Practical 1

To solve partial differential equation

DSolve[eqn, u[x,y], {x,y}]- to solve a PDE for u[x,y].
The general solutions to partial differential equations involve arbitrary functions. These functions are labeled as C[i]. DSolve works for PDEs having two independent variables.

First-Order PDEs - Linear and Quasi-Linear PDEs

A first-order PDE for an unknown function u(x,y) is said to be linear if it can be expressed in the form

$$a(x, y)\frac{\partial u(x, y)}{\partial x} + b(x, y)\frac{\partial u(x, y)}{\partial y} + c(x, y)\,u(x, y) = d(x, y)$$

The PDE is said to be quasilinear if it can be expressed in the form

$$a(x, y, u(x, y))\frac{\partial u(x, y)}{\partial x} + b(x, y, u(x, y))\frac{\partial u(x, y)}{\partial y} = c(x, y, u(x, y))$$

Q1. Solve the first order linear partial differential equation

$y\,u_x - x\,u_y = 0$

Solution:

```
In[∘]:= eqn = y D[u[x, y], x] - x D[u[x, y], y] == 0;
        DSolve[eqn, u[x, y], {x, y}]
```

Out[∘]= $\left\{\left\{u[x, y] \to c_1\left[\frac{1}{2}\left(x^2 + y^2\right)\right]\right\}\right\}$

Q2. Solve the first order linear partial differential equation

$x\,u_x - y\,u_y = 0$

Solution:

```
In[∘]:= eqn1 = x D[u[x, y], x] - y D[u[x, y], y] == 0;
        DSolve[eqn1, u[x, y], {x, y}]
```

Out[∘]= $\{\{u[x, y] \to c_1[x\,y]\}\}$

Q. Solve the first order linear partial differential equation

$2\,u_x + 3\,u_y + u = 0$

Solution:

```
In[∘]:= eqna = 2 D[u[x, y], x] + 3 D[u[x, y], y] + u[x, y] == 0;
        DSolve[eqna, u[x, y], {x, y}]
```

Out[∘]= $\left\{\left\{u[x, y] \to e^{-x/2}\,c_1\left[-\frac{3x}{2} + y\right]\right\}\right\}$

Second-Order PDEs

The general form of a linear second-order PDE
is

$$a \frac{\partial^2 u}{\partial^2 x} + b \frac{\partial^2 u\,(x,y)}{\partial x\, \partial y} + c \frac{\partial^2 u}{\partial^2 y} + d \frac{\partial u}{\partial x} + e \frac{\partial u}{\partial y} + fu = g$$

Here u=u(x,y), and a, b, c, d, e, f, and g are functions of x and y only-they do not depend on u. If g = 0, the equation is said to be **homogeneous.** *DSolve* can find the general solution for a homogeneous linear second-order PDE of the form

$$a \frac{\partial^2 u}{\partial^2 x} + b \frac{\partial^2 u\,(x,y)}{\partial x\, \partial y} + c \frac{\partial^2 u}{\partial^2 y} = 0.$$

Here, a,b, and c are constants

Q3. Solve the second order linear partial differential equation

$u_{yy}+ u=0$

Solution:

```
In[ ]:= eqn2 = D[u[x, y], {y, 2}] + u[x, y] == 0;
DSolve[eqn2, u[x, y], {x, y}]
Out[ ]= {{u[x, y] → Cos[y] ℂ1[x] + Sin[y] ℂ2[x]}}
```

Q4. Solve the Laplace’s equation

```
In[ ]:= LaplaceEqn =  D[u[x, y], {x, 2}] + D[u[x, y], {y, 2}] == 0;
DSolve[LaplaceEqn, u[x, y], {x, y}]
Out[ ]= {{u[x, y] → ℂ1[ⅈ x + y] + ℂ2[-ⅈ x + y]}}
```

Q5. Solve the second order linear partial differential equation

$3\, u_{xx} + u_{xy} + 5\, u_{yy}=0$

Solution:

```
In[ ]:= eqn3 = 3 D[u[x, y], {x, 2}] + D[u[x, y], x, y] + 5 D[u[x, y], {y, 2}] == 0;
DSolve[eqn3, u[x, y], {x, y}]
```

Out[ ]= $\left\{\left\{u[x,y] \to ℂ_1\left[\frac{1}{6}\left(-1+ i\sqrt{59}\right)x+y\right]+ℂ_2\left[\frac{1}{6}\left(-1-i\sqrt{59}\right)x+y\right]\right\}\right\}$

Q. Solve the second order linear partial differential equation

$x^2\, u_{xx} - y^2\, u_{yy}=0$

Solution:

In[ ]:= DSolve$\left[x^2\, D[u[x, y], \{x, 2\}] - y^2\, D[u[x, y], \{y, 2\}] == 0, u[x, y], \{x, y\}\right]$

Out[ ]= DSolve$\left[-y^2\, u^{(0,2)}[x, y] + x^2\, u^{(2,0)}[x, y] == 0, u[x, y], \{x, y\}\right]$

Practicals-2

Solution and plotting of Cauchy problem for first order PDEs

Problem-:1 **Obtain the solution of the linear equation**

$$u_x - u_y = 1,$$

with the Cauchy data u(x,0) = x^2**.**

Plot the integral surface with in the range {x, -4, 4} & {y, -5, 5}.

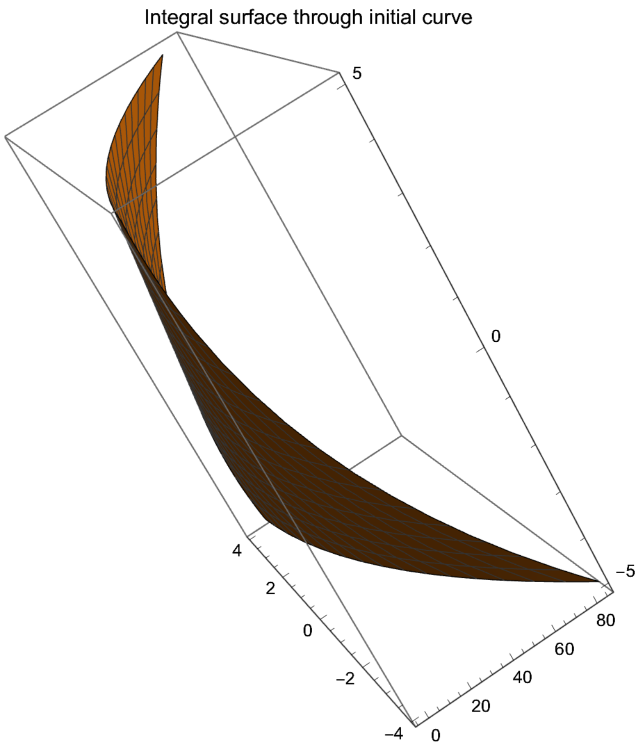
Solution-:

In[60]:=
```
pde = D[u[x, y], x] - D[u[x, y], y] == 1
sol = DSolve[{pde, u[x, 0] == x^2}, u[x, y], {x, y}]
g1 = ParametricPlot3D[{0, t^2, 0}, {t, -2, 2}];
g2 = Plot3D[u[x, y] /. sol, {x, -4, 4}, {y, -5, 5},
  PlotLabel → "Integral surface through initial curve"]
Show[g1, g2]
```

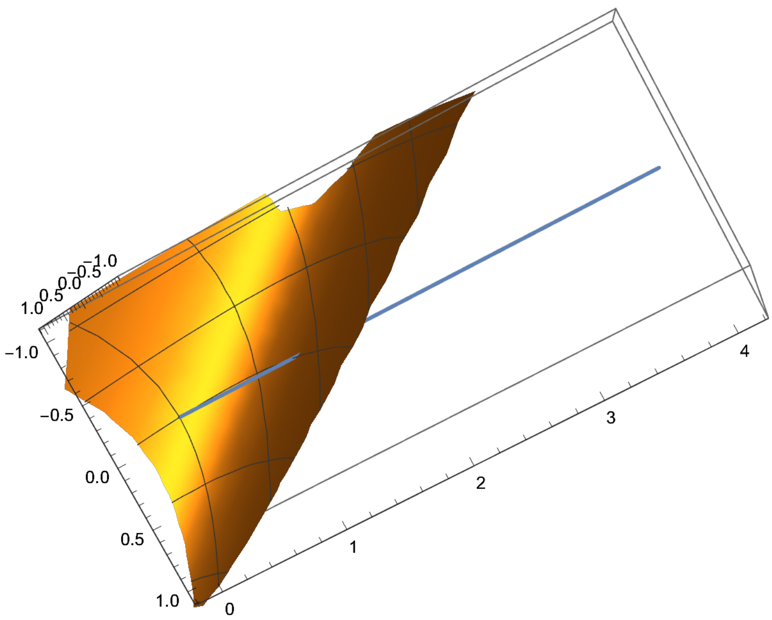
Out[60]= $-u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$

Out[61]= $\{\{u[x, y] \to x^2 - y + 2\,x\,y + y^2\}\}$

Out[63]=



Out[64]=



Problem-2: Find the solution of the equation

$$y u_x - 2\,x y u_y = 2xu,$$

with the Cauchy data $u(0,y) = y^3$.

Plot the integral surface with in the range {x, -7, 7} & {y, -5, 5}.

Solution:-

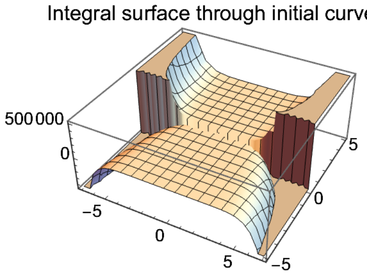
```
h = y * D[u[x, y], x] - 2 * x * y * D[u[x, y], y] == 2 * x * u[x, y]
```

```
sol3 = DSolve[{h, u[0, y] == y^3}, u[x, y], {x, y}]
Plot3D[u[x, y] /. sol3, {x, -7, 7}, {y, -5, 5},
 PlotLabel → "Integral surface through initial curve"]
```

$$-2\,x\,y\,u^{(0,1)}[x, y] + y\,u^{(1,0)}[x, y] == 2\,x\,u[x, y]$$

$$\left\{\left\{u[x, y] \to \frac{\left(x^2 + y\right)^4}{y}\right\}\right\}$$

Integral surface through initial curve



Problem-:3 Determine the integral surfaces of the equation

$$u_x + u_y = u^2,$$

with the data $x+y = 0$, $u=1$.

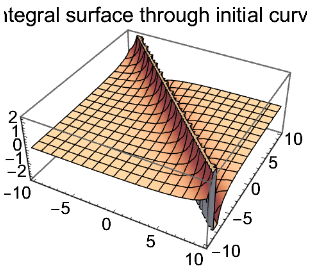
Plot the integral surface with in the range {x, -10, 10} & {y, -10, 10}.

Solution:-

```
Eqn = D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y]
DSolve[{Eqn, u[x, -x] == 1}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -10, 10}, {y, -10, 10},
 PlotLabel → "Integral surface through initial curve"]
```

$$u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == u[x, y]^2$$

$$\left\{\left\{u[x, y] \to -\frac{2}{-2 + x + y}\right\}\right\}$$



Problem:-4 **Obtain the solution of the linear equation**

$$u_x + u_y = 1,$$

with the Cauchy data u(x, 2x) = x^3**.**

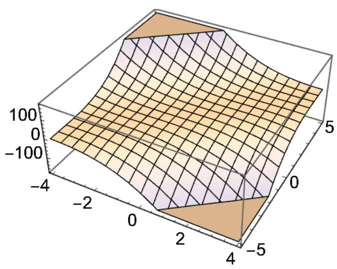
Plot the integral surface with in the range {x, -4, 4} & {y, -5, 5}.

Solution:-

```
D[u[x, y], x] + D[u[x, y], y] == 1
sol = DSolve[
  {D[u[x, y], x] + D[u[x, y], y] == 1, u[x, 2 x] == x^3}, u[x, y], {x, y}]
Plot3D[u[x, y] /. sol, {x, -4, 4}, {y, -5, 5}]
```

$$u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$$

$$\left\{\left\{u[x, y] \to 2\,x - x^3 - y + 3\,x^2\,y - 3\,x\,y^2 + y^3\right\}\right\}$$



Problem-5: Find the solution of the equation

$$u_x + y * u_y = 0,$$

with the Cauchy data u(0, y) = 4 * y**.**

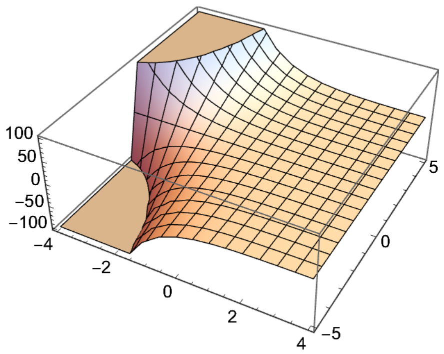
Plot the integral surface with in the range {x, -4, 4} & {y, -5, 5}.

Solution:-

```
pde5 = D[u[x, y], x] + y * D[u[x, y], y] == 0
sol5 = DSolve[{pde5, u[0, y] == 4 y}, u[x, y], {x, y}]
Plot3D[u[x, y] /. sol5, {x, -4, 4}, {y, -5, 5}]
```

$y\, u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 0$

$\{\{u[x, y] \to 4\, e^{-x}\, y\}\}$



Problem-:6 Determine the integral surfaces of the equation

$$u_x + u_y = u^2,$$

with the data u(x,0)=tanh(x).

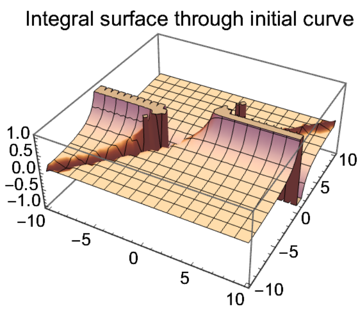
Plot the integral surface with in the range {x, -10, 10} & {y, -10, 10}.

Solution:-

```
pde7 = D[u[x, y], x] + D[u[x, y], y] == u[x, y] * u[x, y]
sol7 = DSolve[{pde7, u[x, 0] == Tanh[x]}, u[x, y], {x, y}]
Plot3D[u[x, y] /. %, {x, -10, 10}, {y, -10, 10},
 PlotLabel → "Integral surface through initial curve"]
```

$u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == u[x, y]^2$

$$\left\{\left\{u[x, y] \to \frac{1}{-y + \text{Coth}[x - y]}\right\}\right\}$$



Problem-:7 Determine the integral surfaces of the equation

$$u_x + u_y = 5^* u^2,$$

with the data u(x, 0) = x^2.

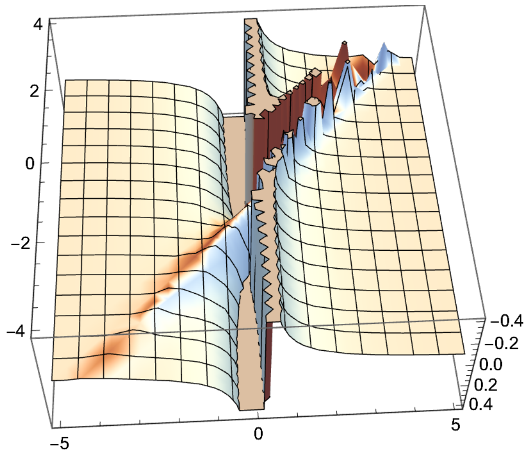
Plot the integral surface with in the range {x, -4, 4} & {y, -5, 5}.

Solution:-

```
pde8 = D[u[x, y], x] + D[u[x, y], y] == 5 * u[x, y] * u[x, y]
sol8 = DSolve[{pde8, u[x, 0] == x^2}, u[x, y], {x, y}]
Plot3D[u[x, y] /. sol8, {x, -4, 4}, {y, -5, 5}]
```

$$u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 5\,u[x, y]^2$$

$$\left\{\left\{u[x, y] \to -\frac{(x-y)^2}{-1 + 5 x^2 y - 10 x y^2 + 5 y^3}\right\}\right\}$$



Practical 2

Plotting the Characteristics for the first order PDE

Q1. Plot the Characteristic curves of the equation: $(u - y)\, u_x + y u_y = x+y$ ....
(1)

Solution: The characteristic equations are:

$$\frac{dx}{u-y} = \frac{dy}{y} = \frac{du}{x+y}$$

On taking I + III = II, we get

(u+x)/y=C1

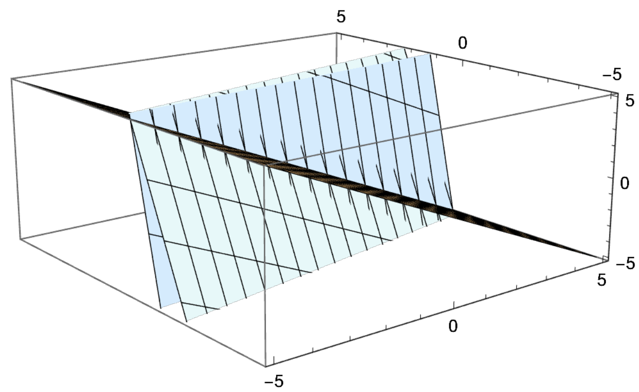
On taking I + II=III, we get

(x+y)^2-u^2=C2

where C1 and C2 are constants.

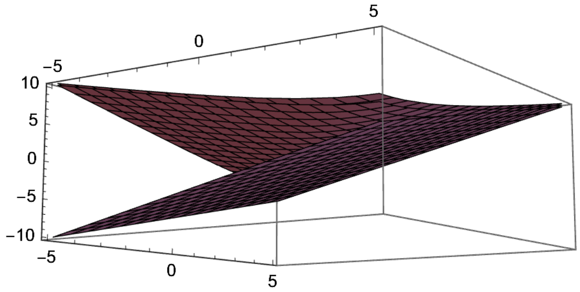
We plot the characteristic curves of the equation (1) for C1 and C2 equal to 0,5,10.

```
f0 = Plot3D[-x, {x, -5, 5}, {y, -5, 5}, PlotPoints → 10];

f1 = Plot3D[5 * y - x, {x, -5, 5}, {y, -5, 5}, PlotPoints → 10];

f2 = Plot3D[10 * y - x, {x, -5, 5}, {y, -5, 5}, PlotPoints → 10];

g1 = Show[f0, f1, f2]
```




```
h0 = Plot3D[x + y, {x, -5, 5}, {y, -5, 5}, PlotPoints → 10];

h1 = Plot3D[Sqrt[(x + y) ^ 2 + 5], {x, -5, 5}, {y, -5, 5}, PlotPoints → 10];

h2 = Plot3D[Sqrt[(x + y) ^ 2 + 10], {x, -5, 5}, {y, -5, 5}, PlotPoints → 10];
```

```
g2 = Show[h0, h1, h2]
```



Q2. Plot the Characteristic curves of the equation : $xu_x + yu_y = u$ ... . (2)

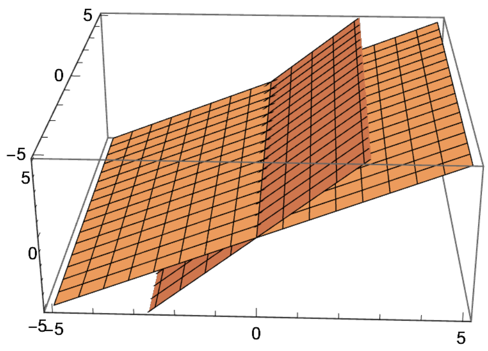
Solution : The characteristic equations are :

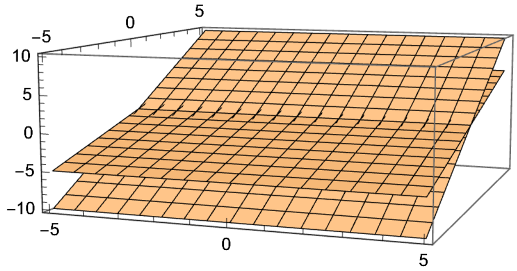
$$\frac{dx}{x} = \frac{dy}{y} = \frac{du}{u}$$

From these equations we get: x/u = C1 and y/u = C2 where C1 and C2 are constants
We plot the characteristic curves of (2) by taking C1 and C2 as 1 and 2.

```
f0 = Plot3D[x, {x, -5, 5}, {y, -5, 5}, PlotPoints → 10];

f1 = Plot3D[2 x, {x, -5, 5}, {y, -5, 5}, PlotPoints → 10];

h0 = Plot3D[2 y, {x, -5, 5}, {y, -5, 5}, PlotPoints → 10];

h1 = Plot3D[y, {x, -5, 5}, {y, -5, 5}, PlotPoints → 10];

g1 = Show[f0, f1];


g2 = Show[h0, h1];
```

```
Show[GraphicsArray[{g1, g2}]]
```





Q3. Plot the Characteristic curves of the equation : $x^2\, u_x + y^2\, u_y =$ (x + y) u ... . (3)

Solution : The characteristic equations are :

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{du}{(x+y)\,u}$$

From the first of these equations , we get: x^{-1} - y^{-1}= C1, ...(i)
where C1 is an arbitrary constant.

Also, $\frac{dx - dy}{x^2 - y^2} = \frac{du}{(x+y)\,u}$

or

$\frac{d(x-y)}{x-y} = \frac{du}{u}$.

This gives

$\frac{x-y}{u}$ = C2, ...(ii)

where C2 is constant.

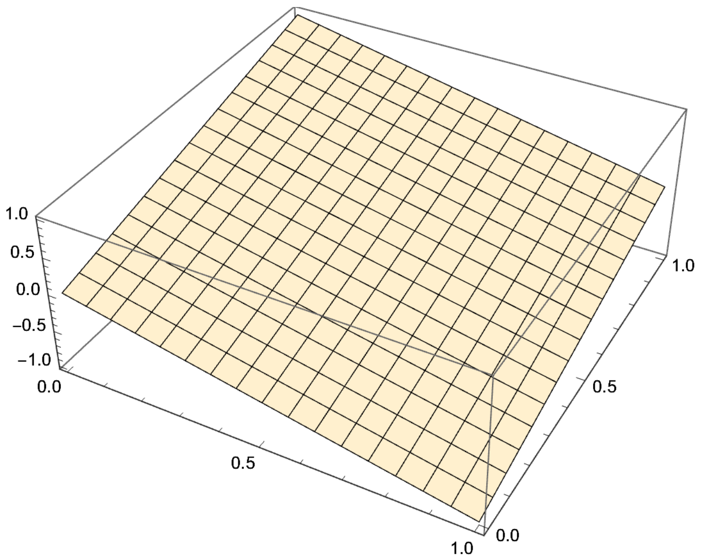
Further (i) and (ii) give

$\frac{xy}{u}$ = C3,.....(iii)

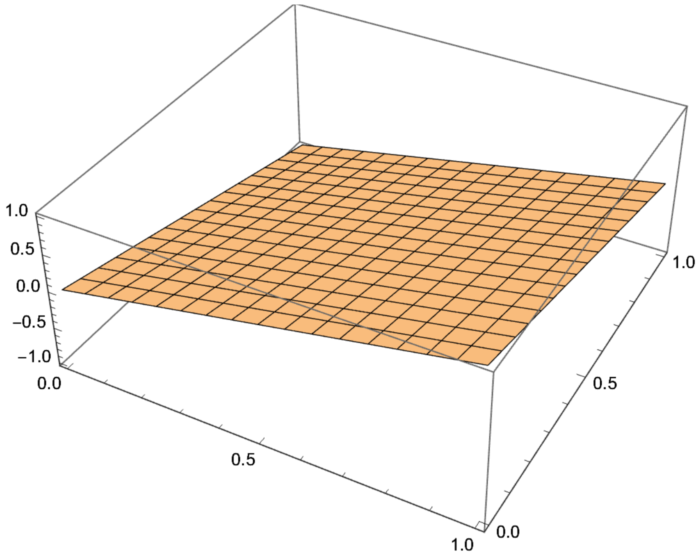
where C3 is a constant.

We plot the characteristic curves (ii) and (iii) by taking C2 and C3 as -1,1,2.

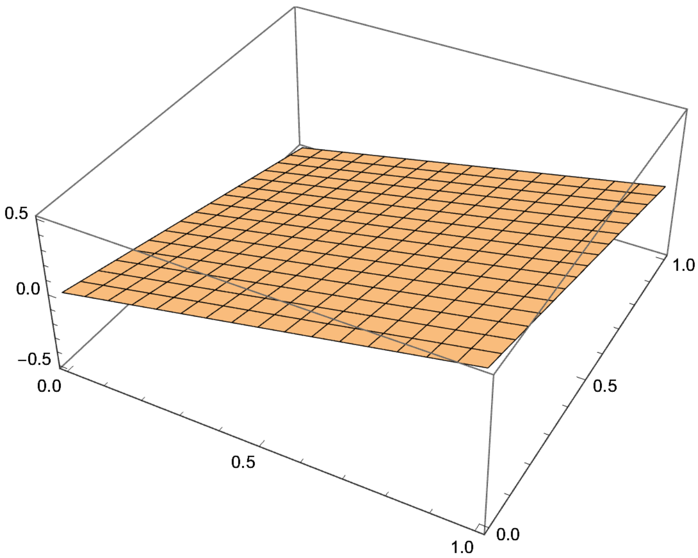
```
f0 = Plot3D[-x + y, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10]
```



```
f1 = Plot3D[x - y, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10]
```

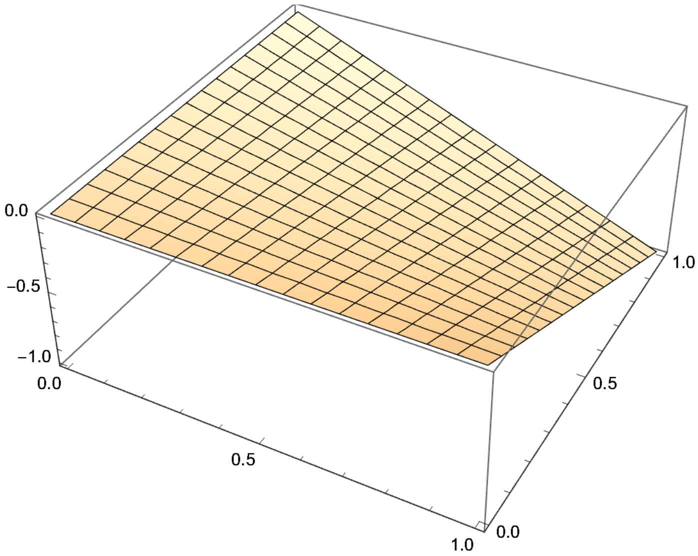



```
f2 = Plot3D[(x - y)/2, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10]
```

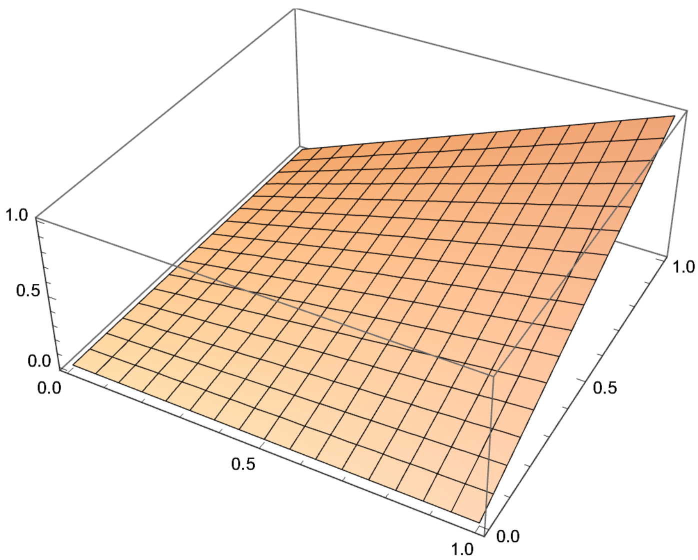



```
g1 = Show[f0, f1, f2];
```

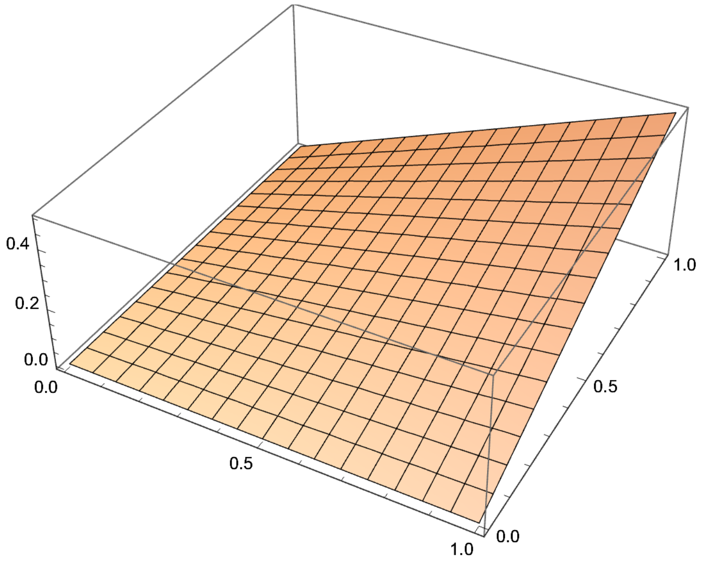
```
h0 = Plot3D[-x y, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10]
```




```
h1 = Plot3D[x y, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10]
```

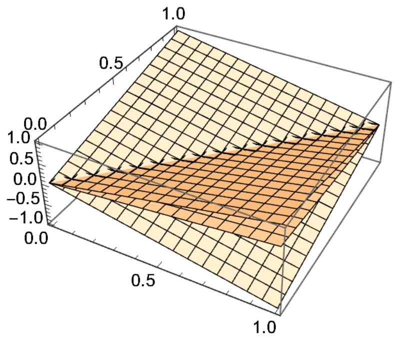


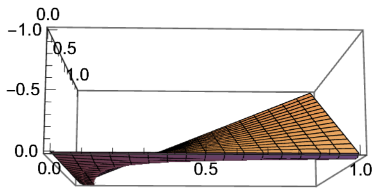
```
h2 = Plot3D[x y / 2, {x, 0, 1}, {y, 0, 1}, PlotPoints → 10]
```




```
g2 = Show[h0, h1, h2];
```

```
Show[GraphicsGrid[{{g1}, {g2}}]]
```





Q1. Plot the Characteristic curves of the equation: $u_x - u_y = 1$ ....

Solution: The characteristic equations are:

$$\frac{dx}{1} = \frac{dy}{-1} = \frac{du}{1}$$

On taking I = III, we get

In[45]:=
```
sol1 = DSolve[u'[x] == 1, u[x], x]
```

Out[45]= $\{\{u[x] \to x + c_1\}\}$

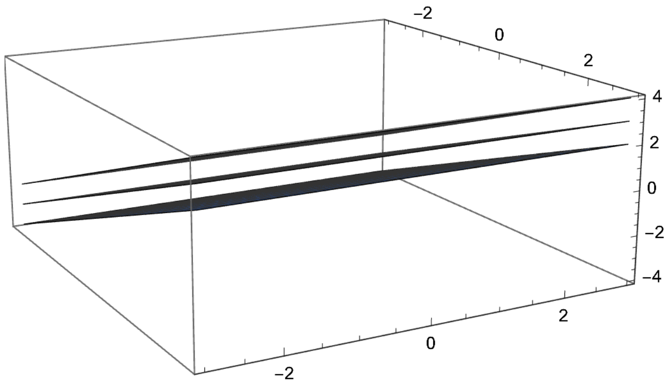
On taking II = III, we get

In[47]:=
```
sol2 = DSolve[u'[y] == -1, u[y], y]
```

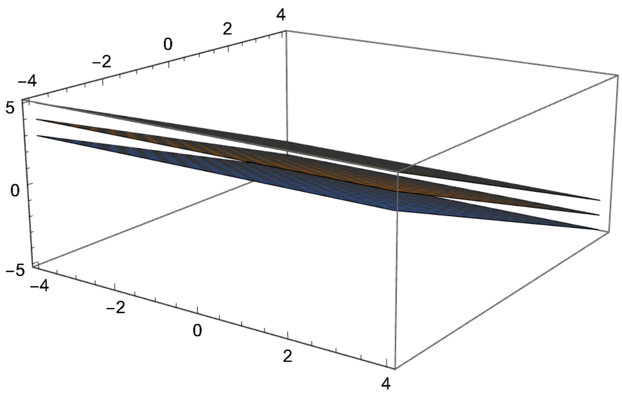
Out[47]= $\{\{u[y] \to -y + c_1\}\}$

In[50]:=
```
f1 = Plot3D[{x, x - 1, x + 1}, {x, -3, 3}, {y, -3, 3}]
f2 = Plot3D[{-y, -y - 1, -y + 1}, {x, -4, 4}, {y, -4, 4}]
```

Out[50]=

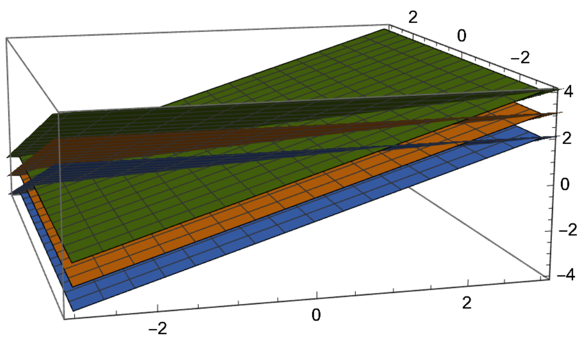


Out[51]=



In[52]:=
```
Show[f1, f2]
```

Out[52]=



Q2. Plot the Characteristic curves of the equation : $u\,u_x + u_y = 1$ . . . .

Solution : The characteristic equations are :

$$\frac{dx}{u} = \frac{dy}{1} = \frac{du}{1}$$

On taking II = III, we get

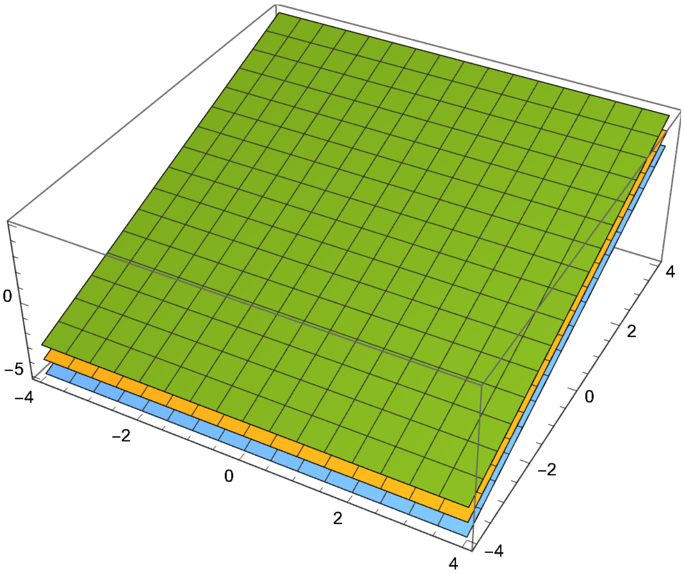
In[53]:=
```
sol1 = DSolve[u'[y] == 1, u[y], y]
```
Out[53]= $\{\{u[y] \to y + c_1\}\}$

On taking I= III, we get

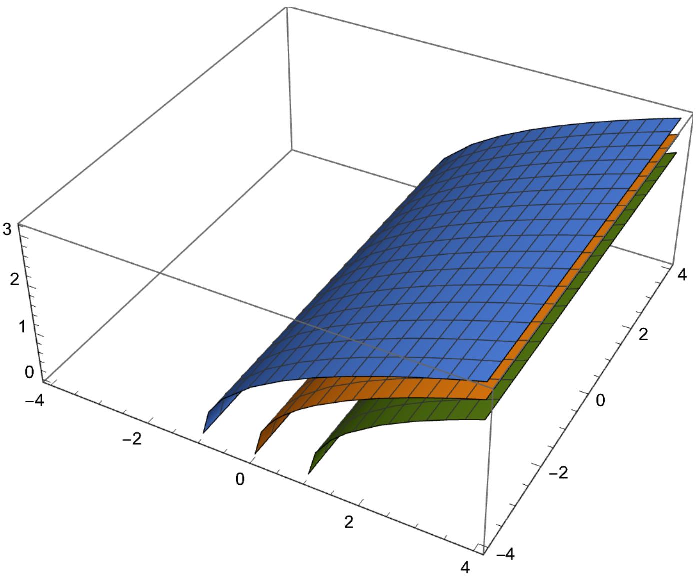
In[54]:=
```
sol2 = DSolve[u'[x] × u[x] == 1, u[x], x]
```
Out[54]= $\left\{\left\{u[x] \to -\sqrt{2}\ \sqrt{x + c_1}\right\}, \left\{u[x] \to \sqrt{2}\ \sqrt{x + c_1}\right\}\right\}$

In[84]:=
```
f1 = Plot3D[{y, y - 1, y + 1}, {x, -4, 4}, {y, -4, 4}]
f2 = Plot3D[{√2 √x, √2 √(x + 1), √2 √(x - 1)}, {x, -4, 4}, {y, -4, 4}]
```

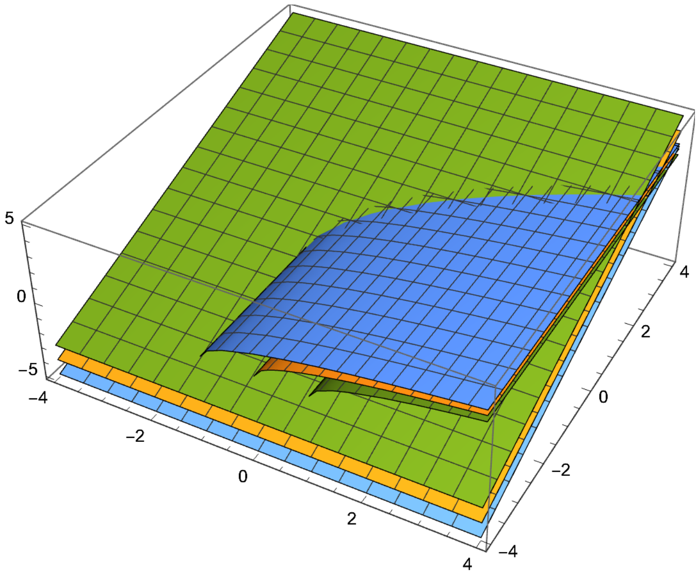
Out[84]=



Out[85]=



In[86]:=
```
Show[f1, f2]
```

Out[86]=


Q3. Plot the Characteristic curves of the equation: $(u - y)\,u_x + y u_y = x+y$ .... (1)

Solution: The characteristic equations are:

$$\frac{dx}{u-y} = \frac{dy}{y} = \frac{du}{x+y}$$

On taking I + III = II, we get

In[76]:=
```
Clear[x, y, u, w];
```

In[77]:=
```
sol1 = DSolve[w'[y] == w[y]/y, w[y], y]
```

Out[77]= $\{\{w[y] \to y\, c_1\}\}$

In[78]:=
```
w = u + x /. sol1
```

Out[78]= $\{u + x\}$

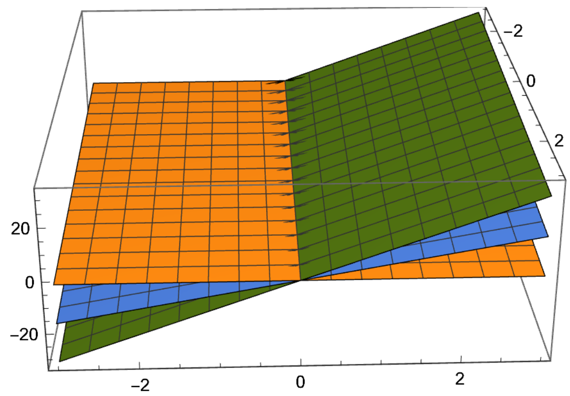
On taking I + II = III, we get

In[81]:=
```
Clear[x, y, u, w];
sol2 = DSolve[w'[u] × w[u] == u, w[u], u]
```

Out[82]= $\left\{\left\{w[u] \to -\sqrt{u^2 + 2\, c_1}\right\}, \left\{w[u] \to \sqrt{u^2 + 2\, c_1}\right\}\right\}$

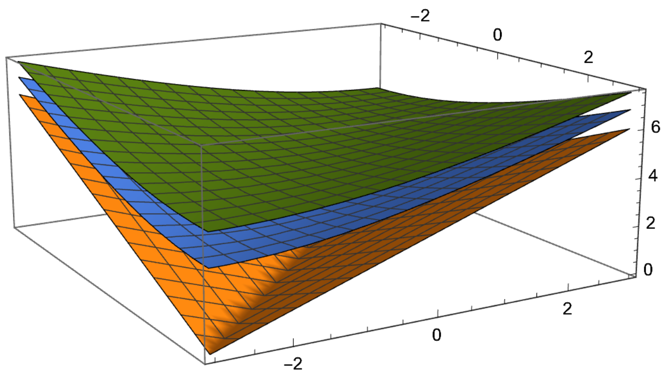
In[83]:=
```
w = u + x /. sol2
```
Out[83]= $\{u + x, u + x\}$

In[80]:=
```
f1 = Plot3D[{-x, -x + 5 y, -x + 10 y}, {x, -3, 3}, {y, -3, 3}]
```
Out[80]=



In[87]:= f2 = Plot3D$\left[\left\{\sqrt{(x+y)^2}, \sqrt{(x+y)^2+10}, \sqrt{(x+y)^2+20}\right\}, \{x, -3, 3\}, \{y, -3, 3\}\right]$

Out[87]=



Practical 4

Solution of vibrating string problem using D'Almebert formula with initial conditions

Problem-:1 Solve the initial value problem

$u_{tt} = u_{xx}$, $\quad -\infty < x < \infty, \quad t > 0,$

$u(x, 0) = \sin x$, $\quad -\infty < x < \infty,$

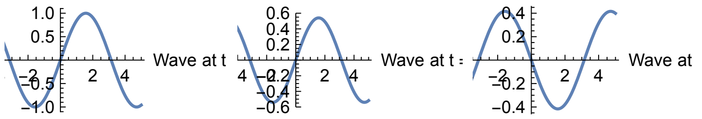
$u_t(x, 0) = 0$, $\quad -\infty < x < \infty$

In[11]:=
```
c = 1;
f[x_] := Sin[x];
g[x_] := 0;
u[x_, t_] := 1/2 (f[x + c t] + f[x - c t]) + 1/(2 c) Integrate[g[s], {s, x - c t, x + c t}]
Print[u[x, t]]
h0 = Plot[Evaluate[u[x, 0]], {x, -5, 5},
   PlotRange → All, PlotLegends → "Wave at t =0"];
h1 = Plot[Evaluate[u[x, 1]], {x, -5, 5},
   PlotRange → All, PlotLegends → "Wave at t =1"];
h2 = Plot[Evaluate[u[x, 2]], {x, -5, 5},
   PlotRange → All, PlotLegends → "Wave at t =2"];
Show[GraphicsArray[{h0, h1, h2}]]
Plot3D[u[x, t], {x, -3, 3}, {t, 0, 4}]
```

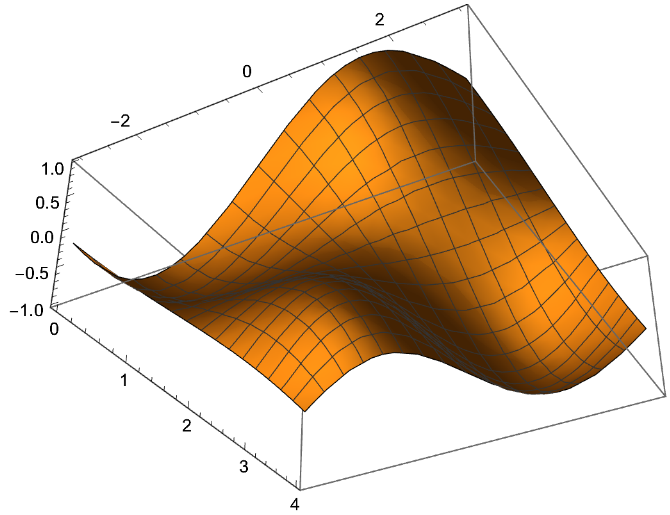
$\frac{1}{2}(-\mathrm{Sin}[t-x]+\mathrm{Sin}[t+x])$

GraphicsArray : GraphicsArray is obsolete. Switching to GraphicsGrid.

Out[19]=



Out[20]=



Problem-:2 Solve the initial value problem

$u_{tt} = 4\, u_{xx}, \qquad -\infty < x < \infty, \quad t > 0,$

$u(x, 0) = e^{-x^2} \sin x, \qquad -\infty < x < \infty,$

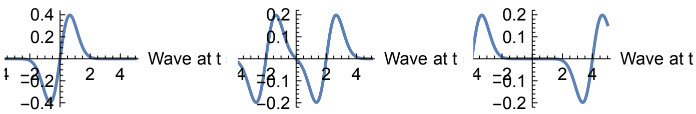
$u_t(x, 0) = 0, \qquad -\infty < x < \infty$

In[21]:=
```
c = 2;
f[x_] := Exp[-x^2] Sin[x];
g[x_] := 0;
u[x_, t_] := 1/2 (f[x + c t] + f[x - c t]) + 1/(2 c) Integrate[g[s], {s, x - c t, x + c t}]
Print[u[x, t]]
h0 = Plot[Evaluate[u[x, 0]], {x, -5, 5},
   PlotRange → All, PlotLegends → "Wave at t =0"];
h1 = Plot[Evaluate[u[x, 1]], {x, -5, 5},
   PlotRange → All, PlotLegends → "Wave at t =1"];
h2 = Plot[Evaluate[u[x, 2]], {x, -5, 5},
   PlotRange → All, PlotLegends → "Wave at t =2"];
Show[GraphicsArray[{h0, h1, h2}]]
Plot3D[u[x, t], {x, -3, 3}, {t, 0, 4}]
```

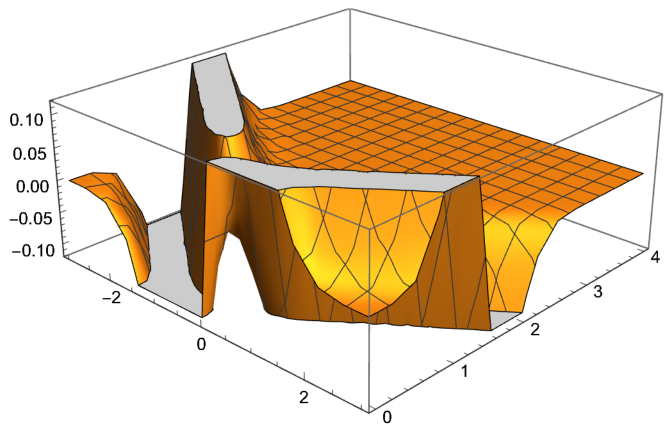
$$\frac{1}{2}\left(-e^{-(-2t+x)^2}\,\text{Sin}[2t-x]+e^{-(2t+x)^2}\,\text{Sin}[2t+x]\right)$$

GraphicsArray : GraphicsArray is obsolete. Switching to GraphicsGrid.

Out[29]=



Out[30]=



Problem-:3 Solve the initial value problem

$$u_{tt} = 2\,u_{xx}, \qquad -\infty < x < \infty, \quad t > 0,$$

$$u\,(x,0) = \begin{cases} 0 & x < -1 \\ 1 & -1 \le x \le 1, \\ 0 & x > 1 \end{cases} \qquad -\infty < x < \infty,$$

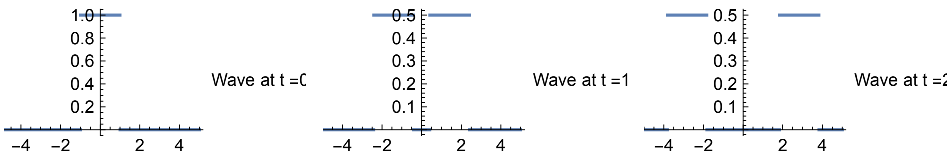
$$u_t\,(x,0) = \sin x, \qquad -\infty < x < \infty$$

In[31]:=
```
c = √2;
f[x_] := Piecewise[{{0, x < -1}, {1, -1 ≤ x ≤ 1}, {0, x > 1}}];
g[x_] := 0;
u[x_, t_] := 1/2 (f[x + c t] + f[x - c t]) + 1/(2 c) Integrate[g[s], {s, x - c t, x + c t}]
Print[u[x, t]]
h0 = Plot[Evaluate[u[x, 0]], {x, -5, 5},
   PlotRange → All, PlotLegends → "Wave at t =0"];
h1 = Plot[Evaluate[u[x, 1]], {x, -5, 5},
   PlotRange → All, PlotLegends → "Wave at t =1"];
h2 = Plot[Evaluate[u[x, 2]], {x, -5, 5},
   PlotRange → All, PlotLegends → "Wave at t =2"];
Show[GraphicsArray[{h0, h1, h2}]]
Plot3D[u[x, t], {x, -3, 3}, {t, 0, 4}]
```

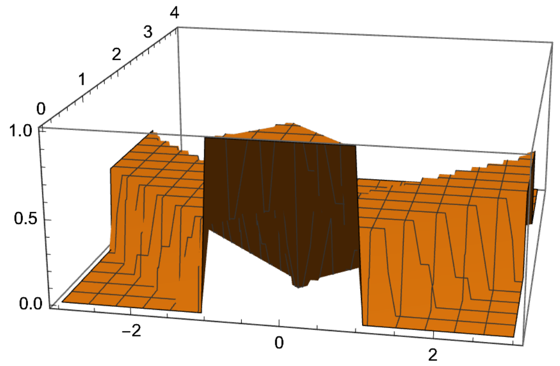
$$\frac{1}{2}\left(\left(\begin{cases}0 & -\sqrt{2}\,t+x<-1\\ 1 & -1\le-\sqrt{2}\,t+x\le 1\\ 0 & \text{True}\end{cases}\right)+\left(\begin{cases}0 & \sqrt{2}\,t+x<-1\\ 1 & -1\le\sqrt{2}\,t+x\le 1\\ 0 & \text{True}\end{cases}\right)\right)$$

GraphicsArray : GraphicsArray is obsolete. Switching to GraphicsGrid.

Out[39]=



Out[40]=



Problem-:4 Solve the initial value problem

$u_{tt} = 2\,u_{xx}, \qquad -\infty < x < \infty, \quad t > 0,$

$u\,(x, 0) = \sin x, \qquad -\infty < x < \infty,$

$u_t\,(x, 0) = \cos x, \qquad -\infty < x < \infty$

Problem-:5 Solve the initial value problem

$u_{tt} = \pi\,u_{xx}, \qquad -\infty < x < \infty, \quad t > 0,$

$u\,(x, 0) = 0, \qquad -\infty < x < \infty,$

$u_t\,(x, 0) = e^{-x^2}, \qquad -\infty < x < \infty$

Practical : –(5)

Solution of Heat Equations

Problem-:1(a) $u_t - u_{xx} = 0, \quad 0 < x < 5, \quad t > 0,$

$u(x, 0) = 0, \quad 0 \leqslant x \leq 5,$

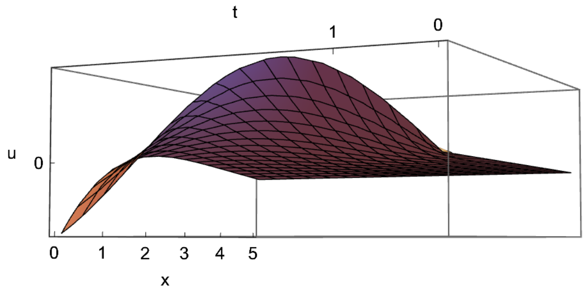
$u(0, t) = \text{Sin}(t) \quad t \geq 0$

$u(5, t) = 0, \quad t \geq 0.$

```
eqn1a = {∂_t u[x, t] - ∂_{x,x} u[x, t] == 0,
  u[x, 0] == 0, u[0, t] == Sin[t], u[5, t] == 0}
sol1a = u[x, t] /. NDSolve[eqn1a, u[x, t],
    {x, 0, 5}, {t, 0, 10}, PrecisionGoal → 3]  [[1]]
Plot3D[sol1a, {x, 0, 5}, {t, 0, 4}, AxesLabel → {"x", "t", "u"},
 Ticks → {{0, 1, 2, 3, 4, 5}, {0, 1}, {-3, 0}}]
```

$\{u^{(0,1)}[x, t] - u^{(2,0)}[x, t] == 0, u[x, 0] == 0, u[0, t] == \text{Sin}[t], u[5, t] == 0\}$

InterpolatingFunction[{{0., 5.}, {0., 10.}}, <>][x, t]



Problem-:1(b) $u_t - u_{xx} = 0, \quad 0 < x < 20, \quad t > 0,$

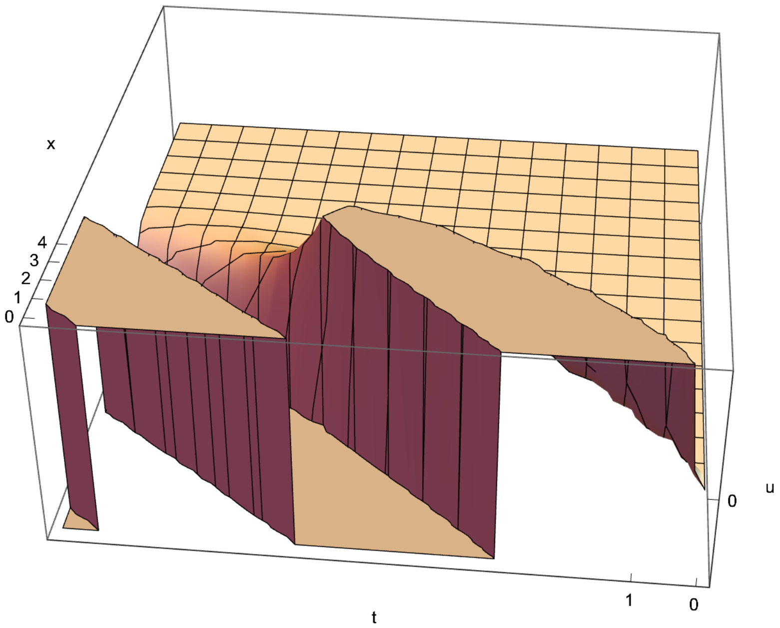
$u(x, 0) = 0, \quad 0 \leqslant x \leq 20,$

$u(0, t) = t^2 * \text{Sin}(t) \quad t \geq 0$

$u(20, t) = 0, \quad t \geq 0.$

```
eqn1b = {∂t u[x, t] - ∂x,x u[x, t] == 0,
   u[x, 0] == 0, u[0, t] == t^2 * Sin[t], u[20, t] == 0}
sol1b = u[x, t] /. NDSolve[eqn1b, u[x, t],
     {x, 0, 20}, {t, 0, 10}, PrecisionGoal → 3][[1]]
Plot3D[sol1b, {x, 0, 20}, {t, 0, 10}, AxesLabel → {"x", "t", "u"},
  Ticks → {{0, 1, 2, 3, 4}, {0, 1}, {-3, 0}}]
```

```
{u^(0,1)[x, t] - u^(2,0)[x, t] == 0, u[x, 0] == 0, u[0, t] == t^2 Sin[t], u[20, t] == 0}

InterpolatingFunction[{{0., 20.}, {0., 10.}}, <>][x, t]
```



Problem-:1(c) $u_t - u_{xx} = 0,\quad 0 < x < 20,\quad t > 0,$

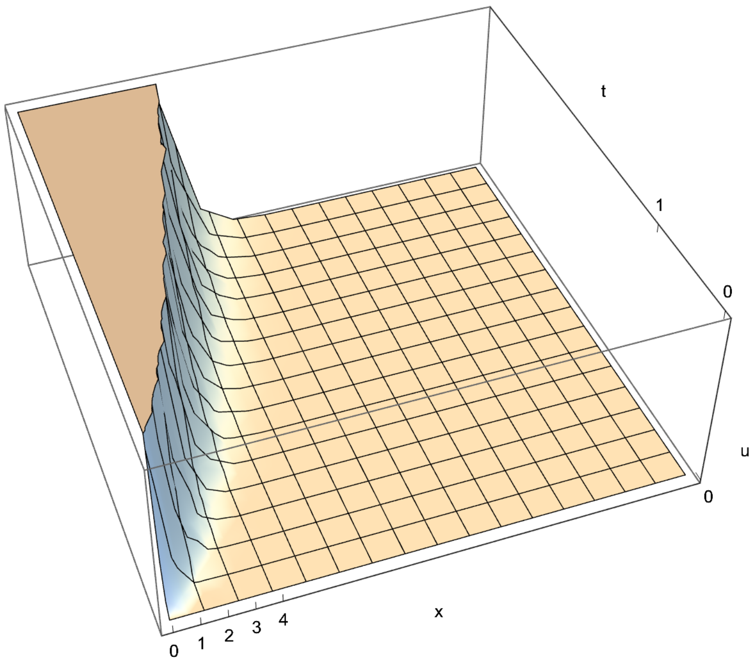
$u(x, 0) = 0,\quad 0 \leqslant x \leq 20,$

$u(0, t) = t^2\quad t \geq 0$

$u(20, t) = 0,\quad t \geq 0.$

```
eqn1c =
 {∂t u[x, t] - ∂x,x u[x, t] == 0, u[x, 0] == 0, u[0, t] == t^2, u[20, t] == 0}
sol1c = u[x, t] /. NDSolve[eqn1c, u[x, t],
    {x, 0, 20}, {t, 0, 10}, PrecisionGoal → 3][[1]]
Plot3D[sol1c, {x, 0, 20}, {t, 0, 4}, AxesLabel → {"x", "t", "u"},
 Ticks → {{0, 1, 2, 3, 4}, {0, 1}, {-3, 0}}]
```

```
{u^(0,1)[x, t] - u^(2,0)[x, t] == 0, u[x, 0] == 0, u[0, t] == t^2, u[20, t] == 0}

InterpolatingFunction[{{0., 20.}, {0., 10.}}, <>][x, t]
```



************End************

Problem-:1(d) $u_t = u_{xx}$, $\quad 0 < x < \pi, \quad t > 0,$

$u(x, 0) = \sin^2 x, \quad 0 \leqslant x \leq \pi,$

$u(0, t) = 0 \quad t \geq 0$

$u(\pi, t) = 0, \quad t \geq 0.$