Robotics

PS03 Solution

Problem 1

Given a DC-motor with

- no-load speed N_0: 6,000 rpm
- stall torque T_s: 0.5 gr·cm

What is the maximum mechanical power of the motor (in Watt)?

Problem 1

maximum power

$$P^{\max} = 1/4 \cdot \omega^{\max} \cdot T^{\max}$$

in Watt: $$W = V \cdot A = \frac{kg \cdot m^2}{s^3} = \frac{N \cdot m}{s}$$

T in Nm, ω in rad/s

(note: $rad = 1m/1m = 1$)

Problem 1

6,000 rpm

(rotations aka revolutions aka rounds per minute)

$$1\, rotation = 2\pi\, rad$$

$$1\, rpm = 1/60 \cdot 2\pi \frac{rad}{\mathrm{sec}}$$

$$\Rightarrow 6000 rpm = 100 \cdot 2\pi \frac{rad}{sec} = 200\pi \frac{1}{s} = 200\pi Hz$$

Problem 1

torque „unit“: often as „weight“ times length

- proper physical weight is a force F_w
- F_w = mass · gravity value (g)

$$F_w = m \cdot g \qquad\qquad g = 9.8\frac{m}{s^2}$$

$$1N = 1kg \cdot 1\frac{m}{s^2}$$

Problem 1

„torque“: often as ***mass*** times length

- convert to *kg*
- multiply mass (colloquially often called „weight“) with gravity value g to get a proper force (in N)

$$
\begin{aligned}
F_w &= 0.5gr \cdot 9.8\, {}^{m}\!/_{s^2} \\
&= 5 \cdot 10^{-1} \cdot 10^{-3} kg \cdot 9.8 \frac{m}{s^2} \\
&= 5 \cdot 10^{-4} kg \cdot 9.8\, {}^{m}\!/_{s^2} \\
&= 4.9 \cdot 10^{-3} kg \cdot {}^{m}\!/_{s^2} \\
&= 4.9 \cdot 10^{-3} N
\end{aligned}
$$

Problem 1

torque: often as mass times ***length***

- convert to *m*

$$T_{informal} = 0.5gr \cdot cm$$

$$\begin{aligned} T &= F_w \cdot l \\ &= 4.9 \cdot 10^{-3} N \cdot 1cm \\ &= 4.9 \cdot 10^{-3} N \cdot 10^{-2} m \\ &= 4.9 \cdot 10^{-5} Nm \end{aligned}$$

Problem 1

Note: US/UK often even imperial units for “torque”

- 1 in·lb ≈ 0.113 Nm
- 1 ft·lb ≈ 1.356 Nm
- 1 in·oz ≈ $7.062 \cdot 10^{-3}$ Nm

- 1 Nm ≈ 8.851 in·lb
- 1 Nm ≈ 0.737 ft·lb
- 1 Nm ≈ 141.59 in·oz

Problem 1

maximum power with proper SI-units

$$P^{max} = \frac{1}{4} \cdot \omega^{max} \cdot T^{max}$$

$$= \frac{1}{4} \cdot 200\pi \frac{1}{s} \cdot 4.9 \cdot 10^{-5} Nm$$

$$= \frac{1}{4} \cdot 2\pi \cdot 4.9 \cdot 10^{-3} \frac{Nm}{s}$$

$$= 7.696902 \cdot 10^{-3} \frac{Nm}{s}$$

$$= 7.696902 \cdot 10^{-3} W$$

Problem 2

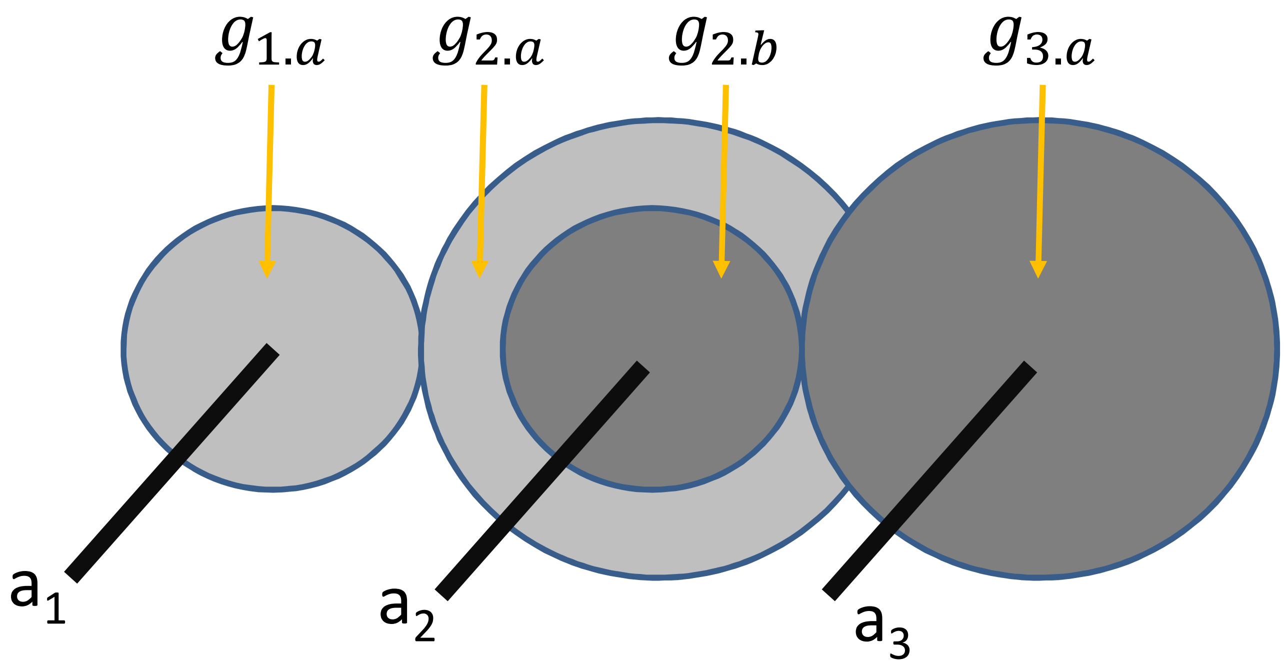
Given a spur gear train with 3 axes a_1 to a_3. On each axis a_i is a gear or two gears $g_{i.x}, x \in \{a, b\}$ with following numbers of teeth

- $g_{1.a}$: 10
- $g_{2.a}$: 20
- $g_{2.b}$: 50
- $g_{3.a}$: 100

Gear $g_{1.a}$ drives $g_{2.a}$, $g_{2.b}$ drives $g_{3.a}$. Given input speed or torque on a_1 calculate the according output on a_3

- input torque a_1: 10 Nm; output torque a_3?
- input speed a_1: 100 rpm; output speed a_3?

Problem 2: gear train

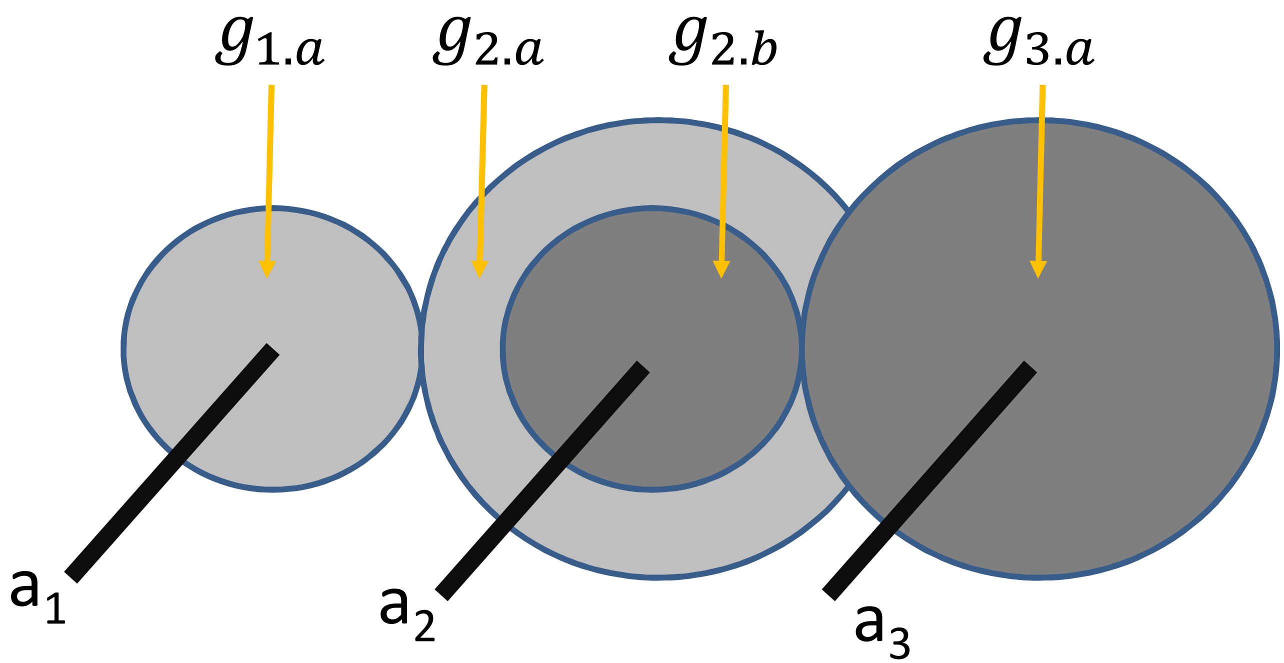


$$z_{1.a} = 10$$
$$z_{2.a} = 20$$
$$z_{2.b} = 50$$
$$z_{3.a} = 100$$

$$-\frac{z_{1.a}}{z_{2.a}} = -\frac{10}{20} = -\frac{1}{2} \quad (GR_1 = 1:2)$$

$$-\frac{z_{2.b}}{z_{3.a}} = -\frac{50}{100} = -\frac{1}{2} \quad (GR_2 = 1:2)$$

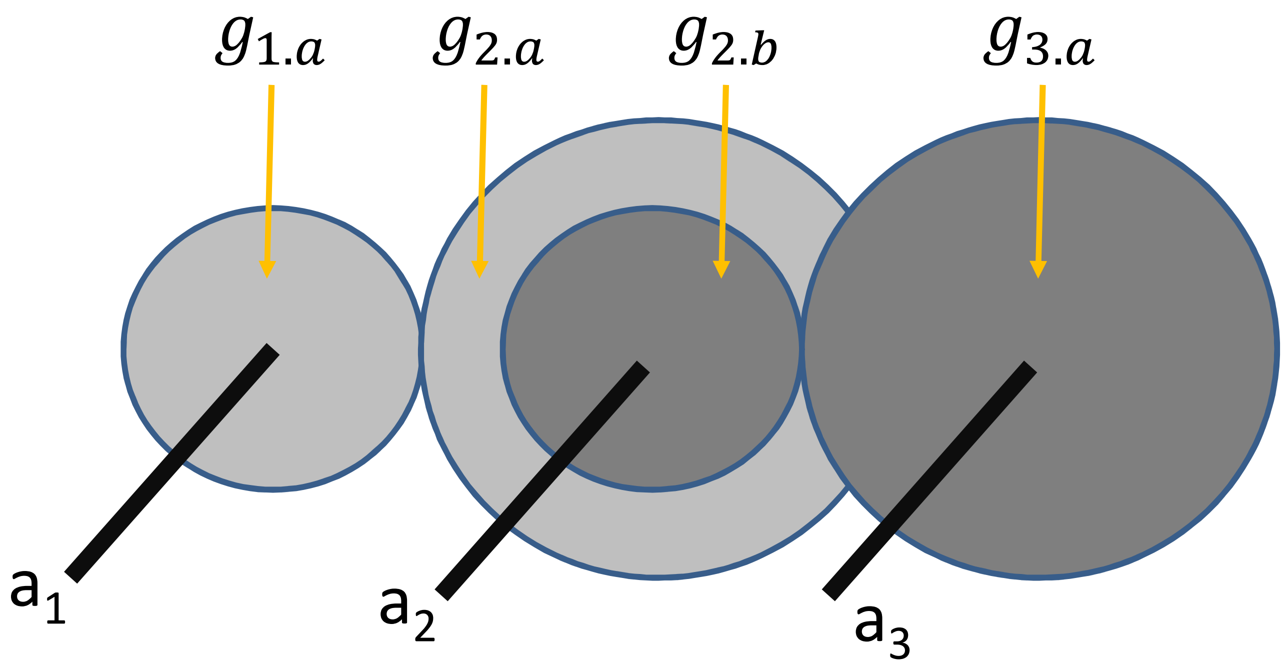
Problem 2: gear train



$$N_3 = -\frac{1}{2} \cdot N_2 = -\frac{1}{2} \cdot -\frac{1}{2} \cdot N_1 = \frac{1}{4} \cdot N_1$$

$$T_3 = \left(-\frac{1}{2}\right)^{-1} \cdot T_2 = \left(-\frac{1}{2} \cdot -\frac{1}{2}\right)^{-1} \cdot T_1 = 4 \cdot T_1$$

Problem 2: gear train



$$N_3 = \frac{1}{4} \cdot 100\, rpm = 25\, rpm$$

$$T_3 = 4 \cdot 10\, Nm = 40\, Nm$$

Problem 3

Given

1. a worm gear G_w with
   - $z_w = 100$ teeth on the worm wheel
2. a planetary gear G_p with
   - 4 planet gears
   - $z_p = 50$ teeth on each planet
   - $z_i = 200$ teeth on the internal gear
   - $z_s = 10$ teeth on the sun

What is the gear ratio of G_w, respectively of G_p?

Problem 3

worm gear $GR_w = \frac{1}{z_w} = \frac{1}{100}$ $(= 1:100)$

notes

- axes perpendicular to each other
  (sense of direction depends on the worm)
- high GR, but also high friction losses
  (significant amount of sliding friction)

Problem 3

planetary gear

- number of planets does not matter
- number of planet teeth does not matter (idler)

$$GR_p = \frac{N_C}{N_S} = \frac{z_S}{z_I + z_S} = \frac{10}{210} = \frac{1}{21}$$

(sun: input, carrier: output)