We want to show: $2\,E_X E_{Y|X}[(f(x) - E[Y|X])(E[Y|X] - Y)|X] = 0$

We start by using the fact that both $f(x)$ & $E[Y|X]$ are independent of Y, and thus we can "pull them out" of $E_{Y|X}[\cdot|X]$ by linearity

$$\Rightarrow \quad E_{Y|X}[(f(x) - E[Y|X])(E[Y|X] - Y)|X] = (f(x) - E[Y|X])\, E_{Y|X}[E[Y|X] - Y|X]$$

Similarly, since $E[Y|X]$ does not depend on Y & by linearity of $E[\cdot]$, we have

$$E_{Y|X}[(E[Y|X] - Y)|X] = E_{Y|X}[\underbrace{E[Y|X]}_{\text{indep of } Y}|X] - E_{Y|X}[Y|X]$$

$$= E[Y|X] - E[Y|X] = 0$$

$$2\,E_X E_{Y|X}[(f(x) - E[Y|X])(E[Y|X] - Y)|X]$$

$$= 2\,E_X[(f(x) - E[Y|X]) \cdot 0] = 2E_X[0] = 0$$