Problem 1

(10 points)

Compute the derivative of the following functions directly from the definition

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

a) $f(x) = x^3$. **(4 points)**

b) $f(x) = \sqrt{(x)}$. **(4 points)**

c) $f(x) = x$. **(1 points)**

d) $f(x) = c$ with some constant c. **(1 points)**

Problem 2

(14 points)

Compute the derivatives of the following functions

a) $f(x) = \frac{x^2}{b-3x^2}$ where b is a constant **(2 points)**

b) $g(t) = \cos(\omega t + \phi) + \sin(\omega t + \phi)$ where ω and ϕ are constants **(2 points)**

c) $h(s) = \cos(s^2 + s) + \sin(s/2)$ **(2 points)**

d) $j(x) = \ln(x^{a^2} + x^{-a^2})$ where a is a constant
Note: You can use $(\ln x)' = 1/x$ from the lecture **(2 points)**

e) $k(x) = \ln(x^a + b^x)$ where a and b are constants **(2 points)**

f) $l(x) = x^2 \exp(-x^2)$ **(2 points)**

g) $m(x) = x^{x^2}$ **(2 points)**
Note for e) and g): You cannot directly work with something of the form a^x (with some a) but only with something of the form e^{cx} (with some c). Transform the function accordingly before differentiation.

Problem 3

(6 points)

Use the definition of the derivative, $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, to show that the function $f(x) = |x|$ is not differentiable at $x = 0$.