Real Numbers, Slope and Line Equations

Calculus I

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Week 1

Outline for the Day

1. Real Numbers
   - Concept of Real Numbers
   - Properties of Real Numbers
2. Line Equations
   - Slope
   - Finding Line Equations
   - Parallel and Perpendicular Lines

Some Elementary Sets

- **Natural Number**, denoted by N (often styled as $\mathbb{N}$), is the set

$$\{1, 2, 3, \cdots\}$$

- **Integers**, denoted by Z (often styled as $\mathbb{Z}$), is the set

$$\{\cdots, -2, -1, 0, 1, 2, \cdots\}$$

- **Rational Number**, denoted by Q (often styled as $\mathbb{Q}$), is the set that contains **all** integral fractions. That is,

$$\mathbb{Q} := \left\{ \frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0 \right\}$$

Example: $2 \in \mathbb{Q}$ and $-0.4 \in \mathbb{Q}$ since $2 = \dfrac{2}{1}$ and $-0.4 = \dfrac{-4}{10}$

Irrational Numbers

- **Irrational Number** contains all the **non-rational** numbers, i.e. the ones that cannot be expressed as a ratio of two integers.
- It doesn't have any designated notation, but because it acts as the **opposite** of rational number set, it's often written as $\mathbb{Q}^C$, the complement of set $\mathbb{Q}$.
- Examples: the number π, e and $\sqrt{2}$ are elements of the irrational numbers. $22/7$, albeit a very good one, is just an approximation of π—hence it won't make π rational.
- **All repeating decimals** are rational, and **all nonrepeating decimals** are irrational.

Concept of Real Numbers

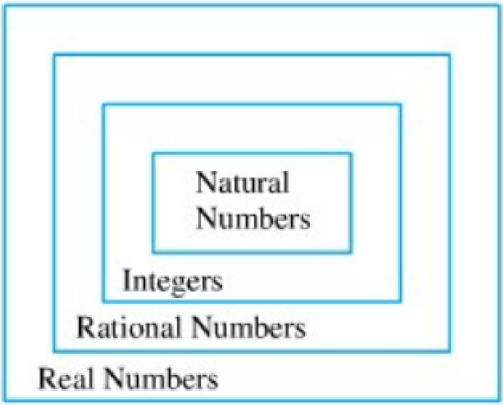

Figure: Real Numbers and its subsets

Some Properties of Real Numbers

- $\mathbb{R}$ is a **completely ordered** set, meaning all elements of $\mathbb{R}$ are *comparable*. Mathematically, if $a, b \in \mathbb{R}$, then exactly one of $a < b, a = b$, or $a > b$ must be true. This property is also called the trinity axiom.
- $\mathbb{R}$ is **dense**, meaning we cannot state two *consecutive* real numbers (unlike natural, integers). Mathematically, if $a, b \in \mathbb{R}$, then there's always a real number c such that

  $$a < c < b$$

  A trivial value of c is the *middle value* between a and b, that is $c = \dfrac{a+b}{2}$.

Smallest Positive Real Numbers

Cool trivia: there's no such number called the 'smallest positive real number'. Let $a \in \mathbb{R}$ is such number, then we will have $\frac{a}{2} \in \mathbb{R}$ that satisfies $0 < \frac{a}{2} < a$ hence contradicting the assumption that a is the smallest positive real number.

Slope or Gradient

- The Slope (or Gradient) is a universal measure of **how steep** a line is.
- It is defined by the ratio of its 'rise' and 'run', often denoted as m.

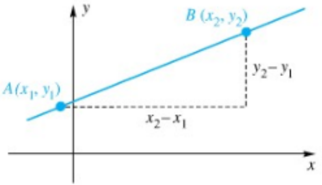

Figure: Visualization of Gradient

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Line Equations

- Generally we need two things to **uniquely define** a line equation: a gradient of the line (m) and a point that lies in said line (x_1, y_1).
- Formula used to find a line equation:

$$y - y_1 = m(x - x_1)$$

Line Equations

Example: Find the line equation that passes both $(-1, 3)$ and $(4, 2)$.

On Parallel and Perpendicular Lines

Let g and h be a couple of straight lines and m_g and m_h as their slopes respectively.If g is parallel to h, then $m_g = m_h$If g is perpendicular to h, then $m_g \times m_h = -1$

Example

Find line equations that passes through $(-1, 3)$ if:

1. it's parallel with the line $x + 3y - 4 = 0$
2. it's perpendicular to $y = -\frac{1}{4}x - \frac{5}{4}$

Exercise

Calculus 9th Edition: Problem Set 0.3 (page 22)

1. Do all odd numbered problems from 23-38
2. Do 39, 41, 42 and 44.