4 oct 2021

HW #4

Calc & Linear Algebra.

Tuesday

Problem 1

$(x^2 + h^2 + 2hx)(x+h)$

$x^3 + 2hx^2 + hx^2 + h^2x + 2h^2x + h^3$

$= x^3 + 3hx^2 + 3h^2x + h^3$

a) $f(x) = x^3$

$$\lim_{h \to 0} \frac{(x-h)^3 - x^3}{h}$$

$$= \frac{\cancel{x^3} + 3hx^2 + 3h^2x + h^3 \cancel{-x^3}}{h}$$

$$= \frac{\cancel{h}(3x^2 + 3hx + h)}{\cancel{h}}$$

$$\lim_{h \to 0} 3x^2 + 3(0)(x) + 0$$

$$= \boxed{3x^2}$$

b) $$\lim_{h \to 0} \frac{\sqrt{x+h} \cancel{+} - \sqrt{x}}{h}$$

$$\frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$\lim_{h \to 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$\lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$\frac{1}{\sqrt{x+0} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

c) $f(x) = x$

$$\frac{x+h-x}{h}$$

$$\lim_{h \to 0} \frac{h}{h} \quad 1$$

$$= \boxed{1}$$

d) $f(x) = c$

$$\lim_{h \to 0} \frac{c-c}{h}$$

$$\lim_{x \to 0} \frac{0}{h}$$

$$= 0$$

Problem 2

a) $$f(x) = \frac{x^2}{b - 3x^2}$$

using $$\frac{d}{dx}\frac{u}{v} = \frac{u'v - uv'}{v^2}$$

$$f'(x) = \frac{\frac{d}{dx}x^2 \cdot (b-3x^2) - x^2 \frac{d}{dx}(b-3x^2)}{(b-3x^2)^2}$$

$$= \frac{2x(b-3x^2) - x^2(-6x)}{(b-3x^2)^2}.$$

$$= \frac{2bx \cancel{- 6x^3} \cancel{+ 6x^3}}{(b-3x^2)^2}$$

$$\boxed{= \frac{2bx}{(b-3x^2)^2}}$$

b) $g(t) = \cos(\omega t + \phi) + \sin(\omega t + \phi)$

$g'(t) = (-\sin(\omega t + \phi) \times \omega) + (\cos(\omega t + \phi) \times \omega)$

$= \omega\left(\cos(\omega t + \phi) - \sin(\omega t + \phi)\right)$

c) $h(s) = \cos(s^2 + s) + \sin\left(\frac{s}{2}\right)$

$h'(s) = \left(-\sin(s^2+s) \times (2s+1)\right) + \cos\left(\frac{s}{2}\right) \times \frac{1}{2}$

$= \frac{1}{2}\cos\left(\frac{s}{2}\right) - 2s\sin(s^2+s) - \sin(s^2+1)$

d) $j(x) = \ln\left(x^{a^2} + x^{-a^2}\right)$

$= \frac{1}{x^{a^2} + x^{-a^2}} \times a^2 x^{a^2-1} + -a^2 x^{-a^2-1}$

$= \frac{a^2 x^{2a^2} - a^2}{x^{1+2a^2} + x}$

e) $k(x) = \ln(ax^a + b^x)$

$$k'(x) = \frac{1}{x^a + b^x} \times (ax^{a-1} + \ln(b) \times b^x)$$

$$= \frac{ax^{a-1} + b^x \ln(b)}{ax^a + b^x}$$

f) $l(x) = x^2 e^{-x^2}$

$$uv = u'v + uv'$$

$$l'(x) = 2x(e^{-x^2}) + x^2 e^{-x^2} \times -2x$$

$$= 2xe^{-x^2} - 2x^3 e^{-x^2}$$

g) $m(x) = x^{x^2}$ $\qquad x = e^{\ln(x)}$

$$\frac{d}{dx} = (e^{\ln(x)})^{x^2}$$

$$\frac{d}{dx}(e^{\ln(x) x^2}).$$

$$= e^{\ln(x).x^2} \times \frac{1}{x} \times x^2 + \ln(x) + 2x$$

$$= x^{x^2+1} + (2x^{x^2+1} \cdot \ln(x)).$$

Problem 3

$f(x) = |x|$

$$\lim_{h \to 0} \frac{|x+h| - |x|}{h}$$

$$\lim_{h \to 0} \frac{|x+h| - |x|}{h} \times \frac{|x+h| + |x|}{|x+h| + |x|}$$

$$\lim_{h \to 0} \frac{(|x+h|)^2 - (|x|)^2}{h(|x+h| + |x|)}$$

$$\lim_{h \to 0} \frac{(x+0)^2 - (x)^2}{0(|x+0| + |x|)}$$

$$\lim_{h \to 0} \frac{x^2 - x^2}{0}$$

$= 0$ hence proven that limit does not exist and function is not differentiable at $x=0$.