Modern Control

Project: Ball & Beam System (Phase 2)

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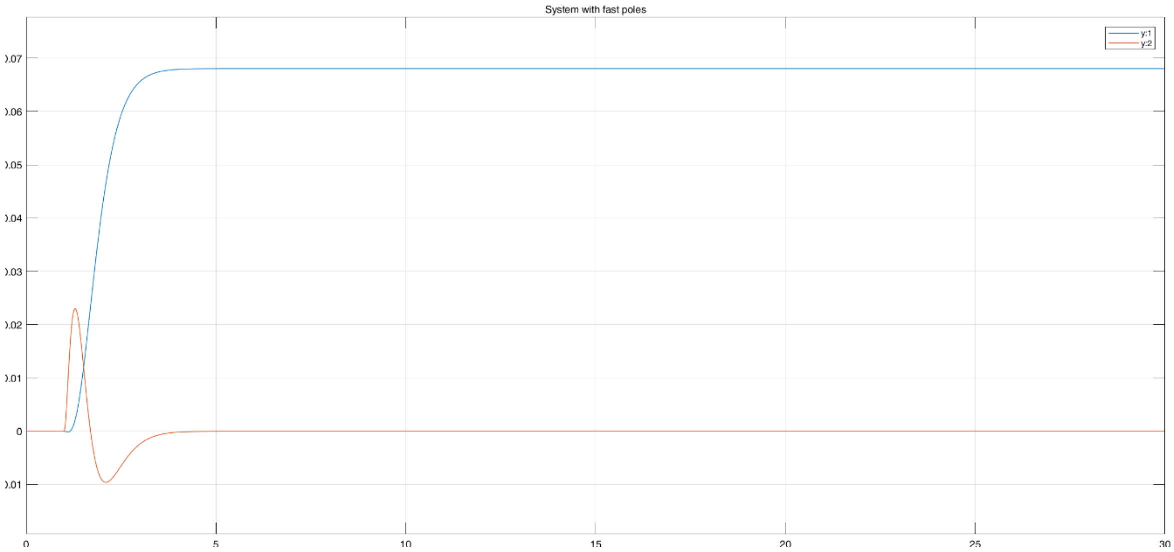
Questions

1

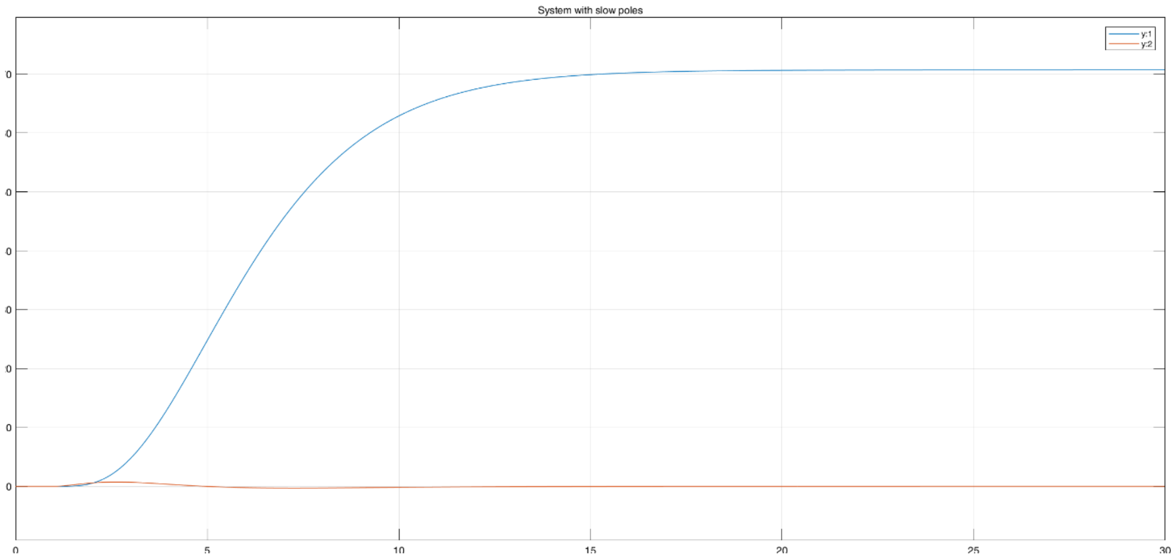
In this part, we consider -0.5,-0.7,-0.9,-1.1 for slow poles and -3,-4,-5,-6 for fast poles.

$$K_{fast} = [20.0993\,,13.9591\,,34.2198\,,4.9373]$$




```
fast_inf = struct with fields:
        RiseTime: 0.0024
    SettlingTime: 0.0681
     SettlingMin: 27
     SettlingMax: 30
       Overshoot: 0
      Undershoot: 0
            Peak: 30
        PeakTime: 0.0681
```

$$K_{slow} = [5.1497\,,0.0771\,,0.9613\,,0.4268]$$

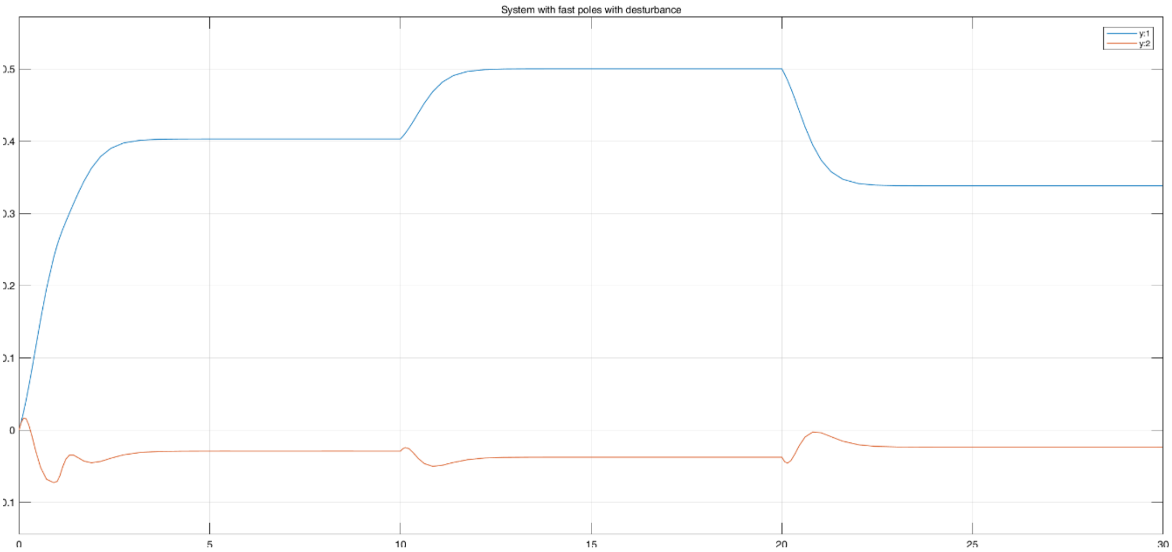


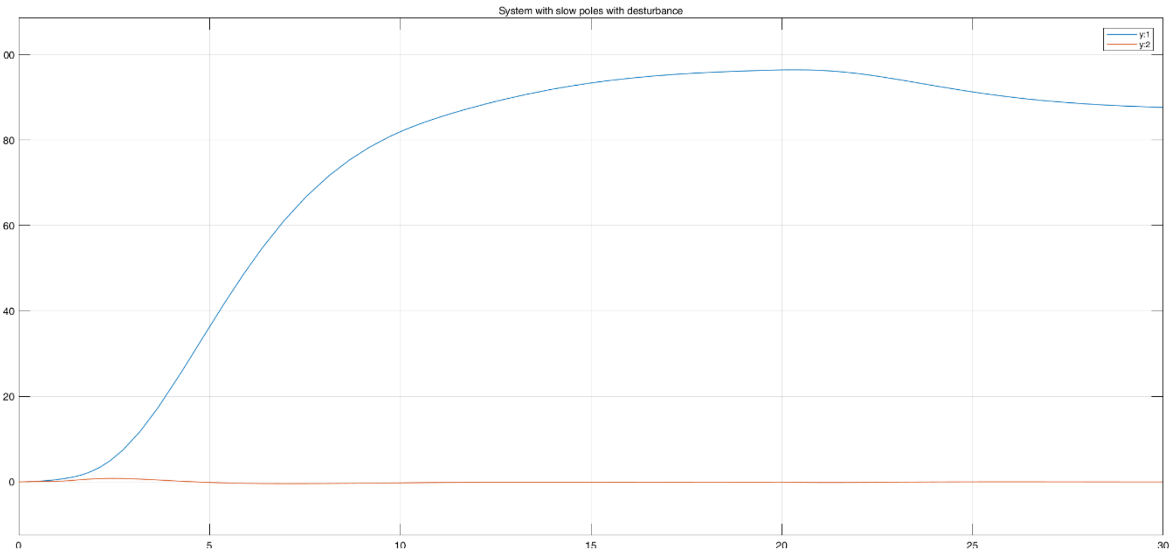
```
slow_inf = struct with fields:
        RiseTime: 65.9480
    SettlingTime: 70.7068
     SettlingMin: 27
     SettlingMax: 30
       Overshoot: 0
      Undershoot: 0
            Peak: 30
        PeakTime: 70.7070
```

As can be seen from both systems the slower poles move smoother and take more time to converge, the problem is that when we change the poles the final value of output will change too (except in the systems in which the final value converges to zero), so if the system is slow final value will be large and if the system desing with fast poles the final value will be small in this example is about 0.07.

2

In this part, we first add disturbance with *sample time=10s* to the system, and we see that systems can’t remove disturbance and their final values change over time.





The final value changes every time with the amount of disturbance is given.

3

In this part, we only choose position as output, and the new matrix is:

$$\begin{bmatrix}\dot{x}\\ \dot{q}\end{bmatrix} = \begin{bmatrix}A & 0\\ -C & 0\end{bmatrix}\begin{bmatrix}x\\ q\end{bmatrix} + \begin{bmatrix}B\\ 0\end{bmatrix}u + \begin{bmatrix}0\\ I\end{bmatrix}$$

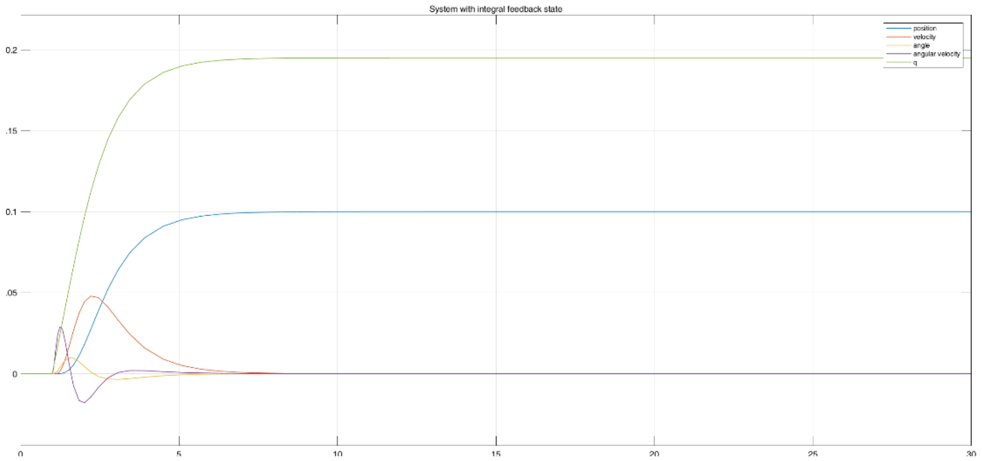
$$\bar{A} = \begin{bmatrix}0 & 1 & 0 & 0 & 0\\ -0.378 & 0 & 7.0147 & 0.0343 & 0\\ 0 & 0 & 0 & 1 & 0\\ 18.9 & 0 & -0.3797 & -1.713 & 0\\ -1 & 0 & 0 & 0 & 0\end{bmatrix} \;\&\; \bar{B} = \begin{bmatrix}0\\ -0.0699\\ 0\\ 3.4965\\ 0\end{bmatrix}$$

Now we check the controllability of the new matrix and the matrix is full rank so we can control it.

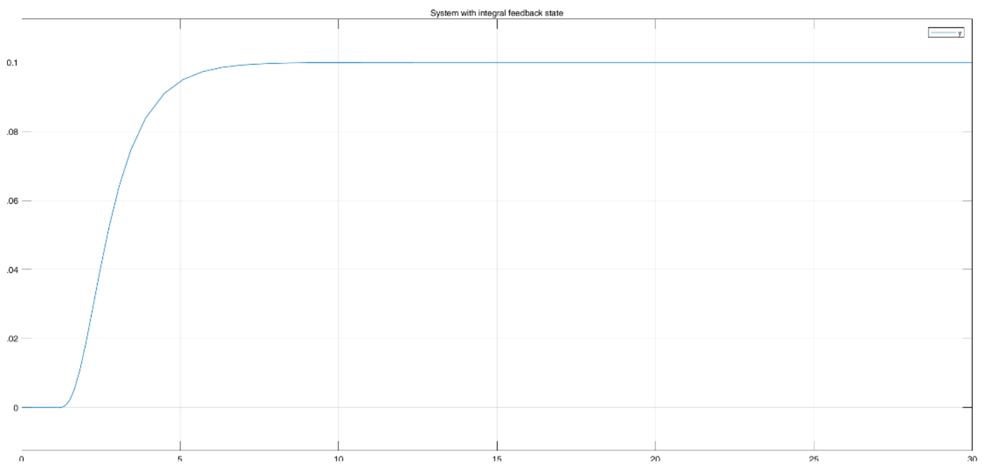
Now again we choose poles like before P= [-3, -4 , -5,-6,-1] and

K=[34.05 , 18.85 , 39.64 , 5.32 , -14.69]

System's states are depicted below:



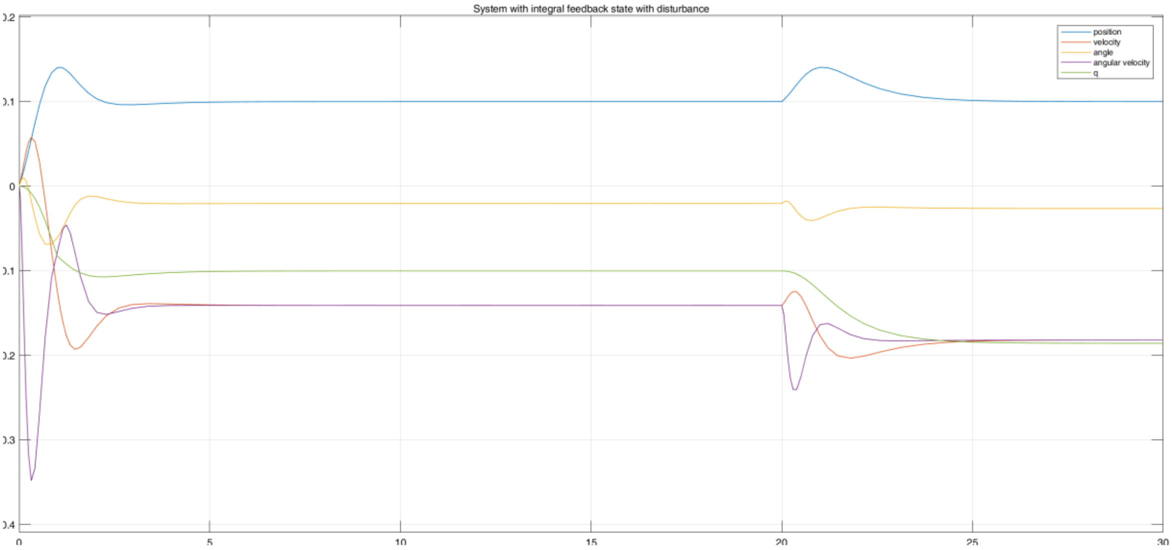
System's response is depicted below:



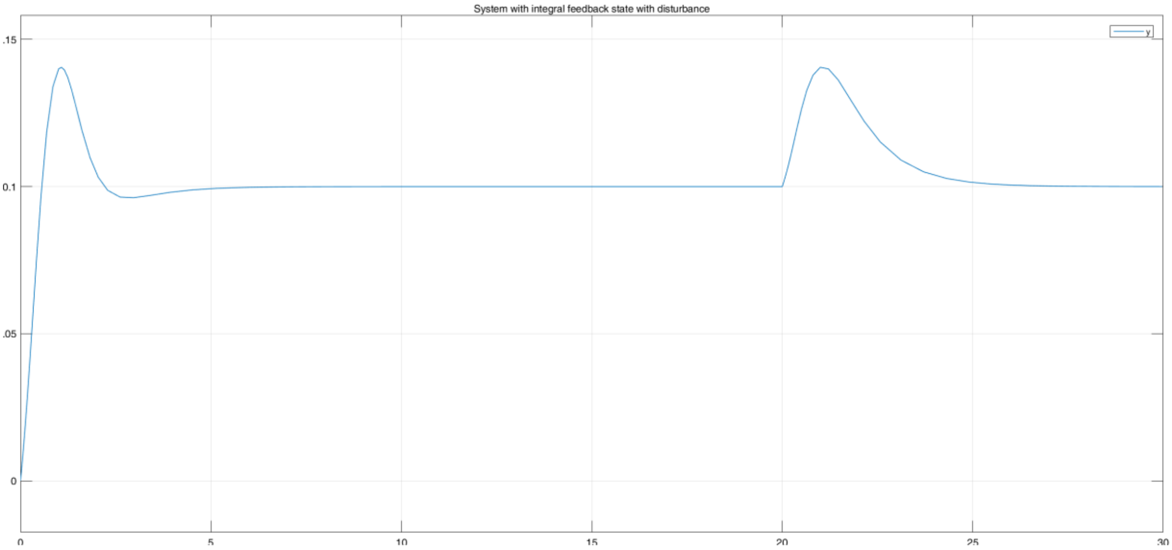
4

In this part, we add disturbance which changes every 20 seconds.

System’s states are depicted below:



System’s response is depicted bwlow:



As is shown, every time the disturbance is added, the system tries to track the input and remove the disturbance.

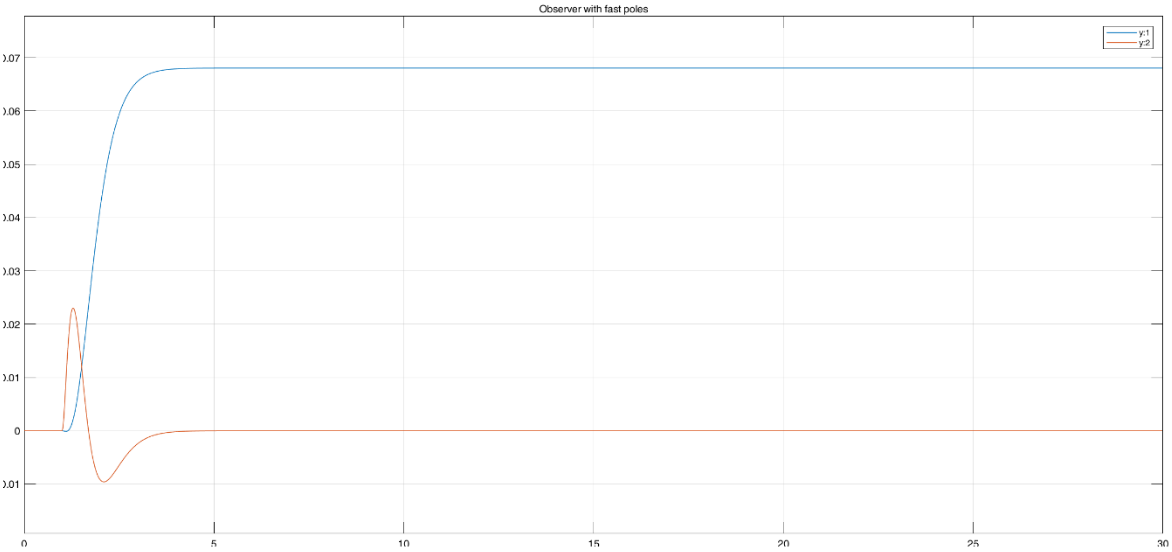
5

Luenberger feedback gain is written as follows:

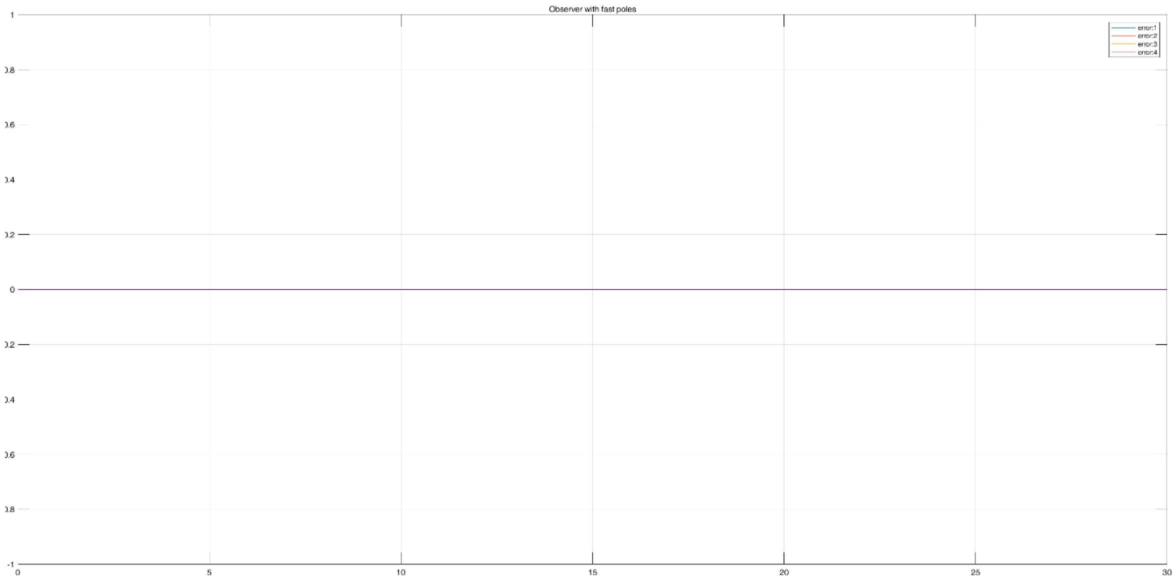
$$L \approx \begin{bmatrix} 15.9 & 0.88 \\ 58.95 & 13.4 \\ 0.468 & 10.38 \\ 21.22 & 14.5 \end{bmatrix}$$

Where observer poles are $[-4, -6, -8, -10]$.

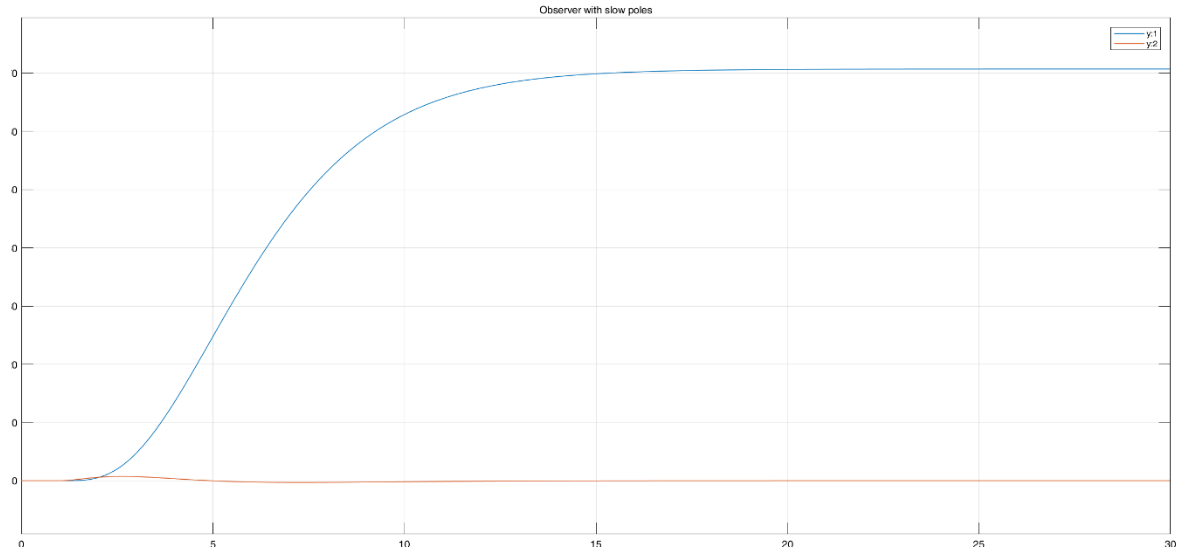
System's states with estimated state feedback with aforementioned fast poles are depicted below:



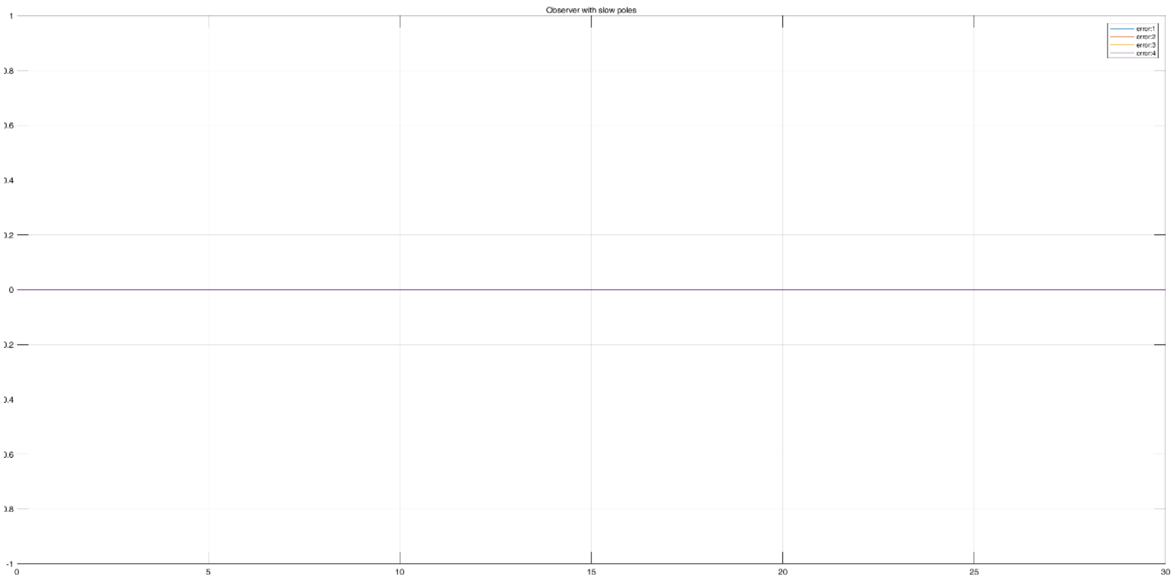
The $error = X - \hat{X}$ is depicted below:



System’s states with estimated state feedback with aforementioned slow poles are depicted below:



The $error = X - \hat{X}$ is depicted below:



Same problem that was mentioned in first question still remains.

6

$n = 4, l = 2$

$F = \begin{bmatrix} -6 & 0 \\ 0 & -8 \end{bmatrix}, L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, T_{2\times 4} = ?$

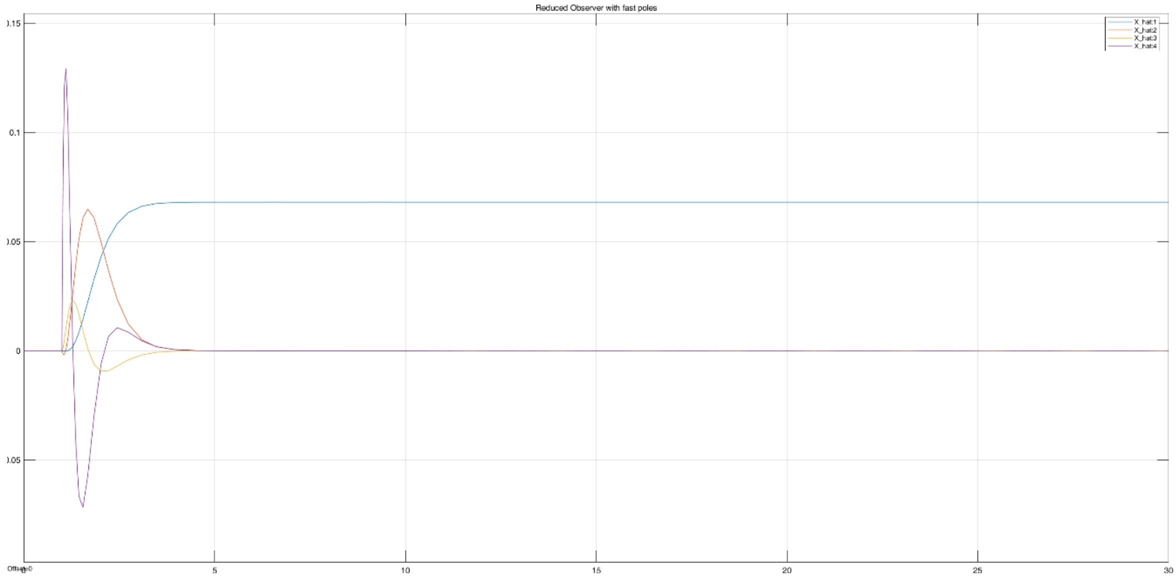
$Co(F, L) \rightarrow Full\ rank$

$TA - FT = LC \rightarrow T = \begin{bmatrix} 0.1908 & -0.0318 & 0.0367 & -0.0083 \\ 0.0482 & -0.006 & 0.1293 & -0.0205 \end{bmatrix}$

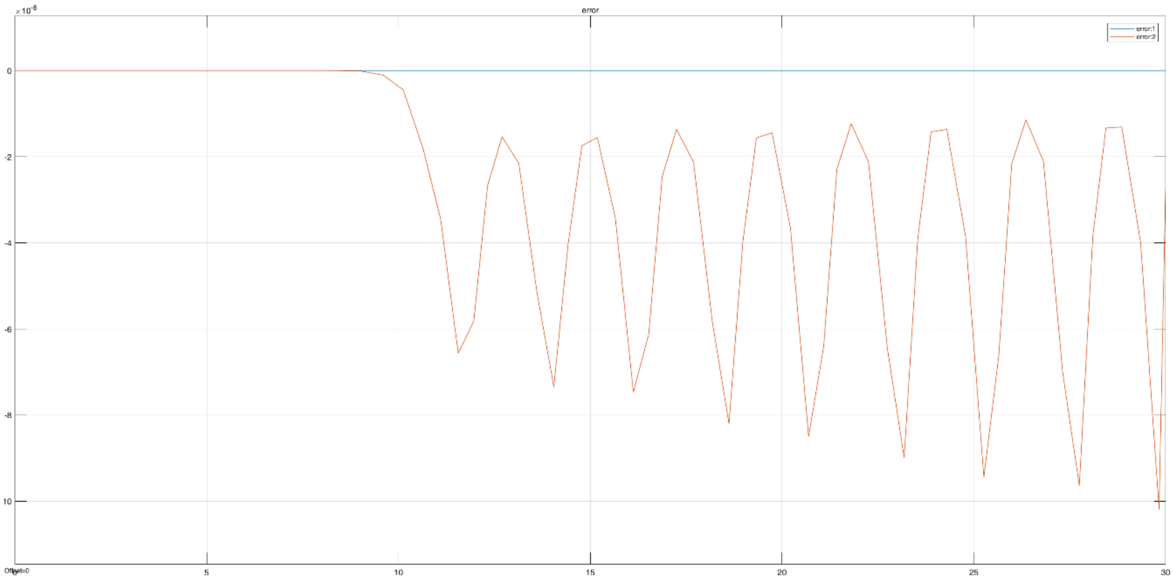
$\dot{z} = Fz + TBu + Ly$

$\hat{x} = \begin{bmatrix} C \\ T \end{bmatrix}^{-1} \begin{bmatrix} y \\ z \end{bmatrix}$

System's states with reduced estimated state feedback with fasts poles are depicted below.

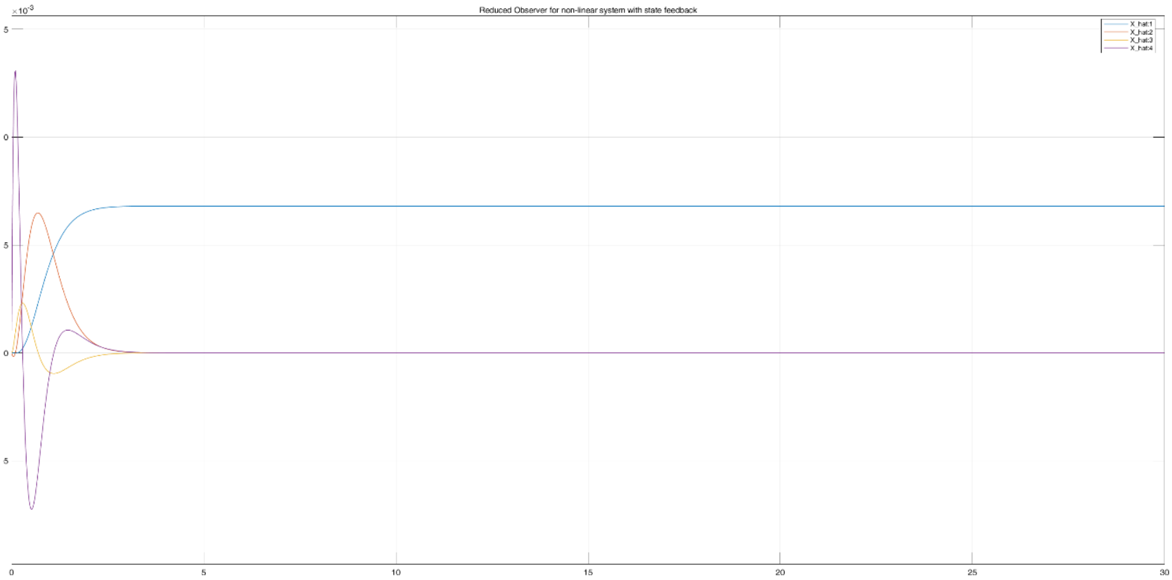


Also the error signal which is calculated from $error = z - TX$ is depicted below.

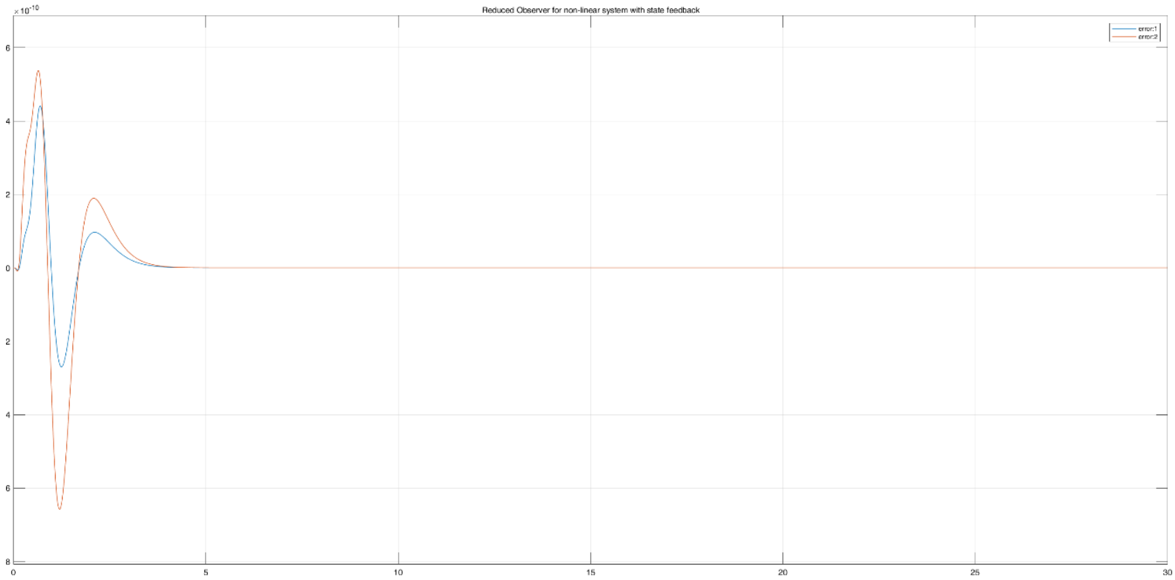


7

Non-linear system's states with reduced estimated state feedback with fasts poles are depicted below.



Also error signal which is calculated from $error = z - TX$ is depicted below.

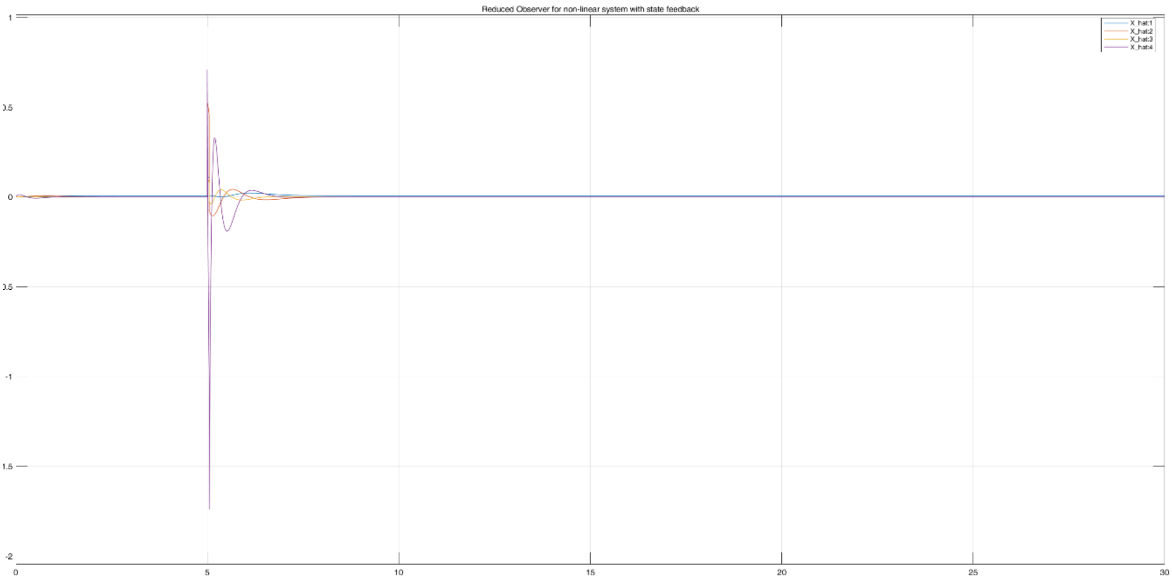


8

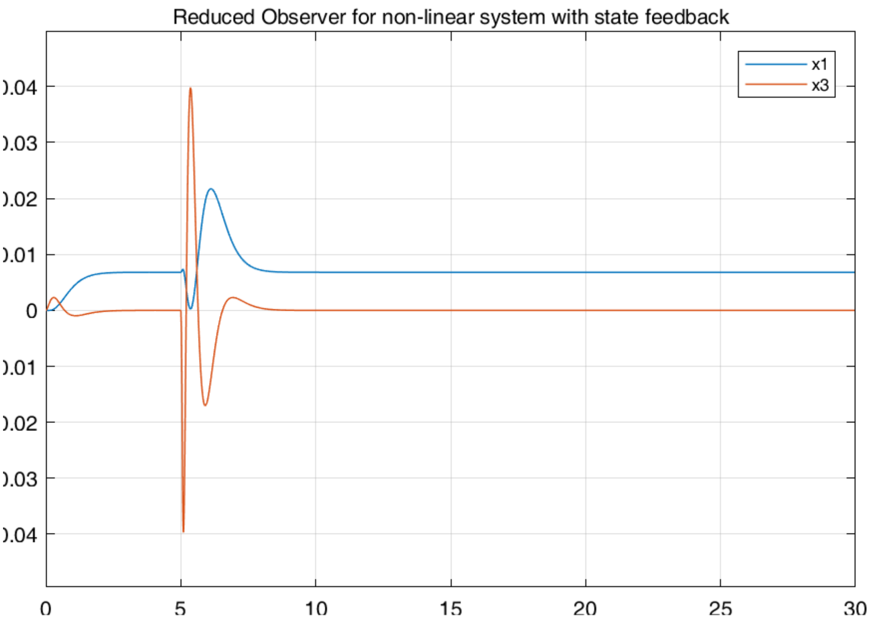
Non-linear system's states with reduced estimated state feedback with fasts poles are depicted below.

A single interval square signal is added as disturbance with the following properties:

$$\begin{cases} Amplitude = 0.1 \\ Delay = 5s \\ Width = 0.05 \end{cases}$$

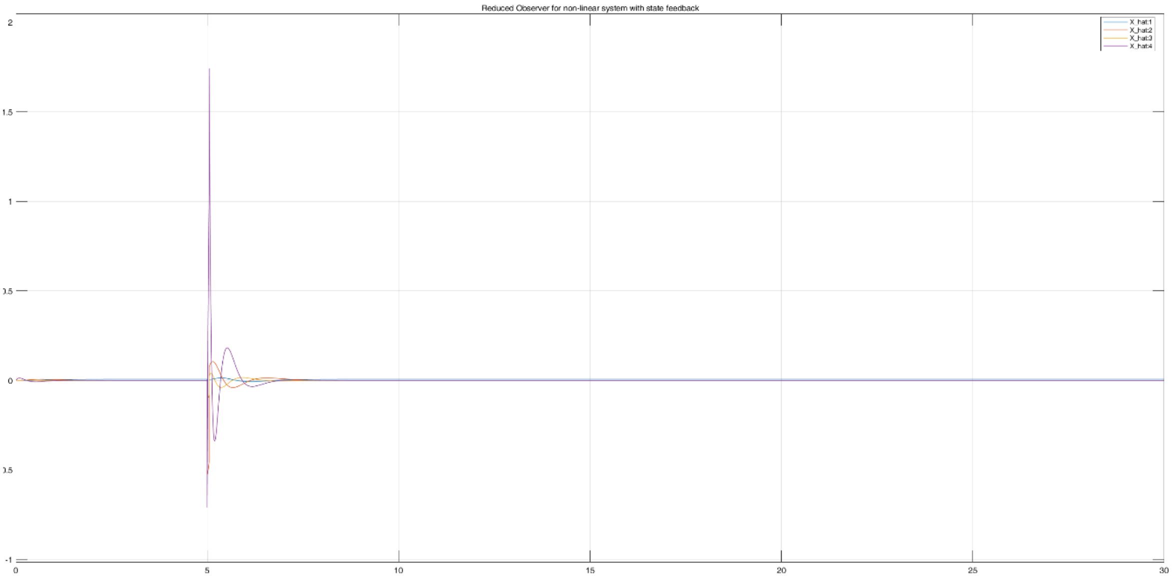


Also output signals are depicted below.



Moreover, results are shown for another single interval square signal as disturbance with the following properties.

$$\begin{cases} Amplitude = -0.1 \\ Delay = 5s \\ Width = 0.05 \end{cases}$$



Also output signals are depicted below.