Camera Calibration

3D Computer Vision

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Hartley, R., Zisserman, A. (2003). Multiple View Geometry in Computer Vision. New York, NY, USA: Cambridge University Press. ISBN: 0521540518

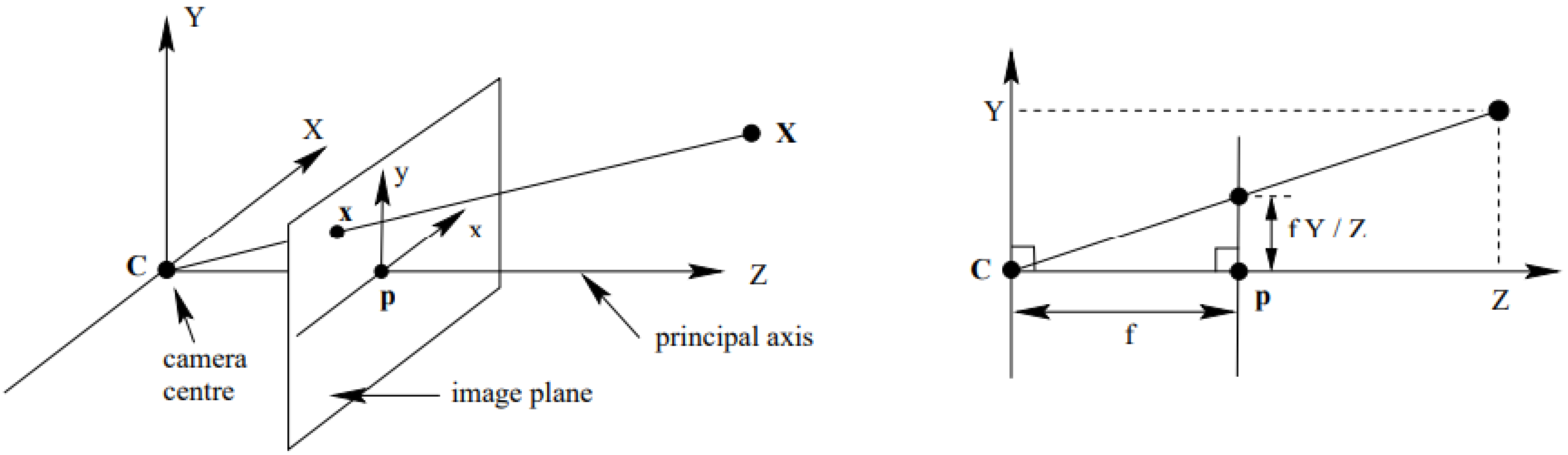
Lines and Points: Homogeneous Representation

1. The line $ax + by + c = 0$ can be represented by $(a, b, c)^T$.
2. The line $(ka)x + (kb)y + (kc) = 0$ is the same line so it also can be be represented by $(a, b, c)^T$.
3. An equivalence class of vectors under this equivalence relationship is known as a **homogeneous** vector.
4. A point $x = (x, y)^T$ lies on the line $l = (a, b, c)^T$ if and only if $ax + by + c = 0$.
5. This may be written in terms of an inner product of vectors representing the point as $(x, y, 1)(a, b, c)^T = l(x, y, 1) = 0$
6. An arbitrary homogeneous vector representative of a point is of the form $x = (x_1, x_2, x_3)$, representing the point $(\frac{x_1}{x_3}, \frac{x_2}{x_3}, 1)$.

Properties of Homogeneous Representation

1. The point x lies on the line l if and only if $x^T l = 0$.
2. The intersection of two lines l and l' is the point $x = l \times l'$.
3. The line through two points x and x' is $l = x \times x'$.
4. Projective transformation of points: $x' = Hx$.
5. Projective transformation of lines: $l' = H^{-T} l$.

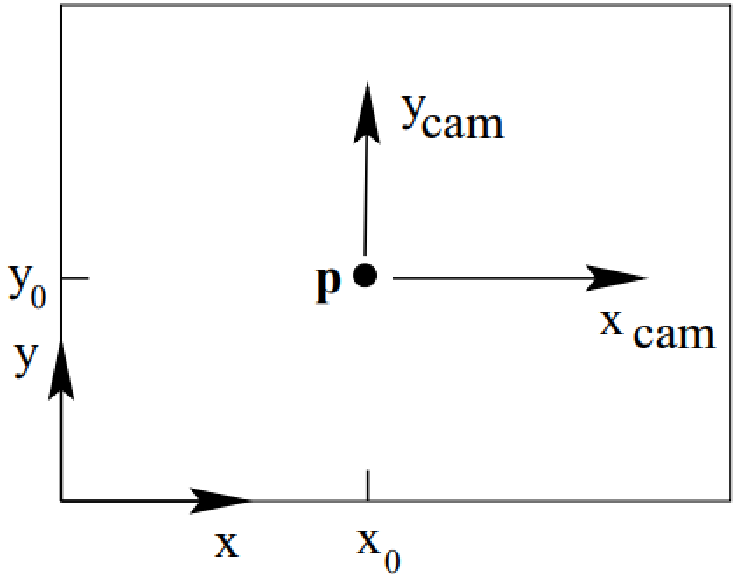
Camera Model: Focal Length



$$(X, Y, Z)^{\top} \mapsto (fX/Z, fY/Z, f)^{\top} \mapsto (fX/Z, fY/Z)$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & & & 0 \\ & f & & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

Camera Model: Camera Center



$$(X, Y, Z)^{\top} \mapsto (\frac{fX}{Z} + p_x, \frac{fY}{Z} + p_y)^T$$

$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & & p_x & 0 \\ & f & p_y & 0 \\ & & 1 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

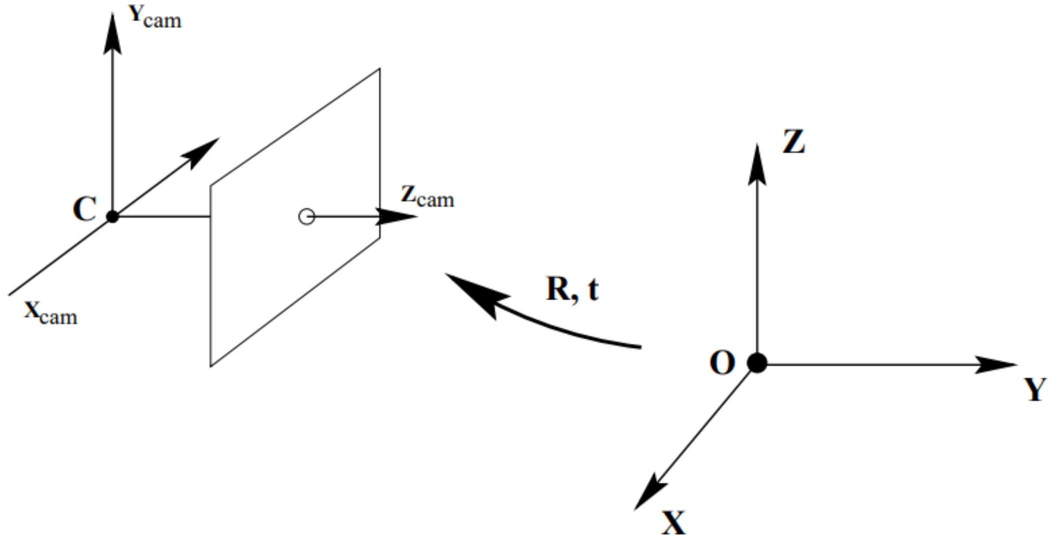
Camera Model: CCD cameras

$$\begin{array}{ccccc} x_I & & & & X_C \\ \begin{pmatrix} f_x \mathrm{X} + \mathrm{Z} p_x \\ f_y \mathrm{Y} + \mathrm{Z} p_y \\ \mathrm{Z} \end{pmatrix} & = & \begin{bmatrix} f_x & & p_x & 0 \\ & f_y & p_y & 0 \\ & & 1 & 0 \end{bmatrix} & & \begin{pmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \\ 1 \end{pmatrix} \end{array}$$

$$\begin{pmatrix} f_x \mathrm{X} + \mathrm{Z} p_x \\ f_y \mathrm{Y} + \mathrm{Z} p_y \\ \mathrm{Z} \end{pmatrix} = [K|\mathbf{0}] \begin{pmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \qquad Z \begin{pmatrix} \dfrac{f_x \mathrm{X} + \mathrm{Z} p_x}{Z} \\ \dfrac{f_y \mathrm{Y} + \mathrm{Z} p_y}{Z} \\ 1 \end{pmatrix} = [K|\mathbf{0}] \begin{pmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \\ 1 \end{pmatrix}$$

World Coordinate



$$X_C = [R \mid t]X_w$$

Camera Model

$$x_I = [K|\mathbf{0}][R|t]X_w$$

$$x_I = K\underbrace{[R|t]X_w}_{X_C}$$

$$x_I = PX_w$$

Finding P: Camera Calibration

$$x_I = PX_w$$

$$K \qquad T = [R|t]$$

One Last Thing! Camera Distortion

$$x_I = PX_w$$

Radial distortion

$$x_{distorted} = x(1 + k_1 r^2 + k_2 r^4 + k_3 r^6) y_{distorted} = y(1 + k_1 r^2 + k_2 r^4 + k_3 r^6)$$

tangential distortion

$$x_{distorted} = x + [2p_1 xy + p_2(r^2 + 2x^2)] y_{distorted} = y + [p_1(r^2 + 2y^2) + 2p_2 xy]$$

A Flexible New Technique for Camera Calibration

Abstract

We propose a flexible new technique to easily calibrate a camera. It is well suited for use without specialized knowledge of 3D geometry or computer vision. The technique only requires the camera to observe a planar pattern shown at a few (at least two) different orientations. Either the camera or the planar pattern can be freely moved. The motion need not be known. Radial lens distortion is modeled. The proposed procedure consists of a closed-form solution, followed by a nonlinear refinement based on the maximum likelihood criterion. Both computer simulation and real data have been used to test the proposed technique, and very good results have been obtained. Compared with classical techniques which use expensive equipment such as two or three orthogonal planes, the proposed technique is easy to use and flexible. It advances 3D computer vision one step from laboratory environments to real world use.

Index Terms— Camera calibration, calibration from planes, 2D pattern, absolute conic, projective mapping, lens distortion, closed-form solution, maximum likelihood estimation, flexible setup.

Homography Estimation

$$s\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathrm{K}[R|T]\begin{bmatrix} X \\ Y \\ \mathrm{Z} \\ 1 \end{bmatrix}$$

$$s\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathrm{K}\begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{r}_3 & \mathrm{t} \end{bmatrix}\begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

$$= \underbrace{\mathrm{K}\begin{bmatrix} \mathrm{r}_1 & \mathrm{r}_2 & \mathrm{t} \end{bmatrix}}_{\mathrm{H}}\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}.$$

Ax = b

Camera Intrinsic Parameters

$$[\mathrm{h}_1 \quad \mathrm{h}_2 \quad \mathrm{h}_3] = \lambda A [\mathrm{r}_1 \quad \mathrm{r}_2 \quad \mathrm{t}]$$

$$\downarrow$$

$$\mathrm{h}_1^T \mathrm{K}^{-T} \mathrm{K}^{-1} \mathrm{h}_2 = 0$$
$$\mathrm{h}_1^T \mathrm{K}^{-T} \mathrm{K}^{-1} \mathrm{h}_1 = \mathrm{h}_2^T \mathrm{K}^{-T} \mathrm{K}^{-1} \mathrm{h}_2$$

$$\downarrow$$

$$\text{Cholesky decomposition} \longleftarrow B = \mathrm{K}^{-T} \mathrm{K}^{-1} \longrightarrow$$

$$\mathbf{b} = [B_{11}, B_{12}, B_{22}, B_{13}, B_{23}, B_{33}]^T$$

$$\mathbf{v}_{ij} = [h_{i1}h_{j1}, h_{i1}h_{j2} + h_{i2}h_{j1}, h_{i2}h_{j2}, \\ h_{i3}h_{j1} + h_{i1}h_{j3}, h_{i3}h_{j2} + h_{i2}h_{j3}, h_{i3}h_{j3}]^T$$

$$\downarrow$$

$$\boldsymbol{V}_{\mathbf{2n \times 6}} * \boldsymbol{b}_{\mathbf{6 \times 1}} = \mathbf{0}$$

Camera Extrinsic Parameters

$$[\mathrm{h}_1 \quad \mathrm{h}_2 \quad \mathrm{h}_3] = \lambda \mathrm{A}[\mathrm{r}_1 \quad \mathrm{r}_2 \quad \mathrm{t}]$$

$$\begin{aligned}
\mathbf{r}_1 &= \lambda \mathbf{K}^{-1}\mathbf{h}_1 \\
\mathbf{r}_2 &= \lambda \mathbf{K}^{-1}\mathbf{h}_2 \\
\mathbf{r}_3 &= \mathbf{r}_1 \times \mathbf{r}_2 \\
\mathbf{t} &= \lambda \mathbf{K}^{-1}\mathbf{h}_3
\end{aligned}$$

Dealing with Radial Distortion

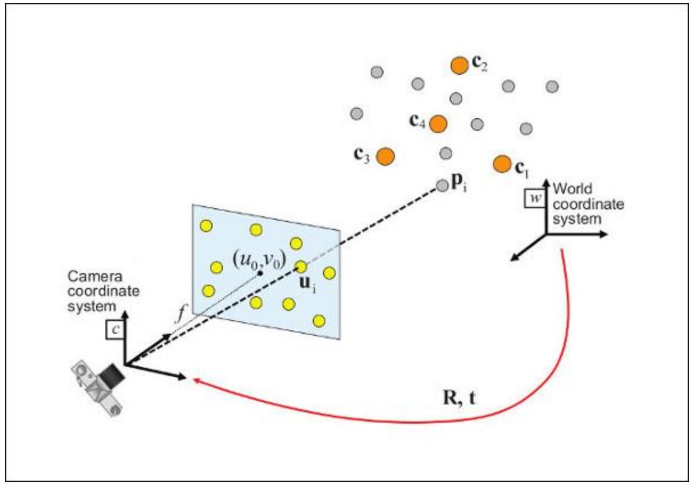
$$\sum_{i=1}^{n}\sum_{j=1}^{m}\|\mathbf{m}_{ij}-\breve{\mathbf{m}}(\mathbf{A},k_1,k_2,\mathbf{R}_i,\mathbf{t}_i,\mathtt{M}_j)\|^2$$

Applications: Perspective-n-Point

$$s\begin{bmatrix}u\\v\\1\end{bmatrix} = \mathrm{K}[R|T]\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}$$

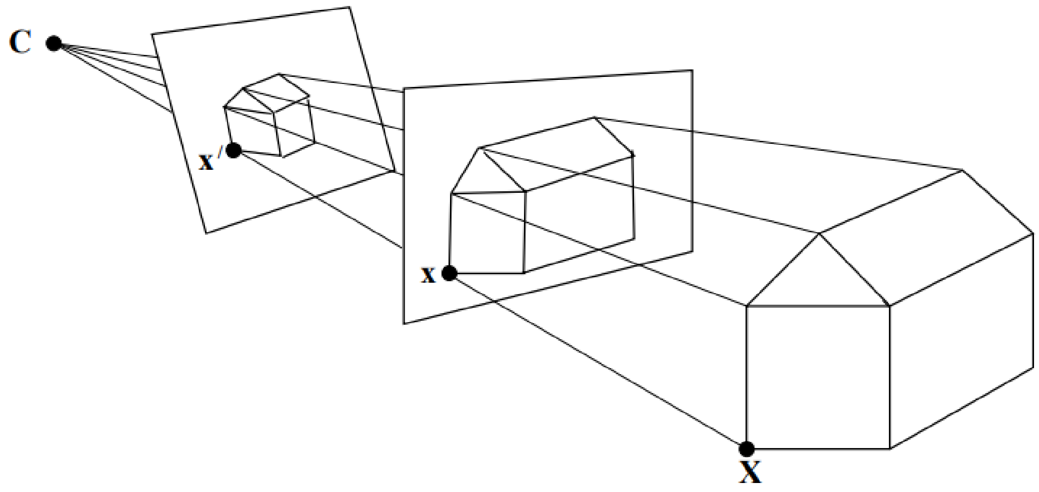
known

camera pose



Applications: Perspective-n-Point

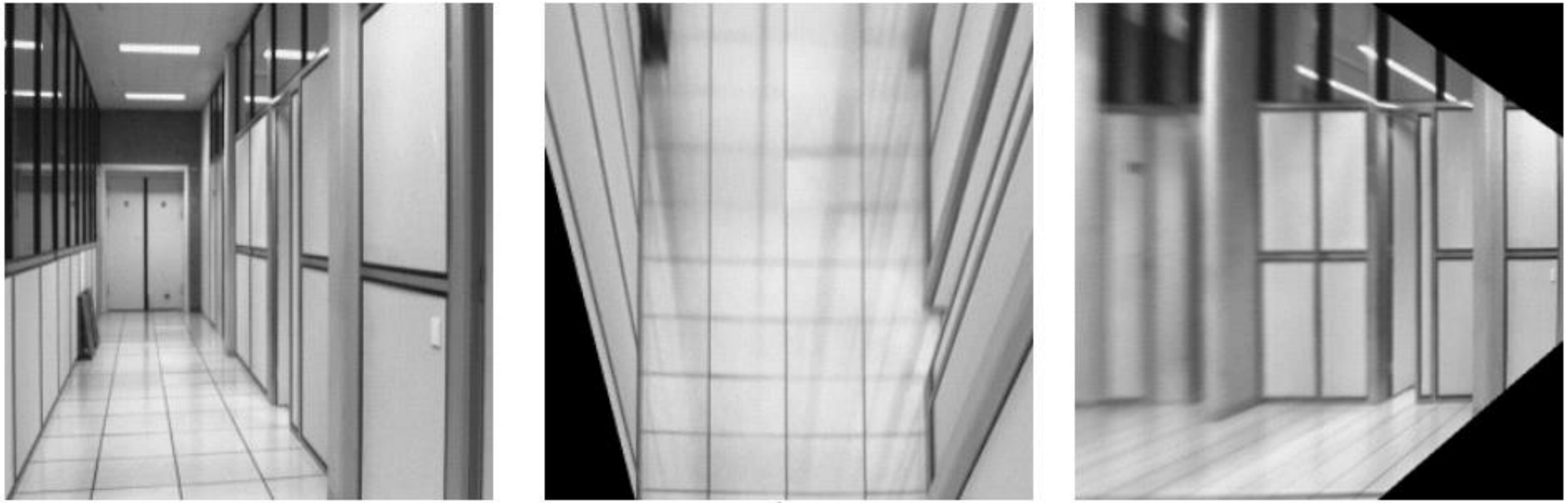
The Importance of the Camera Center



$$\mathbf{x}' = \mathrm{P}'\mathbf{X} = (\mathrm{K}'\mathrm{R}')(\mathrm{K}\mathrm{R})^{-1}\mathrm{P}\mathbf{X} = (\mathrm{K}'\mathrm{R}')(\mathrm{K}\mathrm{R})^{-1}\mathbf{x}$$

The Importance of the Camera Center

The Importance of the Camera Center



a b c

Multiview Stereo