# Fractal dimensions in Esporte Clube Bahia's crest. Analytic and computer-assisted calculation

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**Abstract**

Bahia Sport Club's (Esporte Clube Bahia) badge was designed in the 1930s and presents self-similar fractal patterns. This text characterizes its geometry. The motive is composed by squares and circles alternately inscribed in each other. Self-similarity dimension and empirical measures of Minkowski-Bouligand box-counting dimension were calculated. A recursive algorithm is provided to plot the crest.

## 1 Introduction

The crest (Figure 1) was designed in the 1930s.

"Under the slogan of 'Born to Win', Esporte Clube Bahia emerged in 1931. Historically, it was Raimundo Magalhães who created the tricolor badge in the late 1930s." Marcio Luis F. Nascimento, in On why Bahia's crest is unique.[3]

In its self-similar pattern, squares and circles are alternately inscribed in each other. The design starts with an inner square presenting red horizontal stripes. This square is inscribed in a circle, which is also inscribed in an intermediate blue square. These 3 shapes compose the left upper quarter in a new red striped square, forming the loop.

### 1.1 Self-similarity dimension

Self-similarity dimension for exactly self-similar objects with different scaling factors is formally defined[1, 2].

Let $n$ be the number of scaled down pieces in the construction of an exactly self-similar fractal and let $s_1, ..., s_n$ be the scaling factors (some of them can be equal). Then, the self-similarity dimension $D$s is the solution to:

$$\sum_{i}^{n} S_i^D = 1$$


Figure 1: Bandeira oficial do clube.

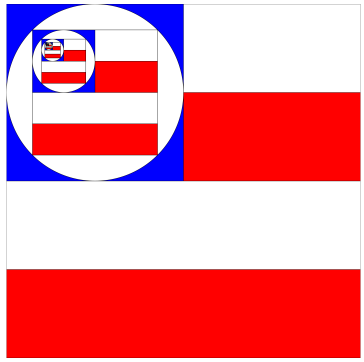

Figure 2: Computer generated crest.

# 2 Methods for empirical measures

A home-brewed recursive function was written to plot the crest using Julia (v1.6.3) with packages Cairo.jl(v1.0.5) and Compose.jl(v0.9.3). Code is available in the **Appendix** section and also in a public repository ( fargolo/ecb_fractal ) .

Black and white contour only plots with resolution ranging from 32x32 to 8192x8192px were written to PNG (Portable Network Graphics) files. Then, open-source packages were used to calculate box-counting dimension from the 2D images. Namely, BoxCount and Fractal-Dimension Python(v3.8.10) libs.

# 3 Results

## 3.1 Crest's proportions

We do not consider the outer circle bearing the club's name for calculations. It circumscribes the repeating motive, which is described as follows.

A (1) square flag of side $L_0$, whose (2) upper left quarter is a square of side $L_{top} = L_0/2$, containing one (3) inscribed circle of radius $R_1$, which in turn circumscribes (4) a second square flag of side $L_1$.

Then, motives (2) and (3) are repeated in the new square of side $L_1$.

We can use basic Euclidian geometry to show that the side of the second square $L_1$ is given by $L_0\frac{\sqrt{2}}{4}$:

First, notice that circle $R_1$ is inscribed in the $L_{top}$ square. Therefore, $R_1$ corresponds to

$$R_1 = L_{top}/2 = L_0/4$$

.

The diagonal ($\mathrm{Diag}_1 = L_1\sqrt{2}$) of the flag inscribed in the inner circle corresponds to the diameter: ($2R_1$):

$$2R_1 = \mathrm{Diag}_1 = L_1\sqrt{2} \implies R_1 = \frac{L_1\sqrt{2}}{2}$$

.

Then, we may find the ratio between $L_0$ and $L_1$.

$$\frac{L_1\sqrt{2}}{2} = L_0/4$$

$$L_1 = \frac{2L_0}{4\sqrt{2}} = \frac{L_0}{2\sqrt{2}} = \frac{L_0\sqrt{2}}{4}$$

Therefore, the side $L_0$ is decreased by the factor $\frac{1}{2\sqrt{2}}$(or $\frac{\sqrt{2}}{4}$) every two copies. And the area $A_1$ is $(\frac{L_0}{2\sqrt{2}})^2 = \frac{A_0}{8}$.

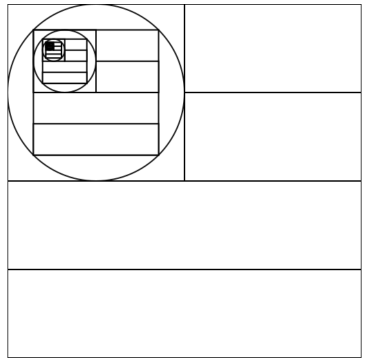

Figure 3: Computer generated crest (countours only).

### 3.2 Self-similarity dimension

Solving the equation for D:

$$\sum_{i}^{n} S_i^D = 1$$

The outer square has 4 line segments with length $L$, 1 large internal horizontal stripe (length $L$) and 2 smaller internal horizontal stripes ($L/2$). Closing a square in the top left section requires 2 more segments ($L/2$). Finally, the inscribed circle has 1 segment ($2\pi L/4$)

$$4(\frac{L_1}{L_0})^D + 1(\frac{L_1}{L_0})^D + 2\frac{1}{2}(\frac{L_1}{L_0})^D + 2\frac{1}{2}(\frac{L_1}{L_0})^D + 2\pi\frac{1}{4}(\frac{L_1}{L_0})^D = 1$$

$$7(\frac{L_1}{L_0})^D + \frac{\pi}{2}(\frac{L_1}{L_0})^D = 1$$

$$(7 + \frac{\pi}{2})(\frac{L_1}{L_0})^D = 1$$

For $D \in R$ and $L_1/L_0 = \sqrt{2}/4$.

$$D = -(log(4) - 2log(14 + \pi))/log(8) \sim 2.066$$

### 3.3 Computer assisted calculation of Minkowski-Bouligand dimension in ECB's crest

Empirical values for computer assisted calculation of Minkowski-Bouligand dimension$d$ are available at Table 1.

Table 1 - Minkowski-Bouligand dimension

| Resolution \| | BoxCount | Software Fractal-Dimension |
|---|---|---|
| 32px | 1.89 | 1.91 |
| 64px | 1.79 | 1.89 |
| 128px | 1.69 | 1.77 |
| 256px | 1.60 | 1.66 |
| 512px | 1.51 | 1.57 |
| 1024px | 1.45 | 1.49 |
| 2048px | 1.39 | 1.43 |
| 4096px | 1.35 | 1.38 |
| 8192px | NA | 1.34 |

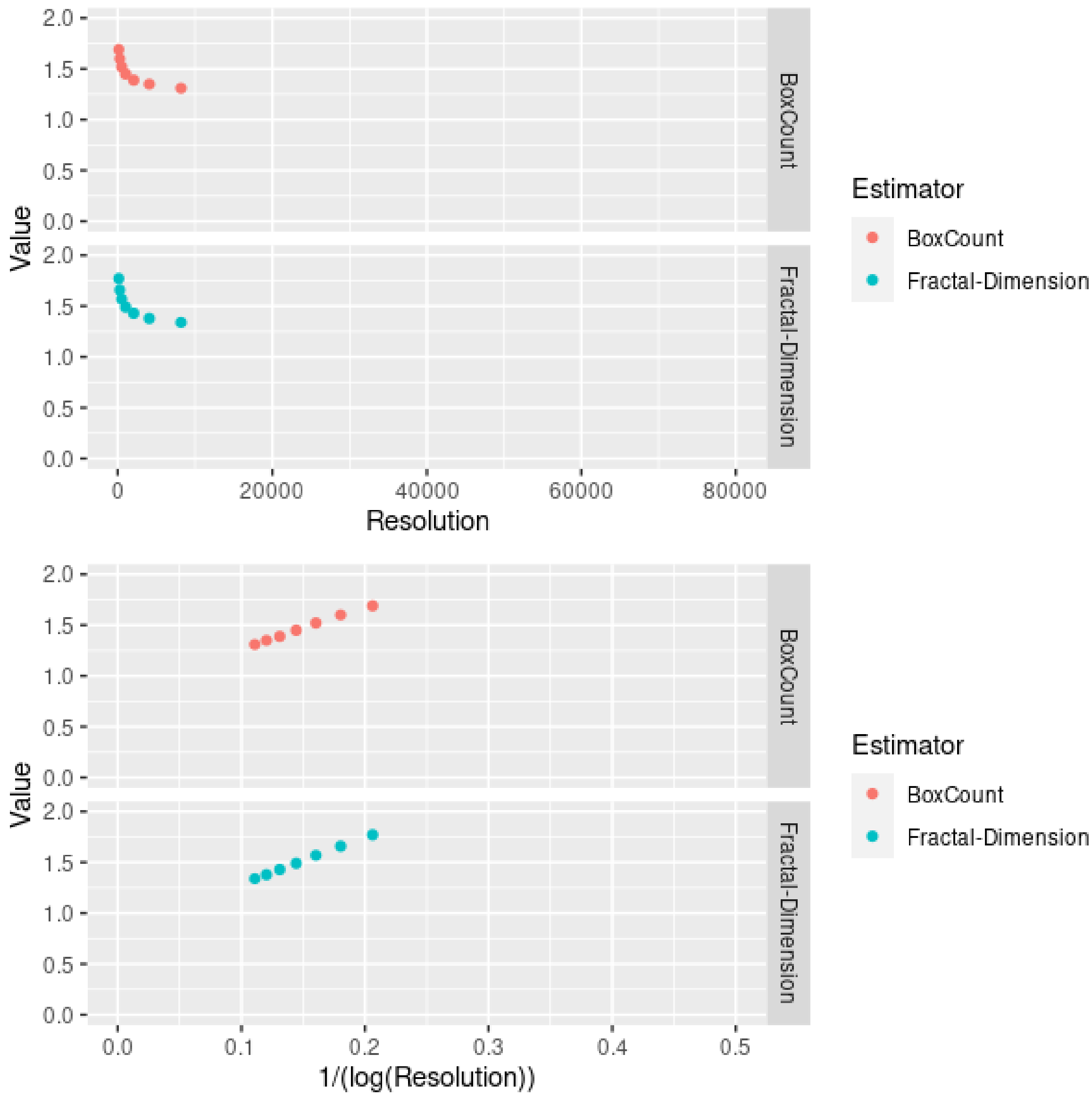


Figure 4: Resolution vs. Empirical MB dimension

## 4 Appendix

Julia code for plot.

```julia
using Cairo, Compose

function plot_ecb_crest(n)
    L_0 = 1
    L_1 = L_0*1/2
    R_1 = L_1*1/2
    L_2 = L_0/(2*sqrt(2))
    xy_new = R_1 - L_2/2

    if n == 0
        compose(context(),
        (context(0,0,1/2,1/2), #Top-left section
            Compose.rectangle(),
            Compose.circle(1/2,1/2,1/2),
            Compose.fill("white"),fillopacity(0.1),Compose.stroke("black")),
        (context(), # Outward rectangles (red filled in original)
            Compose.rectangle(0,3/4,1,1/4),Compose.rectangle(1/2,1/4,1/2,1/4),
            Compose.stroke("black"),Compose.fill("white")),
        (context(), # Outer square
            Compose.rectangle(),
        fill("white"),Compose.stroke("black")))
    else
        t = plot_ecb_crest(n-1)
        compose(context(),
            (context(xy_new,xy_new,L_2,L_2),t), # Repeat motive in smaller section
            plot_ecb_crest(0)) # Plot motive in outer space
    end
end
```

## References


[1] Jens Feder. "The fractal dimension". In: *Fractals*. Springer, 1988, pp. 6–30.

[2] Jonathan David Klotzbach. *New methods for estimating fractal dimensions of coastlines*. Florida Atlantic University, 1998.

[3] Marcio Luis F. Nascimento. *Por que o escudo do Bahia É único?* Sept. 2014. URL: https://www.correio24horas.com.br/noticia/nid/marcio-luis-f-nascimento-por-que-o-escudo-do-bahia-e-unico/.