Tutorial 5
Oct 13

#1 **Def**: Random Variable 'X' has exponential distribution with paramater 'λ' if its pmf is of the form.

$$f_X(t) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0 \\ 0 & \text{Else} \end{cases}$$

Its CDF can be computed as:

$$F_X(t) = \int_0^{\infty} f_X(t)\, dt = \begin{cases} 1 - e^{-\lambda t} & t \geq 0 \\ 0 & \text{Else} \end{cases}$$

#1 **Def**: For RV 'X' taking positive values, we say X to be memoryless if

$$\forall t, s \geq 0: \quad \mathbb{P}[x \geq s + t \mid x \geq s] = \mathbb{P}[x \geq t]$$

Theorem: For non-negative RV 'X', then X has exponential distribution iff it is memoryless.

Exponential => Memoryless

Recall $\mathbb{P}[x \geq t] = 1 - F_X(t) = e^{-\lambda t}$

$$\mathbb{P}[x \geq s + t \mid x \geq s] = \frac{\mathbb{P}[x \geq s + t]}{\mathbb{P}[x \geq s]} = \frac{e^{-\lambda(t+s)}}{e^{-\lambda s}} = e^{-\lambda t}$$

Memoryless => Exponential:

Assume X to be positive, continuous and memoryless.

define $h(t) := \mathbb{P}[h \geq t]$ -- ①

than memoryless conditions becomes $h(t+s) = h(t)h(s)$ --- ②

define $h(1) = a \Rightarrow h(n) = a^n$ (using 2)

Can deduce for any rations $r \in \mathbb{Q}$: $h(r) = a^r$

using continuity of X:

$$\forall t \in \mathbb{R}: \ h(t) = a^t = e^{-\lambda t} \quad \text{with } \lambda = -\log a$$

$\square$

Problem 3[1+3+1 points]. An auditorium has 25 rows. Row 1 has 11 seats, 2 has 12, and 25 has 35 seats. One seat is randomly selected in a randomly selected row.

- What is the probability of selecting seat 15 if it is selected in row 20?
- What is the probability that row 20 is selected given that seat 15 is selected?
- Compare the result with the probability of selecting row 20 without information on the seat selection.

Note that: no. of seats in Row k = $k+10$

1) $\mathbb{P}(S15 \mid R20) = 1/30$

2) $$\mathbb{P}(R20 \mid S15) = \frac{\mathbb{P}(S15 \mid R20)\,\mathbb{P}(R20)}{\sum_{k=5}^{25} \mathbb{P}(S15 \mid Rk)\,\mathbb{P}(Rk)}$$

no seat 15 on rows 1, 2, 3, 4.

$$= \frac{1/30 \cdot 1/25}{\sum_{k=5}^{25} \frac{1}{10+k} \cdot \frac{1}{25}} = \frac{1/30}{\sum_{k=5}^{25} \frac{1}{10+k}}$$

≈ 0.0872

3) $P(R20 \text{ without any information on seat}) = \frac{1}{25} = 0.04$

notice how $0.04 > 0.0372$