



COMSATS UNIVERSITY ISLAMABAD

Subject:

Linear Algebra

Assignment 1

Submitted by:

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Question 1

Goods	Services
0.2	0.7 → Goods
0.8	0.3 → Services

$$\begin{array}{r} 0.2G + 0.7S = 1G \\ 0.8G + 0.3S = 1S \end{array}$$

$$-0.80 + 0.75 = 0$$

$$0.84 - 0.75 = 0$$

$$\begin{bmatrix} -0.8 & 0.7 & 0 \\ 0.8 & -0.7 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} -0.8 & 0.7 & 0 \\ 0 & 0 & 0 \end{bmatrix} R_2 + R_1 \rightarrow R_2$$

g	s	
① -0.875	0	
0	0	∴ S is free

$$x = \begin{bmatrix} C \\ S \end{bmatrix} = \begin{bmatrix} 0.875s \\ s \end{bmatrix} \Rightarrow s \begin{bmatrix} 0.875 \\ 1 \end{bmatrix} \quad (\text{Parametric form})$$

$$\therefore G = 0.8755 \quad \therefore \text{if } S = 1000 \quad \therefore G = \frac{75}{8}$$

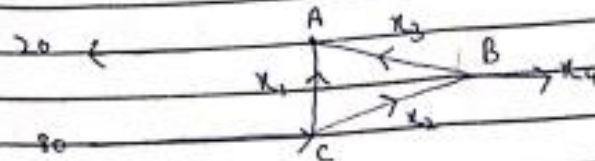
QUESTION 2

If we change the currency from dollars to yen; the structure of the equation will be "unaffected" because the ratio is going to be the same.

$$\frac{1.75}{8}$$

QUESTION 11

Q - Find general flow pattern of the network shown. Assuming all values are non-negative, what is the largest possible x_3 value?



Node	Flow IN	Flow OUT
A	$x_1 + x_3$	20
B	x_2	$x_3 + x_4$
C	80	$x_1 + x_2$
Total	80	$x_4 + 20$

$$x_1 + x_3 = 20 \rightarrow (i)$$

$$x_2 = x_3 + x_4 \rightarrow (ii)$$

$$80 = x_1 + x_2 \rightarrow (iii)$$

$$80 = x_1 + 20 \rightarrow (iv)$$

$$\boxed{x_1 = 60}$$

1	0	1	0	20
0	1	-1	-1	0
1	1	0	0	80
0	0	0	1	60

$$= \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & -1 & 1 & 0 & -60 \\ 0 & 0 & 0 & 1 & 60 \end{array} \right] \quad R_1 - R_3 \rightarrow R_3$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & -60 \\ 0 & 0 & 0 & 1 & 60 \end{array} \right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 20 \\ 0 & 1 & -1 & 0 & 60 \\ 0 & 0 & 0 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$x = \begin{cases} x_1 = 20 - x_3 \rightarrow \text{eqn (i)} \\ x_2 = 60 + x_3 \\ x_3 \text{ is free} \\ x_4 = 60 \end{cases}$$

$\therefore 0$ is largest possible value for x_3 bcoz in eqn (i) if we take $x_3 = 21$ then x_1 would be $-ve$ which is against the condition of this question.

QUESTION 3

(a)

Chemicals

Fuel

Machinery

0.2

0.8

0.4

→ Chemicals

0.3

0.3

0.4

→ Fuel

0.5

0.1

0.5

→ Machinery

(3b)

$$0.2C + 0.8F + 0.4M = 1C$$

$$0.3C + 0.1F + 0.4M = 1F$$

$$0.5C + 0.1F + 0.2M = 1M$$

$$\therefore -0.8C + 0.8F + 0.4M = 0$$

$$0.3C - 0.9F + 0.4M = 0$$

$$0.5C + 0.1F - 0.8M = 0$$

Augmented Matrix :-

$$\left[\begin{array}{ccc|c} -0.8 & 0.8 & 0.4 & 0 \\ 0.3 & -0.9 & 0.4 & 0 \\ 0.5 & 0.1 & -0.8 & 0 \end{array} \right]$$

multiplying by 10

$$\left[\begin{array}{ccc|c} -8 & 8 & 4 & 0 \\ 3 & -9 & 4 & 0 \\ 5 & 1 & -8 & 0 \end{array} \right]$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & -1/2 & 0 \\ 3 & -9 & -4 & 0 \\ 5 & 1 & -8 & 0 \end{array} \right] \quad -\frac{1}{8}R_1 \rightarrow R_1$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & -1/2 & 0 \\ 0 & 6 & -11/2 & 0 \\ 0 & -6 & 11/2 & 0 \end{array} \right] \quad \begin{array}{l} 3R_1 - R_2 \rightarrow R_2 \\ 5R_1 - R_3 \rightarrow R_3 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & -1/2 & 0 \\ 0 & 6 & -11/2 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad R_2 + R_3 \rightarrow R_2$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & -1/2 & 0 \\ 0 & 1 & -11/12 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \frac{R_2}{6} \rightarrow R_2$$

$$\begin{array}{c} C \quad F \quad M \\ = \left[\begin{array}{ccc|c} 1 & 0 & -1.417 & 0 \\ 0 & 1 & -0.917 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad R_2 + R_1 \rightarrow R_1 \end{array}$$

$\therefore M$ is free variable

$$x = \begin{bmatrix} C \\ F \\ M \end{bmatrix} = \begin{bmatrix} 1.417M \\ 0.917M \\ M \end{bmatrix} \Rightarrow M = \begin{bmatrix} 1.417 \\ 0.917 \\ 1 \end{bmatrix}$$

(3c)

$$M = 100$$

$$C = 1.417M$$

$$= 1.417(100)$$

$$\boxed{\text{Chemicals} = 141.7}$$

$$F = 0.917M$$

$$= 0.917(100)$$

$$\boxed{\text{Fuel} = 91.7}$$

Q No. 8



$$\text{H}_2\text{O} \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} (x_1), \text{CaCO}_3 \begin{bmatrix} 0 \\ 3 \\ 1 \\ 1 \end{bmatrix} (x_2), \text{H}_2\text{O} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} (x_3), \text{Ca} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} (x_4), \text{CO}_2 \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix} (x_5)$$

$$\begin{bmatrix} 3 & 0 & -2 & 0 & 0 & 0 \\ 1 & 3 & -1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix} R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 & 0 \\ 3 & 0 & -2 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix} R_2 + (-3)R_1$$

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 & 0 \\ 0 & -9 & 1 & 0 & 6 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix} R_3 + R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 & 0 \\ 0 & -9 & 1 & 0 & 6 & 0 \\ 0 & 0 & 1 & -9 & 6 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \end{bmatrix} 9R_4 + R_2$$

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 & 0 \\ 0 & -9 & 1 & 0 & 6 & 0 \\ 0 & 0 & 1 & -9 & 6 & 0 \\ 0 & 0 & 1 & 0 & -3 & 0 \end{bmatrix} R_4 + (-1)R_3$$

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 & 0 \\ 0 & -9 & 1 & 0 & 6 & 0 \\ 0 & 0 & 1 & -9 & 6 & 0 \\ 0 & 0 & 0 & 9 & -9 & 0 \end{bmatrix} R_4 = R_4/9$$

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 & 0 \\ 0 & -9 & 1 & 0 & 6 & 0 \\ 0 & 0 & 1 & -9 & 6 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$R_3 + 9(R_4)$

$$\begin{bmatrix} 1 & 3 & -1 & 0 & -2 & 0 \\ 0 & -9 & 1 & 0 & 6 & 0 \\ 0 & 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$R_1 + R_3$

$$\begin{bmatrix} 1 & 3 & 0 & 0 & -5 & 0 \\ 0 & -9 & 0 & 0 & 9 & 0 \\ 0 & 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$R_2 = R_2 / -9$

$$\begin{bmatrix} 1 & 3 & 0 & 0 & -5 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$R_1 - 3R_2$

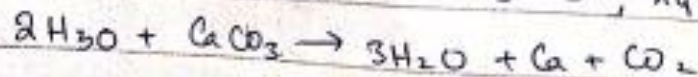
$$\begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

General Equations:-

$$x_1 = 2x_5, \quad x_2 = x_5, \quad x_3 = 3x_5, \quad x_4 = x_5$$

take $x_5 = 1$ so

$$x_1 = 2, \quad x_2 = 1, \quad x_3 = 3, \quad x_4 = 1$$



Q No. 9



$$(x_1) \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + H_2O (x_2) \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} + (x_3) \begin{bmatrix} -1 \\ 0 \\ -3 \\ -3 \end{bmatrix} + H_2S (x_4) \begin{bmatrix} 0 \\ -1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix} \xrightarrow{2R_2 - 3R_1} \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

$$R_2 \leftrightarrow R_3 \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

$$2R_1 - (R_2) \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & -3 & 2 & 0 \end{bmatrix}$$

$$R_4 + R_3 \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_3/3 \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 + R_3 \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & 0 & -4 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 + R_3 \begin{bmatrix} 2 & 0 & 0 & -2/3 & 0 \\ 0 & 2 & 0 & -0.4 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

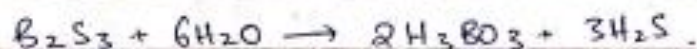
$$R_2/2 \begin{bmatrix} 2 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1/2 \begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

General Solution:-

$$x_1 = 1/3 x_4, \quad x_2 = 2x_4, \quad x_3 = 2/3 x_4$$

$$\text{Take } x_4 = 3 \quad x_1 = 1 \quad x_2 = 6 \quad x_3 = 2.$$



Q.No.10

$$PbNO_3 \begin{bmatrix} 1 \\ 6 \\ 0 \\ 0 \\ 0 \end{bmatrix} + CrM_2O_8 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 0 \end{bmatrix} + Pb_3O_4 \begin{bmatrix} -3 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix} + Cr_2O_3 \begin{bmatrix} 0 \\ 0 \\ -2 \\ 0 \\ -3 \end{bmatrix} + MnO_2 \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ -2 \end{bmatrix} + NO \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$R_2 + (-6)R_1 \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -18 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{bmatrix}$$

$$R_2 \leftrightarrow R_3, \quad R_4 + (-2)(R_1) \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -18 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 4 & 0 & -1 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{bmatrix}$$

$$R_5 - 8R_2 \quad \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -18 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & -1 & -1 & 0 \\ 0 & 0 & -4 & 13 & -2 & -1 & 0 \end{bmatrix}$$

$$9R_5 - 2R_3 \quad \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -18 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & -1 & 0 \\ 0 & 0 & 0 & 117 & -18 & -1 & 0 \end{bmatrix}$$

$$R_4/4 \quad \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -18 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1/4 & 0 \\ 0 & 0 & 0 & 117 & -18 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & 0 & 0 & -22/45 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & 0 & -11/45 & 0 \\ 0 & 0 & 0 & 0 & 1 & -44/45 & 0 \end{bmatrix}$$

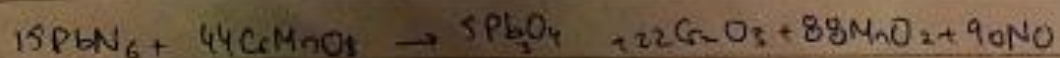
General Solution:-

$$x_1 = 1/6 x_6, \quad x_2 = 22/45 x_6, \quad x_3 = 1/18 x_6$$

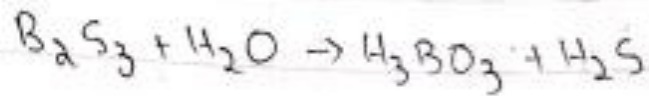
$$x_4 = 11/45 x_6, \quad x_5 = 44/45 x_6$$

$$x_6 = \text{free} \quad x_6 = 90$$

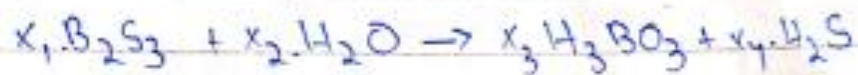
$$x_1 = 15 \quad x_2 = 44 \quad x_3 = 5 \quad x_4 = 22 \quad x_5 = 88$$



Question 5:-



Order:- Boron
sulfur
hydrogen
oxygen



$$x_1 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ -3 \\ -3 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ -1 \\ -2 \\ 0 \end{bmatrix} = 0$$

augmented matrix:-

$$\left[\begin{array}{ccccc} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array} \right]$$

$$= \left[\begin{array}{ccccc} 1 & 0 & -1/2 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array} \right] \rightarrow \frac{1}{2} R_1$$

$$= \left[\begin{array}{ccccc} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array} \right] R_2 - 3R_1 \rightarrow R_2$$

$$= \left[\begin{array}{ccccc} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \end{array} \right] \begin{array}{l} R_2 \leftrightarrow R_4 \\ \text{swap} \end{array}$$

$$= \begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \end{bmatrix} \quad R_3 - 2R_2 \rightarrow R_3$$

$$= \begin{bmatrix} 1 & 0 & -1/2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 3/2 & -1 & 0 \end{bmatrix} \rightarrow \frac{1}{3} R_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} R_2 + 3R_3 \rightarrow R_2 \\ R_4 - 3/2 R_3 \rightarrow R_4 \\ R_1 + 1/2 R_3 \rightarrow R_1 \end{array}$$

$$x_1 - 1/3 x_4 = 0 \Rightarrow x_1 = 1/3 x_4$$

$$x_2 - 2x_4 = 0 \Rightarrow x_2 = 2x_4$$

$$x_3 - 2/3 x_4 = 0 \Rightarrow x_3 = 2/3 x_4$$

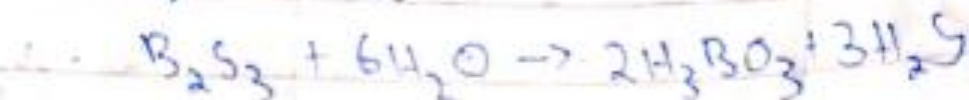
$$\text{taking } x_4 = 3$$

$$x_1 = 1/3 (3) = 1$$

$$x_2 = 2(3) = 6$$

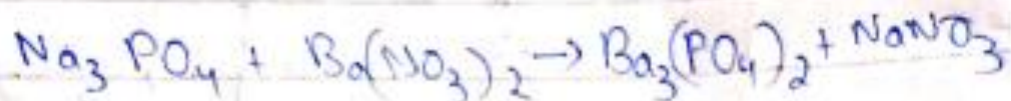
$$x_3 = 2/3 (3) = 2$$

Now equation is:-



The deduction

Q6:-



orderlist:-

sodium
phosphorus
oxygen
barium
nitrogen

[]

$$x_1 \begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \\ 0 \end{bmatrix}$$

$$+ x_2 \begin{bmatrix} 0 \\ 0 \\ 6 \\ 1 \\ 2 \end{bmatrix}$$

$$= x_3 \begin{bmatrix} 6 \\ 2 \\ 8 \\ 3 \\ 0 \end{bmatrix}$$

$$+ x_4 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix}$$

augmented matrix

$$\left[\begin{array}{ccccc} 3 & 0 & 0 & -1 & 0 \\ 1 & 0 & -2 & 0 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right]$$

$$= \left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 4 & 6 & -8 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right] \quad \begin{array}{l} R_1 \leftrightarrow R_2 \\ \text{swap} \end{array}$$

$$= \left[\begin{array}{ccccc} 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 6 & 0 & -3 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{array} \right] \quad \begin{array}{l} R_2 - 3R_1 \rightarrow R_2 \\ R_3 - 4R_1 \rightarrow R_3 \end{array}$$

$$= \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 6 & 0 & -3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 2 & 0 & -1 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_4$
swap

$$= \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 18 & -3 & 0 \\ 0 & 0 & 6 & -1 & 0 \\ 0 & 0 & 6 & -1 & 0 \end{bmatrix}$$

$R_3 - 6R_2$
 $\rightarrow R_3$

$R_5 - 2R_2$
 $\rightarrow R_5$

$$= \begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & -3/18 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rightarrow \frac{1}{18} R_3$

$R_4 \leftrightarrow R_5 \rightarrow R_4$

$R_5 - R_4 \rightarrow R_5$

$$= \left[\begin{array}{ccccc|l} 1 & 0 & 0 & -\frac{1}{3} & 0 & R_1 + 2R_3 \rightarrow R_1 \\ 0 & 1 & 0 & -\frac{1}{2} & 0 & R_2 + 3R_3 \rightarrow R_2 \\ 0 & 0 & 1 & -\frac{1}{6} & 0 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \end{array} \right]$$

$$x_1 - \frac{1}{3}x_4 = 0 \Rightarrow x_1 = \frac{1}{3}x_4$$

$$x_2 - \frac{1}{2}x_4 = 0 \Rightarrow x_2 = \frac{1}{2}x_4$$

$$x_3 - \frac{1}{6}x_4 = 0 \Rightarrow x_3 = \frac{1}{6}x_4$$

Take ~~$x_4 = 3$~~ ~~$x_4 = 2$~~

~~$$x_1 = \frac{1}{3} \cdot 3 = 1$$~~

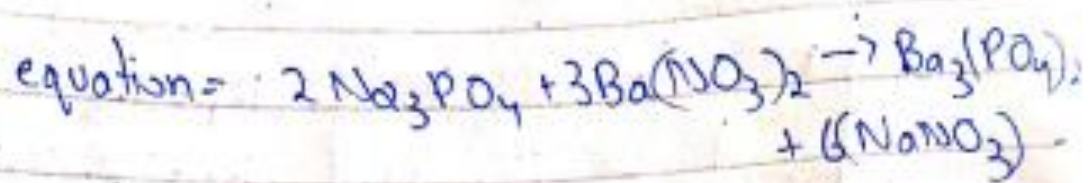
~~$$x_2 = \frac{1}{2} \cdot 2 = 1$$~~

Take $x_4 = 6$

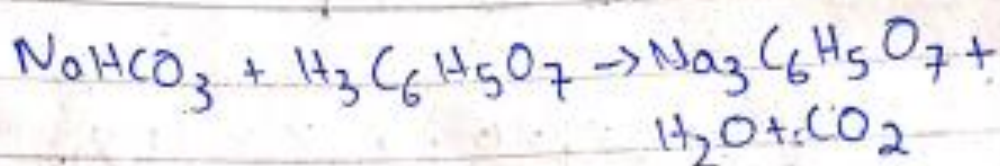
$$x_1 = \frac{1}{3} \cdot 6 = 2$$

$$x_2 = \frac{1}{2} \cdot 6 = 3$$

$$x_3 = \frac{1}{6} \cdot 6 = 1$$



Question 7:-



order :- Sodium
Hydrogen
Carbon
Oxygen

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 8 \\ 6 \\ 7 \end{bmatrix} = x_3 \begin{bmatrix} 3 \\ 5 \\ 6 \\ 7 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$$+ x_5 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix}$$

augmented matrix :-

$$\left[\begin{array}{cccccc|c} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 1 & 8 & -5 & -2 & 0 & 0 & 0 \\ 1 & 6 & -6 & 0 & -1 & 0 & 0 \\ 3 & 7 & -7 & -1 & -2 & 0 & 0 \end{array} \right]$$

$$= \left[\begin{array}{cccccc|c} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 8 & -2 & -2 & 0 & 0 & 0 \\ 0 & 6 & -3 & 0 & -1 & 0 & 0 \\ 0 & 7 & 2 & -1 & -2 & 0 & 0 \end{array} \right]$$

$$R_2 - R_1 \rightarrow R_2$$

$$R_3 - R_1 \rightarrow R_3$$

$$R_4 - 3R_1 \rightarrow R_4$$

$$= \left[\begin{array}{cccccc|c} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 & 0 \\ 0 & 6 & -3 & 0 & -1 & 0 & 0 \\ 0 & 7 & 2 & -1 & -2 & 0 & 0 \end{array} \right] \rightarrow \frac{1}{8} R_2$$

$$= \left[\begin{array}{cccccc|c} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 & 0 \\ 0 & 6 & -3/2 & 3/2 & -1 & 0 & 0 \\ 0 & 7 & 15/4 & 3/4 & -2 & 0 & 0 \end{array} \right]$$

$$R_3 - 6R_2 \rightarrow R_3$$

$$R_4 - 7R_2 \rightarrow R_4$$

$$= \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 1 & -1/4 & -1/4 & 0 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 15/4 & 3/4 & -2 & 0 \end{bmatrix} \quad -2/3 R_3$$

$$= \begin{bmatrix} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -1/2 & 1/6 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 0 & 9/2 & -27/6 & 0 \end{bmatrix} \begin{array}{l} R_1 + 3R_3 \rightarrow R_1 \\ R_2 + 1/4 R_3 \rightarrow R_2 \\ R_4 - \frac{-4}{15} R_3 \rightarrow R_4 \\ R_4 - \frac{15}{4} R_3 \rightarrow R_4 \end{array}$$

$$= \begin{bmatrix} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -1/2 & 1/6 & 0 \\ 0 & 0 & 1 & -1 & 2/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \quad \rightarrow 2/9 R_4$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \begin{array}{l} R_1 + 3R_4 \rightarrow R_1 \\ R_2 + 1/2 R_4 \rightarrow R_2 \\ R_4 - R_5 \rightarrow R_4 \end{array}$$

equations:

$$x_1 - x_5 = 0 \Rightarrow x_1 = x_5$$

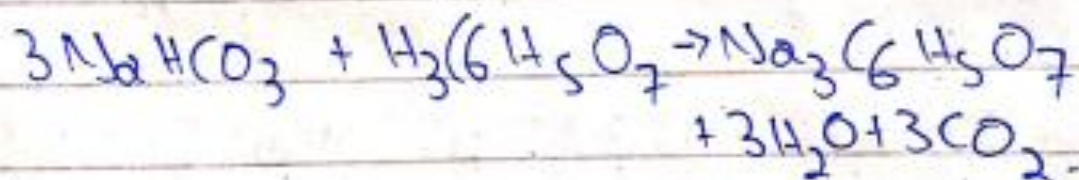
$$x_2 - \frac{1}{3}x_5 \Rightarrow x_2 = \frac{1}{3}x_5$$

$$x_3 - \frac{1}{3}x_5 \Rightarrow x_3 = \frac{1}{3}x_5$$

$$x_4 - x_5 = 0 \Rightarrow x_4 = x_5$$

$$x_5 = 3$$

$$x_1 = 3, x_2 = 1, x_3 = 1, x_4 = 3$$



Question 12.

Node	Flow In	Flow out
A	x_1	$x_3 + x_4 + 40$
B	200	$x_1 + x_2$
C	$x_3 + x_2$	100
D	$x_4 + x_5$	60
Total:-	200	200

equations:

$$x_1 - x_3 - x_4 = 40$$

$$x_1 + x_2 = 200$$

$$x_2 + x_3 - x_5 = 100$$

$$x_4 + x_5 = 60$$

augmented matrix

$$\begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix} \quad R_2 - R_1 \rightarrow R_2$$

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & -1 & -1 & -60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix} \quad R_3 - R_2 \rightarrow R_3$$

$$= \begin{bmatrix} 1 & 0 & -1 & -1 & 0 & 40 \\ 0 & 1 & 1 & 1 & 0 & 160 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{bmatrix} \quad (-1)R_3$$

$$= \begin{bmatrix} 1 & 0 & -1 & 0 & 1 & 100 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} R_1 + R_3 \\ R_2 - R_3 \\ R_5 - R_4 \end{array}$$

$$x_1 - x_3 + x_5 = 100 \Rightarrow x_1 = x_3 - x_5 + 100$$

$$x_2 + x_3 - x_5 = 100 \Rightarrow x_2 = x_5 - x_3 + 100$$

$$x_4 + x_5 = 60 \Rightarrow x_4 = 60 - x_5$$

x_3 and x_5 are free

(b) When $x_4 = 0$, $x_5 = 60$

$$x_1 = 40 + x_3$$

$$x_2 = 160 - x_3$$

(c) The minimum value of x_1 is 40 cars/min
bcuz x_3 cannot be negative.

Question 13

node	Flow in	Flow out
A	$x_2 + 30$	$x_1 + 80$
B	$x_3 + x_5$	$x_2 + x_4$
C	$x_6 + 100$	$x_5 + 40$
D	$x_4 + 40$	$x_6 + 90$
E	$x_1 + 60$	$x_3 + 20$
	230	230

$$x_1 - x_2 = -50$$

$$x_2 - x_3 + x_4 - x_5 = 0$$

$$x_3 - x_6 = 60$$

$$x_4 - x_6 = 50$$

$$x_1 - x_3 = -40$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 60 & 0 \\ 0 & 0 & 0 & 1 & -1 & 50 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & -40 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 1 & 0 & 1 & 0 & 0 & 0 & -46 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 1 & 1 & 0 & 0 & 0 & 10 \end{bmatrix} \begin{array}{l} \\ \\ R_5 - R_1 \rightarrow R_5 \\ \\ \end{array}$$

$$= \begin{bmatrix} 1 & 0 & -1 & 1 & -1 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 2 & 1 & -1 & 0 & 10 \end{bmatrix} \begin{array}{l} R_1 + R_2 \\ R_5 + R_2 \\ \\ \\ \end{array}$$

$$= \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 40 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x_1 = x_3 - 40$$

$$x_2 = x_3 + 10$$

x_3 is free

$$x_4 = x_6 + 50$$

$$x_5 = x_6 + 60$$

x_6 is free

(b) x_1 cannot be negative $x_3 \geq 40$,

$x_2 \geq 50$, x_6 cannot be negative,

$x_4 \geq 50$, $x_5 \geq 60$

minimum flow:

$$x_2 = 50, x_3 = 40, x_4 = 50,$$

$$x_5 = 60 \text{ (when } x_1 = 60 \text{ and } x_6 = 0)$$