On the unicity and location of the orbits homeomorphic to the zeta function: defining conditions for infinite non-trivial zeros from holomorphic spirals.

Unicity and location of the zeta spirals define conditions for non-trivial zeros

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Abstract

The zeta function is defined as the analytic continuation of the Dirichlet series $\zeta(s) = \sum_1^\infty \frac{1}{n^s}, Re(s) > 1$. Riemann hypothesized that besides the trivial zeros, located at the even negative integers of s ($\Re(s) = -2n; n = 1, 2, 3, 4, ...$), all other zeros would be images of $Re(s) = \frac{1}{2}$.

Complex analysis on homeomorphisms of ζ privided several advances. Solutions for the rotation quantities after cosmological equations yield poles identifiable with $Re(\mathit{T}r_0$, the non-trivial zeros. We review methods to identify unique poles using displacement derivatives of the arc θ in $Im(s) = \theta i$ (equivalent to Fesenko poles). However, a formal proof seems elusively connected with number theory and properties of primes.

We show that the critical line's location is actually a direct consequence of preserving connectedness in an infinite lattice generated by the sets and functions occuring in the definition of *zeta*. Ultimately, a problem solved through percolation theory and the subgroup maping symmetries of $\{0, 1, \infty\} \subset P^1(C)$, where $P^1(C)$ is the complex projective line, a Riemann Sphere. Actions by this group stabilize the sets $\{0, 1, \infty\}$ and $\{-\phi, \phi\}(\phi$, the golden ratio) in quotient map $S_3 \to S_2$. The fixed points of the individual 2-cycles difine a unique value for $Re(\mathit{T}r_0) = \frac{1}{2}$.

Ultimately, interpolating numbers of the harmonic series and the natural logarithm in infinitesimal intervals reflects Cantor's theorem on the cardinality of the continuum. Merten's and Brun's theorems proves a lower bound to measure on the interpolation of operators necessary for Riemann ζ existence. This defines target orbits $R_{(s)}$, $s = \frac{1}{2} + \theta i$ as the only element of S satisfying the solution conditions.

Keywords: Riemann hypothesis, zeta function, non-trivial zeros;

Introduction

The zeta function is defined as the analytic continuation of the Dirichlet series $\zeta(s) = \sum_1^\infty \frac{1}{n^s}, Re(s) > 1$. Riemann hypothesized that besides the trivial zeros, located at the even negative integers of $s(\Re(s) = -2n; n = 1, 2, 3, 4, ...)$, all other zeros would be images of $Re(s) = \frac{1}{2}$.

$$\zeta(s) = \frac{\Gamma(1-s)}{2\pi i} \oint_\gamma \frac{u^{s-1}}{e^{-u} - 1}$$

The value of $Re(Tr_0) = \frac{1}{2}$ is a direct consequence of preserving cycles (connectedness) in the topological space under the operations defined for the Riemann ζ. We demonstrate the unique value of $Re(Tr_0)$ can be derived from complex analysis using displacement derivatives of the arc θ of $Im(s) = \theta i$ (Fesenko poles) or through percolation theory on the lattice of the complex plane.

Ultimately, interpolating numbers of the harmonic series and the natural logarithm over infinitesimal intervals reflects Cantor's theorem on the cardinality of the continuum and the diagonal argument. Merten's and Brun's theorems proves a lower bound to measure on the interpolation of operators in Riemann ζ. This identifies the spiral R containing the target orbits $s = \frac{1}{2} + \theta i$ as the only element of S satisfying the solution conditions.

We outline a proof relating the definition of the Riemann Zeta function to the *Riesz-Thorin theorem* on the interporlation of operators and *Cantor's theorem* on the cardinality of the continuum. The assertion holds accepting Euclidian geometry for unitary vector products, including the 5_{th} axiom.

The argument is presented, along with a brief demonstration including non-trivial or unexplored identities. Further discussion ilustrate known identities with peculiar geometrical meaning under the proposed homeomorphisms.

In projective coordinates, the points $0, 1, \infty$ are given by [0:1], [1:1], and [1:0], which explains why their stabilizer is represented by integral matrices.

Homeomorphisms of ζ

S Spirals

For $Re(s)$ in the critical strip, define bijective mappings M of the set $Z = \zeta(s)$ in the complex plane to the superset S of orbits $a \in A$.

We define homeomorphisms M between values of $\zeta(s)$ and the sets of orbits A under the action of group C with multiplication. M defines bijections between orbits and lines with fixed values of $Re(s)$ in $\zeta(s)$, parallel to the imaginary axis.

Each spiral $a \in A$ is a subset of S whose elements are involutes $(C_a(C + Im(s)))$ of circles $(C\cos(\theta), C\sin(\theta))$ with radius identical to other elements in R according to the real part of s $C_{(s)} = Re(s)$. Each element a defines the lines parallel to the imaginary axis, through infinit arcs of sucessive involutions. $C, C \in \mathbb{R}$ and θi, i is the imaginary unit in the perpendicular axis.

This is similar to Zagier approach of constructing invariant functions on the upper half-plane and looking for eigenvalues, which yields the orbit R $\frac{1}{2} + \theta i$ as a candidate solution.

The real line and the complex numbers with absolute value one (circle group S^1) are **connected** Lie groups with dimension one under the operations of addition and multiplication, respectively. The S^1 group is often denoted as $U(1)$ the group of 1×1 1 unitary matrices.

The holomorphism of subsets of spirals defined by a have been demonstrated in Davis solution for the analytic continuation of the Theodorus Spiral. Also in evaluation of Archimedean Zeta Integrals.

The Borel–Carathéodory theorem demonstrates that if this function is analytic on a closed disc of radius R centered at the origin, it is bounded by its real part. This is the case for the ζ function differentiable along the critical strip, $0 < Re(s) < 1$. Therefore, in the critical strip, there is always a real number M bounding the involutions generating elements in a.

Consider also the resulting lattices $lat\mathbb{C}$

We define subsets of $A_{Re(s)}$ of S for complex numbers s with identical $Re(s) = c$ as a function f of s. The orbits with constant real part c, A_c:

$$A_c(s_s) = \{f : c + \theta i | c \in Re(s)\}$$

We can parameterize the equations as:

$$X(\theta) = c(cos\theta + (\theta)sin(\theta)),$$

$$Y(\theta) = c(sin\theta - (\theta)cos(\theta))$$

Given the tangential angle, $\phi(\theta)$.

Derivatives of arc displacement with respect to its length

Altough the problem of finding Lorenz Attractors have been long solved for special context in physics, it has not been subject to analysis until recently.[14] A superset of these geometrical objects have been examined in complex analysis, including Theodorus Spiral, Archimedean Zeta Integrals and Lipschitz curves. Analysis of localy convergent values in Laurent series have successfully been used to solve equations on Lorenz Attractors. [15]

The holomorphic manifold defined above describes the curve's lengths of the circular involute converging to a value given by the integral of the involutes of the circle with radius c and constant rotating angle θ according to ζ arguments. [16] For archimedean spirals assuming polar coordinates as described above, the length of the arc θ is given by the integral $\sqrt{1 - \theta^2}$ along the interval $[0, \theta]$:
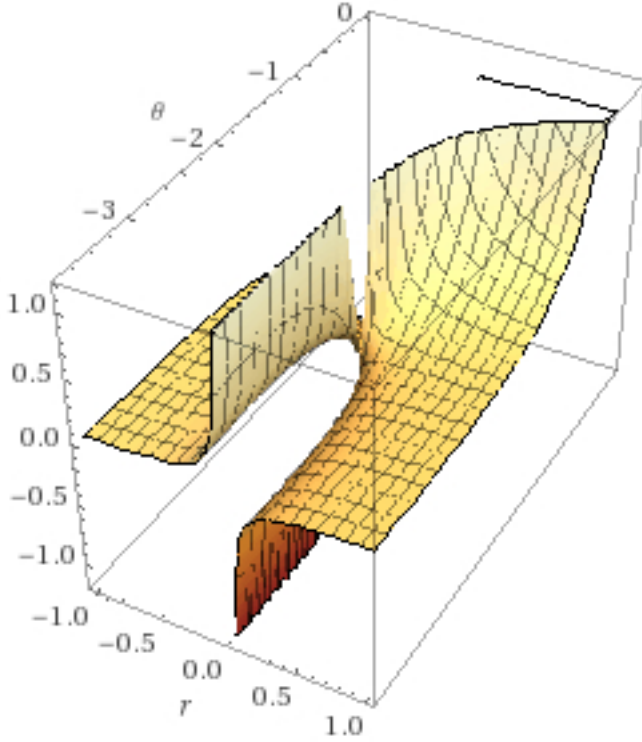
This approach is an application of Fesenko's general framework of studying poles in the Riemann ζ function after the eigenvalues of the zeta integral.[16]

Our **first identity** is:

$$L = c \int_0^\theta \sqrt{1 + \theta^2} d\theta$$

$$= \frac{1}{2}c(\theta\sqrt{1 + \theta^2} + sinh^{-1}\theta)$$

$$= \frac{1}{2}c(\theta\sqrt{1 + \theta^2} + ln(\theta + \sqrt{1 + \theta^2}))$$

One may define space curvature corresponding to Lorenz attactors after c and θ:

$$\kappa(\theta, c) = \frac{2 + \theta}{c(1 + \theta^2)^{3/2}}$$

Each element in a corresponds to an observation of in an infinite rotating orbit periodically towards an attractor (singularity) at successive time steps. The spiral corresponds to the locations over time of a point moving away from a fixed point with a constant speed along a line that rotates with constant angular velocity.[1]1

Lemma 1: For real values in the critical strip, involutions define bijections for sets of $\zeta(s)$ with identical real values

$$L = c \int_0^\theta \sqrt{1+\theta^2} d\theta$$

From the definition of operations in the complex plane, involutions $\vec{C_\theta}(s) = \vec{C_0}(s) + \theta \vec{c'}(s)$ use $c'(s)$ as the unit normal for $\vec{C(0)}$. Hence, increasing values of θ in $Im(s) = \theta i$ defines unique images of parallel spirals in S.

Conditions for infinite non-trivial zeros

For infinite non-trivial zeros, it has been demonstrated that the ϵ distance to the spiral's attractor must converge at unique periodic rate defined by γ, the Euler-Mascheroni constant.

Next lemmas describe the conditions for a set of orbits containing infinite images of $\zeta(s) = 0, s \in \mathbb{T}r\zeta_0$.

Lemma 2: The elements of the unique set of orbits with images containing the infinite non-trivial zeros are the spiral a_0 asymptoptically approaching the real line as $\theta \to \infty$ at a specific rate related to $Re(s)$

Take the converging length L of the arc in a:

$$L = c \int_0^\theta \sqrt{1+\theta^2} d\theta$$

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Considering that the Lagrangian is symmetrical for rotations, we may obtain displacement derivatives to locate Lorenz attractors for an orbit. Again, these could be alternatively done with zeta integrals.

We evaluate the derivative of the length with respect to θ.

$$\omega = \frac{\delta}{\delta\theta}L$$

$$= c\sqrt{1+\theta^2} + sinh^{-1}$$

Now, evaluate the changes in ω after c.

$$\alpha = \frac{\delta}{\delta c}\omega$$

$$= \frac{\delta}{\delta c}c\sqrt{1+\theta^2} + sinh^{-1}$$

$$= \sqrt{1+\theta^2}$$

This derivative determines changes in the ω mappings after each real part in zeta.

The problem is then equivalent to that of predicting a cosmological equations of state for the cosmological fluid [1]3 . Specifically, finding the attractor with periodicity for $\zeta(s) = 0$ as $\lim Im(s) \to \infty$ and $Re(s) \neq -2n, n = 1, 2, 3, 4, ...$

Considering that the Lagrangian is symmetrical for rotations, calculate a higher order derivatives, called jerk (3_{th}order) and snap (4_{th}).[1]3 This is allowed due to surface smoothness (infinite diferentiable attribute). In general relativity formulations, quantities changing orbits according to the density and angular momentum of a singularity.

Specifically, we may calculate values of θ when the 4_{th} order snap makes $Re(\zeta) = 0$ converge to 0 through periodically arcs.
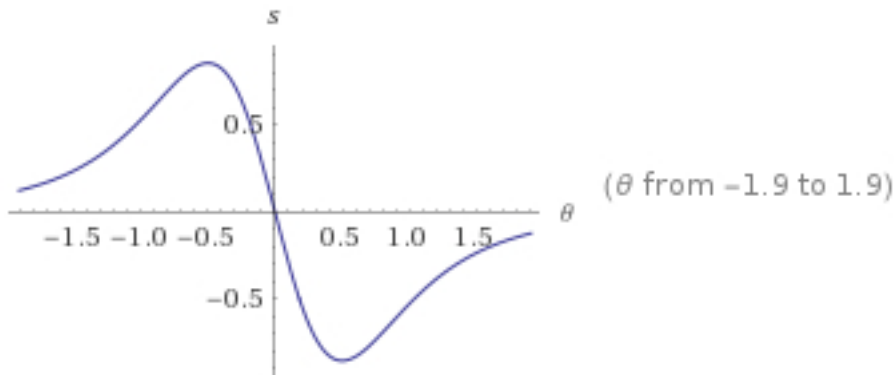
$$\frac{d^4}{d\theta}^4 (\frac{1}{2}(\theta\sqrt{1+\theta^2} + sinh^{-1}(\theta))) =$$

The snap is odd and achieves min. and max. global values at $\theta \in \{\frac{1}{2}, -\frac{1}{2}\}$.

These poles define the $Re(s)$ of elements in A whose orbits define the poles associated Riemann Zeta zeros.

$$= -\frac{3\theta}{(\theta^2+1)^{5/2}}$$

The odd function shows the unique two poles along with the arcs generated in polar coordinates.
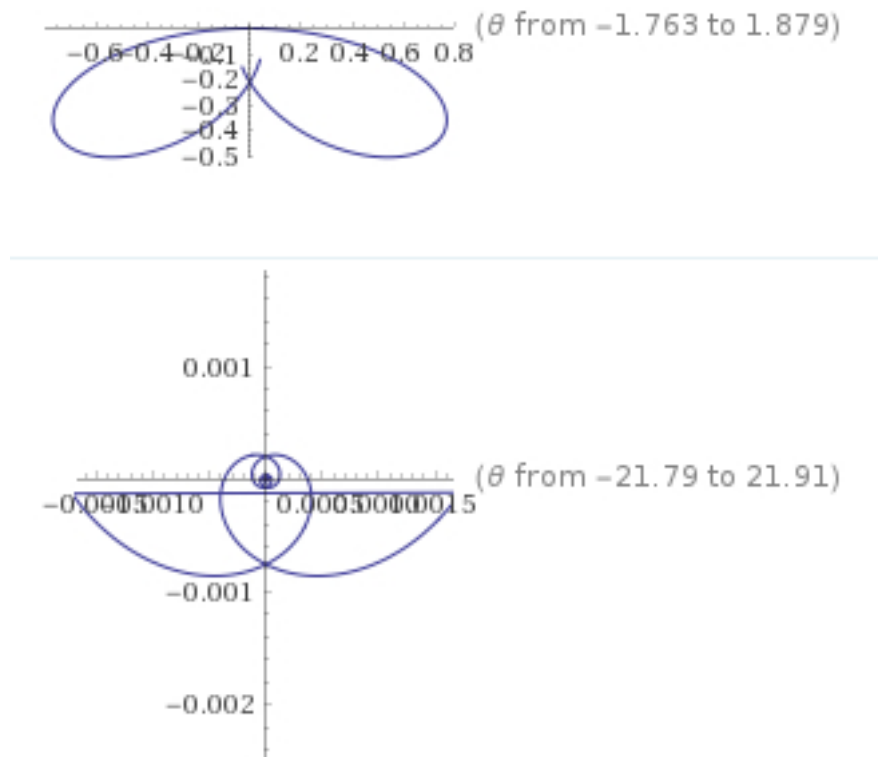

(θ from −1.9 to 1.9)

Figure 1: Plots of 4th derivative (snap) on polar coordinates. The orbits sugest the location of symetrical attractors

By evaluating the orbits after the global extrema of θ maximizing the snap, we find several geometrically intuitive identities. They are associated with intrinsic angular momentum (spin) or certain solutions for field equations general relativity. The Lorenz solutions are equivalent to the ones desbriding conserved angular momentum in white and black holes.

Lemma 3: The Riesz-Thorin theorem on the interporlation of operators bounds the quantities Mertens' third theorem provide the asymptoptic limit for the corvegence rate values associated with this set R_0 of unique orbits.

Ultimately, interpolating numbers of the harmonic series and the natural logarithm over infinitesimal intervals reflects Cantor's theorem on the cardinality of the continuum and the diagonal argument. Merten's and Brun's theorems proves a lower bound to measure on the interpolation of operators in Riemann ζ. This identifies the spiral R containing the target orbits $s = \frac{1}{2} + \theta i$ as the only element of S satisfying the solution conditions.

We outline a proof relating the definition of the Riemann Zeta function to the *Riesz-Thorin theorem* on the interporlation of operators and *Cantor's theorem* on the cardinality of the continuum. The assertion holds accepting Euclidian geometry for unitary vector products, including the 5_{th} axiom.

This is assserted by Merten's second theorem proving the existence of the Meissel-Mertens constant and Brun's theorem on the convergence the sum of reciprocals of two primes.

Suppose one of the solution set of orbits a_0 is located at the critical strip $Re(s) = \frac{1}{2}$.

It has previously been show that there are infinitely many non-trivial zeros along the critical line, $Re(s) = 1/2$.

The identity between the Euler-Mascheroni constant, the harmonic series and the natural logarithm identifies the spiral R containing the target orbits $s = 1/2 + \theta i$ as the only element of S satisfying the conditions. This is based on Perseval's theorem on the unitary characteristic of the Fourier transform or its generalized version, the Plancharel theorem.

Ultimately, the Hausdorff–Young inequality reflects the cardinal inequality between rational numbers in the harmonic series and uncountably infinite real numbers in the natural logarithm $C(\mathbb{Q}) = C(\mathbb{N}) = \aleph \neq C(\mathbb{R}) = 2\aleph$.

The value of $Re(\cancel{T}r) = \frac{1}{2}$ is a direct consequence of topology preservation of the sets under operations and sets defined in the *zeta*. Ultimately, the Hausdorff–Young inequality reflects the cardinal inequality between rational numbers in the harmonic series and uncountably infinite real numbers in the natural logarithm $C(\mathbb{Q}) = C(\mathbb{N}) = \aleph \neq C(\mathbb{R}) = 2\aleph$.

We may verify that α is an even function with a global minimum and two complex roots: $\alpha_0 \in \{i, -i\}$, are singularities. Values of $\theta = \alpha_0$ indicate the periods of the arc. The global minimum is $\alpha(\theta) = 1, \theta = 0$ the uncurved orbits along the real line $\zeta(s), s \in \mathbb{R}$.

In order to evaluate

The nex derivative ,

$$c \int_0^\theta \sqrt{1 + \theta^2} d\theta$$

In the set S of spirals R, consider a potentially non empty subset S' of curves whose involutions define arcs constinously rotated towards the non-trivial values of $Re(\zeta(s)) = 0$. That is, tho se with real part not in the set of negative even intergers $s \in \cancel{T}r\zeta_0 \rightarrow Re(s) \notin Tr_{\zeta_0}, Re(Tr_{\zeta_0}) = -2n, n = 1, 2, 3, 4, ...$
plex sequences.

It was proved by Franz Mertens that, if the series $\sum_{n=0}^{\infty} a_n \sum_{n=0}^{\infty} a_n$ converges to A and $\sum_{n=0}^{\infty} b_n \sum_{n=0}^{\infty} b_n$ converges to B, and at least one of them converges absolutely, then their Cauchy product converges to AB.
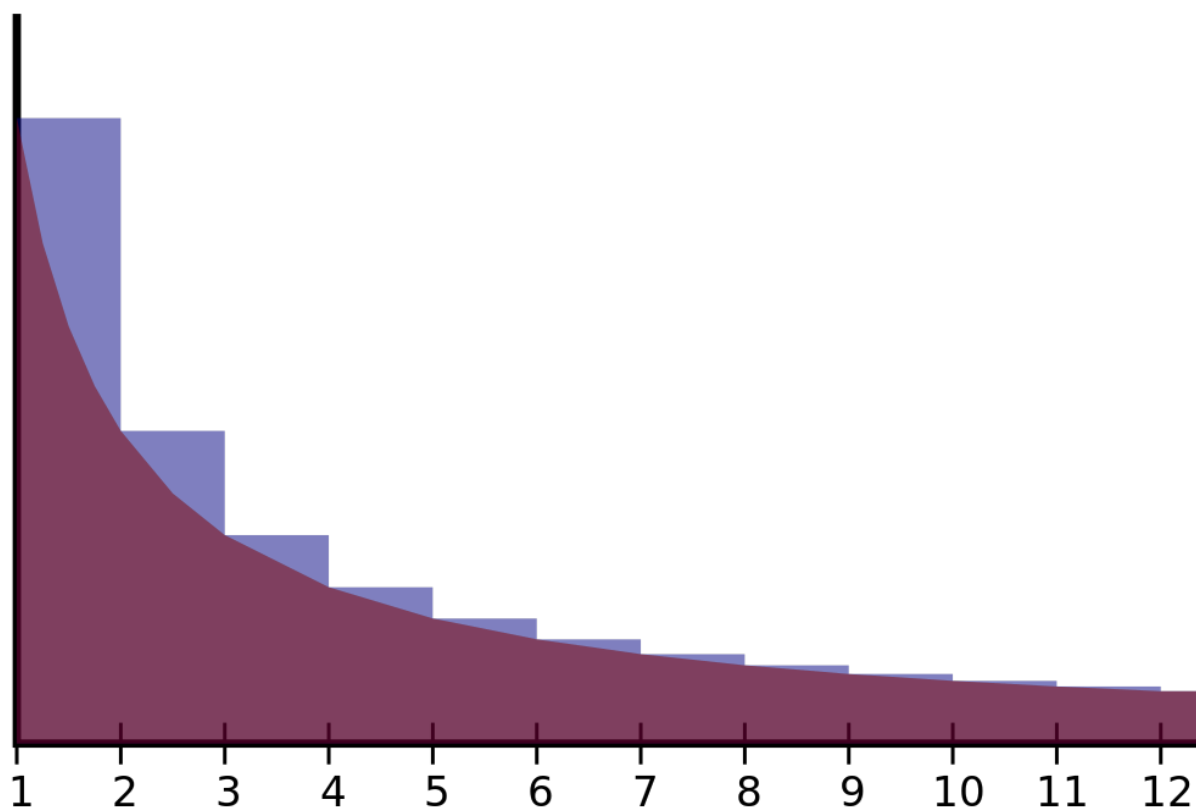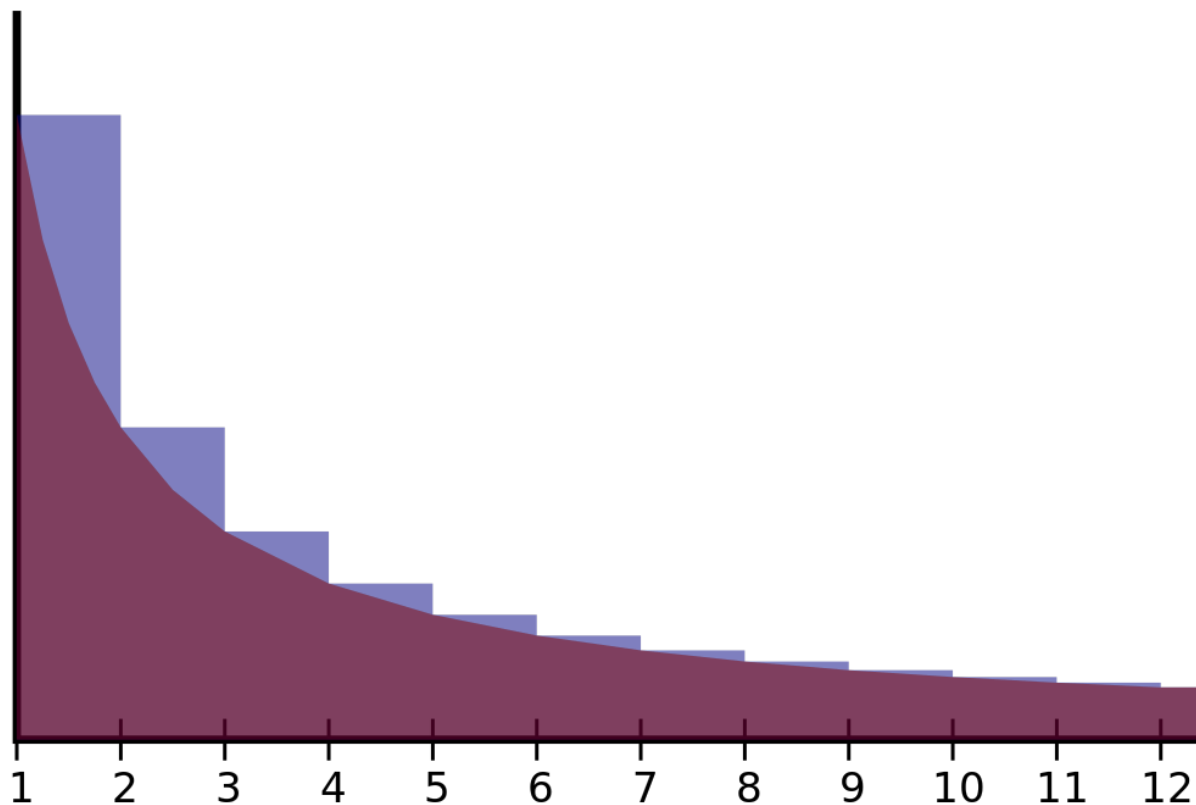
Figure 2: Trocar por diagrama de vetores!!!

Figure 3: Trocar por diagrama de vetores!!!

Orbits with real parts equal to one half

**Theorem for unicity and location of non-tr Harmonic series = discrete vectorial (grid ~ cardinality) sum with $1/n$ rate

Mertens' theorem

Let (an)n >= 0 and (bn) n>= 0 be real or complex sequences. It was proved by Franz Mertens that, if the series $\sum_{n=0}^{\infty} a_n \sum_{n=0}^{\infty} a_n$ converges to A and $\sum_{n=0}^{\infty} b_n \sum_{n=0}^{\infty} b_n$ converges to B, and at least one of them converges absolutely, then their Cauchy product converges to AB.

Orbits with real parts equal to one half

Theorem for unicity and location of non-trivial zeros: Among the orbits associated with $Re(\zeta(s)) = 0$, the ones containing zeros not in the trivial period $(Re(s) = -2n, n \in \mathbb{N})$ are the images of $zeta(s), Re(s) = \frac{1}{2}$.

The argument is as follows:

The value of $Re(s_{non_trivial}) = \frac{1}{2}$ is a direct consequence of the Hausdorff–Young inequality, reflecting the cardinal inequality between rational numbers in the harmonic series and uncountably infinite real numbers in the natural logarithm $C(\mathbb{Q}) = C(\mathbb{N}) = \aleph \neq C(\mathbb{R}) = 2\aleph$.

@ Hadamard three-lines theorem @ Hadamard three-circles theorem

The spiral converges towards the unique attractor fixed at $Re(\zeta s) = 0$. The total rotation angle is defined by the second moment when $s \to \infty$. The identity between the Euler-Maschero https://www.wolframalpha.com/input/?i=zeta+of+(1%2F2+%2B+theta*i) https://www.wolframalpha.com/input/?i=zeta+of+(1+%2B+2theta*i) https://www.wolframalpha.com/input/?i=laurent+series+of+(zeta+of+(1+%2B+2theta*i))

As said, the second moment is related to the scaping velocity by the length of the spiraled arc, which has converging series or analytical extension:

$$L = \frac{1}{n^{2Re(s)}}$$

$$= \zeta(2Re(s))$$

.

If the resulting vector $\vec{v_s}$ makes angle $\theta + \frac{\pi}{2}$ with the origin of the real axis, then

$$Im(\vec{v_s}) = \vec{v_s} * cos(\theta)i$$

Since we are considering zeros at the real axis, the projection on the real axis is $cos(\theta + \frac{\pi}{2})$, the opposite of $sin\theta$. Diagram:

Identites:

$$Re(\vec{v_s}) = \vec{v_s} * -sin(\theta)$$

.

$$Im(\vec{v_s}) = \vec{v_s} * cos(\theta)i$$

$$s = \vec{v_s} * -sin(\theta) + \vec{v_s} * cos(\theta)$$

The projection $Re(s)$ will be 0 when:

$$s = \vec{v_s} * -sin(\theta) + \vec{v_s} * cos(\theta)$$

which is $s(cos(\theta) - sin(\theta))$ or $(\frac{1}{2} - \frac{i}{2})e^{-i\theta}s + (\frac{1}{2} + \frac{i}{2})e^{i\theta}s$ or

$$s = \sqrt{(2)}\vec{s}sin(\pi/4 - \theta)$$

$$= \sqrt{(2)}\vec{s} - sin(\theta)$$

$$= \sqrt{(2)}Re(s)$$

$$Re(s) = \frac{s}{\sqrt{(2)}}$$

Taking the derivatives with respect to Re(s) and Im(s):

$$\frac{d}{da}\frac{a+bi}{\sqrt{(2)}} = 1/\sqrt{(2)}$$

$$\frac{d}{db}\frac{a+bi}{\sqrt{(2)}} = 1/\sqrt{(2)}$$

Notice roots in

$$\theta = \pi n - \frac{3\pi}{4}$$

n in Z.

The first moment is M_1. If the n_{th} line segment length is given by $\frac{1}{n^s}$, we evaluate the moment around the real axis by the sum the distances $M_1 = \sum_1^\infty \frac{1}{n^s}$.

When $\zeta(s)$ approaches 0, that the vectorial sum approaches the center of the real axis. This is equal to rotate the object enough to cancel its projection on the real axis.

The first moment is M_1. If the n_{th} line segment length is given by $\frac{1}{n^s}$, we evaluate the moment around the real axis by the sum the distances $M_1 = \sum_1^\infty \frac{1}{n^s}$.

Consider the series $\zeta(s), Re(s) = 2, Im(s) = 0$. The corresponding segmented object's length converges to:

$$\zeta(2) = \sum_{n=1}^\infty \frac{1}{n^2}$$

$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ...$$

$$\zeta(2) = \frac{\pi^2}{6}$$

The identity by Euler which solved the Basel problem.

We shall now consider our series with imaginary numbers:

$$\zeta(s) = \sum_{n=1}^\infty \frac{1}{n^s}, Re(s) > 1, Im(s) \neq 0$$

Consider if the imaginary part of the previous series, $\zeta(2)$, was $Im(s) = \frac{\pi}{3}i$, then:

$$\zeta(2 + \frac{pi}{3}) = \sum_{n=1}^\infty \frac{1}{n^{2+\frac{\pi}{3}i}}$$

$$= \sum_{n=1}^\infty \frac{1}{n^2} * \frac{1}{n^{\frac{\pi}{3}i}}$$

By the definition, this is the dot product ani constant, the harmonic series and the natural logarithm identifies the spiral with orbits $s = 1/2 + \theta i$ as the only element of S_0.

The value of $Re(s_{non-trivial}) = \frac{1}{2}$ is a direct consequence of cardinal inequality between rational numbers in the harmonic series $C(\mathbb{Q}) = C(\mathbb{N}) = \aleph)$ and uncountably infinite real numbers.

It could also be demonstrated from Kolmogorov axioms by defining the characteristic function of probability functions defined after elements of S.

For a distribution of mass or probability on a bounded interval, the collection of all the moments (of all orders, from 0 to ∞) uniquely determines the distribution (Hausdorff moment problem).

There are three named classical moment problems: the Hamburger moment problem in which the support of μ is allowed to be the whole real line; the Stieltjes moment problem, for $[0, \infty)$; and the Hausdorff moment problem for a bounded interval, which without loss of generality may be taken as $[0, 1]$. The uniqueness of μ in the Hausdorff moment problem follows from the Weierstrass approximation theorem, which states that polynomials are dense under the uniform norm in the space of continuous functions on $[0, 1]$. For the problem on an infinite interval, uniqueness is a more delicate question; see Carleman's condition, Krein's condition and Akhiezer (1965).

Laurent Series is applicable in bounded intervals.

These are involutions using line segments parallel to the imaginary axis. The arc's length is parameterized as:

$$\vec{X} = \vec{C}_\kappa = \vec{c}(\theta) - \frac{\vec{c'}(\theta)}{|\vec{c'}(\theta)|} \int_\kappa a^\theta |\vec{c'}(\omega)| d\omega$$

The integral over s represents the length of the free part of the string in the interval $[0, t]$ and the vector prior to that is the tangent unitvector. :

$$C_a(s) = \int_{R^d} Re(y) Im(x - y) ds$$

$$= \int_{R^d} Re(x - y) Im(y) dy$$

Axiomatic basis and geometrical intuitions

We examine axiomatic systems under which the conclusion holds and give geometrical demonstrations for other previously described identities.

This is a lower bound for obtaining new prime numbers on N by infinitesimal subtractions between sets with cardinality $2^{\mathbb{N}_\kappa}$ and $\mathbb{N}$.

If the attractor is unique, the principal component converges to a constant, the Euler Mascheroni, which happens to be the case.
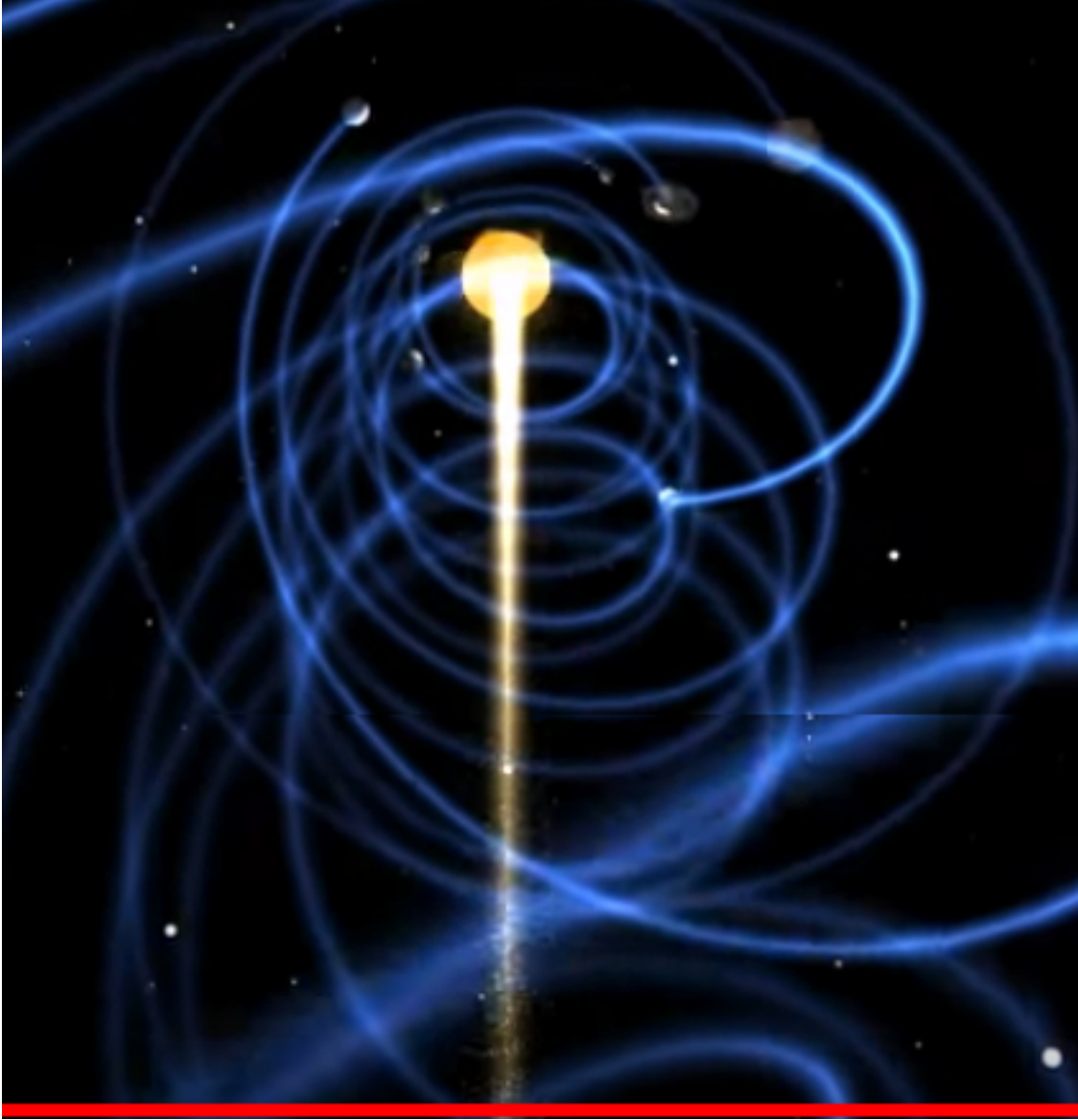
The first and the second moments of $f(s), s = \frac{1}{2} + \theta i$ are associated with the orbit containing the non-trivial zeros of ζ through the angle of the arc drawn towards an attractor at the center of a spiral.

As a consequence of polar coordinate operations defined by the product i as ortoghonal rotatiom, the second moment at $\zeta(s_n), M_2$, has a derivative with respect to $Im(s)$ which equals to double of the derivative at the previous moment. $2\frac{d}{dx}\zeta(Re(s_{n-1}) + \frac{\pi}{2}Im(s_{n-1}))$.

The spiral's arc is the convolution of line segments by θ and should compensate their second moments M_2 for the rotations to be centered on the imaginary axis.

Assumptions

We outline a proof based on quasiprobability functions in the C^0 complex vector space and its nondegenerate quadradic form (Clifford algebra $Cl_C = 0$).

Quasiprobability functions behave as probability functions, although Kolmogorov axioms I and III are relaxed, allowing for negative probabilities. Therefore, σ-additivity might be also violated for subsets of events E_i,

when

$$P(\cup E_i) < 0$$

.

Consider the length of a line segment composed by summing infinite line segments, each of them smaller than the previous.

Consider the infinite series defined by $\zeta(s)$ for $Re(s) > 1$.

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, Re(s) > 1, Im(s) = 0$$

This is the composite size length of infinite line segments (or vectors), each of them smaller than the previous by a ratio of $\frac{\frac{1}{(n+1)^s}}{\frac{1}{n^s}} = \frac{n^s}{(n+1)^s}$.

A physical analogy could be drawn to that of an infinitely sectioned fish rod. As if we started with $l_1 = \frac{1}{1^s}$ and added progressively smaller pieces $(l_2 = \frac{1}{1^s} + \frac{1}{2^s}), (l_3 = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s}, l_4 = ...)$ to its end.

The total length asymptotically approaches $\zeta(s)$.

Notice that $\zeta(s)$ geometrically describes a related spiral, obtained with the n sucessive cross products $Re(s) \times Im(s)$ for $\lim_{n \to \infty}$. $Re(s)$ is a real number and $Re(s)$ lies in the complex space.

Quasiprobability function

Intuition

Consider the scneraio of the infinitely sectioned fish rod described above being bent, by being pulled ortogonally to a given attractor. Whe the total segment is deformed, each line forms the same angle θ with the next one according to the cross product described above.

Consider the orbit formed by the last point in each segment in a spiral towards the attractor.

The vectorial decomposition of the spiral is $z_{s_n} = z_{s_{n-1}} + z_{s_{n-2}} + ... + z_{s_1}$.

Events

Consider each orbit as an event. We define the probability of s the value $\zeta(s)$ according to the analytic continuation:

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s)$$

The probability associated with an event E is $E(Re(\zeta(s)))$ which is the projection of $\zeta(s)$ over the real axis. This quantity is different, but related to $Re(s)$.

Ploting the vectors converging to the spiral is the same as $z_{s_n} = z_{s_{n-1}} + z_{s_{n-2}} + ... + z_{s_1}$.

This spiral can be seen as the orbit S spiraling outside of the attractor with scape velocity $\vec{v}$ given by the Hamiltonian of the system and its respective conjugate momentum. For circular polar coordinates, the angular momentum.

If the n_{th} line segment length is given by $\frac{1}{n^s}$, we evaluate the first moment M_1 relative to the real axis by the sum $M_1 = \sum_1^{\infty} \frac{1}{n^{Re(s)}}$.

The real vectors have a total length that equals to the first moment, M_1, $L = M_1 = \zeta(Re(s))$.

Also, consider the second moment (angular momentum), M_2, given by squaring the length of each line segment $M_2 = \sum_i^{\infty} Re(s_i)^2$

Figure 4: Photo by Alan Bishop, December 3, 2015, Unsplash,East Sooke Park, Sooke, Canada

Each orbit is defined by the cross products of progressively smaller vectors rotated by the same angle with a smaller magnitude.

Uniqueness of orbits

Every involution on an odd number of elements has at least one fixed point.

Considering the scape velocity scenario previously described, the angle spiraled out of the attractor is the one associated with the angular momentum:
1. Each real value of $Re(s)$ results in a different value of M_1, given by $\zeta(Re(s))$.
2. The second moment then converges to:$\lim_{n\to\infty} M_2 = \sum_{n=1}(\frac{1}{n^{Re(s)}})^2 = \zeta(2Re(s))$.
The first and the second moment determine a unique ratio for each pair $\zeta(2Re(s))$ and $\zeta(Re(s))$. Every $Re(s)$ determines a unique family of orbits.

We hereby show that the scenario in which $Re(s) = \frac{1}{2}$ is the one containing non-trivial zeta zeros.

Non-trivial zeros of zeta

When $\zeta(s)$ approaches 0, the vectorial sum approaches the center of the real axis. This is equal to rotate the object enough to cancel its projection on the real axis.

In that case, the spiral has its center on 0. The value $(Im(\zeta(s))$ as the projection of a vector $\vec{v_s}$ onto the imaginary axis.

The cross product $Re(\zeta(s)) \times Im(\zeta(s))$ defines the converging spiral. The related second moment is related to the scaping velocity by the length of the spiralad arch (L), which converges to $\zeta(s)$.

https://www.wolframalpha.com/input/?i=zeta+of+(1%2F2+%2B+theta*i) https://www.wolframalpha.com/input/?i=zeta+of+(1+%2B+2theta*i) https://www.wolframalpha.com/input/?i=laurent+series+of+(zeta+of+(1+%2B+2theta*i))

As said, the second moment is related to the scaping velocity by the length of the spiraled arc, which has converging series or analytical extension:
$$L = \frac{1}{n^{2Re(s)}}$$
$$= \zeta(2Re(s))$$

.

If the resulting vector $\vec{v_s}$ makes angle $\theta + \frac{\pi}{2}$ with the origin of the real axis, then
$$Im(\vec{v_s}) = \vec{v_s} * cos(\theta)i$$

Since we are considering zeros at the real axis, the projection on the real axis is $cos(\theta + \frac{\pi}{2})$, the opposite of $sin\theta$. Diagram:

Identites:
$$Re(\vec{v_s}) = \vec{v_s} * -sin(\theta)$$

.

$$Im(\vec{v_s}) = \vec{v_s} * cos(\theta)i$$

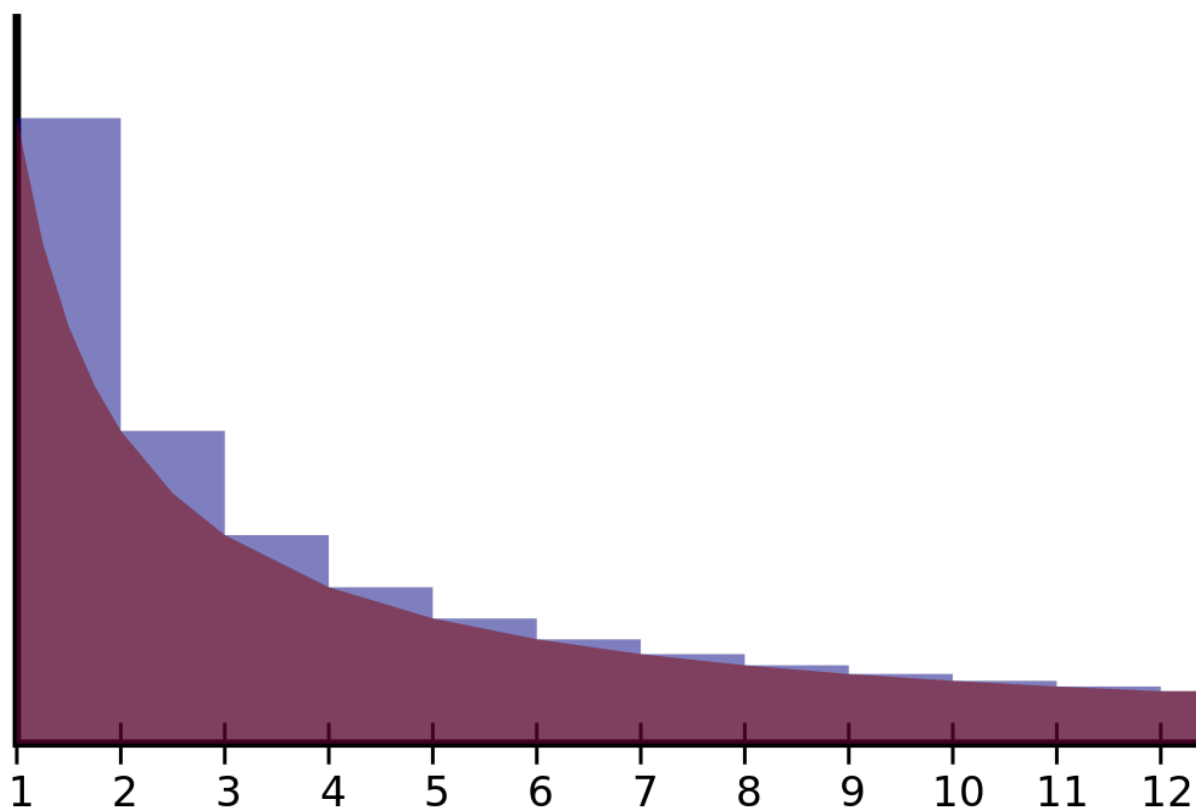$$s = \vec{v_s} * -sin(\theta) + \vec{v_s} * cos(\theta)$$

Figure 5: Trocar por diagrama de vetores!!!

The projection $Re(s)$ will be 0 when:

$$s = \vec{v_s} * -sin(\theta) + \vec{v_s} * cos(\theta)$$

which is $s(cos(\theta) - sin(\theta))$ or $(\frac{1}{2} - \frac{i}{2})e^{-i\theta}s + (\frac{1}{2} + \frac{i}{2})e^{i\theta}s$ or

$$s = \sqrt{(2)}\vec{s}sin(\pi/4 - \theta)$$

$$= \sqrt{(2)}\vec{s} - sin(\theta)$$

$$= \sqrt{(2)}Re(s)$$

$$Re(s) = \frac{s}{\sqrt{(2)}}$$

Taking the derivatives with respect to Re(s) and Im(s):

$$\frac{d}{da} \frac{a + bi}{\sqrt{(2)}} = 1/\sqrt{(2)}$$

$$\frac{d}{db} \frac{a + bi}{\sqrt{(2)}} = 1/\sqrt{(2)}$$

Notice roots in

$$\theta = \pi n - \frac{3\pi}{4}$$

n in Z.

The first moment is M_1. If the n_{th} line segment length is given by $\frac{1}{n^s}$, we evaluate the moment around the real axis by the sum the distances $M_1 = \sum_1^\infty \frac{1}{n^s}$.

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Consider the series $\zeta(s), Re(s) = 2, Im(s) = 0$. The corresponding segmented object's length converges to:

$$\zeta(2) = \sum_{n=1}^\infty \frac{1}{n^2}$$

$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + ...$$

$$\zeta(2) = \frac{\pi^2}{6}$$

The identity by Euler which solved the Basel problem.

We shall now consider our series with imaginary numbers:

$$\zeta(s) = \sum_{n=1}^\infty \frac{1}{n^s}, Re(s) > 1, Im(s) \neq 0$$

Consider if the imaginary part of the previous series, $\zeta(2)$, was $Im(s) = \frac{\pi}{3}i$, then:

$$\zeta(2 + \frac{pi}{3}) = \sum_{n=1}^\infty \frac{1}{n^{2 + \frac{\pi}{3}i}}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n^2} * \frac{1}{n^{\frac{\pi}{3}i}}$$

By the definition, this is the dot product among vectors with lengths determined by the generating series $\zeta(2)$ and $\zeta(\frac{\pi}{3}i)$.

$$\zeta(2 + \frac{\pi}{3}) = \zeta(\vec{2}) \cdot \zeta(\frac{\vec{\pi}}{3}i) = | \zeta(2) || \zeta(\frac{\pi}{3}i) | \cos(\theta)$$

The value of
$$\zeta(a + bi)$$
converges to the dot product of vectors due to the sum $a + bi$ in the exponent turning the expression into a product of perpendicular vectors:

$$\zeta(a + bi) = \sum_{1}^{\infty} \frac{1}{n^{a+bi}}$$

$$= \sum_{1}^{\infty} \frac{1}{n^a} \frac{1}{n^{bi}}$$

$\zeta(\vec{a})$, superposing the real axis and magnitude $\sum_{1}^{\infty} \frac{1}{n^a}$, and perpendicular to $\zeta(\vec{b}i)$, superposing the imaginary axis with magnitude $\sum_{1}^{\infty} \frac{1}{n^{bi}}$.

Through vector addition, the next segment form and angle θ with the continuation of the previous one. The fist segment crosses the origin with angle θ

$$z = e^{\theta i} \rightarrow = cos(\theta) + sin(\theta)i$$

$z_{s_n} = z_{s_{n-1}} + z_{s_{n-2}} + ... + z_{s_1}$

Each rod is uniquely determined by its momentums. The second momentum is: $M_2 = \sum_{1}^{\infty} (\frac{1}{n^s})^2$. Each $Re(s)$ determines a family of rods. We hereby show that the scenario in which $Re(s) = \frac{1}{2}$ is the one containing non-trivial zeta zeros.

Non-trivial zeros

Assume the line along $Re(s) = \frac{1}{2}$ defines the atractor of an envelope defined by the orbits of ζ in the critical strip $0 < s < 1$ as a function of s. It follows that this spiral is the one in which rotations from the vectorial decompostion cancel.

The second moment M_2(e.g: moment of inertia) of the spiral formed by $\zeta(s)$ is then given by the generating series
$$\zeta(2s) = \sum_{1}^{\infty} \frac{1}{n^{2s}}$$
.

The trivial orbits intersecting the real axis are the ones in which the second moment increases by an equal factor to its angle of rotation.

These are given by $\zeta(-2ks), k = 1, 2, 4, ...$, as a direct consequence of
the trivial period for $\sin\left(\frac{\pi s}{2}\right)$ in the analytic continuation. $\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s)$

Although this is only definable for the analytic continuation, the intuition holds for the series. We could also show that multiplying s by negative multiples of 2 is the same as:

$$s = -(a + bi) \rightarrow 2s = 2k * -(a + bi)$$

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Figure 6: Photo by Alan Bishop, December 3, 2015, Unsplash,East Sooke Park, Sooke, Canada

.

$$\sum_1^\infty \frac{1}{n^2 a} * \frac{1}{n^2 bi}$$

For $Re(s) > 1$ it is easy to visualize that large dimensions of $Re(s)$ result in orbits with large segments which do not rotate enough to turn the opposite way. The analytical continuation project those orbits symmetrically, emerging with associated angles in the left quadrants.

The operation changes the dot product by squaring both parts at each weigthed product of ortogonal parts.

$$\vec{s'}^2 = \vec{Re(s)}^2 + \vec{Im(s)}^2$$

Hence, the mapping

$$f : \zeta(s) \to \zeta(2s), Re(s) < 0, Im(s) = 0$$

. preserves the angle θ with the center $\zeta(-2s)$ when the geodesics of the system have no null curvature.

Riemann Hypothesis

The next (non-trivial) zeros are supposed to be related to the ζ function when $Re(s) = \frac{1}{2}$ and several seemenly related identities have been shown.

Excluding the trivial zeros, $\zeta(s) = 0$ only when $0 < s < 1$. The envelope of spirals centered at the orbit $\zeta(\frac{1}{2} + bi)$ is called critical strip.

The orbit $\zeta(\frac{1}{2} + bi)$ is the vectorial decomposition of the orbit, or the sum of rod segments.

$$\sum_1^\infty \frac{1}{n^{(\frac{1}{2}+bi)}}$$

How to find the adequate orbit? In order for extra zeros to happen in the orbit, the net vectorial sum projection on the real axis must be 0.

Consider s along the interval $S = 0 < 1/2 + bi < 1$. No rotation, $Im(s) = 0$, is associated with the divergent series:

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{6}} + ...$$

This is the sum of line segments with lenghts $\sum_1^\infty \frac{1}{sqrtn}$. And the analytic continuation of ζ, which would reflect the first moment related to the real axis:

$$\zeta(1/2) = -(\sqrt{2}+1)\sqrt{\frac{2}{\pi}} \int_0^\infty \frac{\sqrt{x}\,dx}{\cosh^2 x}$$

$$\sim -1.46035450880958$$

The second moment of the line segments is:

$$M_2 = \sum_1^\infty \frac{1}{n^{\frac{1}{2}}}^2$$

$$= \sum_1^\infty \frac{1}{n}$$

Therefore, the M_2 associated with 0 rotation is that of is the diverging harmonic series. Its truncation (harmonic number) can be expressed analytically as:
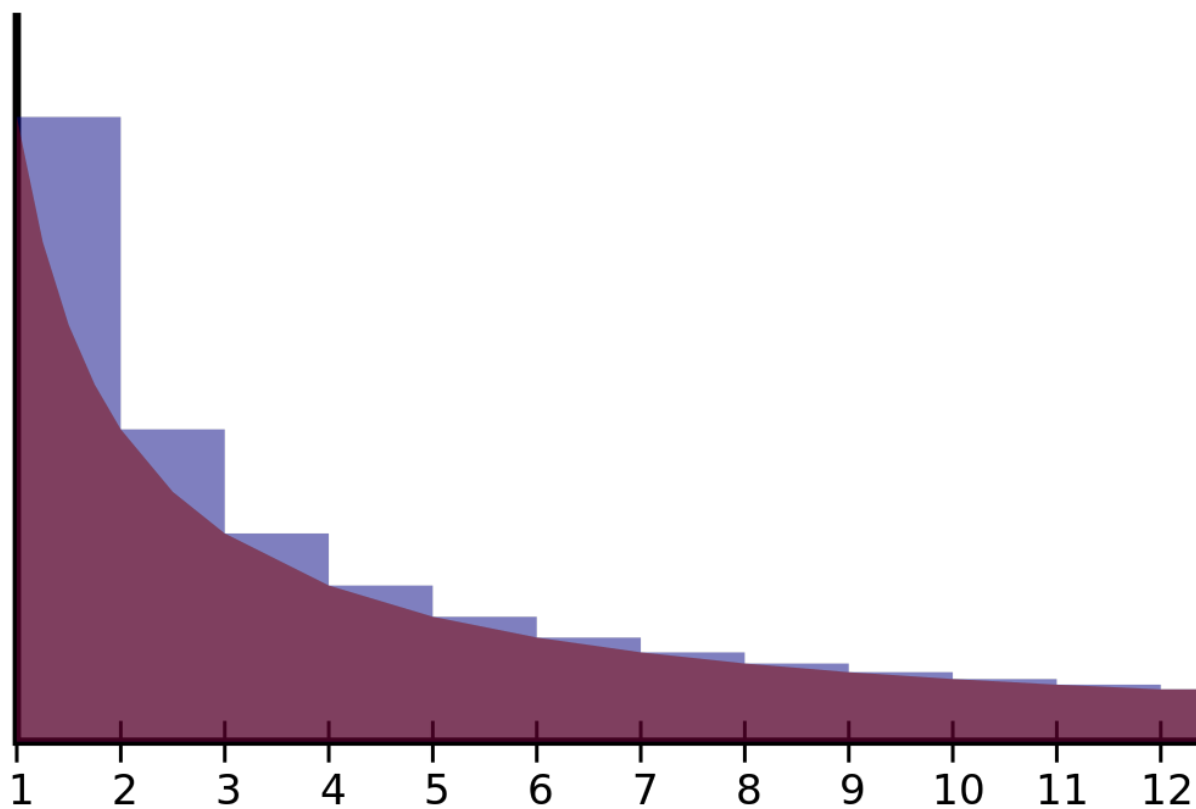
$$H = \gamma + \psi(n+1)$$

Figure 7: Differences between harmonic series and natural logarithm (blue area) converge to Euler-Mascheroni constant, William Demchick (Kiwi128@Wikipedia)

where γ is the Euler-Mascheroni constant and $\psi(x)$ is the logarithmic derivative of the gamma function. The digamma,

$$\frac{d}{ds}ln\Gamma(s)$$

$$=\frac{\Gamma(s)'}{\Gamma(s)}$$

γ is the pole of first order, its principal value exists and is equal to the Euler–Mascheroni constant, the limit of $-ln(n) + \sum_{k=1}^{n} 1/k$ as n increases to infinity. Also,

$$\gamma = \int_0^\infty (-\frac{1}{x} + \frac{1}{floor(x)})dx$$

And

$$\psi(s+1) = \psi(s) + \frac{1}{s}$$

$$=\frac{\Gamma(s)'}{\Gamma(s)} + \frac{1}{s}$$

Notice that the moment of inertia M_2 for large values of s given by:

$$M_2 \sim H = \gamma + \psi(s+1)$$

Where γ is limit of an integral and $\psi(s+1) = \psi(s) + \frac{1}{s}$:

$$= \int_0^s (-\frac{1}{s} + \frac{1}{floor(s)})ds + \frac{d}{ds}ln\Gamma(s) + 1/s$$

Taking the derivative of the second moment with respect to s

$$\frac{d}{ds}ln(x) = 1/x, \frac{d}{dx}x^{-1} = \frac{-1}{x^2} \rightarrow$$

$$= \frac{d}{ds}\int_0^s (-\frac{1}{s} + \frac{1}{floor(s)}) + \frac{d^2}{ds}ln\Gamma(s) + \frac{d}{ds}\frac{1}{s}$$

The second derivative of the Γ function is the first derivative of $\psi(s)$. Its Laurent series at $s = 0$ and ∞:

$$1/s^2 + \pi^2/6 + s\psi(s)'' + \frac{\pi^2 s^2}{30} + O(s^3)$$

$$1/s + 1/2s^2 + 1/6x^3 - 1/30s^5 + O((1/x)^6)$$

For $s \sim 0$:
$$M_2 \sim (-\frac{1}{s} + \frac{1}{floor(s)}) + 1/s^2 + \pi^2/6 + s\psi(s)'' + \frac{\pi^2 s^2}{30} + O(s^3) - 1/s^2$$

$$= (-\frac{1}{s} + \frac{1}{floor(s)}) + \pi^2/6 + s\psi(s)'' + \frac{\pi^2 s^2}{30} + O(s^3)$$

For $s \sim \infty$:
$$M_2 \sim (-\frac{1}{s} + \frac{1}{floor(s)}) + 1/s + 1/2s^2 + 1/6x^3 - 1/30s^5 + O((1/x)^6) - 1/s^2$$

$$= (-\frac{1}{s} + \frac{1}{floor(s)}) + 1/s + 1/6x^3 - 1/30s^5 + O((1/x)^6)$$

$$= (-\frac{1}{s} + \frac{1}{floor(s)}) + \frac{d}{ds}\frac{1}{\Gamma(s)} - s^{-2}$$

$$= (-\frac{1}{s} + \frac{1}{floor(s)}) + \frac{d}{ds}\frac{1}{\Gamma(s)} - s^{-2}$$

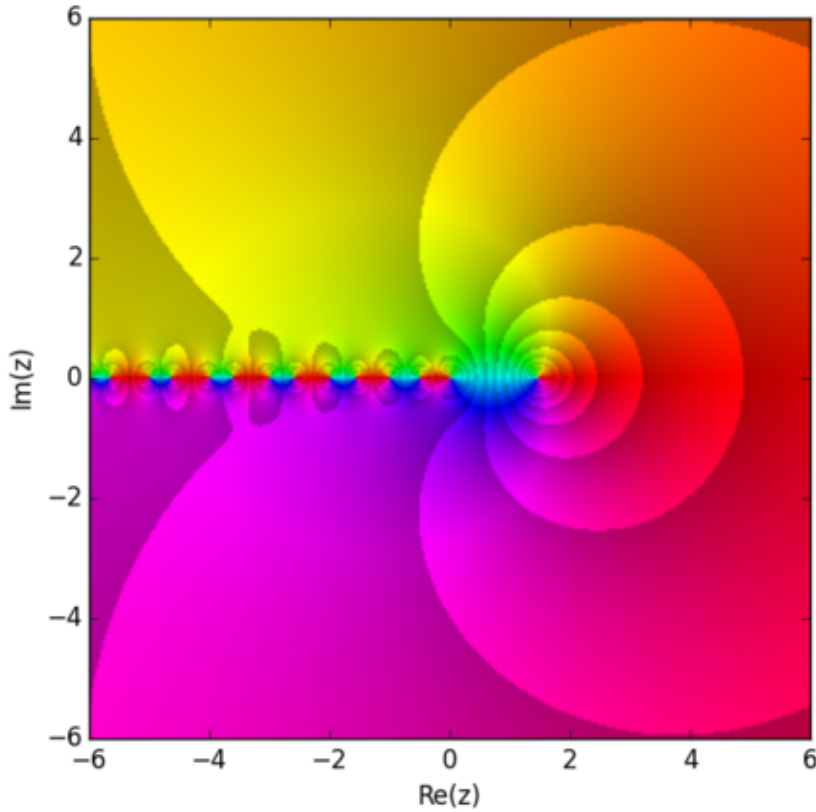$$= \int_0^s (-\frac{1}{s} + \frac{1}{floor(s)})dx + \psi(s) + \frac{1}{s}$$

The value is given by the logarithmic derivative of the gamma function until that moment

$$psi(s)$$

added with s-1$\frac{1}{s}$

Hence, the moment is given

The digamma is the derivative logarithmic of the gamma function. Considering products of successive rotations, the gamma function continues the factorials to real and complex numbers.

This shows that

$$M_2$$

can be approximated by a linear combination of a constant (γ) and

Each value of s determine unique moments for the line segments. Considering the dot products,

For the atractor of an orbit to be centered at zero, the second moment totally compensate the rotation by vectorial sums.

$$M_2(s) = \sum_1^\infty \frac{1}{n^{\frac{1}{2}}}^2$$

components must be co-periodic. The line segments should decrease their size
First moment The second moment must increase periodically at a spefic rate.

Consider the

$$\zeta(s) = \sum_1 \infty \frac{1}{n^s}$$

$$\zeta(2s) = \sum_1^\infty \frac{1}{n^2 s}$$

.

$\zeta(0) = \frac{-1}{2}$.

The non-trivial zeros. The $Im(s) =$ Consider that a hypothetical fish rod is being pull.

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