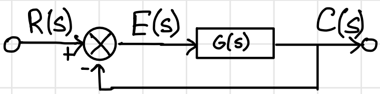
Block Diagrams and Feedback Loops



a) $G(s) = \dfrac{5}{s-2}$ $\qquad T(s) = \dfrac{C(s)}{R(s)}$

$C(s) = E(s) \cdot G(s) \qquad E(s) = R(s) - C(s) \Rightarrow C(s) = [R(s) - C(s)] \cdot G(s)$

$\Rightarrow C(s)[1 + G(s)] = R(s) \cdot G(s) \Rightarrow \dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)}$

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{5/(s-2)}{1+5/(s-2)} = \frac{5}{s-2+5} = \frac{5}{s+3}$$

$R(s) = \dfrac{1}{s} \qquad C(s) = T(s) \cdot R(s) = \dfrac{5}{s+3} \cdot \dfrac{1}{s} = \dfrac{5}{s(s+3)}$

$$E(s) = R(s) - C(s) = \frac{1}{s} - \frac{5}{s(s+3)} = \frac{s+3}{s(s+3)} - \frac{5}{s(s+3)} = \frac{s-2}{s(s+3)}$$

$$C(s) = \frac{5}{s(s+3)} = \frac{5}{3}\left(\frac{1}{s} - \frac{1}{s+3}\right) \longrightarrow c(t) = \frac{5}{3}\left(e^{-0t} - e^{-3t}\right) = \frac{5}{3}\left(1 - e^{-3t}\right)$$

$$E(s) = \frac{s-2}{s(s+3)} = \frac{5}{3}\frac{1}{s+3} - \frac{2}{3}\frac{1}{s} \longrightarrow e(t) = \frac{5}{3}e^{-3t} - \frac{2}{3}e^{-0t} = \frac{5}{3}e^{-3t} - \frac{2}{3}$$

$\lim\limits_{t\to\infty} c(t) = \lim\limits_{t\to\infty} \dfrac{5}{3}\left(1 - e^{-3t}\right) = \dfrac{5}{3} \qquad \lim\limits_{t\to\infty} e(t) = \lim\limits_{t\to\infty}\left(\dfrac{5}{3}e^{-3t} - \dfrac{2}{3}\right) = -\dfrac{2}{3}$

Yes, you can by using Final Value Theorem

$$\lim_{t\to\infty} c(t) = \lim_{s\to 0} s\,C(s) = \lim_{s\to 0} s \cdot \frac{5}{s(s+3)} = \lim_{s\to 0} \frac{5}{s+3} = \frac{5}{3}$$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} s\,\frac{s-2}{s(s+3)} = \lim_{s\to 0} \frac{s-2}{s+3} = \frac{-2}{3}$$

b) For $G(s) = \frac{1}{s+3}$

The steps are the same.

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{1}{s+4}$$

$$C(s) = R(s) \cdot T(s) = \frac{1}{s(s+4)} \qquad c(t) = \frac{1}{4}\left(1 - e^{-4t}\right)$$

$$E(s) = R(s) - C(s) = \frac{s+3}{s(s+4)} \qquad e(t) = \frac{1}{4} e^{-4t} + \frac{3}{4}$$

$$\lim_{t\to\infty} c(t) = \lim_{s\to 0} sC(s) = \frac{1}{4} \qquad \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \frac{3}{4}$$