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Homework 5

Calculus and Linear Algebra
Due: October 13, 2021

Problem 1

(10 points)

Use implicit differentiation to find an equation of the tangent line to the graph of the given equation at the given point.

a) $xy^2 = 3x + y$ at point $(2, 2)$. **(4 points)**

b) $y^{1/2}x^{3/2} + xy^{1/3} = 12$ at point $(2, 8)$. **(4 points)**

c) Show for a) that you get the same tangent if you differentiate with respect to y instead of x. In this case you'll get a slope $\mathrm{d}y/\mathrm{d}x$ and you'll need to use an appropriate line equation. **(2 points)**

Problem 2

(10 points)

a) A balloon is filled at a rate of $0.001\pi\,\mathrm{m}^3$ per second. At what rate is the radius of the balloon increasing when the radius is $20\,\mathrm{cm}$? Be aware of units! **(5 points)**

b) An airplane flying horizontally at a height of $8000\,\mathrm{m}$ with a speed of $500\,\mathrm{m/s}$ passes directly above an observer on the ground. What is the rate of increase of distance to the observer $1\,$minute later? **(5 points)**

Problem 3

(10 points)

a) Show that

$$\frac{\mathrm{d}\arccos(x)}{\mathrm{d}x} = -\frac{1}{\sqrt{1-x^2}}$$

(The function $y = \arccos(x)$ is the (locally) inverse function of $x = cos(y)$.) **(4 points)**

Find all critical points (points where $f'(x) = 0$) for the following functions, and characterize whether they correspond to a local minimum, a local maximum, or neither.

b) $f(x) = 2x^3 - 6x + 9$ **(2 points)**

b) $g(x) = 2x^3 + 6x + 9$ **(2 points)**

b) $h(t) = \sin(\omega t)$ with constant $\omega \neq 0$ **(2 points)**