Assignment #3

Maximum likelihood and Bayesian estimation

How TA evaluates your assignments:

Report: half of your score will be graded proportional to the quality of your report. You should provide a distinct section for each problem, include the desired outputs and explain what you've done. Don't forget to discuss your results as well. It is not necessary to accommodate your source codes in your reports unless you want to refer to them. Compactness, expressiveness and neatness are of high importance.

Source Code: create an m-file for any problem and write all your codes there. If a problem consists of several sub-problems, separate them by comments in your code. Finally, name your m-files according to the number of the problems.

As you have to upload you submission electronically, it is of high interest to prepare your reports using Microsoft Office tools or Latex. However, scanned handwritten solutions are also acceptable as long as they are readable, neat and expressive.

What to hand in:

You must submit your report (.pdf) and source codes (m-files) for each assignment. Zip all your files into an archive file and use the following template to name it:

HW3_XXXXX.zip

where XXXXX must be replaced with your student ID. Your file size must not be bigger than 20MB. Send your files to *mohammadhme@gmail.com* with a subject of PR961_HW3_ XXXXX (replace XXXXX with your student number).

The Due Date for This Assignment is: Aban 28th

Problems:

1. Let $\{x_k\}, k = 1,2,\dots,N$ denote independent training from one of the following densities. Obtain the Maximum Likelihood estimate of θ in each case.
   a. $f(x_k;\theta) = \frac{x_k}{\theta^2}\exp\left(-\frac{x_k^2}{2\theta^2}\right)$ $\quad x_k \geq 0 \quad \theta > 0 \quad$ Rayleigh Density
   b. $f(x_k;\theta) = \sqrt{\theta}x_k^{\sqrt{\theta}-1}$ $\quad 0 \leq x_k \leq 1 \quad \theta > 0 \quad$ Beta Density

2. Let x have uniform density
$$f_x(x|\theta) \sim U(0,\theta) = \begin{cases} \frac{1}{\theta} & 0 \leq x \leq \theta \\ 0 & otherwise \end{cases}$$
   a. Suppose that n samples $D = \{x_1, x_2, \dots, x_n\}$ are drawn independently according to $f_x(x|\theta)$. Show that the maximum likelihood estimate for θ is $\max[D]$, i.e., the value of the maximum element in D.
   b. Suppose that $n = 5$ points are drawn from the distribution and the maximum value of which happens to be $\max_k x_k = 0.6$. Plot the likelihood function $f_x(D|\theta)$ in the range $0 \leq \theta \leq 1$. Explain in words why you do not need to know the values of other four points.

3. Following problems from the reference book [1]:
   Chapter3: Q1, Q3, Q4, Q5

Computer Projects:

Implement the following projects. Make sure that your codes are well-commented. Code-only submissions will gain at most 33/100 pts. So, do not forget to provide a compact, well-documented and informative submission.

1. Implement the following computer exercises from the reference book [1]

   Chapter 3:
   - Q1, Q2, Q3
   - Bonus project: Q11

Reference:

[1] Duda, Richard O., Peter E. Hart, and David G. Stork. Pattern classification. John Wiley & Sons, 2nd Edition.