Report Practical(Assignment 3)

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Abstract

This report includes our solutions to the problems of the 3rd practical assignment. It consists of three sections: In Section 1 we implement the original GAN and WGAN with gradient penalty along with some experimentation. Section 2 is about the implementation and experimentation of the VAEs. The last section (3) includes experimentation and evaluation of the generative models' ability to generate realistic-looking images. In each case, our code is uploaded to the following Github repository [1]

Problem 1

In this problem we are using a GAN discriminator to estimate the Jensen Shannon divergence (JSD) and a the Wasserstein Distance (WD). We implement the discriminator/critic using a multi-layer perceptron that we can initialize to either have an output $y \in [0, 1]$ (sigmoid activation) or an output $y \in \mathbb{R}$ (no activation). The architecture consists of two hidden layers with 64 and 128 output units respectively. We use ReLU activation function in the hidden layers.

1. In this section, we implement a function to estimate the JSD. Using our MLP with sigmoid activation at the output we optimize the following objective function:

$$\arg\max_{\theta} \left\{ \log 2 + \frac{1}{2}\mathbf{E}_{x\sim p}[\log(D_\theta(x))] + \frac{1}{2}\mathbf{E}_{y\sim q}\log(1 - D_\theta(y))] \right\}$$

At its optimum, the objective function estimates the JSD between the distributions given by p and q. An overview of the implementation can be seen below. The full code is available in the file **density_estimation.py** under our github repository [1].

```

#Implementation of the JSD

optimizer.zero_grad()
p = torch.cat((p1,torch.rand(batch_size,1)),1).to(device)
q = torch.cat((phi*torch.ones(batch_size,1),t
                        orch.rand(batch_size,1)),1).to(device)
Dp = net(p)
Dq = net(q)
loss = -(math.log(2.) + (1/2.)*torch.mean(torch.log(Dp)) +
```

```
                                                    (1/2.)*torch.mean(torch.log(1-Dq)))
loss.backward()
optimizer.step()
```

2. In this section, we implement a function to estimate the WD. In this case we use our MLP without activation function, so it has an output of a real valued scalar. Here we optimize the following objective function:

$$\arg\max_{\theta} \mathbf{E}_{x\sim p}[T_\theta(x)] - \mathbf{E}_{y\sim q}[T_\theta(y)] - \lambda \mathbf{E}_{z\sim r}(||\nabla_z T_\theta(z)||_2 - 1)^2.$$

   r is the distribution over $z = ax + (1-a)y$, where $x \sim p$, $y \sim q$ and $a \sim U[0, 1]$.

   At it's optimum, this objective function estimates the WD between the distributions given by p and q. The following portion of the code shows the implementation of the optimization. The full code is also available in our repository [1]

```

#Implementation of the WD

optimizer.zero_grad()
p = torch.cat((p1,torch.rand(batch_size,1)),1).to(device)
q = torch.cat((phi*torch.ones(batch_size,1),
                        torch.rand(batch_size,1)),1).to(device)
Dp = net(p)
Dq = net(q)
#  gradient penalty
a = torch.rand(batch_size, 1).expand(batch_size,2).to(device)
r = a*p + (1-a)*q
r.requires_grad = True
Dr = net(r)
gradients = torch.autograd.grad(outputs=Dr, inputs=r,
                grad_outputs=torch.ones(batch_size,1).to(device),
                create_graph=True, retain_graph=True, only_inputs=True)[0]
loss = -(torch.mean(Dp) - torch.mean(Dq) -
          gp_coeff*torch.mean((gradients.norm(2, dim=1) - 1) ** 2))
loss.backward()
optimizer.step()
```

3. Here, we have trained the above neural network with 21 combinations of $p \sim U(0, Z)$, and $q \sim U(\phi, Z)$, where $Z \sim U(0, 1)$ and ϕ is a value in the interval $[-1, 1]$. The size of the distribution is 512, and the models were trained for 5000 iterations using an SGD optimizer. For every value of ϕ we generate a distribution and measure its distance to $p \sim U(0, Z)$. We implement this for WD and JSD and plot their losses.

   The full code is given in the file **density_estimation.py** under our github repository [1].

   The following figures show the estimated JSD for the 21 values of ϕ :

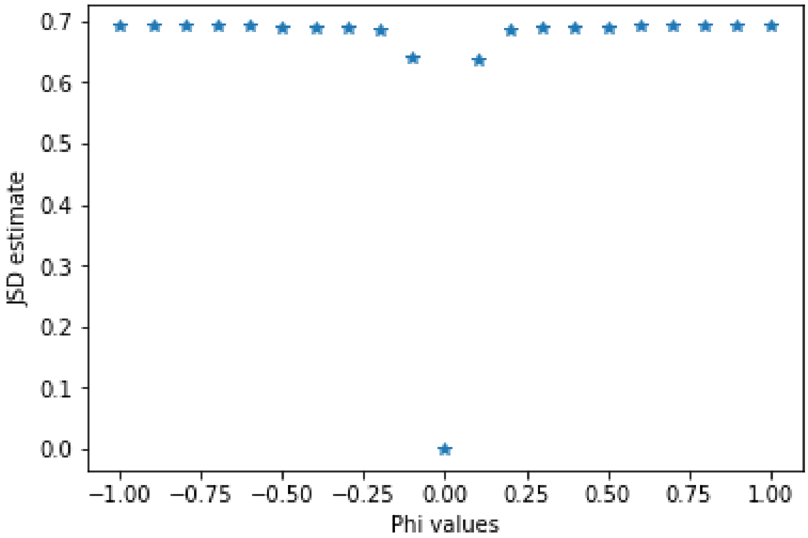

Figure 1: JSD estimation

The estimated WD for the 21 value of ϕ is shown below:

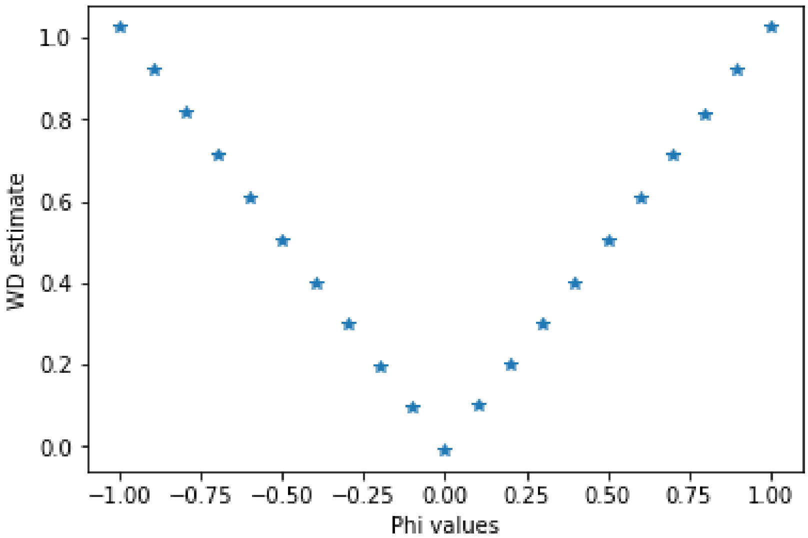

Figure 2: WD estimation

4. In this section we estimate the unknown density f_1 using the approximation $f_0(x)D(x)/(1-D(x))$ (proven in Question 5 from the theoretical part), where f_0 is a known distribution (assumed 1-dimensional standard Gaussian in this question). The full code is provided in the file **density_estimation.py** under our github repository [1].

Using the above neural network (discriminator), we minimize the following function:

$$loss = -(torch.mean(torch.log(Dx)) + torch.mean(torch.log(1 - Dy))),$$

where Dx is the feedforward of f_1 and Dy is the feedforward of f_0.

The following figures show the discriminator's output and the estimated density:

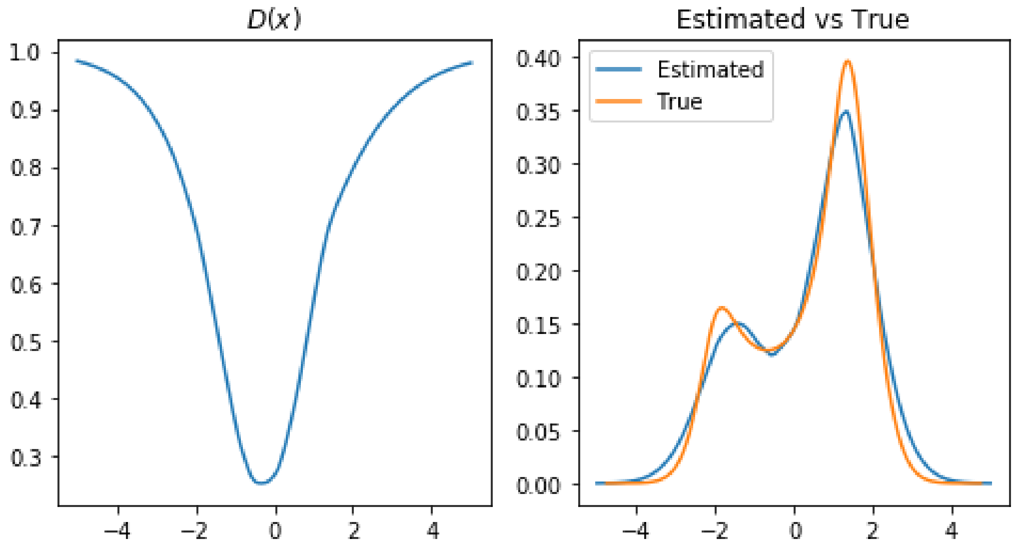

Figure 3: (left) Discriminator output (Right)Estimated f_1

Problem 2

A. Training VAE

We used the given architecture and ADAM with the provided learning rate. After training the model for 20 Epochs, we achieved an average value of ELBO of -93.58 on validation set. It is clearly higher than the reference value provided in question.

B.1 Evaluating log-likelihood with VAE

Here we implement the Importance Sampling procedure that takes as parameters the trained model, an array of x_i and an array of samples z_{ik} from the distribution $q(z|x_i)$. The procedure returns an array of log-likelihood $\log p(x_i)$ of the size of the mini-batches.The code snippet below demonstrates our implementation.

```
def loss_IS(model, true_x, z):

  #Loop over the elements i of batch
  M = true_x.shape[0]

  #Save logp(x)
  logp_x = np.zeros([M])

  #Get mean and std from encoder
  #2 Vectors of 100
  mu, logvar = model.encode(true_x.to(device))
```

```
  std = torch.exp(0.5*logvar)

  K = 200

  #Loop over tha batch
  for i in range(M):
    #z_ik
    samples = z[i,:,:]

    #Compute the reconstructed x's from sampled z's
    x = model.decode(samples.to(device))

    #Compute the p(x_i|z_ik) of x sampled from z_ik
    #Bernoulli dist = Apply BCE
    #Output an array of losses
    true_xi = true_x[i,:,:].view(-1, 784)
    x = x.view(-1, 784)

    p_x = true_xi * torch.log(x) + (1.0-true_xi) * torch.log(1-x)
    p_x = torch.sum(-p_x, dim=1)

    s = std[i, :].view([std.shape[1]])
    m = mu[i, :].view([std.shape[1]])

    q_z = multivariate_normal.pdf(samples.cpu().numpy(),mean=m.cpu().numpy(), cov=np.diag(s.cpu().numpy()**2))

    ##p(z_ik) follows a normal dist with mean 0/variance 1
    #(64, 100)
    #Normally distributed with loc=0 and scale=1
    std_1 = torch.ones(samples.shape[1])
    mu_0 = torch.zeros(samples.shape[1])

    p_z = multivariate_normal.pdf(samples.cpu().numpy(),mean=mu_0.cpu().numpy(), cov=np.diag(std_1.cpu().numpy()**2))

    #Multiply the probablities
    #marginal_likelihood += (p_x * p_z)/q_z
    #Use logsumexp trick to avoid very small prob

    logp_x[i] = np.log((1.0/K) * np.sum(np.exp(np.log(p_x.cpu().numpy()) + np.log(p_z) - np.log(q_z))))

  return logp_x
```

B.2 The evaluation of the training model using the ELBO:

a. Validation: -93.58

b. Test: -93.63

The evaluation of the training model using the log-likelihood:

a. Validation: $* - 93.58$

b. Test: $* - 43.63$

Below is a sample of the obtained images generated by the trained model:

Figure 4: A sample of generated images

Problem 3

We have used in this problem a similare architecture for the VAE's decoder and the GAN's generator which is an MLP with 6 layers, as shown in this code snippet:

```
class Generator(nn.Module):
    def __init__(self):
        super(Generator, self).__init__()

        self.model = nn.Sequential(
            nn.Linear(latent_dim, 128),
            nn.ReLU()
            nn.Linear(128, 256),
            nn.ReLU()
            nn.Linear(256, 512),
            nn.ReLU()
            nn.Linear(512, 1024),
            nn.ReLU()
            nn.Linear(1024, 2048),
            nn.ReLU()
            nn.Linear(2048, int(np.prod(img_shape))),
            nn.Tanh()
        )

    def forward(self, z):
        img = self.model(z)
        img = img.view(img.shape[0], *img_shape)
        return img
```

We have decided to go with this architecture, after having tested several architectures for both models, including convolutionnal neural network. We noticed that the VAE model works fine with a convolutional architecture whereas the GAN have not got good result with that kind of architecture.

A. **Qualitative Evaluations**

1. Visual samples We have generated differents samples from both models as we can see below in Figures 5 and 6. We notice that the images generated by the VAE model are very clear but a little blurry, whereas the images generated by the GAN model are more deversified, and seem more realistic.

Figure 5: Samples generated with VAE

@FARRIS PUT HERE SOME SAMPLES

Figure 6: Samples generated with GAN.

2. Learning the disentangled representation in the latent space The following figures show how the GAN has learned a desentagled representation:

Figure 7: Samples by perturbating dimensions 6 and 96

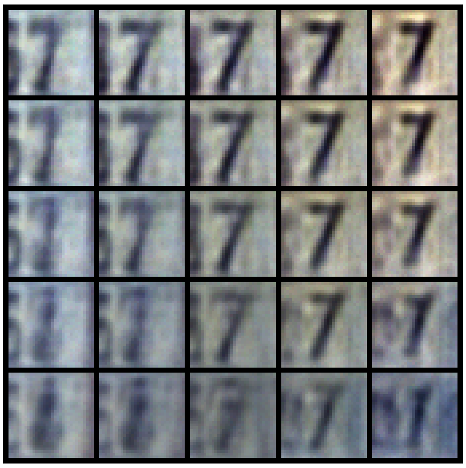

Figure 8: Samples by perturbating dimensions 25 and 75

TO BE CONTINUED BY FARRIS FOR VAE

3. Interpolation in the data space and in the latent space:
TBD BY JORGE FOR GAN AND FARRIS FOR VAE

B. **Quantitative Evaluations**

1. We have used the provided functions to extract the representations of the images. We compute the Frechet Inception Distance by estimating the mean and covariance of the generator's/decoder's distribution. The calculation steps are explained in the following code snippet:

```
def calculate_fid_score(sample_feature_iterator,
                        testset_feature_iterator):
    gen_features = np.array([])
    test_features = np.array([])

    print("Extracting the features ...")
    #sample_feature_iterator is a generator of a minibatch of features images,
    #that is the last conv2d layer of the classifier of 512 features

    #For generated images
    gen_size=0
    for i in sample_feature_iterator: #iterate over minibatch images
        #Now let's get the activation of the images
        gen_features = np.vstack([gen_features,i.reshape(1,512)])  \
        if gen_features.size else i.reshape(1,512)
        gen_size+=1
        if gen_size==1000: break
    gen_features = gen_features.T

    #For test images
    test_size=0
    for i in testset_feature_iterator: #iterate over test images
        test_features= np.vstack([test_features,i.reshape(1,512)])
        test_size+=1
        if test_size==1000: break
```

```python
    test_features = test_features.T

    print("Estimating the mean ...")
    #Estimating the mean of the generated images
    mu_gen = np.mean(gen_features,axis=1).reshape(512,1)

    #Estimating the mean of the test images
    mu_test = np.mean(test_features,axis=1).reshape(512,1)

    print("Estimating the variance ...")
    # We use the unbiased variance estimate which
    #is given by (X-mu)(X-mu)^T/(n-1)
    gen_centered  = gen_features - mu_gen
    test_centered = test_features - mu_test

    sigma_gen = np.matmul(gen_centered, gen_centered.T) / (gen_size - 1)
    sigma_test = np.matmul(test_centered, test_centered.T) / (test_size - 1)

    print("Calculating the sqrt of cov matrices product ...")
    # The sqrt of a matrix A needs A to be symetric, but if A, and B
    # are sysmetric A.B is not symeyric necessarly.
    # To solve that we use this trick:
    # sqrt(sigma1 sigma2) = sqrt(A sigma2 A), where A = sqrt(sigma1)
    # the covariance matrix are by definition symetric

    # to prevent negative values in the cov product
    eps = np.eye(512) * 1e-5

    root_sigma_gen = linalg.sqrtm(sigma_gen + eps)
    sigmas_prod = np.matmul(root_sigma_gen,np.matmul(sigma_test, root_sigma_gen))
    # given np.matmul(root_sigma_gen,np.matmul(sigma_test, root_sigma_gen)) is symetric:
    root_sigmas_prod = linalg.sqrtm(sigmas_prod + eps)

    print("Calculating the FID score ...")
    # Calculating the trace
    trace = np.trace(sigma_test + sigma_gen - 2.0 * root_sigmas_prod)

    # Calculate the squared norm between means
    squared_norm = np.sum((mu_test - mu_gen)**2)

    # Calculate the fid score
    fid = squared_norm + trace

    return fid
```

2. We sampled 1000 images from each generative models and calculate the FID-score as instructed. The results are:
   - For the GAN, the FID score is: 29526.37
   - For the VAE, the FID score is: 51355.12

   This metric confirme our ascertainment that the GAN is more realistic than the VAE, given the ground truth given by the provided classifier.

References

[1] Github repository for assignment 3
https://github.com/faresbs/Representation-Learning.git