

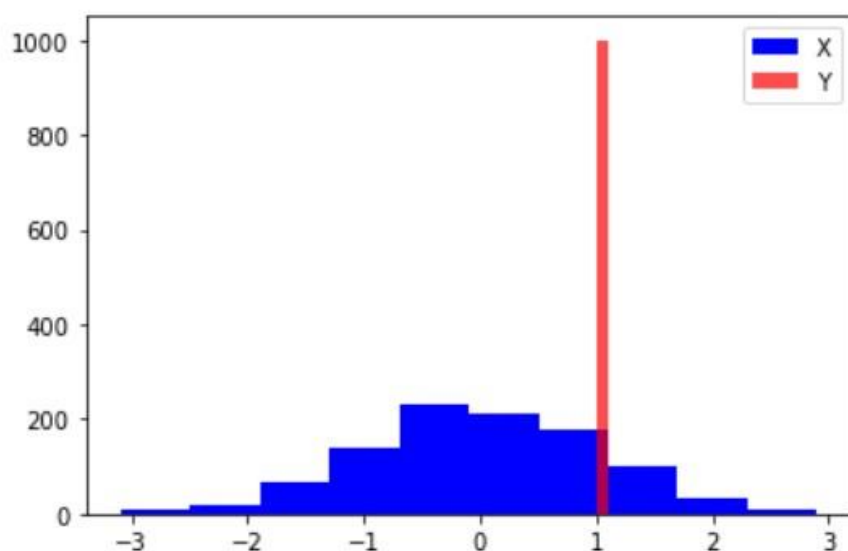
$$\begin{aligned}
 X &\sim N(\mu_1, \sigma_1^2) \quad Y \sim N(\mu_2, \sigma_2^2) \quad Z = X + Y \quad f_X(x) = \frac{1}{\sigma_1 \sqrt{2\pi}} e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}} \\
 f_Y(y) &= \frac{1}{\sigma_2 \sqrt{2\pi}} e^{-\frac{(y-\mu_2)^2}{2\sigma_2^2}} \quad X+Y \rightarrow f_{X+Y}(z) = ? \\
 \text{Sol: } f_{X+Y}(z) &= P(X+Y \leq z) = \iint_{x+y \leq z} f_{XY}(x,y) dx dy = \iint_{x+y \leq z} f_X(x) f_Y(y) dx dy = \int_{-\infty}^{\infty} \int_{-\infty}^{z-y} f_X(x) f_Y(y) dx dy \\
 &= \int_{-\infty}^{\infty} F_X(z-y) f_Y(y) dy \xrightarrow{\frac{d}{dz}} f_{X+Y}(z) = \int_{-\infty}^{\infty} f_X(z-y) f_Y(y) dy \\
 f_{X+Y}(z) &= \int_{-\infty}^{\infty} \frac{1}{\sigma_1 \sqrt{2\pi}} e^{-\frac{(z-y-\mu_1)^2}{2\sigma_1^2}} \times \frac{1}{\sigma_2 \sqrt{2\pi}} e^{-\frac{(y-\mu_2)^2}{2\sigma_2^2}} dy = \frac{e^{-\frac{(z-(\mu_1+\mu_2))^2}{2(\sigma_1^2+\sigma_2^2)}}}{\sqrt{2\pi} \sqrt{\sigma_1^2+\sigma_2^2}} \\
 &= N(\mu_1+\mu_2, \sigma_1^2+\sigma_2^2)
 \end{aligned}$$

size X&Y?1000  
Variance X?1  
Average X?0  
Variance Y?0  
Average Y?1

به عنوان مثال:

$X \sim N(0,1)$  &  $Y \sim N(1,0)$  >-  
 $Z \sim N(1,1)$

که به صورت تحلیلی با فرمول دست  
نویس بالا و با عدد حاصل از کد هم  
مطابقت دارد.



Z average is: 1.0029620084215405  
Z Variance is: 0.9968204233456286

فرض

11.199E (5) 11.199E (5)

$$X = R \cos \theta, Y = R \sin \theta \Rightarrow X^2 + Y^2 = R^2 \quad R = \sqrt{x^2 + y^2} \quad f_R(r) = P[-r \leq r \leq r] = 1 - P[r < e^{-\frac{r^2}{2}}]$$

$$f_R(r) = 1 - \int_0^r e^{-\frac{t^2}{2}} dt = 1 - e^{-\frac{r^2}{2}} \quad f_R(r) = \int_0^r t e^{-\frac{t^2}{2}} dt \quad r > 0$$

2

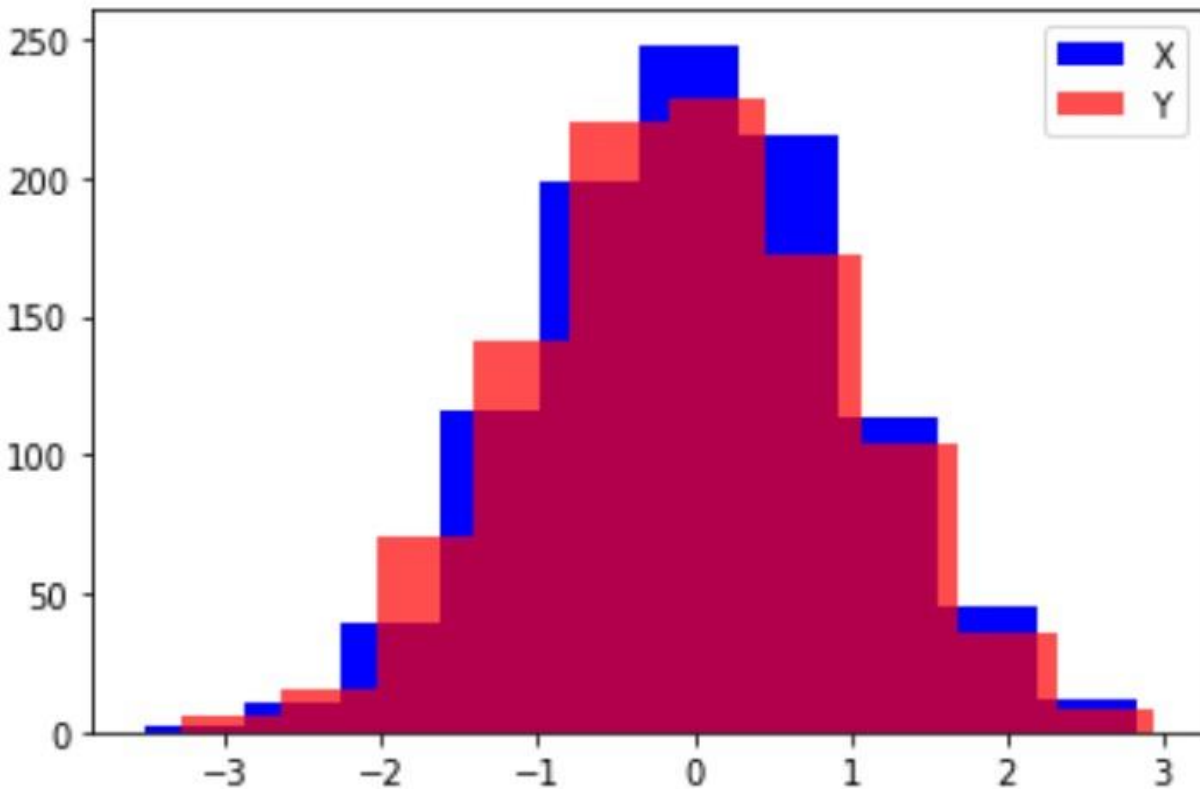
$$X = R \cos \theta, Y = R \sin \theta \rightarrow \frac{Y}{X} = \tan \theta \Rightarrow \theta = \tan^{-1} \left( \frac{Y}{X} \right) \quad X^2 + Y^2 = R^2 \Rightarrow -r \ln r$$

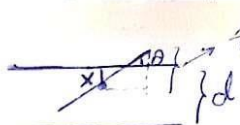
$$r = e^{-\frac{(X^2 + Y^2)}{2}} \quad f_{X,Y}(x,y) = \int \left( \frac{\partial^2 f}{\partial x \partial y} \right) = \int \left[ -x e^{-\frac{(x^2 + y^2)}{2}} - y e^{-\frac{(x^2 + y^2)}{2}} \right]$$

$$= \frac{1}{r^2} \left( \frac{x^2 e^{-\frac{(x^2 + y^2)}{2}}}{x^2 + y^2} + \frac{y^2 e^{-\frac{(x^2 + y^2)}{2}}}{x^2 + y^2} \right) = \frac{1}{r^2} e^{-\frac{(x^2 + y^2)}{2}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \times \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$$

$\downarrow$   $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty}$

size U & V? 1000





$$P\left\{X \leq \frac{l \sin \theta}{d}, 0 \leq \theta \leq \pi\right\} = \int_0^\pi \int_0^{\frac{l \sin \theta}{d}} \frac{r}{\pi d} dx d\theta = \frac{r l}{\pi d}$$

$$f_{X,\theta}(x,\theta) = f_X(x) \cdot f_\theta(\theta) = \frac{r}{\pi d}$$

$$\rightarrow l \leq d : P = \frac{r l}{\pi d} = 0.254$$

needle no? 5000

number of hits: 1272

hit\_probability is: 0.2544

pi Estimation: 3.144654088050314

relative\_error is: 0.0009735361584457095

خروجی کد: