Actuators

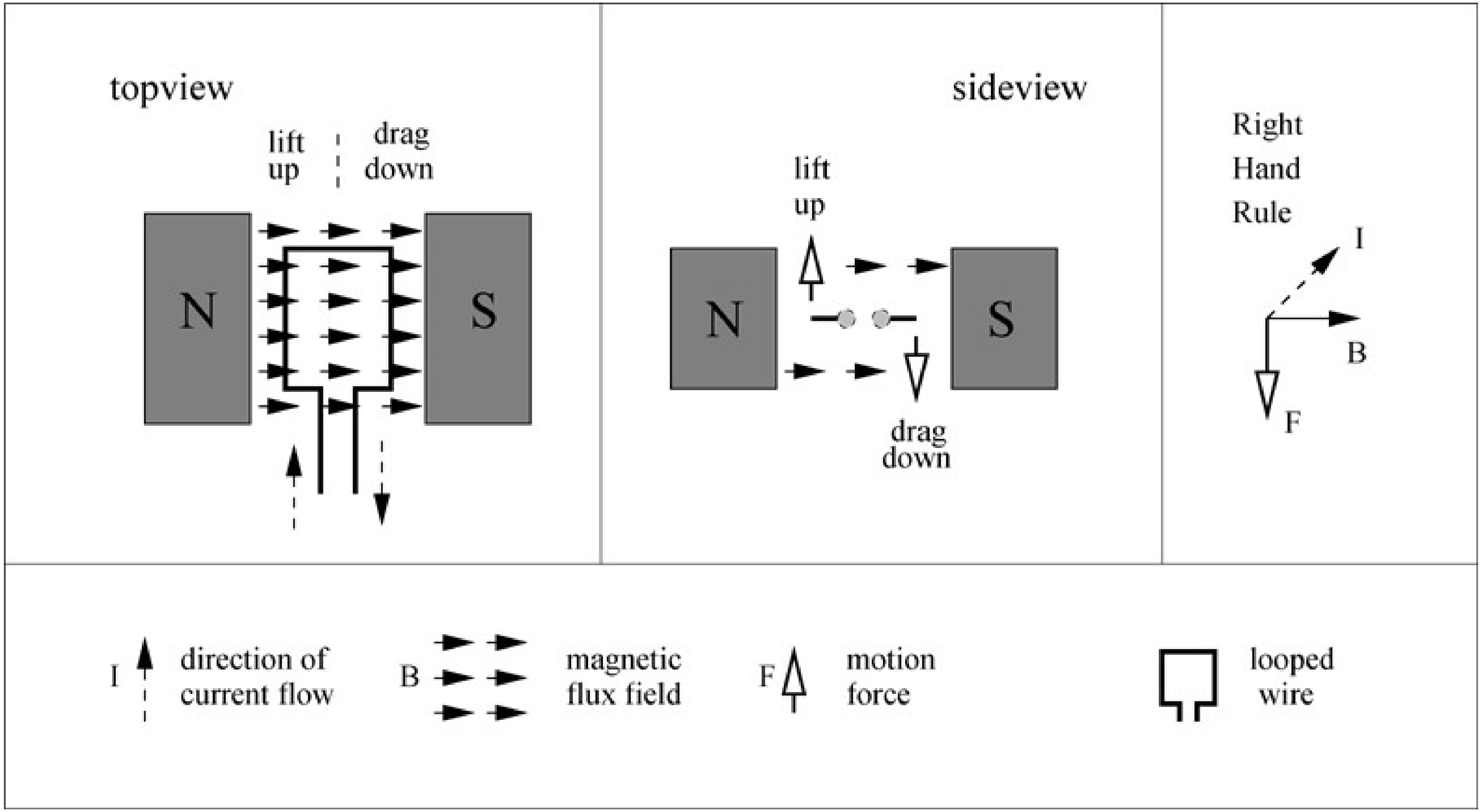
(active joints)

The DC Motor

The Direct Current (DC) Motor

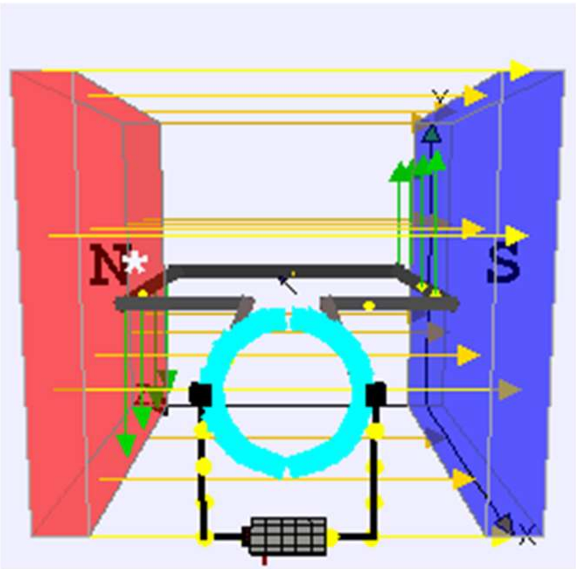
the basic principle:

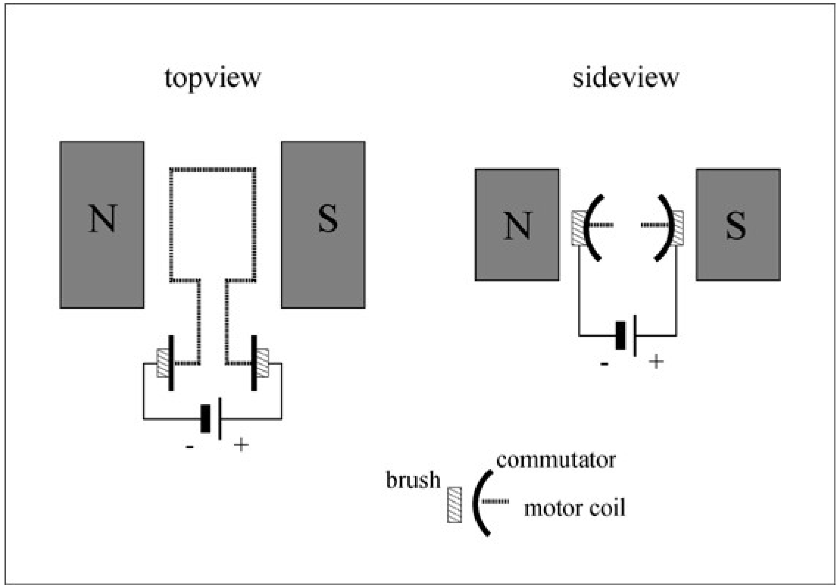
a current flowing through a looped wire in a magnetic field

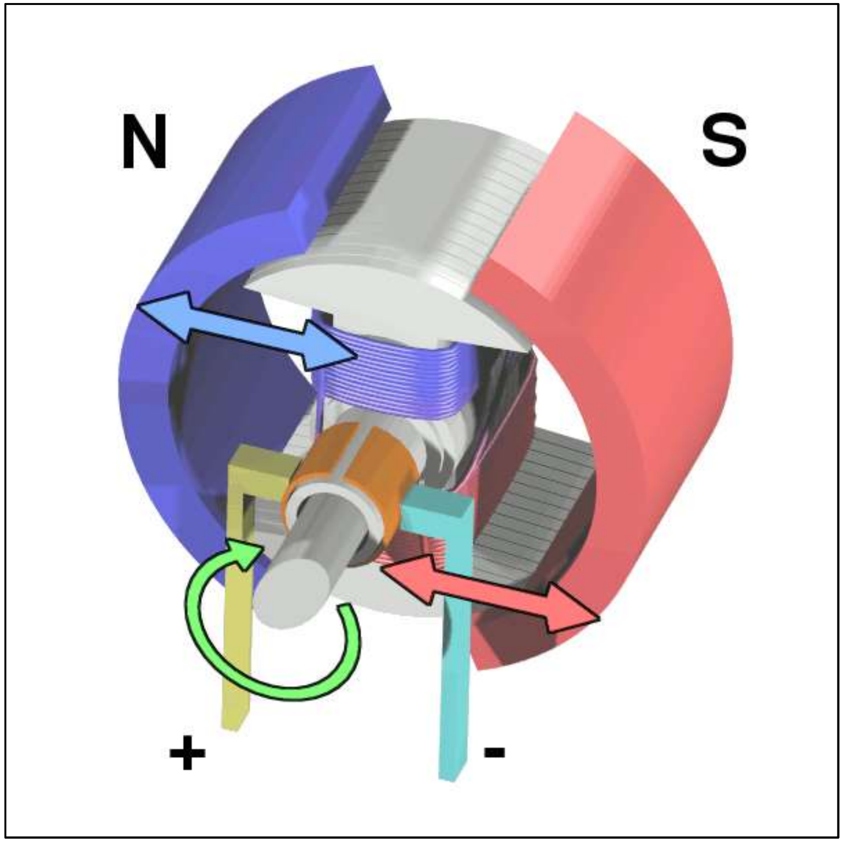


Commutation

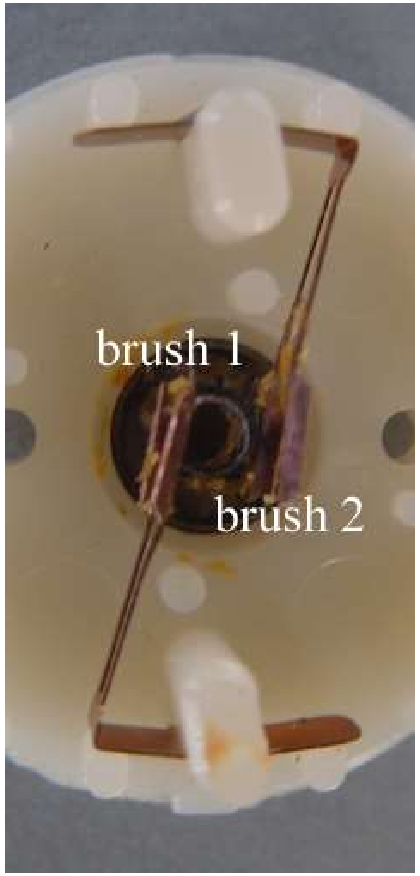
- changes the direction of the current
- mechanical
  - stator
  - rotor with commutator
  - brushes

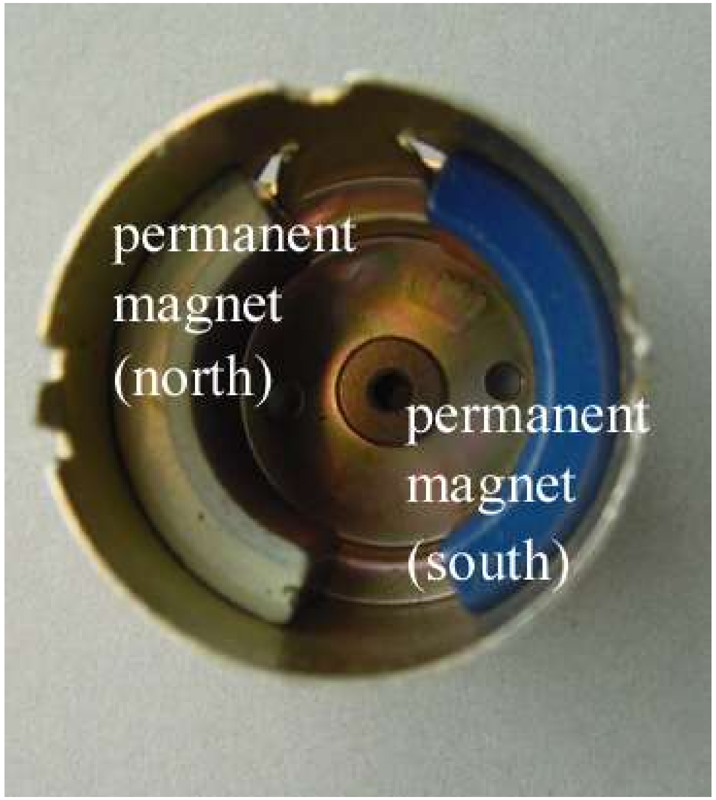




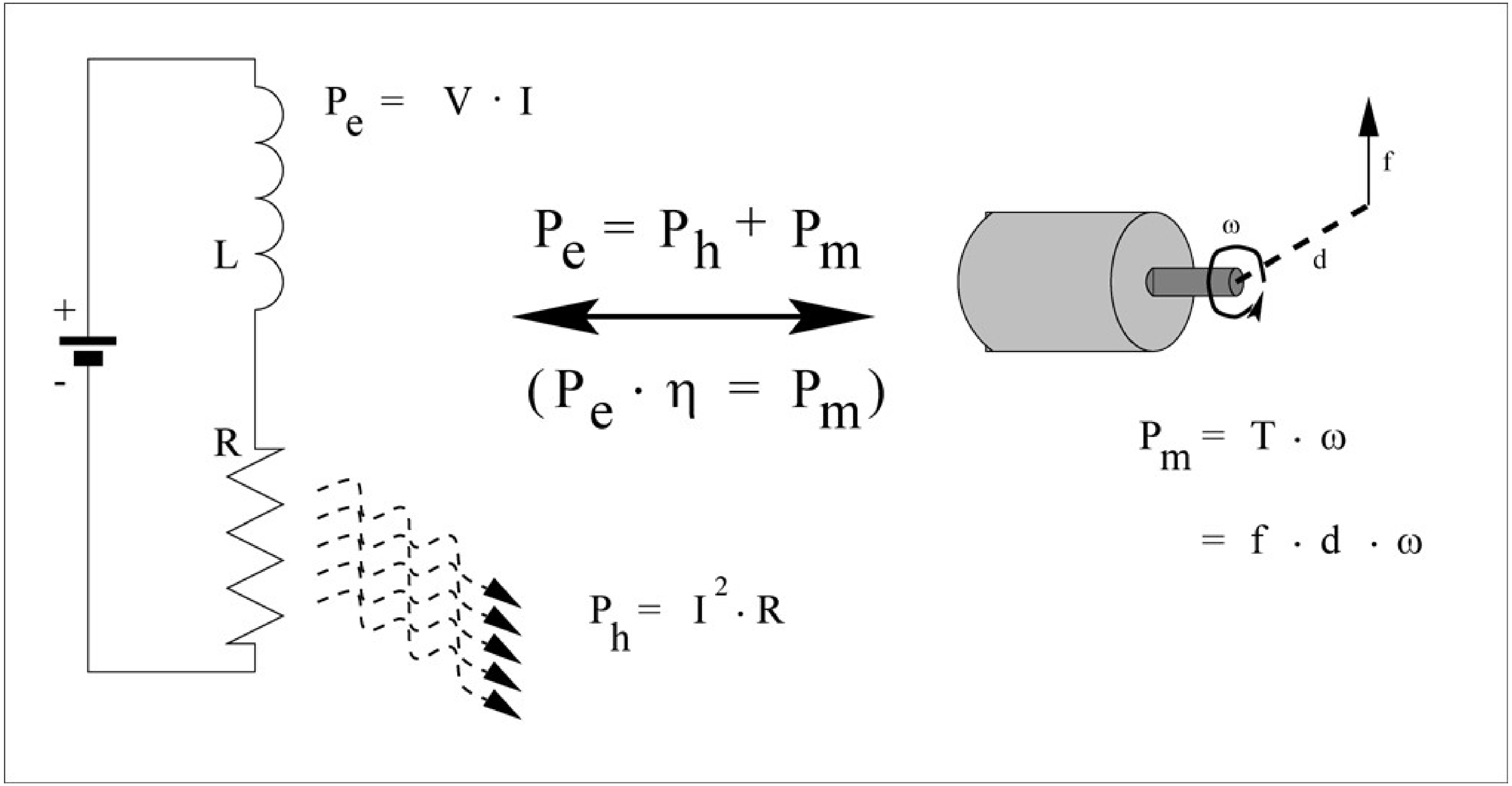


The real thing





DC-motor: Resistor and Inductance



$$
\begin{aligned}
V &= V_{bat} + V_{ind} \\
&= I \cdot R + c' \cdot \omega
\end{aligned}
$$

- induction proportional to change in current, i.e., commutation
- torque proportional to current and magnetic flux

conservation of energy

$$
\begin{aligned}
& P_m &=& \quad P_e - P_h \\
\Longleftrightarrow \quad & T \cdot \omega &=& \quad V \cdot I - I^2 \cdot R \\
\Longleftrightarrow \quad & c'' \cdot I \cdot \omega &=& \quad (I \cdot R + c' \cdot \omega) \cdot I - I^2 \cdot R \\
\Longleftrightarrow \quad & c'' \cdot I \cdot \omega &=& \quad I^2 \cdot R + c' \cdot \omega \cdot I - I^2 \cdot R \\
\Longrightarrow \quad & c'' &=& \quad c'
\end{aligned}
$$

$$
\begin{aligned}
V &= V_{bat} + V_{ind} \\
&= I \cdot R + c' \cdot \omega \\
&= T / c'' \cdot R + c' \cdot \omega
\end{aligned}
$$

- induction proportional to change in current, i.e., commutation
- torque proportional to current and magnetic flux

conservation of energy

$$
\begin{aligned}
& & P_m &= P_e - P_h \\
&\Longleftrightarrow & T \cdot \omega &= V \cdot I - I^2 \cdot R \\
&\Longleftrightarrow & c'' \cdot I \cdot \omega &= (I \cdot R + c' \cdot \omega) \cdot I - I^2 \cdot R \\
&\Longleftrightarrow & c'' \cdot \cancel{I \cdot \omega} &= \cancel{I^2 \cdot R} + c' \cdot \cancel{\omega \cdot I} - \cancel{I^2 \cdot R} \\
&\Longrightarrow & c'' &= c'
\end{aligned}
$$

Maximum Torque and Speed

single constant *c*

$$\omega = -\frac{R}{c^2} \cdot T + \frac{V}{c}$$

$$\begin{aligned} P_m &= \omega \cdot T \\ &= -\frac{R}{c^2} \cdot T^2 + \frac{V}{c} \cdot T \end{aligned}$$

stall torque $\quad \omega = 0 \quad : \quad T_{max} = \frac{c \cdot V}{R}$

no load speed $\quad T = 0 \quad : \quad \omega_{max} = \frac{V}{c}$

Maximum Mechanical Power

$$
\begin{aligned}
& -\frac{2\cdot R}{c^2}\cdot T_{P_m^{max}} + \frac{V}{c} &=& \; 0 \\
\Longleftrightarrow\quad & \frac{2\cdot R}{c^2}\cdot T_{P_m^{max}} &=& \; \frac{V}{c} \\
\Longleftrightarrow\quad & T_{P_m^{max}} &=& \; \frac{1}{2}\cdot\frac{c\cdot V}{R} \\
\Longleftrightarrow\quad & T_{P_m^{max}} &=& \; \frac{1}{2}\cdot T_{max}
\end{aligned}
$$

mechanical power

- as function in *T*
- find zero-value of the 1st derivative
- get the related *T*

$$
\begin{aligned}
& \omega_{P_m^{max}} &=& \; -\frac{R}{c^2}\cdot\frac{1}{2}\cdot\frac{c\cdot V}{R} + \frac{V}{c} \\
\Longleftrightarrow\quad & \omega_{P_m^{max}} &=& \; \frac{V}{c} - \frac{1}{2}\cdot\frac{V}{c} \\
\Longleftrightarrow\quad & \omega_{P_m^{max}} &=& \; \frac{1}{2}\cdot\frac{V}{c} \\
\Longleftrightarrow\quad & \omega_{P_m^{max}} &=& \; \frac{1}{2}\cdot\omega_{max}
\end{aligned}
$$

- use the according *T*
- to find the related *ω*

$$\boldsymbol{P_m^{max} = \frac{1}{4}\cdot T_{max}\cdot \omega_{max}}$$

Maximum Efficiency

current is linear in the torque $I = I_0 + T \cdot \frac{I_S - I_0}{T_{max}}$

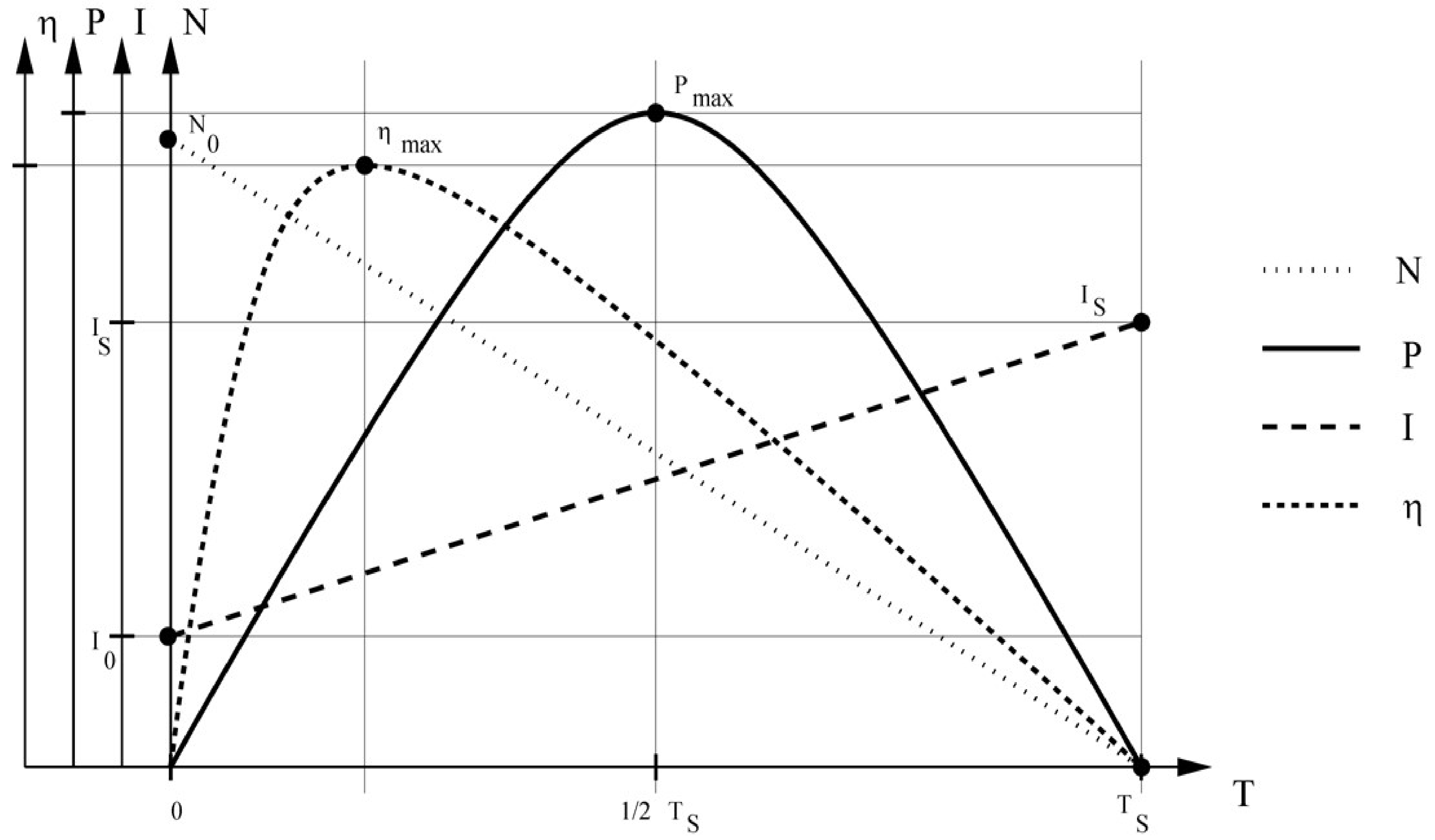
$$\begin{aligned}
\eta &= \frac{P_m}{P_e} \\
\Longleftrightarrow \quad \eta &= \frac{-\frac{R}{c^2} \cdot T^2 + \frac{V}{c} \cdot T}{V \cdot (I_0 + T \cdot \frac{I_S - I_0}{T_{max}})} \\
\Longleftrightarrow \quad \eta &= \frac{\frac{V}{c} \cdot T - \frac{R}{c^2} \cdot T^2}{V \cdot (\frac{I_S - I_0}{T_{max}} \cdot T + I_0)}
\end{aligned}$$

$$\boldsymbol{\eta = \frac{a \cdot T - b \cdot T^2}{c \cdot T + d}}$$

maximum efficiency $\boldsymbol{\eta_{max} = (1 - \sqrt{\frac{I_0}{I_S}})^2}$

Motor Data Graph

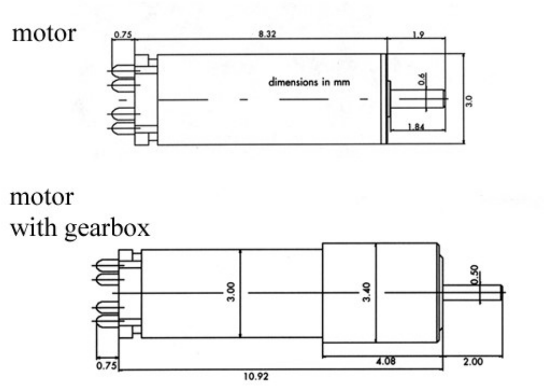
note: the axes are normalized
for illustration purposes



DC-motor Technology

old principle but many recent advances

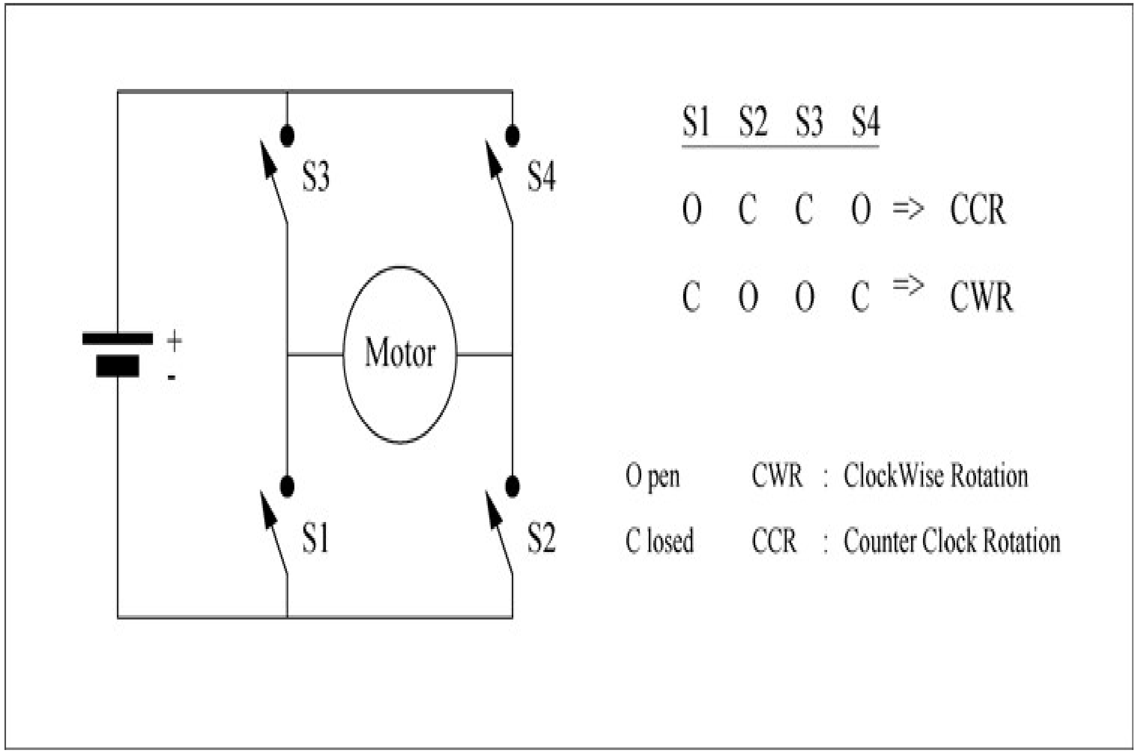
- integration
- efficiency
- combination with electronics
  - mechatronics
  - brushless motors



Direction Control

H-Bridge

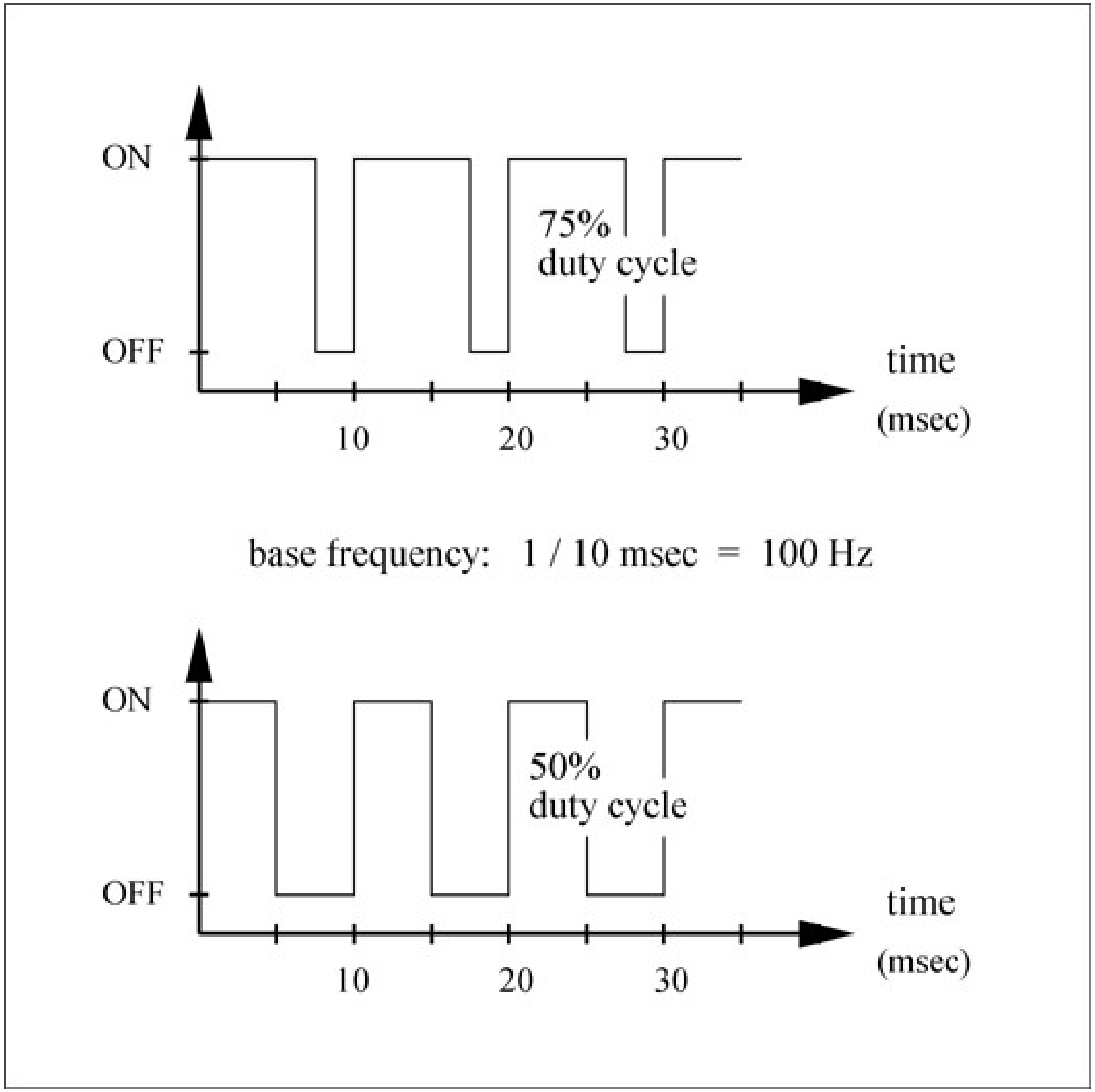
- discrete
  - power-transistors
- integrated
  - wide range available
  - including additional features
    - stall detect
    - heat protection



Speed Control

Pulse Width Modulation (PWM)

- base-frequency
- duty-cycle

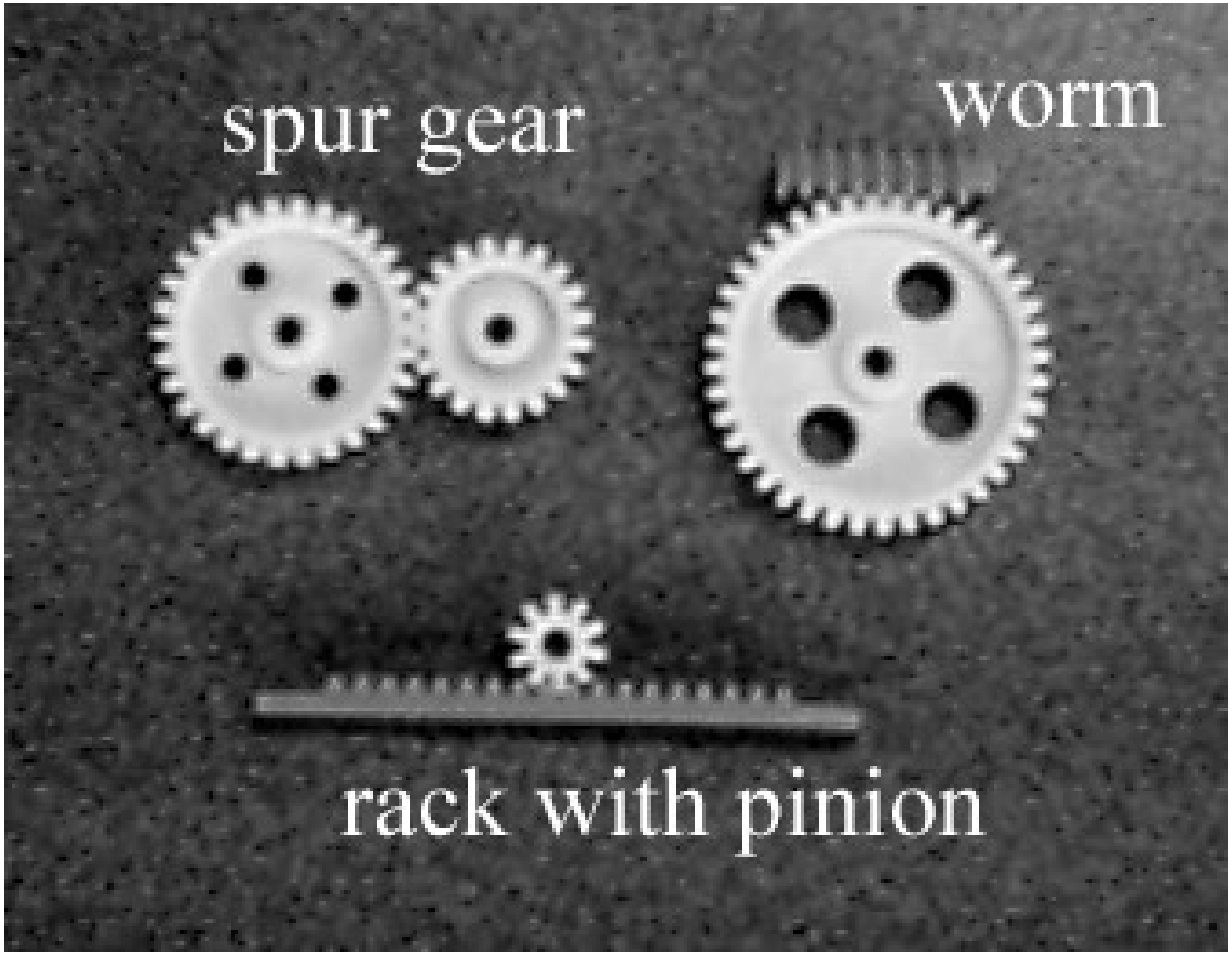


Gears

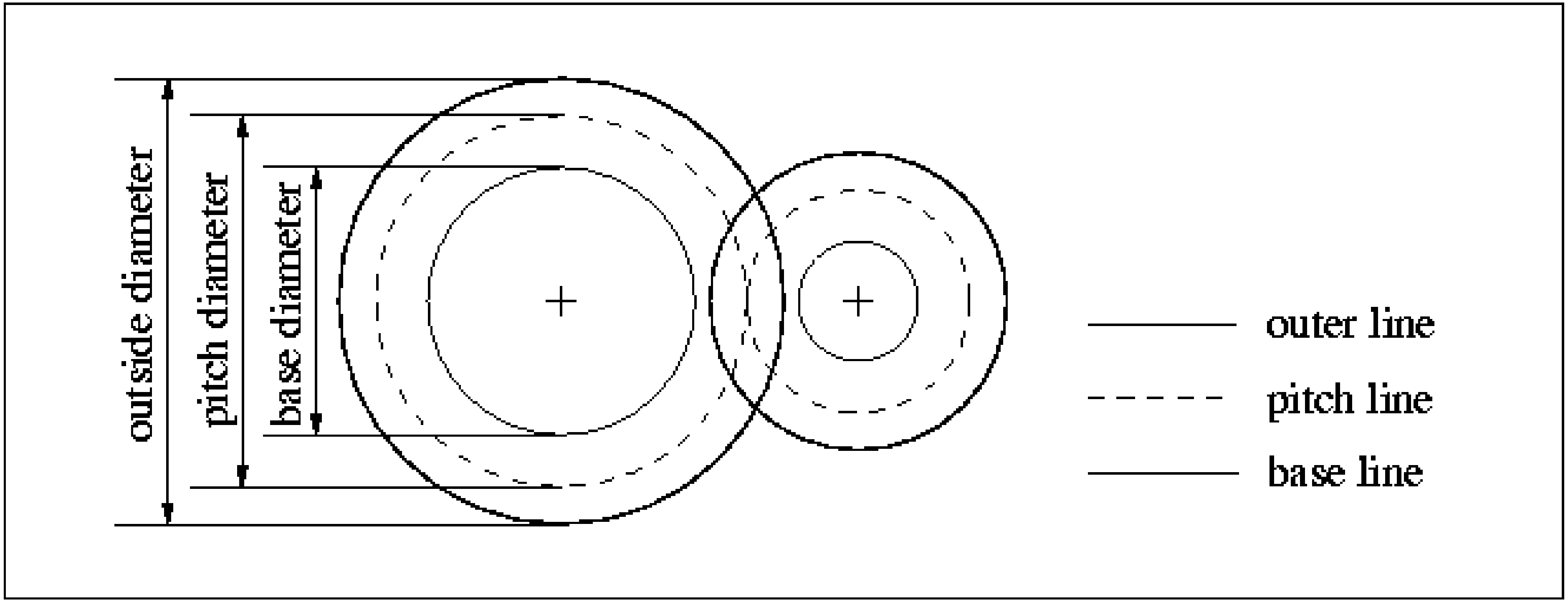
Basics

gears

- converting speed to torque
- changing the direction of motion



The Gear Wheel



- metric system
  - module m
  - pitch diameter (mm) / #teeth
  - common: 0.5 – 10 m
- occasionally (esp. US)
  - diametrical pitch p_d
  - #teeth / pitch diameter (inch)
  - $p_d = 25.4 / m$

Important Formulas

z teeth

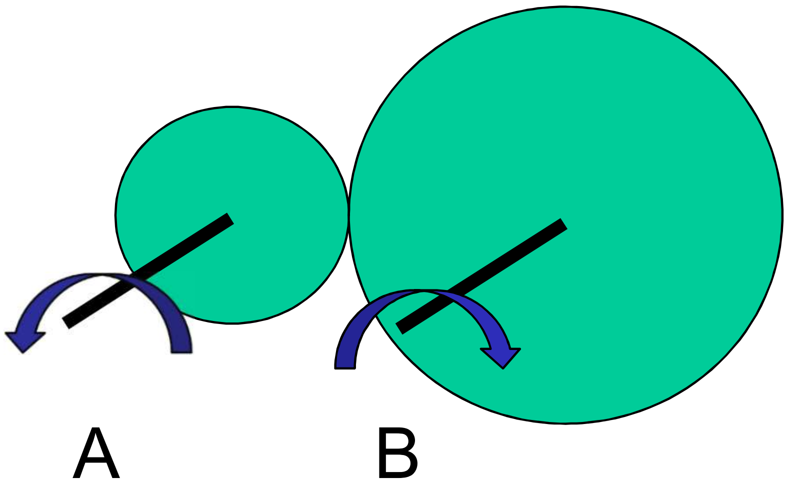
- pitch diameter $D = m \cdot z$
- outer diameter $D_o = D + 2 \cdot m = m \cdot (z + 2)$
- base diameter $D_b = D - 2.5 \cdot m = m \cdot (z - 2.25)$

- distance d_{AB} between two gears A & B

$$d_{AB} = \frac{D_A + D_B}{2} = \frac{m}{2}(z_A + z_B)$$

Gear Ratio

- 1 rotation of A
  - pitch line of A travels the circumference C_A of its pitch line
  - i.e., $D_A = m \cdot z_A \Rightarrow C_A = \pi D_A = \pi \cdot m \cdot z_A$
- B is driven the same distance
  - i.e., the pitch line of B moves by C_A
- i.e., B rotates by the fraction of C_A over its pitch line C_B
  - but in the opposite direction



$$\omega_B = -\frac{C_A}{C_B}\omega_A = -\frac{\pi \cdot m \cdot z_A}{\pi \cdot m \cdot z_B}\omega_A$$

$$= -\frac{z_A}{z_B}\omega_A$$

speed <-> torque conversion

$$T_B = -\frac{z_B}{z_A}T_A$$

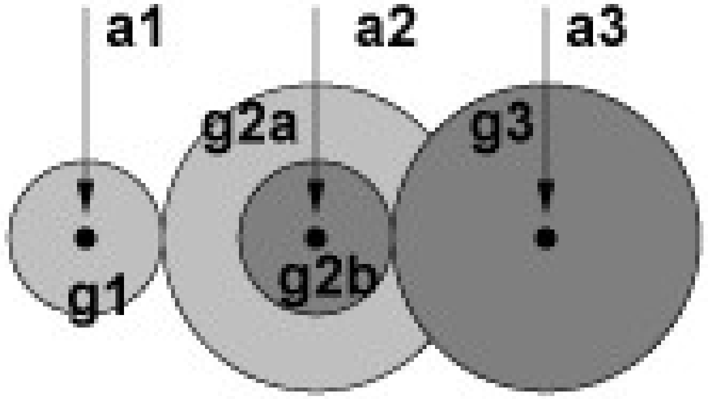
Simple Gear-Train

3 axes:

- axis a1 with gear g1
- axis a2 with gears g2a, g2b
- axis a3 with gear g3

$$\omega(a_2) = -\frac{Z_1}{Z_{2a}} \cdot \omega(a_1)$$

$$\omega(a_3) = -\frac{Z_{2b}}{Z_3} \cdot \omega(a_2)$$
$$= -\frac{Z_{2b}}{Z_3} \cdot -\frac{Z_1}{Z_{2a}} \cdot \omega(a_1)$$



$$T(a_2) = -\frac{Z_1}{Z_{2a}} \cdot T(a_1)$$

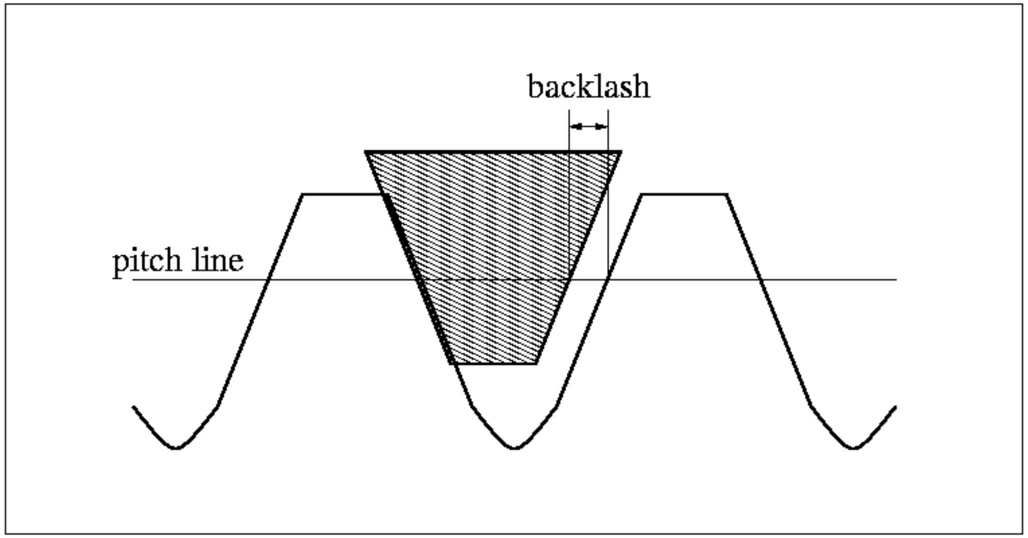
$$T(a_3) = -\frac{Z_3}{Z_{2b}} \cdot T(a_2)$$
$$= -\frac{Z_3}{Z_{2b}} \cdot -\frac{Z_{2a}}{Z_1} \cdot T(a_1)$$

Z_x= #teeth of gear g_x
$\omega(a)$ = angular velocity of axis a
$T(a)$ = torque at axis a

Backlash

spare room

- source of error
  - hysteresis
- but needed
  - otherwise jamming

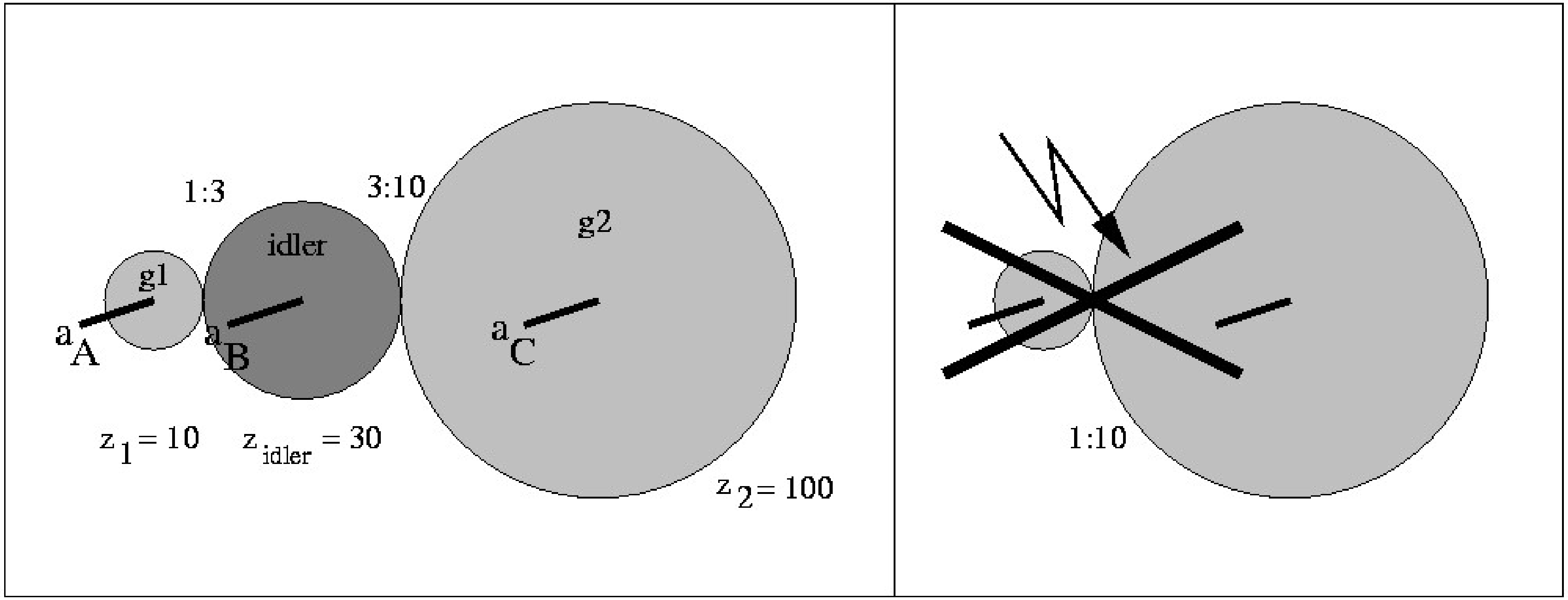


- small backlash only possible with
  - very precise manufacturing
  - small wear-out, i.e., no changes of the teeth

Idler

single gear on one axis

- reverses direction
- displacement
- mediator
- gear-ratio unaffected

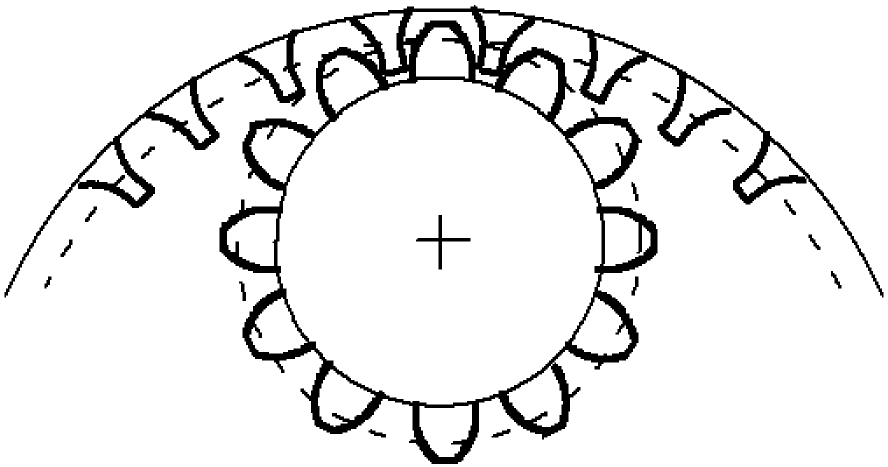


External & Internal Spur

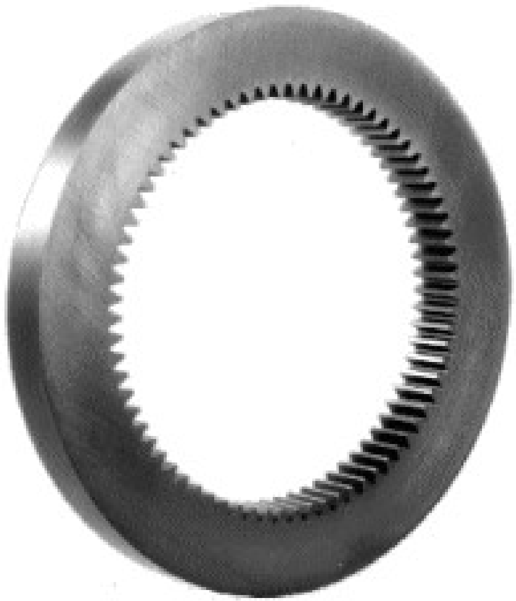
- external
  - convex shape
- internal
  - concave shape
  - different sense of rotation than external

two external:

$$\omega_B = -\frac{z_A}{z_B}\omega_A$$



1 ext. & 1 int.: $\omega_B = \frac{z_A}{z_B}\omega_A$ (no minus)



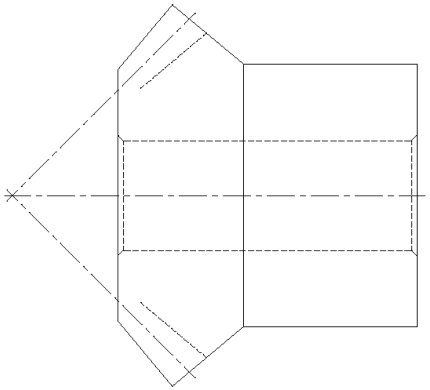
Bevel and Miter Gear

bevel

- for non-parallel axes
- inclined teeth

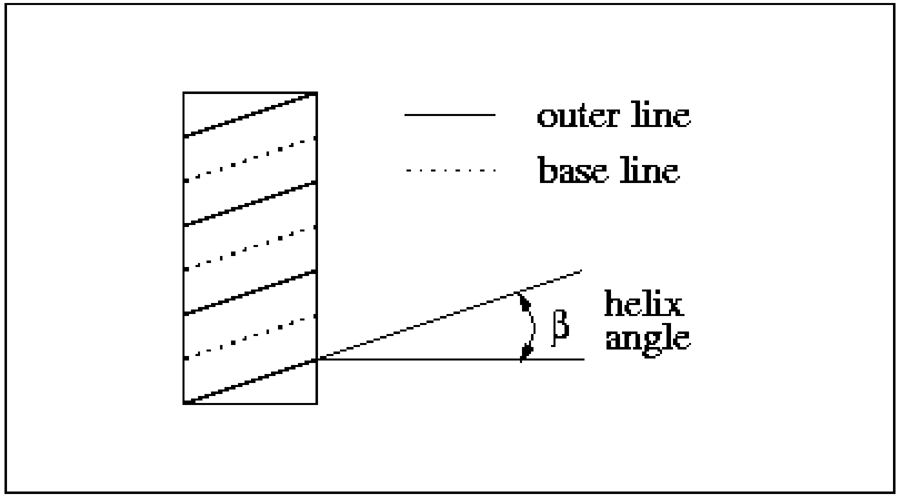
miter

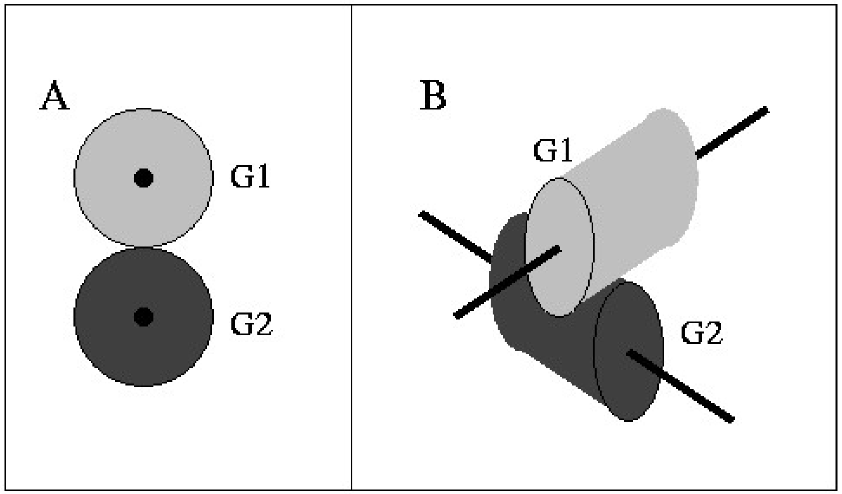
- bevel with GR 1:1



Helical Gear

- teeth cut in angles
  - increased contact surface
  - higher forces
- screw gear
  - helix angle 45^o
  - 90^o transmission

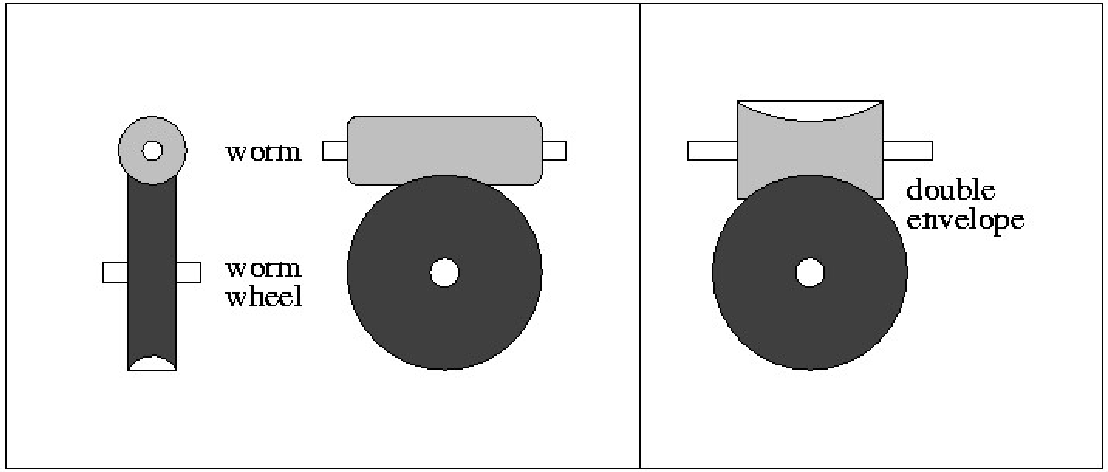




Worm Gear

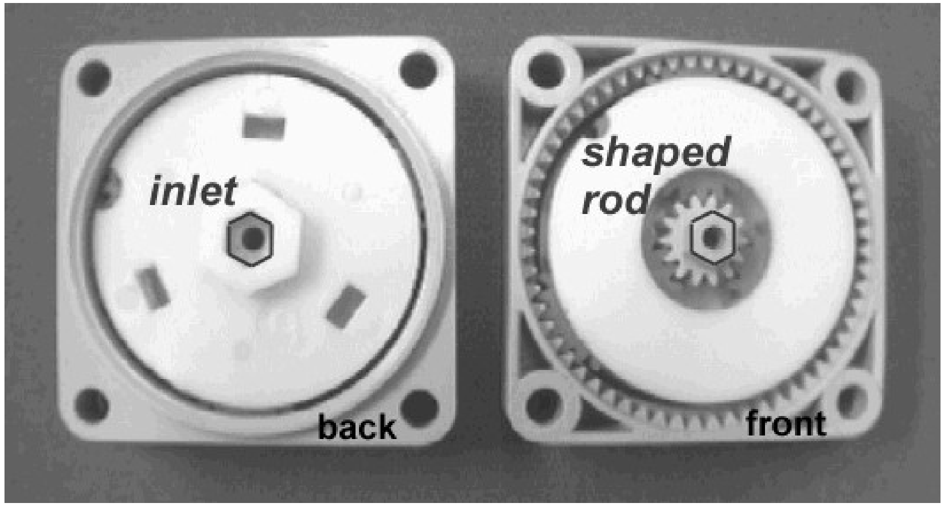
- can lock position
- 90^o intersection
- worm #teeth = 1

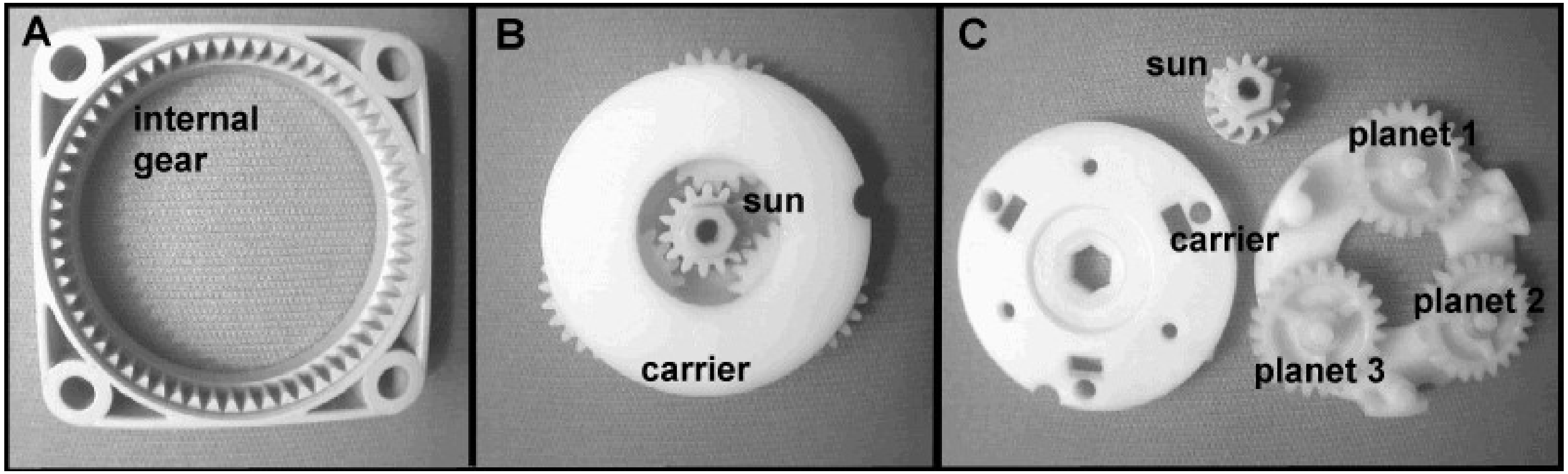
=> high gear ratio



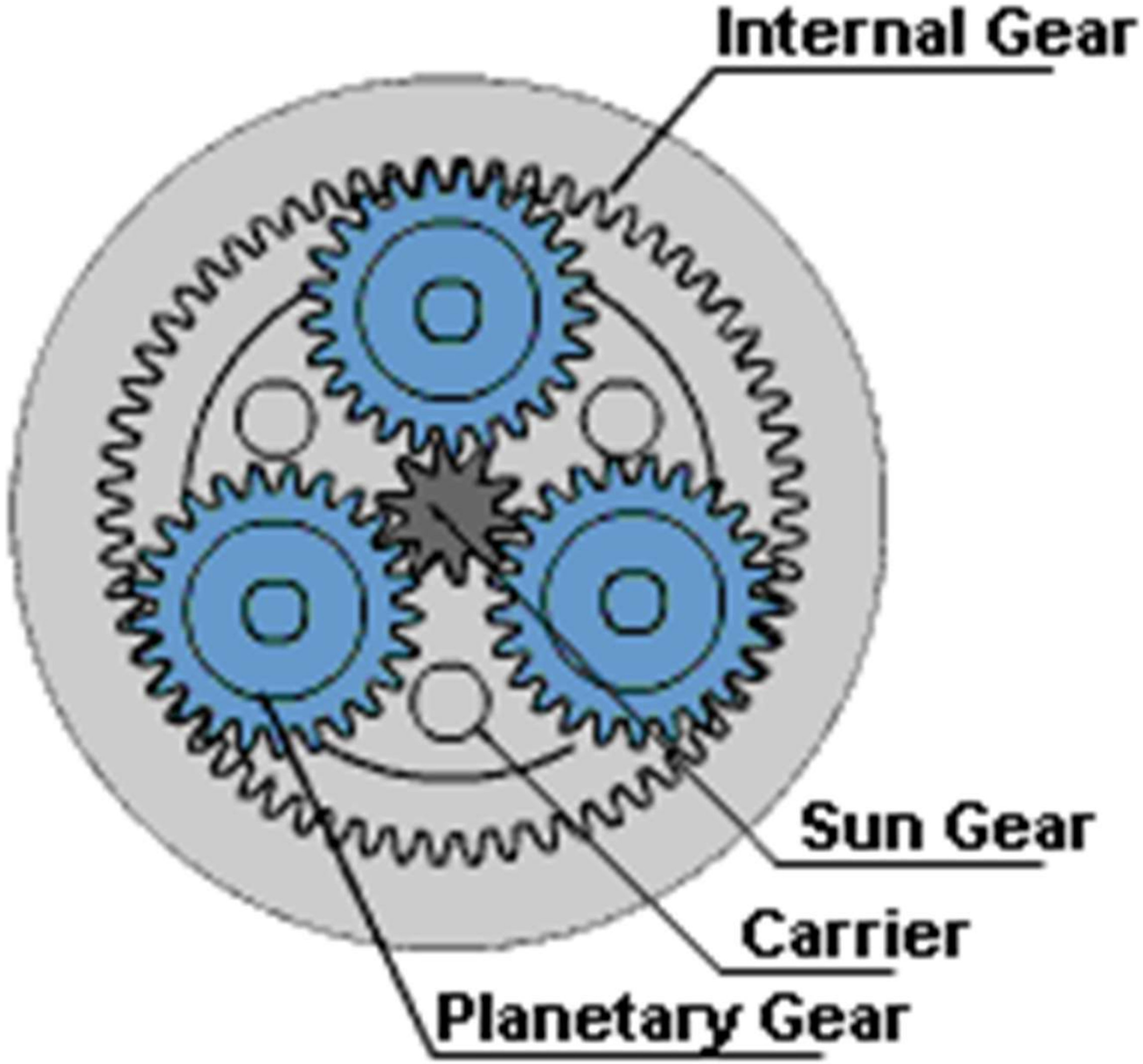
Planetary Gears

- compact
- high gear ratio
- stackable on motor axis





Example



The GR of Planetary Gears

gear-ratio:

$$GR = \frac{N_C}{N_S} = \left(\frac{N_S}{N_C}\right)^{-1} = \frac{z_S}{z_I + z_S}$$ (sun: input, carrier: output)

number of teeth

- internal: z_I
- sun: z_S
- planet: $z_{P=}=(z_I - z_S)/2$

note:

- #teeth proportional to radii
- hence z_P determined by z_I and z_S

The GR of Planetary Gears

Proof:

rotate sun once:

$$N_S = 1,\ N_P = \frac{-z_S}{z_P},\ N_I = \frac{-z_S}{z_I},\ N_C = 0$$

compensate rotation of the housing:

$$N_S = 1 + \frac{z_S}{z_I},\ N_P = \frac{-z_S}{z_P} + \frac{z_S}{z_I},\ N_I = 0,\ N_C = \frac{z_S}{z_I}$$

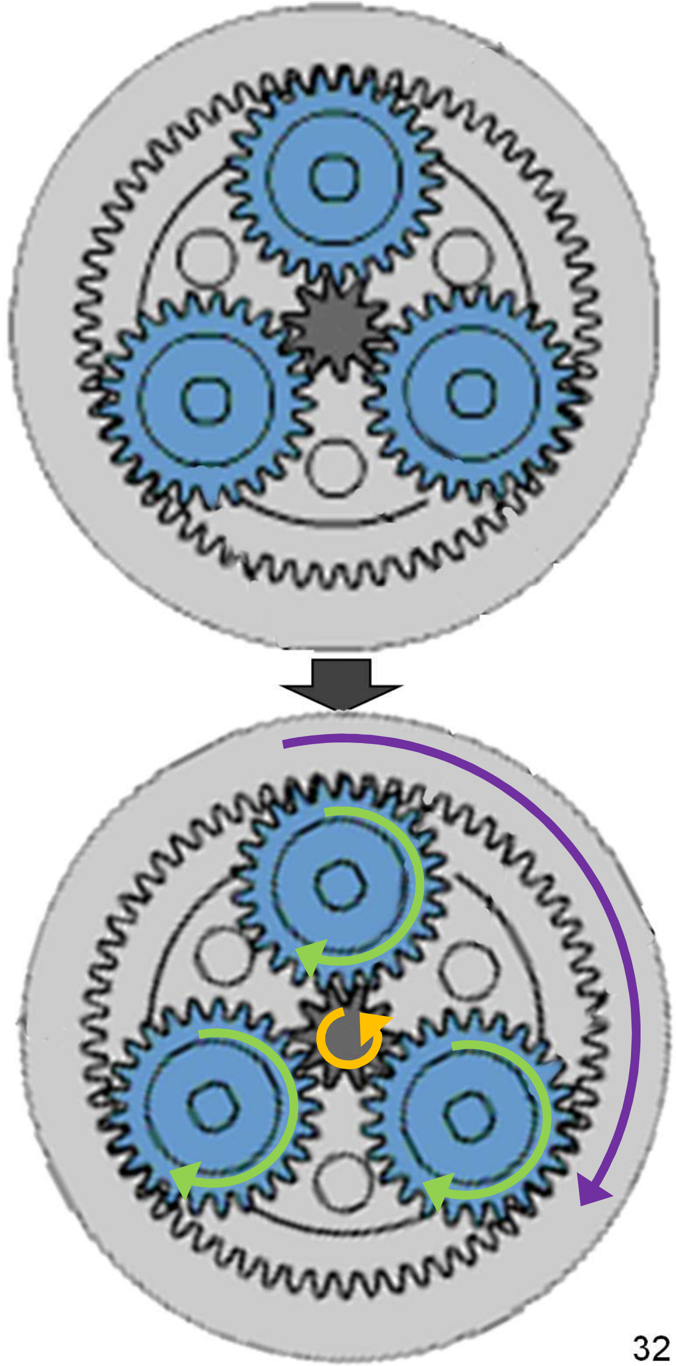
sun over carrier rotations:

$$\frac{N_S}{N_C} = \left(1 + \frac{z_S}{z_I}\right) \cdot \left(\frac{z_S}{z_I}\right)^{-1} = \left(1 + \frac{z_S}{z_I}\right) \cdot \left(\frac{z_I}{z_S}\right) = \frac{z_I}{z_S} + 1 \ \Rightarrow\ GR = \left(\frac{N_S}{N_C}\right)^{-1} = \left(\frac{z_I + z_S}{z_S}\right)^{-1}$$

The GR of Planetary Gears

start:

$$N_S = 0$$
$$N_P = 0$$
$$N_I = 0$$
$$N_C = 0$$



rotate sun once:

$$N_S = 1$$
$$N_P = \frac{-z_S}{z_P}$$
$$N_I = \frac{-z_S}{z_p}\frac{z_p}{z_I} = \frac{-z_S}{z_I}$$
$$N_C = 0$$

The GR of Planetary Gears

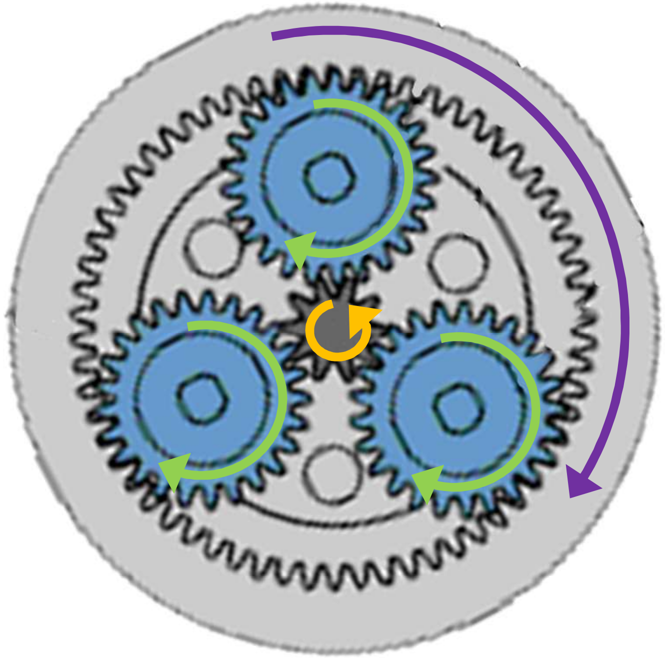
rotate sun once:

$$N_S = 1$$

$$N_P = \frac{-z_S}{z_P}$$

$$N_I = \frac{-z_S}{z_I}$$

$$N_C = 0$$



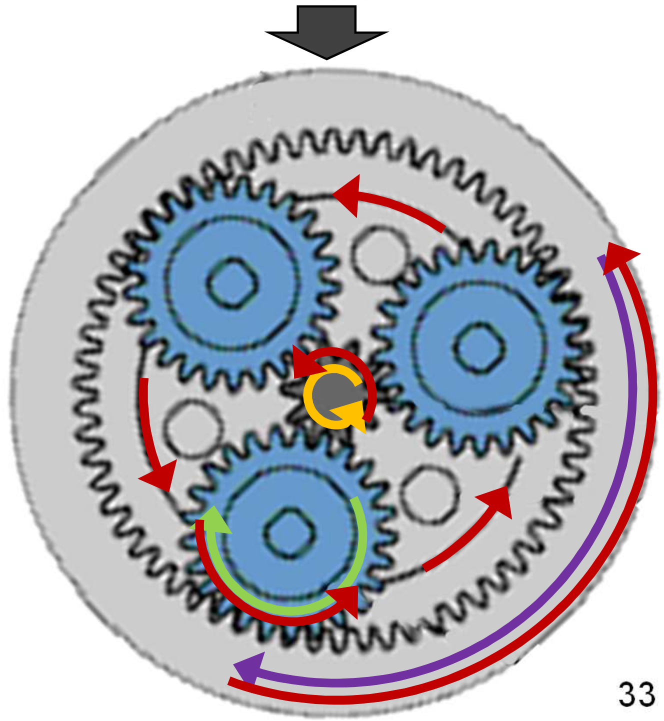
compensate rotation of the housing:

$$N_S = 1 - \frac{-z_S}{z_I}$$

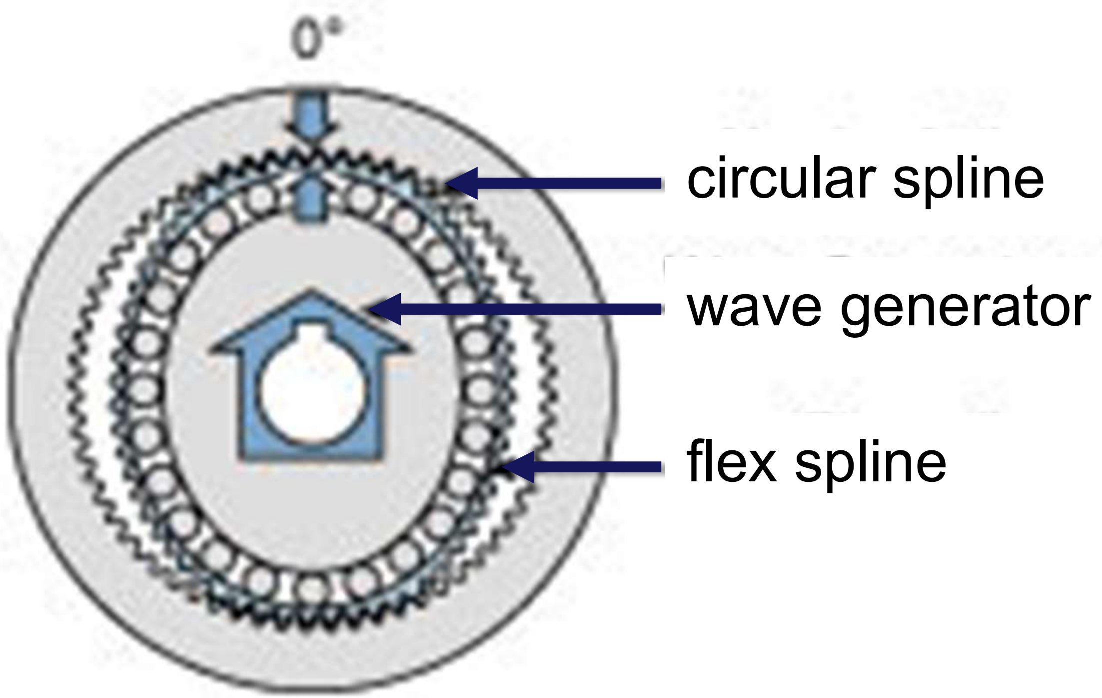
$$N_P = \frac{-z_S}{z_P} + \frac{z_S}{z_I}$$

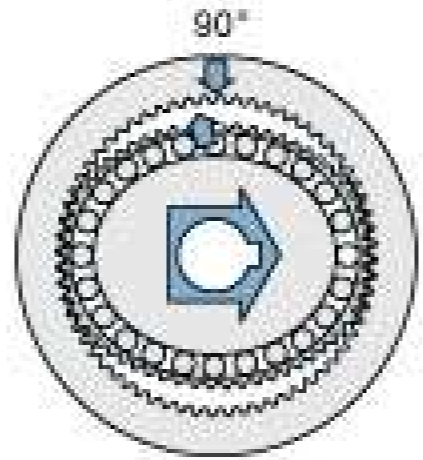
$$N_I = \frac{-z_S}{z_I} + \frac{z_S}{z_I} = 0$$

$$N_C = \frac{z_S}{z_I}$$



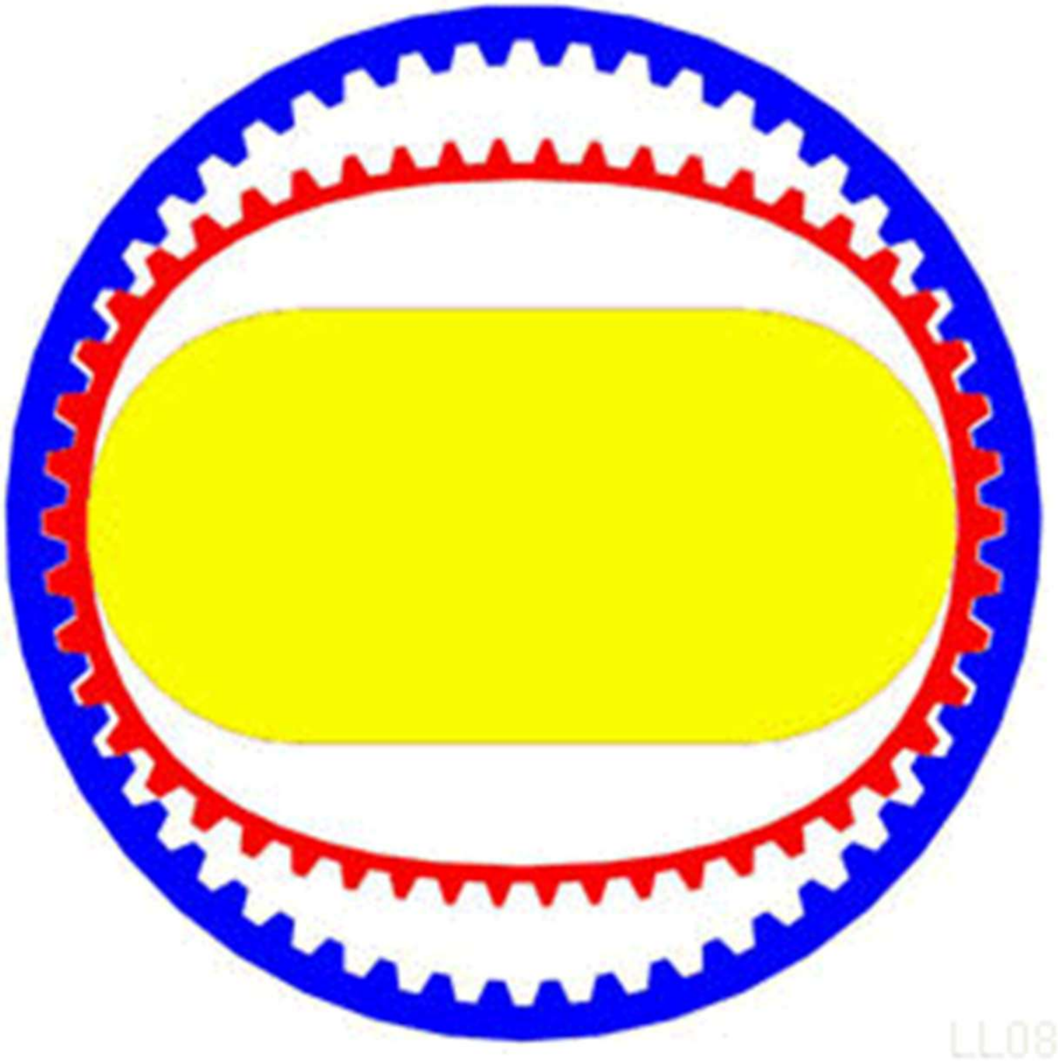
Harmonic Gear





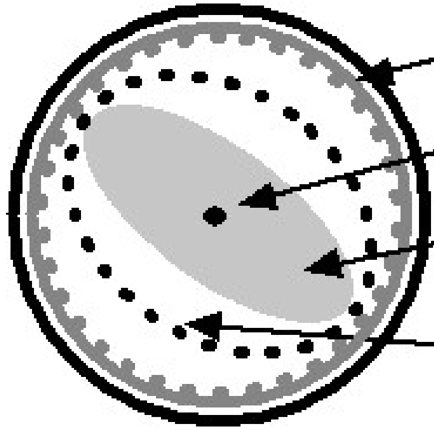
aka harmonic drive

Harmonic Gear



Harmonic Gear

- wave generator
  - rigid, eliptical
- flexible spline
  - z_f teeth
- rigid spline
  - z_r teeth
  - $z_r = z_f + d$



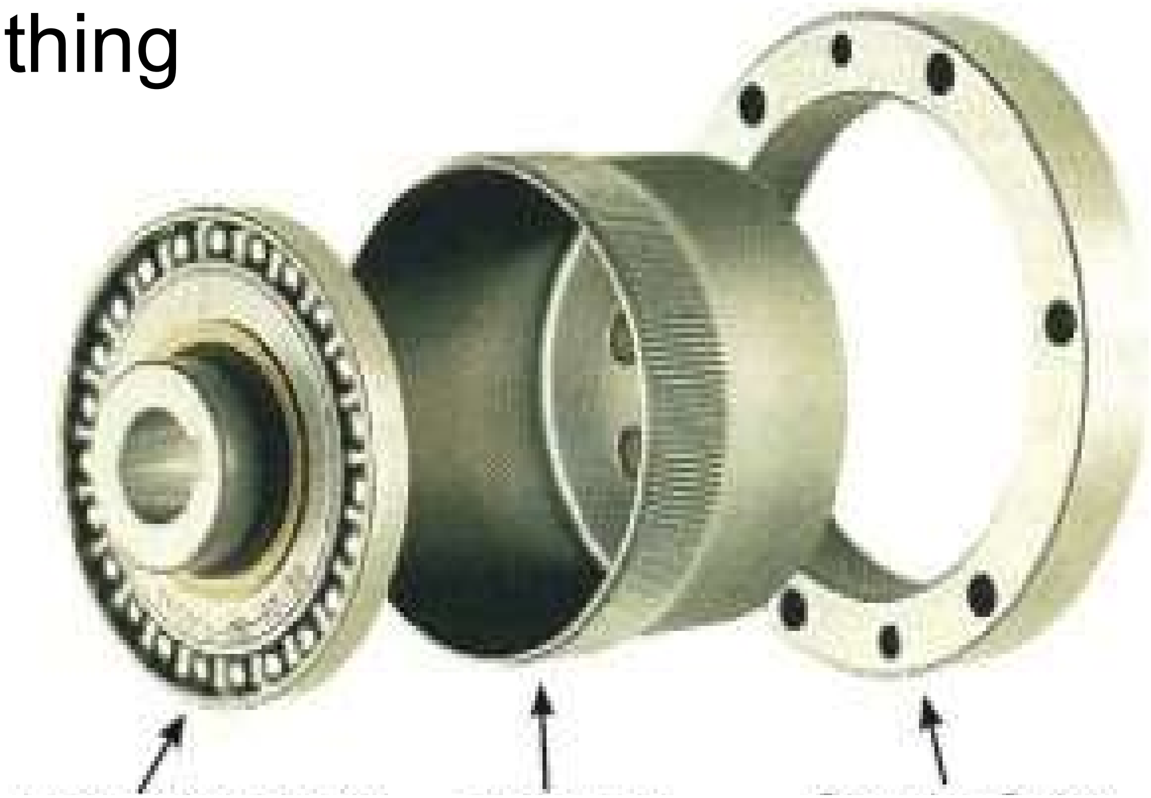
$$GR_{hg} = \frac{-d}{z_r} = -\frac{z_r}{z_f} + 1$$

(typical) example

- $d = 2$
- $z_r = 202$
- $z_f = 200$
- GR = 1:100

Harmonic Gear

the real thing



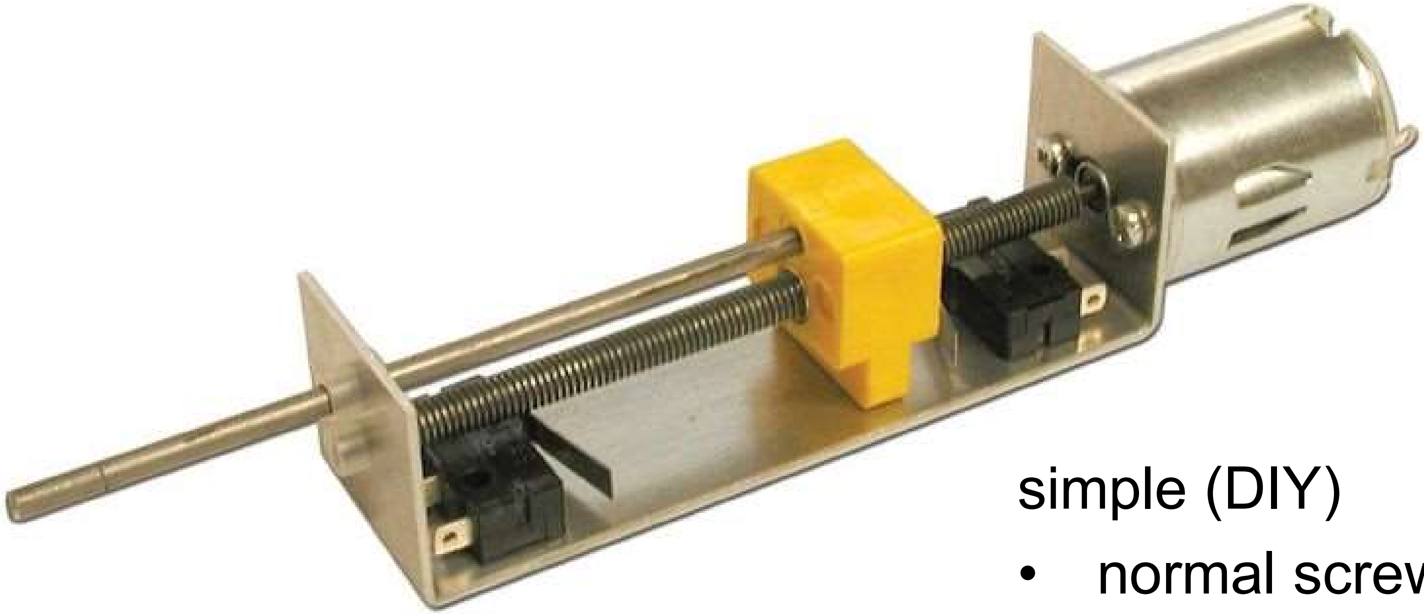
wave generator   flex spline   circular spline

image source: Mitsubishi (joints in the PA-10 robot arm are driven by harmonic gears)

Active Translation

linear actuator

- DC motor with (ball)screw and (ball)nut



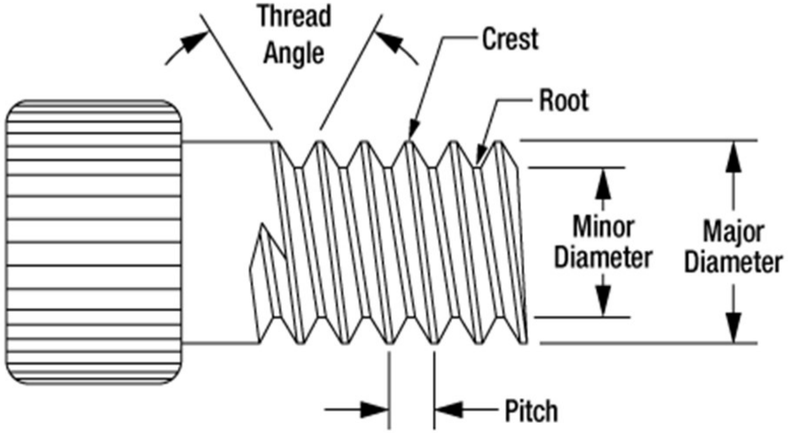
simple (DIY)

- normal screw & nut
- normed screw pitch
- fixed translation/rotation

Active Translation

e.g., metric (M*X*) coarse thread

i.e., standard thread
(there is also fine thread)



Nominal size	**Major diameter**	**Pitch**	Minor diameter	Tap drill size
M6	6.00	1.00	4.92	5.00
M8	8.00	1.25	6.65	6.75
M10	10.00	1.50	8.38	8.50
M12	12.00	1.75	10.11	10.00

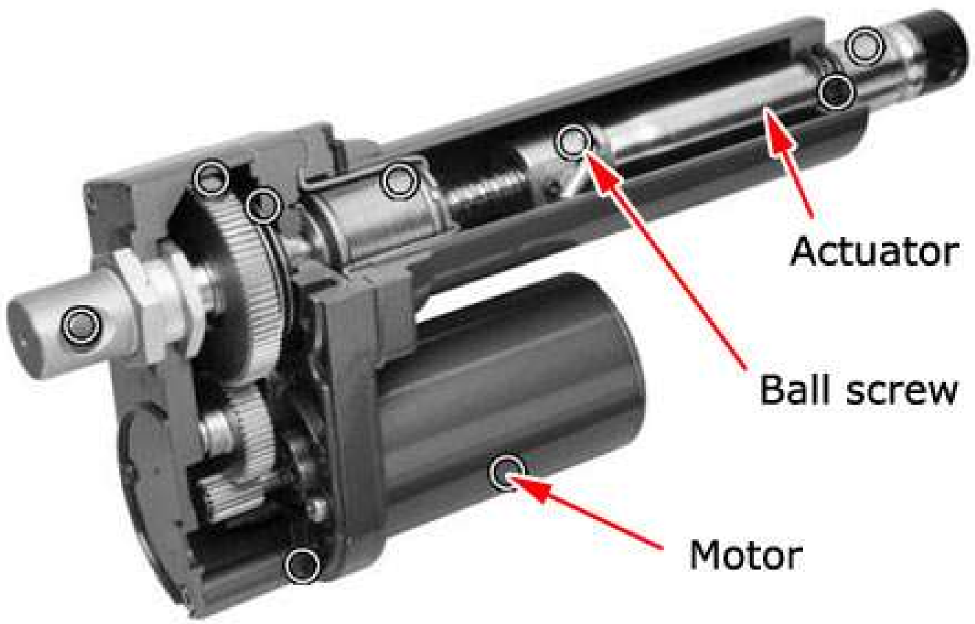
Metric thread

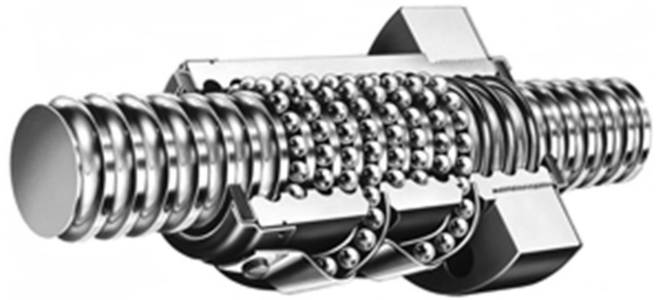
Minor diameter ≈ Tap drill size

Active Translation

linear actuator

- DC motor with (ball)screw and (ball)nut



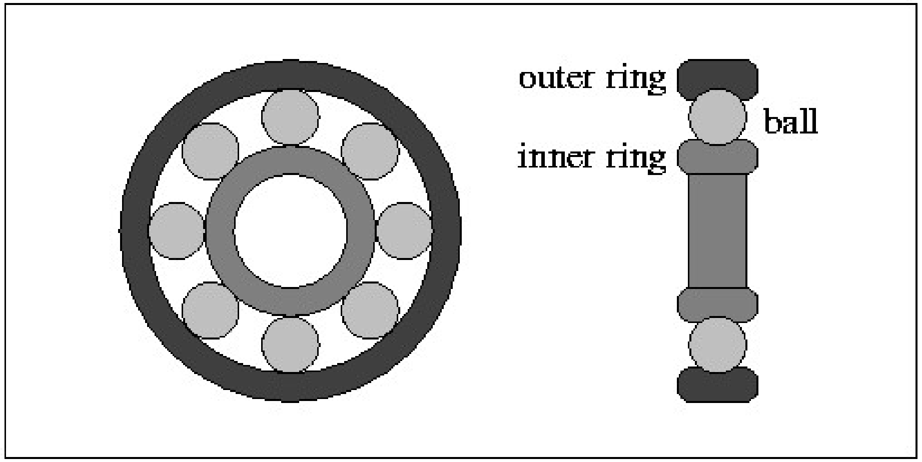


bearings circulating in the nut

- minimize friction
- and distribute forces

Axis Bearings

- support vertical load on an axis
  - minimize friction
  - maximize support
- sliding & ball bearing



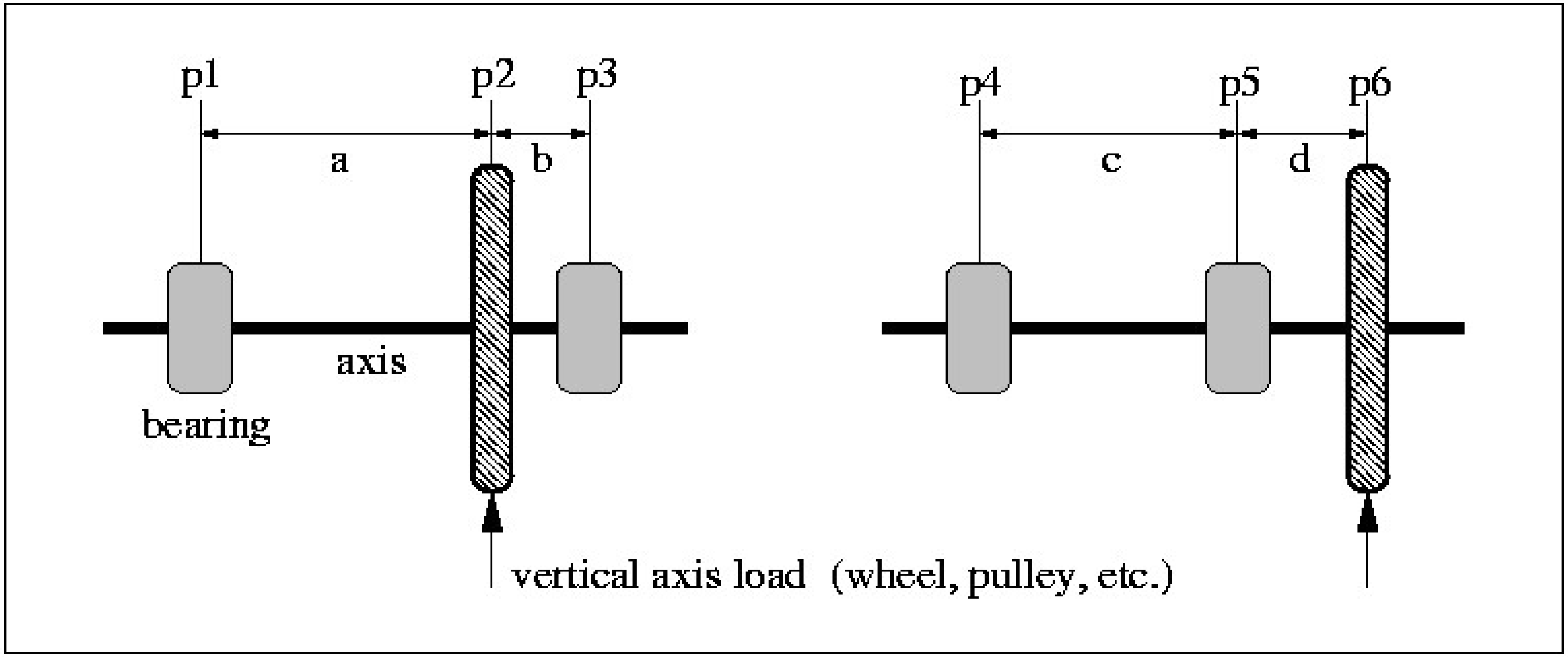
Basic Formulas

$$f_{p_1} = f_{load} \cdot \frac{b}{a+b}$$

$$f_{p_2} = f_{load} \cdot \frac{a}{a+b}$$

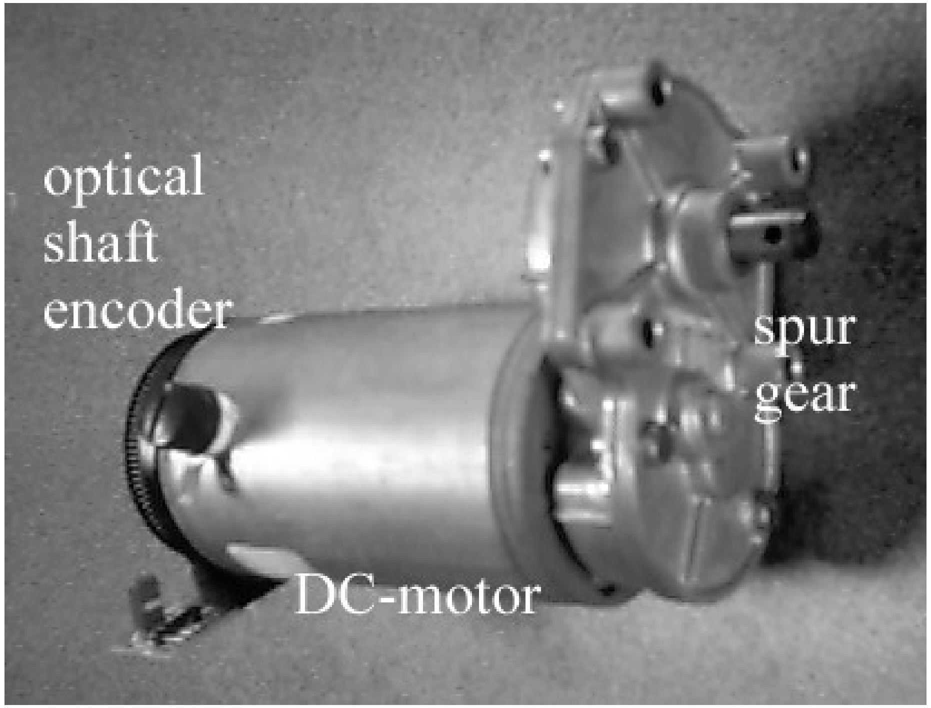
$$f_{p_4} = f_{load} \cdot \frac{d}{c}$$

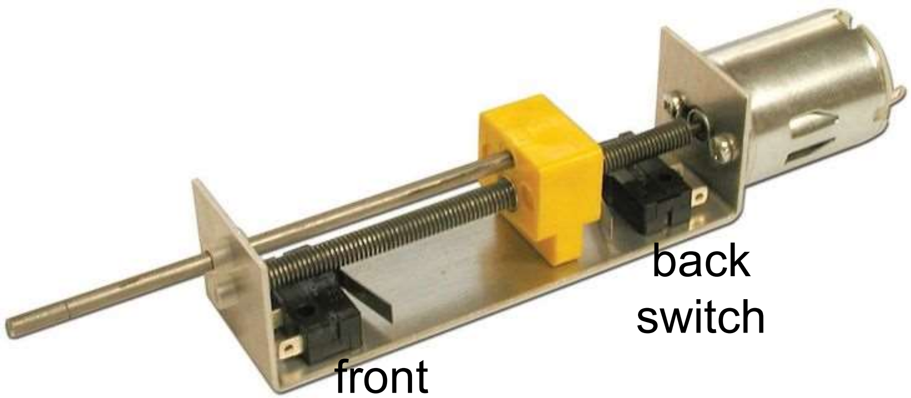
$$f_{p_5} = f_{load} \cdot \frac{c+d}{c}$$



Motor-Units

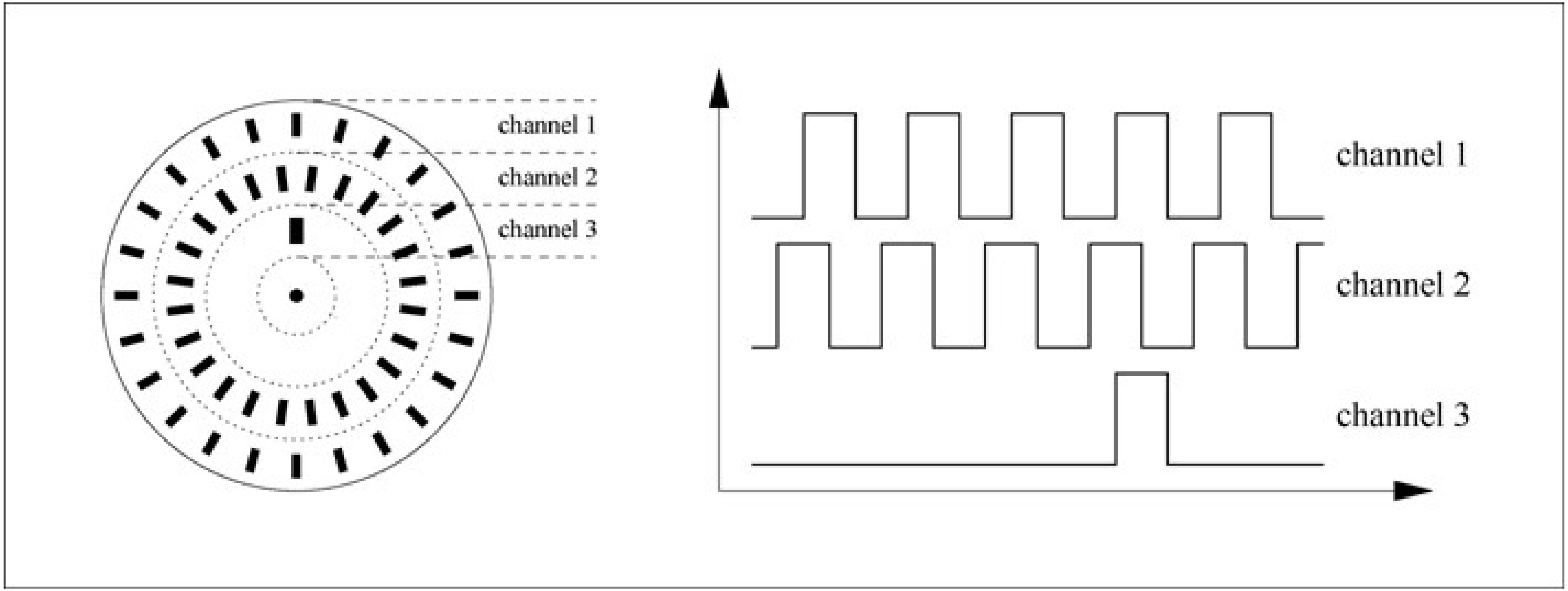
- motor
- gear-box
- position-feedback
  - shaft encoder
    - optical / hall-effect
    - incremental / absolute
  - extreme point





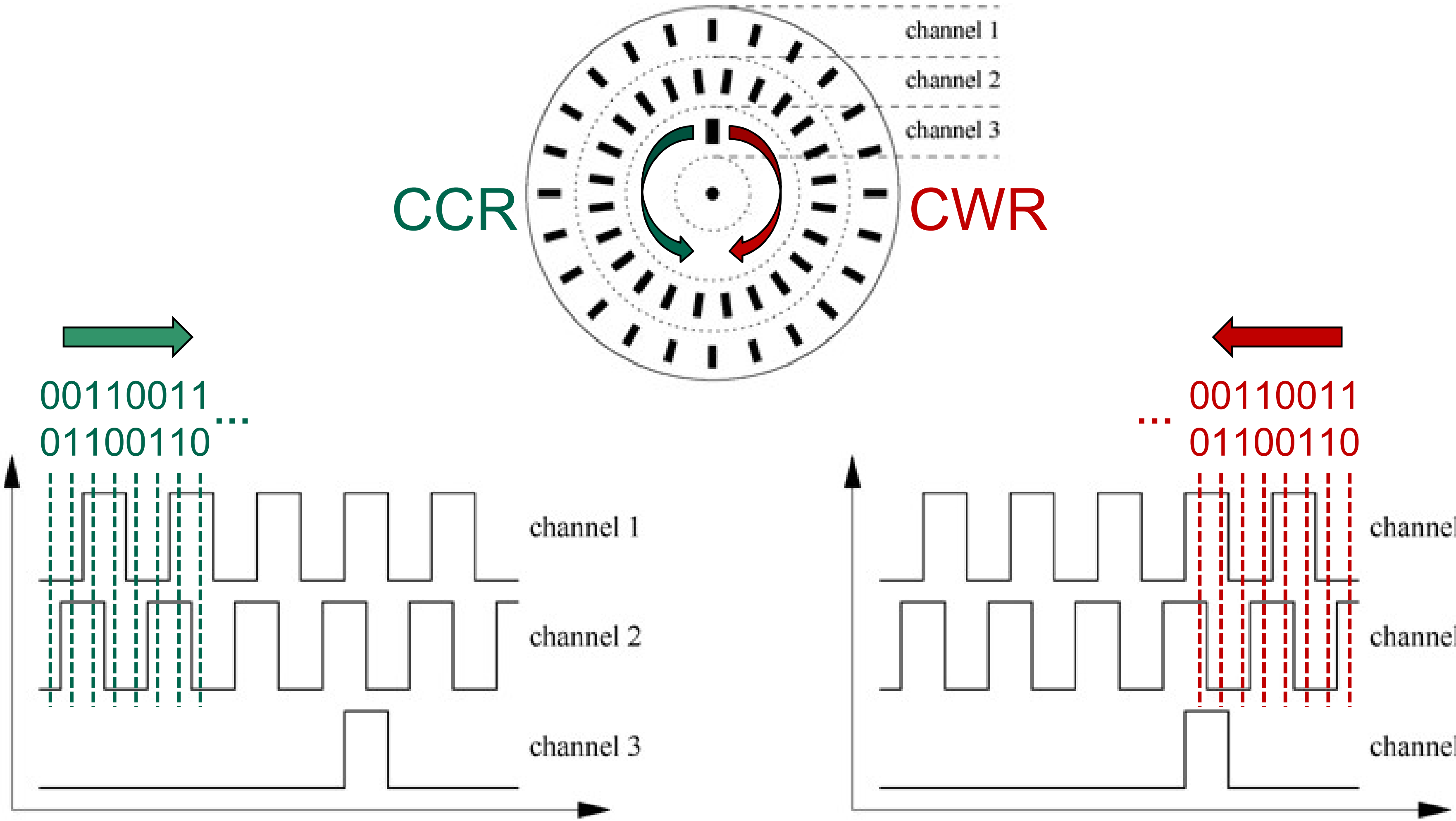
Motion: Incremental Shaft-Encoder

- often 3 channels
- 1x calibration (1 pulse per rotation)
- 2x *n* pulses, 90º phase shift, **quadrature encoder**



Motion: Incremental Shaft-Encoder

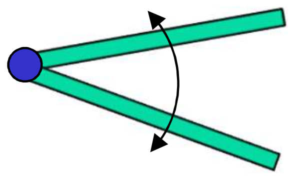
quadrature encoder

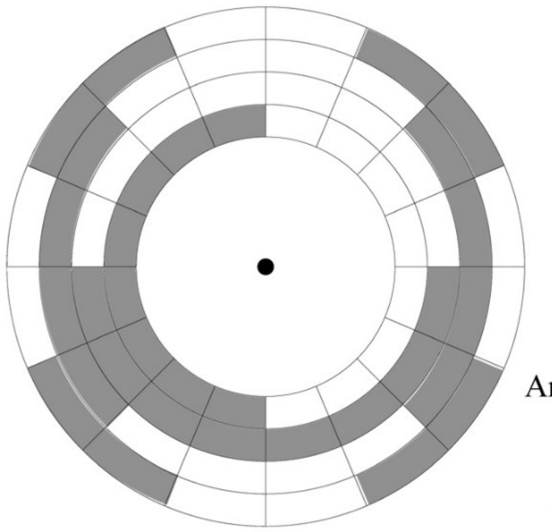


Absolute Shaft-Encoder

to measure

- the absolute orientation
- of a rotational joint





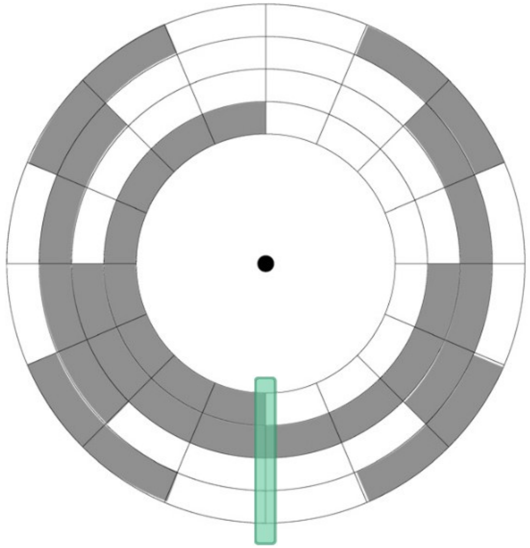
	Gray Code			
0	0	0	0	0
1	0	0	0	1
2	0	0	1	1
3	0	0	1	0
4	0	1	1	0
5	0	1	1	1
6	0	1	0	1
7	0	1	0	0
8	1	1	0	0
9	1	1	0	1
10	1	1	1	1
11	1	1	1	0
12	1	0	1	0
13	1	0	1	1
14	1	0	0	1
15	1	0	0	0

= 1 bit "flips", $0 \to 1 \vee 1 \to 0$

- Gray encoding
- to minimize Hamming distance
- less ambiguity when the marker sensors are between two sectors

Absolute Shaft-Encoder

degrees	binary				Gray			
0.0/360.0	0	0	0	0	0	0	0	0
22.5	0	0	0	1	0	0	0	1
45.0	0	0	1	0	0	0	1	1
67.5	0	0	1	1	0	0	1	0
90.0	0	1	0	0	0	1	1	0
112.5	0	1	0	1	0	1	1	1
135.0	0	1	1	0	0	1	0	1
157.5	0	1	1	1	0	1	0	0
180.0	1	0	0	0	1	1	0	0
202.5	1	0	0	1	1	1	0	1
225.0	1	0	1	0	1	1	1	1
247.5	1	0	1	1	1	1	1	0
270.0	1	1	0	0	1	0	1	0
292.5	1	1	0	1	1	0	1	1
315.0	1	1	1	0	1	0	0	1
337.5	1	1	1	1	1	0	0	0
0.0/360.0	0	0	0	0	0	0	0	0



sensors
in between
two sectors
=>
possible reading of
"left"/"right" sector
for every bit

degrees	Gray			
...	...			
157.5	0	1	0	0
180.0	1	1	0	0
...	...			

2 possible readings:
- 0100
- 1100

degrees	binary			
...	...			
157.5	0	1	1	1
180.0	1	0	0	0
...	...			

16 possible readings:
- 0000
- …
- 1111

Encoder Implementation

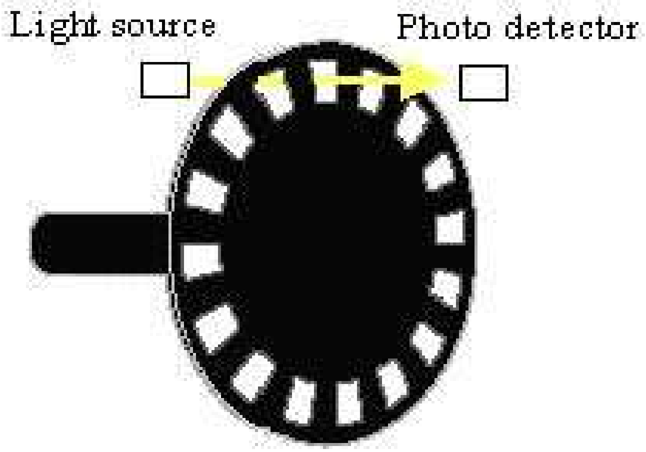
optical

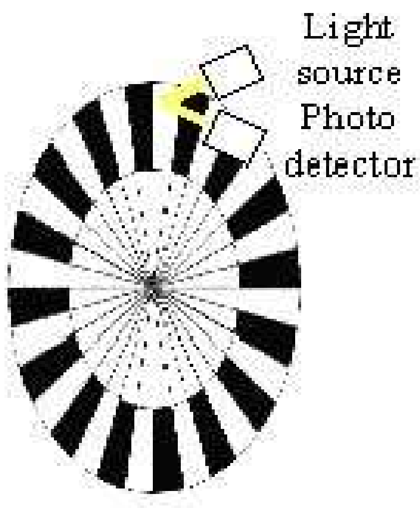
magnetic

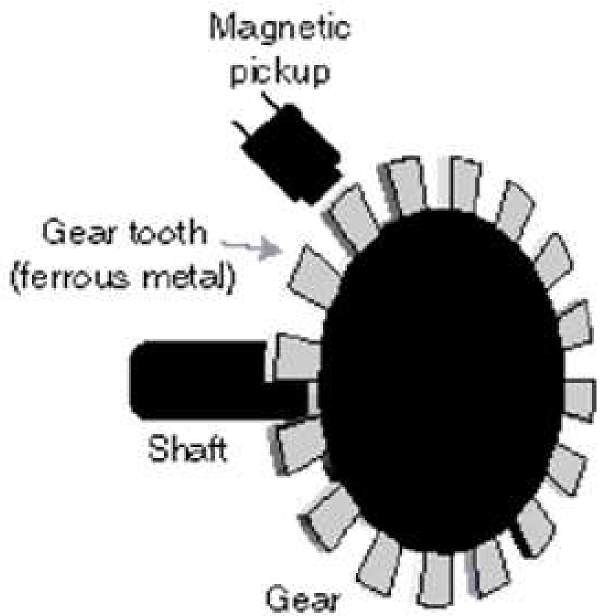
slits

reflective

disk



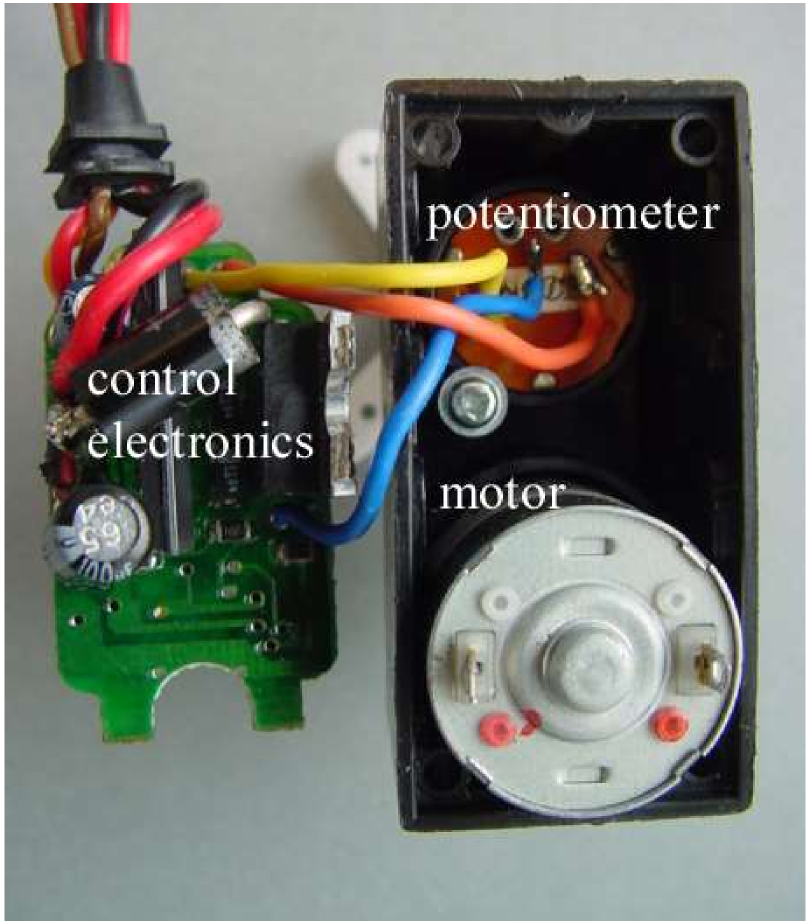




(or magnetic ink dots on disk)

Servo

- DC-motor
- gear-box
- electronics
  - feedback loop
  - via potentiometer (analog)
  - Pulse Code Modulation (PCM)

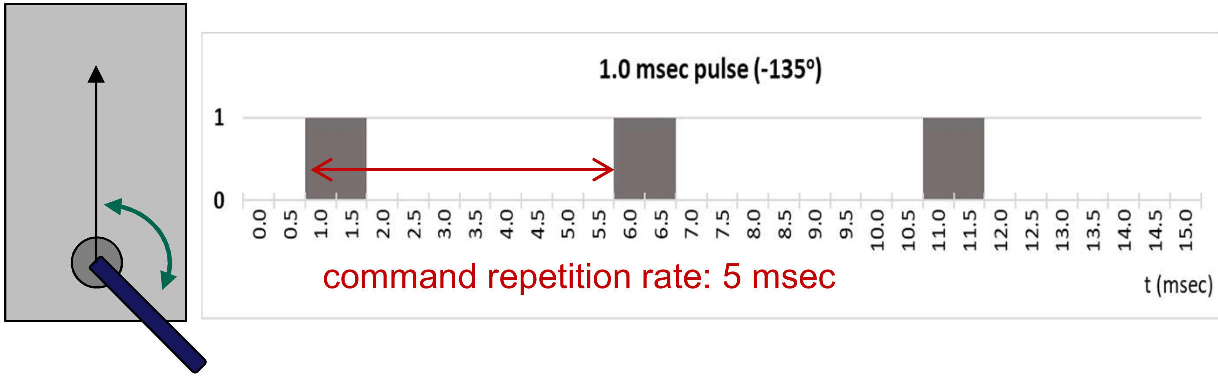


Servo

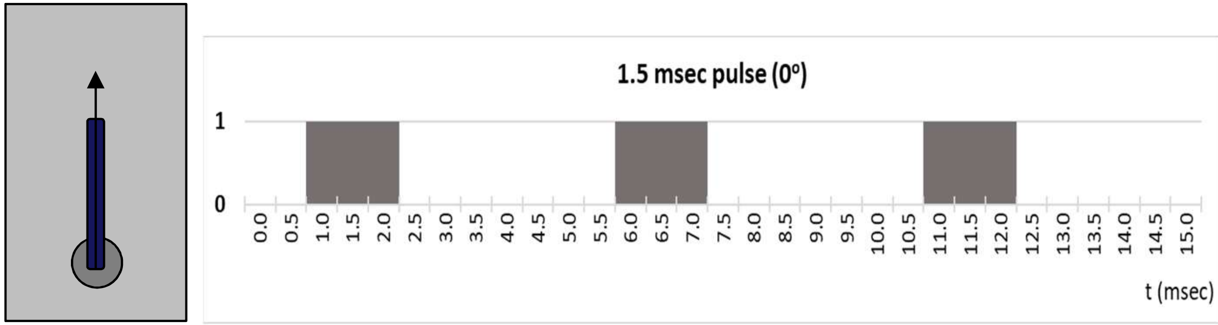
Pulse Code Modulation (PCM)

- on time
  - neutral: 1.5 msec
  - min angle: ca. 1 msec
  - max angle: ca. 2 msec
- command repetition rate
  - to hold position
  - <20-30 msec
- slew rate
  - angular speed

-135°



0°



+135°