1. Model & Solve in Time Domain read comments below

$M\ddot{x} + f_v \dot{x} + Kx = 0 \qquad M = 4 \quad f_v = 0 \quad K = 100$

$x(t) = e^{\lambda t}$

a) $4\lambda^2 e^{\lambda t} + 0 \cdot \lambda e^{\lambda t} + 100 e^{\lambda t} = 0 \Rightarrow e^{\lambda t}(4\lambda^2 + 100) = 0 \Rightarrow$

$\Rightarrow 4\lambda^2 + 100 = 0 \Rightarrow \lambda^2 = -25 \Rightarrow \lambda_1 = 5j \quad \lambda_2 = -5j$

b) $x(t) = a e^{5jt} + b e^{-5jt}$

$x(0) = 10 \qquad \dot{x}(0) = 0$

$x(0) = a + b = 10 \qquad \dot{x}(0) = 5aj - 5bj = 0$

$\Rightarrow a = b$ and $a + b = 10 \Rightarrow a = b = 5$

$\Rightarrow x(t) = 5\left(e^{5jt} + e^{-5jt}\right) = 10 \cdot \frac{e^{j \cdot 5t} + e^{-j \cdot 5t}}{2} = 10\cos(5t)$

2. Laplace Transforms & Solve in frequency domain

$M\ddot{x} + f_v \dot{x} + Kx = 0 \quad M = 4 \quad f_v = 0 \quad K = 100 \quad x(0) = 10 \quad \dot{x}(0) = 0$

$4\ddot{x} + 0 \cdot \dot{x} + 100x = 0 \Rightarrow 4\ddot{x} + 100x = 0$

Laplace Transforms:

$4(-s\,x(0) - \dot{x}(0) + s^2 X(s)) + 100\,X(s) = 0$

$-40s + 4s^2 \cdot X(s) + 100\,X(s) = 0$

$X(s)\left[4s^2 + 100\right] = 40s \qquad \Rightarrow$

$$\Rightarrow \quad X(s) = \frac{40s}{4s^2+100} = \frac{10s}{s^2+25} = \frac{10s}{(s+5j)(s-5j)} =$$

$$= \frac{5}{s+5j} + \frac{5}{s-5j}$$

$$X(s) = 5\left[\frac{1}{s+5j} + \frac{1}{s-5j}\right] \xrightarrow{L} x(t) = 5\left(e^{-5jt} + e^{5jt}\right)$$

$$x(t) = 10\,\frac{e^{-5jt}+e^{5jt}}{2} = 10\cos(5t)$$

Comments:

* Here, I used $j^2=-1$ as a complex number. You can use $i^2=-1$. It does not matter

* It might be useful to remember that

$$L\{\cos(\omega t)\} = \frac{s}{s^2+\omega^2}$$