Robotics
Problem Sheet 8

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Notes

The homework serves as preparation for the exams. It is strongly recommended that you solve them before the given deadline - but you do not need to hand them in. Feel free to work on the problems as a group - this is even recommended.

1 Problem

Given the Gaussian $N(\hat{x}, C)$ representing the estimate of a system state $x = (x_1, x_2)$ and its related uncertainty. At time t, $\hat{x}_t$ and C_t are as follows:

$$\hat{x}_t = (2.1, 3.7)$$

$$C_t = \begin{pmatrix} 0.021 & 0.004 \\ 0.004 & 0.038 \end{pmatrix}$$

The system evolves according to the following function $F()$:

$$F(x) = \begin{pmatrix} sin(x_1) \cdot x_2 \\ cos(x_1) + x_2^2 \end{pmatrix}$$

Use the error propagation law to compute $\hat{x}_{t+1}$ and C_{t+1}.

2 Problem

Given a simple system with a 1D state x that moves proportionally to a system input $u()$, concretely $x_k = x_{k-1} + 5u(k-1)$. Its state, i.e., its 1D location, can be measured with a sensor that behaves linearly, i.e., $z(x) = 0.1x$. Both the motion and the measurement are noisy and can be modeled with zero-mean Gaussians with a variance of $Q = 0.2$, respectively $R = 0.3$.

The system starts at $k = 0$ in state $x = 0$ with no uncertainty. Use a Kalman filter to estimate the system states and the related variances for following inputs and measurements:

k	$u(k-1)$	$z(k)$
1	2.4	1.330
2	1.8	2.031
3	-3.1	-0.370
4	-2.7	-0.588

3 Problem

Given a simple non-linear system with a 1D state x that evolves with input $u()$ as follows $x_k = x_{k-1}^2 + \sin(u(k-1))$. Its state x can be measured with a sensor that also behaves non-linearly with $z(x) = x^3$. Both the motion and the measurement are noisy and can be modeled with zero-mean Gaussians with a variance of $Q = 0.2$, respectively $R = 0.3$.

The system starts at $k = 0$ in state $x = 0$ with no uncertainty. Use an Extended Kalman filter to estimate the system state and the related variance for input $u_0 = \pi/2$ and measurement $z_1 = 1.1$.