

30-11-2021

Week #14 lecture #18

Wednesday

Convolutional Neural Networks (CNNs)

We run a filter throughout an image and each time we get a result. At the end we get a filtered image.

filter could be considered as weights.

After element wise multiplication, & also doing

sum of all these elements. N

Also sum all values in filter n weighted

by sum of filter & replace central element

in image part with this divided result. $\left(\frac{N}{n}\right)$

This is called image smoothing / noise reduction

Usually we copy original image & apply filters on copied image & result is that image

After applying filter to 1 part, then we go to next image part which is after skipping 1 column/pixel to the right

filter.

- ① To smooth Image / Noise reduction.

1	1	1
1	1	1
1	1	1

- ② To remove ~~horizontal~~ horizontal lines / Detect vertical lines.

-1	0	1
-1	0	1
-1	0	1

- ③ To remove Vertical lines / Detect horizontal lines.

-1	-1	-1
0	0	0
1	1	1

In General:-

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

~~Result~~ filtered image is called ~~Convolution~~ Features Map
Example.

22	22	22	22	22
22	22	22	22	22
50	50	50	50	22
22	50	50	50	22
22	22	22	22	22

Image

-1	-1	-1
0	0	0
1	1	1

filter

Applying filter.

$$\begin{aligned}
 & 22 \times -1 + 22 \times -1 + 22 \times -1 \\
 & + 22 \times 0 + 22 \times 0 + 22 \times 0 \\
 & + 22 \times 1 + 50 \times 1 + 50 \times 1
 \end{aligned}
 \quad \begin{aligned}
 & \text{all } 22 = 2 \\
 & \text{all } 50 = 5
 \end{aligned}$$

$$= \boxed{6} \text{ Activation}$$

6		

← features Map

Apply filter to next part

$$\begin{aligned}
 & 22 \times -1 + 22 \times -1 + 22 \times -1 \\
 & + 22 \times 0 + 22 \times 0 + 22 \times 0 \\
 & + 50 \times 1 + 50 \times 1 + 50 \times 1
 \end{aligned}$$

$$= 9$$

6	9	

Apply filter again

$$\begin{aligned} & 22x_{-1} + 22x_{-1} + 22x_{-1} \\ & + 22x_0 + 22x_0 + 22x_0 \\ & + 50x_1 + 50x_1 + 22x_1 \end{aligned}$$

$$= 6$$

6	9	6

Apply filter again Now shifted down

$$\begin{aligned} & 22x_{-1} + 22x_{-1} + 22x_{-1} \\ & + 22x_0 + 50x_0 + 50x_0 \\ & + 22x_1 + 50x_1 + 50x_1 \end{aligned}$$

$$= 6$$

6	9	6
6		

After applying filter to all image we get

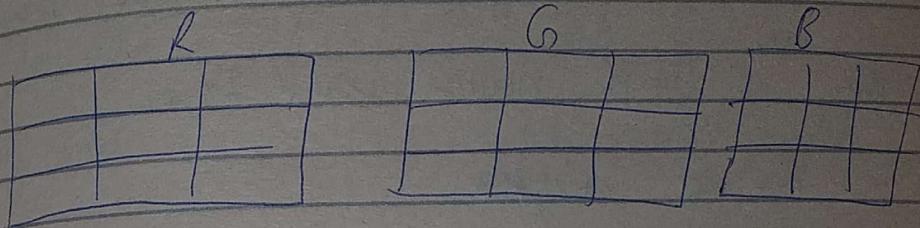
6	9	6
6	9	6
-6	-9	-6

features Map result.

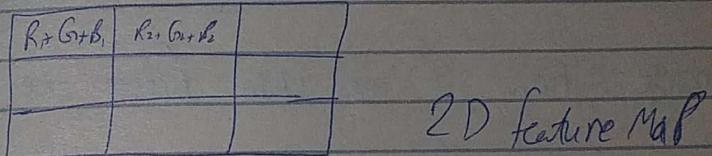
In case of RGB, we have filter size as

$3 \times 3 \times 3$
width \times height \times channels

Now we apply 3 filters on each channel



Now, we have feature Map simply 3×3



After applying filter then we add all values
Activation values for all channels and place in features Map.

Advantages:

- ① In CIFAR dataset we had 3072 features

X_1

X_2

X_{3072}

and if a single neuron attached to all inputs
we had 3072 weights, if we had 200 neurons
then we had 3072×200 weights.

Now, if an image had $200 \times 200 \times 3$ size,
then a single neuron would have
120,000 weights

In all of this, a neuron is looking at the
image, now if we use filter as weights for
a single neuron. Now no. of weights are
drastically reduced.

② In previous case, we were looking at a weight
for a single pixel. Now, a weight is
looking at multiple pixels.

③ Optimization is increased.

How to get size of our feature Map

First we ~~apply~~ have an image \Rightarrow 1D

e.g. $|12|10|15|5|$ image

And have filter $\begin{bmatrix} -1 & 1 \end{bmatrix} \rightarrow$ detects vertical lines

Apply filter $\begin{bmatrix} -1 \\ 1 \end{bmatrix} \rightarrow$ detects horizontal lines

$$|2x-1+10x| = 8 - 2$$

$$|0x-1+15x| = 5$$

$$|8x-1+5x| = -10$$

feature Map $| -2 | 5 | -10 |$

In general, we have odd numbered filter size.

size of feature map:

$$R = S - F + 1$$

↙ Apply this formula
for width & height
separately.

S = size of image

F = size of filter.

e.g. $S = 4$

$$F = 2$$

$$\Rightarrow R = 4 - 2 + 1 \Rightarrow R = 3$$

2D Image feature Map size:

$$S = \frac{5}{3} \times 7$$

$$F = \frac{3}{3} \times 3$$

for width:

$$W = 7 - 3 + 1$$

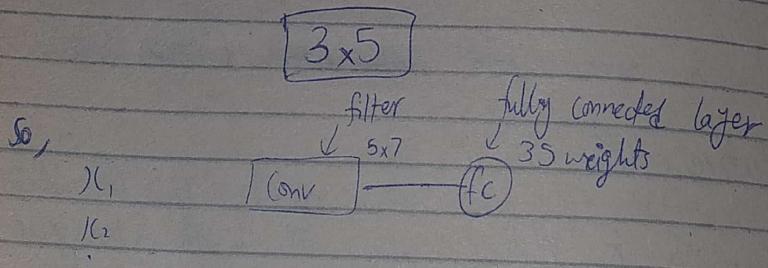
$$\boxed{W = 5}$$

for height:

$$h = 5 - 3 + 1$$

$$\boxed{h = 3}$$

Resultant Image size would be



We initialize convolutional layers in front of NN to reduce dimensions & then at the end we have

fully connected layer.

If we want our feature Map to be same size as

original image size, we do zero padding

zero padding..

Add zero's to original image.

0	0	0	0	0
0				0
0				0
0				0
0	0	0	0	0

1 Zero Padding.

so that our image don't get down sampled

as we have added 1 to left & 1 to right in

so

width size formula becomes.

$$h = I_h - F_h + 1 + 2P \quad P=1 \text{ for above.}$$

$$w = I_w - F_w + 1 + 2P$$

I_h = height of Image
 I_w = width of Image

what

then to Add Padding if filter size change..

as filter size becomes big, our feature Map gets small

we want formula so we get feature map sizes

original image size

Zero padding depends on filter size

$$P = (F-1)/2 \quad \text{for both height & width}$$

$$\text{or } P = \left\lfloor \frac{F-1}{2} \right\rfloor$$

Example:

Image size = 13×13

Filter size = 5×5

Find Padding:

$$P = \frac{5-1}{2}$$

$$P = \frac{4}{2}$$

$$P=2$$

$$h = 13 - 5 + 1 + 2(2)$$

$$= 13$$

$$w = 13 - 5 + 1 + 2(2) = 13$$

if image is 13×12
filter 5×5 $P = 2$

$$h = 13 - 5 + 1 + 2(2) = 13$$

$$w = 12 - 5 + 1 + 2(2) = 12$$

So, image size doesn't affect formulas.

filter size is usually square with same height & width

Jump in filters/stride:

before it was 1 pixel jump to right or below

Now this changes our features map.

① Computation is decreased

② Dimensions are decreased further.

③ Some pixels are not looked at. So weights are only looking at some pixels.

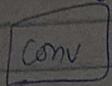
Our formula becomes

$$w = \frac{(I_w - F_w + 2P)}{S} + 1 \quad | \quad h = \frac{(I_h - F_h + 2P)}{S} + 1$$

S = stride

Multiple filters in one convolution layer.

one conv layer may have multiple filter.



$3 \times 5 \times 3$
No. of filters
Filter size
channels

Image shapes to be resized to same shape.