# Robotics

## PS09 Solution

# 1 Problem

Consider a 1-dimensional world where a mobile robot $r$ has a 1-dimensional range sensor that returns the distance $d_o$ to the nearest obstacle. An evidence grid $g(x)$ with log odds is to be used for representing uncertainty in a map of the environment. Concretely, a base two logarithm ($log_2$) is used for the log odds.

The robot is supposed to generate a 1D map over 5 cm with a 1 cm resolution, i.e., $g(x)$ holds the occupancy estimate of the area $[xcm, x + 1cm[$. For the sake of convenience, we assume discrete motions and discrete sensor readings.

Given the robot pose $x_r$ and a sensor reading $d_o$, the conditional probability $P(s = d_o | o@x)$, respectively $P(s = d_o | \neg o@x)$ - or short $P(o@x)$ and $P(\neg o@x)$ - of getting sensor value $d_o$ when there is an obstacle at $x$ ("$o@x$"), respectively free space at coordinate $x$ ("$\neg o@x$") is given as:

- $P(o@x) = 0.9 \Leftrightarrow x = x_r + d_o$
- $P(o@x) = 0.3 \Leftrightarrow x = x_r + d_o \pm 1cm$
- $P(\neg o@x) = 0.8 \Leftrightarrow \forall x : x_r \leq x < x_r + d_o - 1cm$
- for all other $x$ it holds that $P(o@x) = P(\neg o@x)$

No information about the environment is given for the initial state of the map, i.e., $\forall x : P(o@x) = P(\neg o@x)$ as long as there are no sensor readings yet.

- What is the initial map $g()_0$ at time $t = 0$, i.e., the value of all cells $g(x)_0$?
- Suppose the robot starts at coordinate (0) and gets a sensor reading of $d_o = 6$ at $t = 1$. What does the map $g()_1$ look like after this sensor reading is integrated in it?
- At $t = 2$, the robot is moving and it gets to coordinate (3). There, the sensor value is $d_o = 4$. What does the map $g()_2$ look like after this sensor reading is used to update the map?
- At $t = 3$, the robot is still at coordinate (3). The sensor value is now $d_o = 3$. What does the map $g()_3$ look like?
- At $t = 4$, the robot is again still at coordinate (3). The sensor value is again $d_o = 3$. What does the map $g()_4$ look like?

# Problem 1

## log odds

$$\log_2\left(\frac{P(o@(x < x_r + d_o - 1))}{P(\neg o@(x < x_r + d_o - 1))}\right) = \log_2\left(\frac{0.2}{0.8}\right) = -2$$

$$\log_2\left(\frac{P(o@(x = x_r + d_o \pm 1))}{P(\neg o@(x = x_r + d_o \pm 1))}\right) = \log_2\left(\frac{0.3}{0.7}\right) = -1.22239$$

$$\log_2\left(\frac{P(o@(x = x_r + d_o))}{P(\neg o@(x = x_r + d_o))}\right) = \log_2\left(\frac{0.9}{0.1}\right) = 3.169925$$

# Problem 1

$$\log_2\left(\frac{P(o@<d_o-1))}{P(\neg o@<d_o-1))}\right)=-2,\log_2\left(\frac{P(o@d_o\pm 1)}{P(\neg o@d_o\pm 1)}\right)=-1.22239,\log_2\left(\frac{P(o@d_o)}{P(\neg o@d_o)}\right)=3.169925$$

| t= | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | initial map | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | robot@(0), d=6 | -2 | -2 | -2 | -2 | -2 | -1.22 | 3.17 | -1.22 | 0 | 0 |
| 2 | robot@(3), d=4 | -2 | -2 | -2 | -4 | -4 | -3.22 | 1.95 | 1.95 | -1.22 | 0 |
| 3 | robot@(3), d=3 | -2 | -2 | -2 | -6 | -6 | -4.44 | 5.12 | 0.73 | -1.22 | 0 |
| 4 | robot@(3), d=3 | -2 | -2 | -2 | -8 | -8 | -5.67 | 8.29 | -0.5 | -1.22 | 0 |