Practice Problems - 05

Practice problems are supposed to help you digest the content of the lecture. It is important that you manage to solve them on your own. Before you write your solutions, you may of course ask questions, and discuss things. In order to prepare for the exam, already now, try to explicitly write down your solutions – clearly and easy to read. Apply definitions properly, and give explanations for what you are doing. That will help you to understand them later when you prepare for the final exam.

I. Motion

1) Consider a mobile robot that moves in the 2D space. It starts at (0,0). It will move along the y axis for 6 meters. Then it will rotate about the origin of the coordinate system by 45 degrees. Then it will move by a vector $\left(3\sqrt{2}\,,-3\sqrt{2}\right)^T$.
   a) Calculate the full transformation matrix as a combination of the three individual transformations, and show the robot path in a graph.
   b) Considering a bang-bang motion law in the first segment, and considering that the robot took exactly 8 seconds to complete it, calculate the acceleration and maximum velocity, and show them on a graph.

2) Consider a mobile robot that moves in the 2D space. It starts at (0,0). It will move along the y axis for -6 meters. Then it will rotate about the origin of the coordinate system by 45 degrees. Then it will move by a vector $\left(-3\sqrt{2}\,,-3\sqrt{2}\right)^T$.
   a) Calculate the final transformation matrix as a combination of the three individual transformations, and show the robot path in a graph.
   b) Considering a bang-bang motion law in the first segment, and considering that the robot took exactly 10 seconds to complete it, calculate the acceleration and maximum velocity, and show them on a graph.

3) Consider a mobile robot that moves in the 2D space. It starts at (0,0). It will move along the y axis for 8 meters. Then it will rotate about the origin of the coordinate system by 45 degrees. Then it will move by a vector $\left(4\sqrt{2}\,,-4\sqrt{2}\right)^T$.
   a) Calculate the final transformation matrix as a combination of the three individual transformations, and show the robot path in a graph.
   b) Considering a bang-bang motion law in the first segment, and considering that the robot took exactly 8 seconds to complete it, calculate the acceleration and maximum velocity, and show them on a graph.

4) Consider a mobile robot that moves in the 2D space. Axis units are meters. It starts at (0,0), denoted as O. It will move along the y axis for 12 meters, arriving at point A. Then it will rotate about the origin of the coordinate system by 45 degrees, arriving at point B. Then it will move by a vector $\left(0,-6\sqrt{2}\right)^T$ arriving at point C and then by a vector $\left(6\sqrt{2}\,,0\right)^T$ arriving at point D.

a) Considering the vectors $\overrightarrow{BC}$ and $\overrightarrow{CD}$, calculate the dot product and the cross product
b) Considering the vectors $\overrightarrow{OA}$ and $\overrightarrow{CD}$, calculate the sum $\overrightarrow{OA} + \overrightarrow{CD}$ and the difference $\overrightarrow{OA} - \overrightarrow{CD}$
c) Discuss whether the four operations above are commutative
d) Calculate the final transformation matrix as a combination of the four individual transformations, and show the robot path in a graph.
e) Considering a motion with uniform acceleration $a = 1m/s^2$ during the first segment $\overrightarrow{OA}$, calculate the time needed to arrive at A and the velocity.
f) Considering that the robot will continue in the path AB with the same velocity magnitude reached at point A, calculate the angular velocity and the tangential and centripetal accelerations.
g) Considering that the robot will then move on BC and CD with a velocity of magnitude 1m/s, considering that it will smooth out the trajectory, without passing by C (over-fly), considering that the transition time between BC and CD is 3 seconds, calculate the point where the robot leaves the segment BC and the point where it arrives to the segment CD.
h) Express in polar and spherical coordinates the points O, A, and B.

5) Consider a mobile robot that moves in the 2D space. Axis units are meters. It starts at (0,0), denoted as O. It will move along the x axis for 12 meters, arriving at point A. Then it will rotate about the origin of the coordinate system by 45 degrees, arriving at point B. Then it will move by a vector $\left(-6\sqrt{2}, 0\right)^T$ arriving at point C and then by a vector $\left(0, -6\sqrt{2}\right)^T$ arriving at point D.
    a) Considering the vectors $\overrightarrow{BC}$ and $\overrightarrow{CD}$, calculate the dot product and the cross product
    b) Considering the vectors $\overrightarrow{OA}$ and $\overrightarrow{CD}$, calculate the sum $\overrightarrow{OA} + \overrightarrow{CD}$ and the difference $\overrightarrow{OA} - \overrightarrow{CD}$
    c) Discuss whether the four operations above are commutative
    d) Calculate the full transformation matrix as a combination of the four individual transformations, and show the robot path in a graph.
    e) Considering a motion with uniform acceleration $a = 1m/s^2$ during the first segment $\overrightarrow{OA}$, calculate the time needed to arrive at A and the velocity.
    f) Considering that the robot will continue in the path AB with the same velocity magnitude reached at point A, calculate the angular velocity and the tangential and centripetal accelerations.
    g) Considering that the robot will then move on BC and CD with a velocity of magnitude 1m/s, considering that it will smooth out the trajectory, without passing by C (over-fly), considering that the transition time between BC and CD is 3 seconds, calculate the point where the robot leaves the segment BC and the point where it arrives to the segment CD.
    h) Express in polar and spherical coordinates the points O, A, and B.