Practice Problems II - 03

I. Transfer Functions

a) $M\ddot{x} + f_v\dot{x} + Kx = f(t)$

$$Ms^2X(s) + f_v s X(s) + KX(s) = F(s)$$

$$X(s)(Ms^2 + f_v s + K) = F(s)$$

$$\frac{X(s)}{F(s)} = H(s) = \frac{1}{Ms^2 + f_v s + K}$$

b) $$H(s) = \frac{1}{Ms^2 + f_v s + K} = \frac{1}{1s^2 + 5s + 100}$$

$$H(j\omega) = \frac{1}{(j\omega)^2 + 5(j\omega) + 100} = \frac{1}{-\omega^2 + 5j\omega + 100}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(100)^2 + (5)^2}} =$$

c) Put the value of the answer you get in part b) equals to $-\omega^2 + 5j\omega + 100$ and get the value(s) of ω.

II Modelling in frequency domain

$$M_1 = \left[M_1 s^2 + (K_1 + K_2) + (f_{v_1} + f_{v_3}) s\right] X_1(s) - K_2 X_2(s) - f_{v_3} s X_3(s) = 0$$
$$M_2 = \left[M_2 s^2 + K_2 + (f_{v_2} + f_{v_4}) s\right] X_2(s) - K_2 X_1(s) - f_{v_4} s X_3(s) = F(s)$$
$$M_3 = \left[(f_{v_3} + f_{v_4}) s + M_3 s^2\right] X_3(s) - f_{v_3} s X_1(s) - f_{v_3} s X_2(s) = 0$$

$$\begin{pmatrix} M_1 s^2 + (K_1 + K_2) + (f_{v_1} + f_{v_3}) s & -K_2 & -f_{v_3} s \\ -K_2 & M_2 s^2 + K_2 + (f_{v_3} + f_{v_4}) s & -f_{v_4} s \\ -f_{v_3} s & -f_{v_3} s & (f_{v_3} + f_{v_4}) s + M_3 s^2 \end{pmatrix} \begin{pmatrix} X_1(s) \\ X_2(s) \\ X_3(s) \end{pmatrix} = \begin{pmatrix} 0 \\ F(s) \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} X_1(s) \\ X_2(s) \\ X_3(s) \end{pmatrix} = Z^{-1} \begin{pmatrix} 0 \\ F(s) \\ 0 \end{pmatrix}$$

III. Modeling in Frequency Domain

Mesh 1 : $R_1 I_1(s) + L_1 s I_1(s) \quad - L_1 s I_2(s) \quad - R_1 I_3(s) = V(s)$

Mesh 2 : $L_1 s I_2(s) + R_2 I_2(s) + L_2 s I_2(s) \quad - L_1 s I_1(s) - R_2 I_3(s) = V_L(s)$

Mesh 3 : $L_3 s I_3(s) + R_2 I_3(s) + R_1 I_3(s) \quad - R_1 I_1(s) - R_2 I_2(s) = 0$

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\begin{aligned}
(R_1 + L_1 s) I_1(s) \quad & - L_1 s I_2(s) \quad & - R_1 I_3(s) &= V(s) \\
- L_1 s I_1(s) \quad & (L_1 s + R_2 + L_2 s) I_2(s) \quad & - R_2 I_3(s) &= V_L(s) \\
- R_1 I_1(s) \quad & - R_2 I_2(s) \quad & (L_3 s + R_2 + R_1) I_3(s) &= 0
\end{aligned}
$$

$$
\begin{pmatrix}
R_1 + L_1 s & - L_1 s & - R_1 \\
- L_1 s & L_1 s + R_2 + L_2 s & - R_2 \\
- R_1 & - R_2 & L_3 s + R_2 + R_1
\end{pmatrix}
\begin{pmatrix}
I_1(s) \\ I_2(s) \\ I_3(s)
\end{pmatrix}
=
\begin{pmatrix}
V(s) \\ V_L(s) \\ 0
\end{pmatrix}
$$