NIM Genap: 12821046 (Fardhan Indrayesa)

Turunkan $\frac{\partial^2 u}{\partial x^2}$ untuk skema beda mundur orde $O[(\Delta x)^4]$!

Untuk menurunkan formula tersebut, dibutuhkan 5 persamaan dengan masing-masing persamaan ditulis hingga orde keenam.

$$u(x-\Delta x) = u(x) - \frac{\partial u}{\partial x}\frac{\Delta x}{1!} + \frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} - \frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} + \frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} - \frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} + \frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots \quad (1)$$

$$u(x-2\Delta x) = u(x) - 2\frac{\partial u}{\partial x}\frac{\Delta x}{1!} + 2^2\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} - 2^3\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} + 2^4\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} - 2^5\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} + 2^6\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots \quad (2)$$

$$u(x-3\Delta x) = u(x) - 3\frac{\partial u}{\partial x}\frac{\Delta x}{1!} + 3^2\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} - 3^3\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} + 3^4\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} - 3^5\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} + 3^6\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots \quad (3)$$

$$u(x-4\Delta x) = u(x) - 4\frac{\partial u}{\partial x}\frac{\Delta x}{1!} + 4^2\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} - 4^3\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} + 4^4\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} - 4^5\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} + 4^6\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots \quad (4)$$

$$u(x-5\Delta x) = u(x) - 5\frac{\partial u}{\partial x}\frac{\Delta x}{1!} + 5^2\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} - 5^3\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} + 5^4\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} - 5^5\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} + 5^6\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots \quad (5)$$

- Menghilangkan turunan pertama

  Kurangkan persamaan 2 × (1) dengan (2)

$$2u(x-\Delta x) = 2u(x) - 2\frac{\partial u}{\partial x}\frac{\Delta x}{1!} + 2\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} - 2\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} + 2\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} - 2\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} + 2\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots$$

$$u(x-2\Delta x) = u(x) - 2\frac{\partial u}{\partial x}\frac{\Delta x}{1!} + 2^2\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} - 2^3\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} + 2^4\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} - 2^5\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} + 2^6\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots$$

$$2u(x-\Delta x) - u(x-2\Delta x) = u(x) - 2\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} + 6\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} - 14\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} + 30\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} - 62\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots \quad (6)$$

Kurangkan persamaan 3 × (1) dengan (3)

$$3u(x-\Delta x) = 3u(x) - 3\frac{\partial u}{\partial x}\frac{\Delta x}{1!} + 3\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} - 3\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} + 3\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} - 3\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} + 3\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots$$

$$u(x-3\Delta x) = u(x) - 3\frac{\partial u}{\partial x}\frac{\Delta x}{1!} + 3^2\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} - 3^3\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} + 3^4\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} - 3^5\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} + 3^6\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots$$

$$3u(x-\Delta x) - u(x-3\Delta x) = 2u(x) - 6\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} + 24\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} - 78\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} + 240\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} - 726\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots \quad (7)$$

Kurangkan persamaan 4 × (1) dengan (4)

$$4u(x-\Delta x) = 4u(x) - 4\frac{\partial u}{\partial x}\frac{\Delta x}{1!} + 4\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} - 4\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} + 4\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} - 4\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} + 4\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots$$

$$u(x-4\Delta x) = u(x) - 4\frac{\partial u}{\partial x}\frac{\Delta x}{1!} + 4^2\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} - 4^3\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} + 4^4\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} - 4^5\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} + 4^6\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots$$

$$4u(x-\Delta x) - u(x-4\Delta x) = 3u(x) - 12\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} + 60\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} - 252\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} + 1020\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} - 4092\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots \quad (8)$$

Kurangkan persamaan 5 × (1) dengan (5)

$$5u(x-\Delta x) = 5u(x) - 5\frac{\partial u}{\partial x}\frac{\Delta x}{1!} + 5\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} - 5\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} + 5\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} - 5\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} + 5\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots$$

$$u(x-5\Delta x) = u(x) - 5\frac{\partial u}{\partial x}\frac{\Delta x}{1!} + 5^2\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} - 5^3\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} + 5^4\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} - 5^5\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} + 5^6\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots$$

$$5u(x-\Delta x) - u(x-5\Delta x) = 4u(x) - 20\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} + 120\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!} - 620\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!} + 3120\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!} - 15620\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots \quad (9)$$

Didapat empat persamaan tanpa turunan pertama

$$2u(x-\Delta x)-u(x-2\Delta x)=u(x)-2\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+6\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!}-14\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!}+30\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}-62\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (6)$$

$$3u(x-\Delta x)-u(x-3\Delta x)=2u(x)-6\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+24\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!}-78\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!}+240\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}-726\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (7)$$

$$4u(x-\Delta x)-u(x-4\Delta x)=3u(x)-12\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+60\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!}-252\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!}+1020\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}-4092\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (8)$$

$$5u(x-\Delta x)-u(x-5\Delta x)=4u(x)-20\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+120\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!}-620\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!}+3120\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}-15620\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (9)$$

- Menghilangkan turunan ketiga

  Kurangkan persamaan 4 × (6) dengan (7)

$$8u(x-\Delta x)-4u(x-2\Delta x)=4u(x)-8\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+24\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!}-56\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!}+120\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}-248\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots$$

$$3u(x-\Delta x)-u(x-3\Delta x)=2u(x)-6\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+24\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!}-78\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!}+240\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}-726\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots$$

$$5u(x-\Delta x)-4u(x-2\Delta x)+u(x-3\Delta x)=2u(x)-2\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+22\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!}-120\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}+478\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (10)$$

  Kurangkan persamaan 10 × (6) dengan (8)

$$20u(x-\Delta x)-10u(x-2\Delta x)=10u(x)-20\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+60\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!}-140\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!}+300\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}-620\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots$$

$$4u(x-\Delta x)-u(x-4\Delta x)=3u(x)-12\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+60\frac{\partial^3 u}{\partial x^3}\frac{(\Delta x)^3}{3!}-252\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!}+1020\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}-4092\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots$$

$$16u(x-\Delta x)-10u(x-2\Delta x)+u(x-4\Delta x)=7u(x)-8\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+112\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!}-720\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}+3472\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (11)$$

Kurangkan persamaan 20 × (6) dengan (9)

$$40u(x-\Delta x)-20u(x-2\Delta x)=20u(x)-40\frac{\partial^2u}{\partial x^2}\frac{(\Delta x)^2}{2!}+120\frac{\partial^3u}{\partial x^3}\frac{(\Delta x)^3}{3!}-280\frac{\partial^4u}{\partial x^4}\frac{(\Delta x)^4}{4!}+600\frac{\partial^5u}{\partial x^5}\frac{(\Delta x)^5}{5!}-1240\frac{\partial^6u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots$$

$$5u(x-\Delta x)-u(x-5\Delta x)=4u(x)-20\frac{\partial^2u}{\partial x^2}\frac{(\Delta x)^2}{2!}+120\frac{\partial^3u}{\partial x^3}\frac{(\Delta x)^3}{3!}-620\frac{\partial^4u}{\partial x^4}\frac{(\Delta x)^4}{4!}+3120\frac{\partial^5u}{\partial x^5}\frac{(\Delta x)^5}{5!}-15620\frac{\partial^6u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots$$

$$35u(x-\Delta x)-20u(x-2\Delta x)+u(x-5\Delta x)=16u(x)-20\frac{\partial^2u}{\partial x^2}\frac{(\Delta x)^2}{2!}+340\frac{\partial^4u}{\partial x^4}\frac{(\Delta x)^4}{4!}-2520\frac{\partial^5u}{\partial x^5}\frac{(\Delta x)^5}{5!}+14380\frac{\partial^6u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (12)$$

Didapat tiga persamaan tanpa turunan pertama dan ketiga

$$5u(x-\Delta x)-4u(x-2\Delta x)+u(x-3\Delta x)=2u(x)-2\frac{\partial^2u}{\partial x^2}\frac{(\Delta x)^2}{2!}+22\frac{\partial^4u}{\partial x^4}\frac{(\Delta x)^4}{4!}-120\frac{\partial^5u}{\partial x^5}\frac{(\Delta x)^5}{5!}+478\frac{\partial^6u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (10)$$

$$16u(x-\Delta x)-10u(x-2\Delta x)+u(x-4\Delta x)=7u(x)-8\frac{\partial^2u}{\partial x^2}\frac{(\Delta x)^2}{2!}+112\frac{\partial^4u}{\partial x^4}\frac{(\Delta x)^4}{4!}-720\frac{\partial^5u}{\partial x^5}\frac{(\Delta x)^5}{5!}+3472\frac{\partial^6u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (11)$$

$$35u(x-\Delta x)-20u(x-2\Delta x)+u(x-5\Delta x)=16u(x)-20\frac{\partial^2u}{\partial x^2}\frac{(\Delta x)^2}{2!}+340\frac{\partial^4u}{\partial x^4}\frac{(\Delta x)^4}{4!}-2520\frac{\partial^5u}{\partial x^5}\frac{(\Delta x)^5}{5!}+14380\frac{\partial^6u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (12)$$

- Menghilangkan turunan keempat
  Kurangkan persamaan 56 × (10) dengan 11 × (11)

$$280u(x-\Delta x)-224u(x-2\Delta x)+56u(x-3\Delta x)=112u(x)-112\frac{\partial^2u}{\partial x^2}\frac{(\Delta x)^2}{2!}+1232\frac{\partial^4u}{\partial x^4}\frac{(\Delta x)^4}{4!}-6720\frac{\partial^5u}{\partial x^5}\frac{(\Delta x)^5}{5!}+26768\frac{\partial^6u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots$$

$$176u(x-\Delta x)-110u(x-2\Delta x)+11u(x-4\Delta x)=77u(x)-88\frac{\partial^2u}{\partial x^2}\frac{(\Delta x)^2}{2!}+1232\frac{\partial^4u}{\partial x^4}\frac{(\Delta x)^4}{4!}-7920\frac{\partial^5u}{\partial x^5}\frac{(\Delta x)^5}{5!}+38192\frac{\partial^6u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots$$

$$104u(x-\Delta x)-114u(x-2\Delta x)+56u(x-3\Delta x)-11u(x-4\Delta x)=35u(x)-24\frac{\partial^2u}{\partial x^2}\frac{(\Delta x)^2}{2!}+1200\frac{\partial^5u}{\partial x^5}\frac{(\Delta x)^5}{5!}-11424\frac{\partial^6u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (13)$$

Kurangkan persamaan 170 × (10) dengan 11 × (12)

$$850u(x-\Delta x)-680u(x-2\Delta x)+170u(x-3\Delta x)=340u(x)-340\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+3740\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!}-20400\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}+81260\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots$$

$$385u(x-\Delta x)-220u(x-2\Delta x)+11u(x-5\Delta x)=176u(x)-220\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+3740\frac{\partial^4 u}{\partial x^4}\frac{(\Delta x)^4}{4!}-27720\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}+158180\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots$$

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$$465u(x-\Delta x)-460u(x-2\Delta x)+170u(x-3\Delta x)-11u(x-5\Delta x)=164u(x)-120\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+7320\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}-76920\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (14)$$

Didapat dua persamaan tanpa turunan pertama, ketiga, dan keempat

$$104u(x-\Delta x)-114u(x-2\Delta x)+56u(x-3\Delta x)-11u(x-4\Delta x)=35u(x)-24\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+1200\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}-11424\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (13)$$

$$465u(x-\Delta x)-460u(x-2\Delta x)+170u(x-3\Delta x)-11u(x-5\Delta x)=164u(x)-120\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+7320\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}-76920\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (14)$$

- Menghilangkan turunan kelima

  Kurangkan persamaan 61 × (13) dengan 10 × (14)

$$6344u(x-\Delta x)-6954u(x-2\Delta x)+3416u(x-3\Delta x)-671u(x-4\Delta x)=2135u(x)-1464\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+73200\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}-696864\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots$$

$$4650u(x-\Delta x)-4600u(x-2\Delta x)+1700u(x-3\Delta x)-110u(x-5\Delta x)=1640u(x)-1200\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+73200\frac{\partial^5 u}{\partial x^5}\frac{(\Delta x)^5}{5!}-769200\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots$$

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$$1694u(x-\Delta x)-2354u(x-2\Delta x)+1716u(x-3\Delta x)-671u(x-4\Delta x)+110u(x-5\Delta x)=495u(x)-264\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!}+72336\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!}+\ldots \quad (15)$$

Evaluasi persamaan (15) untuk turunan kedua

$$264\frac{\partial^2 u}{\partial x^2}\frac{(\Delta x)^2}{2!} = 495u(x) - 1694u(x-\Delta x) + 2354u(x-2\Delta x) - 1716u(x-3\Delta x) + 671u(x-4\Delta x) - 110u(x-5\Delta x) + 72336\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots$$

$$132\frac{\partial^2 u}{\partial x^2}(\Delta x)^2 = 495u(x) - 1694u(x-\Delta x) + 2354u(x-2\Delta x) - 1716u(x-3\Delta x) + 671u(x-4\Delta x) - 110u(x-5\Delta x) + 72336\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^6}{6!} + \ldots$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{495u(x) - 1694u(x-\Delta x) + 2354u(x-2\Delta x) - 1716u(x-3\Delta x) + 671u(x-4\Delta x) - 110u(x-5\Delta x)}{132(\Delta x)^2} + 548\frac{\partial^6 u}{\partial x^6}\frac{(\Delta x)^4}{6!} + \ldots$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{12}\left[\frac{45u(x) - 154u(x-\Delta x) + 214u(x-2\Delta x) - 156u(x-3\Delta x) + 61u(x-4\Delta x) - 10u(x-5\Delta x)}{(\Delta x)^2}\right] + O[(\Delta x)^4]$$

Jadi, formula untuk turunan kedua skema beda mundur dengan orde akurasi 4 adalah

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{12}\left[\frac{45u(x) - 154u(x-\Delta x) + 214u(x-2\Delta x) - 156u(x-3\Delta x) + 61u(x-4\Delta x) - 10u(x-5\Delta x)}{(\Delta x)^2}\right] + O[(\Delta x)^4]$$