Problem 1

(10 points)

a) Compute the following limit $\lim\limits_{\phi \to 0} \dfrac{1 - \cos^2 \phi}{\phi^2}$. **(5 points)**

b) Find the asymptotes of the graph $\frac{3x^2 - 12x + 9}{x^2 - 5}$. **(5 points)**

Problem 2

(10 points)

Find the limit of the following functions using "Squeeze Law".

a) $\lim_{x \to 0} x^n \cos(\frac{1}{x^n})$ where $n \in \mathbb{N} \setminus \{0\}$ **(5 points)**

b) $\lim_{x \to 0} x^2 e^{\sin(\frac{1}{x})}$ **(5 points)**

Problem 3

(10 points)

Show that the equation
$$\cos(x) = e^x + x + 2$$

has at least one solution over $\mathbb{R}$. **(10 points)**

Hint : Intermediate value theorem.

Bonus: Prove that the function
$$f(x) = e^x + x + 2$$

has only one root **(5 points)**