# **Machine Learning**

**Lecture 7: Neural Networks and Keras** 

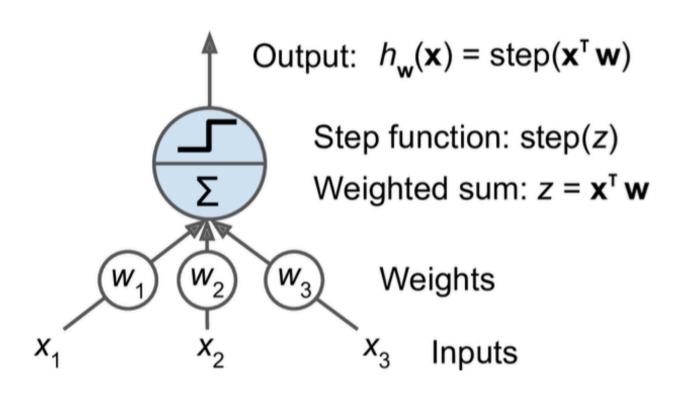
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#### Introduction

- We start this discussion by introducing Artificial Neural Network (ANN) architectures and Multilayer Perceptrons (MLPs)
  - History: ANNs gained popularity in 1940s and 1980s
- Why did deep learning become so popular again?
  - Massive amounts of data
  - Increase in computing power
  - Better understanding of the optimization landscape
- We discuss implementing neural networks using the Keras API
  - Simple high-level API for building and training neural networks
  - Keras is flexible enough to build various neural network architectures
  - Write custom Keras components using its lower-level API for extra flexibility

### Threshold logic unit (TLU)

• TLU computes a weighted sum of its inputs and then applies a step function



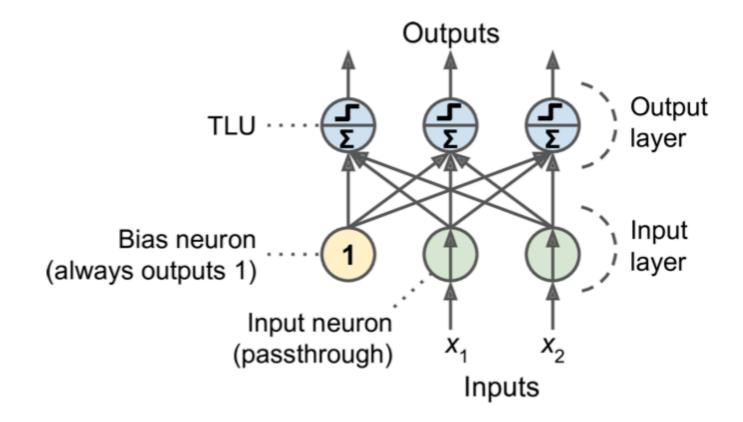
Common step functions

heaviside 
$$(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \ge 0 \end{cases}$$
  $sgn(z) = \begin{cases} -1 & \text{if } z < 0 \\ 0 & \text{if } z = 0 \\ +1 & \text{if } z > 0 \end{cases}$ 

• Just like logistic regression and linear SVM

#### **Perceptron**

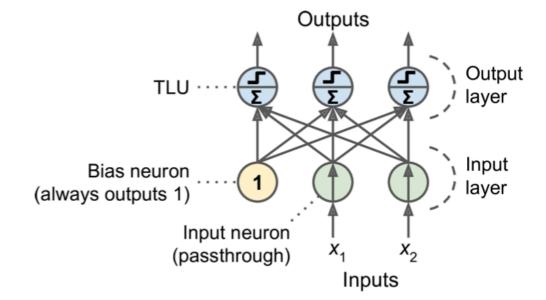
- A perceptron is composed of a single layer of TLUs, with each TLU connected to all the inputs (neurons in the previous layer)
- Example: 2 input neurons, 1 bias neuron, and 3 output neurons



Called fully connected or dense layer

# Computing the outputs

- We can find a compact representation of outputs using linear algebra
  - Number of instances: *n*, number of features: *d*
  - Number of neurons: *m*

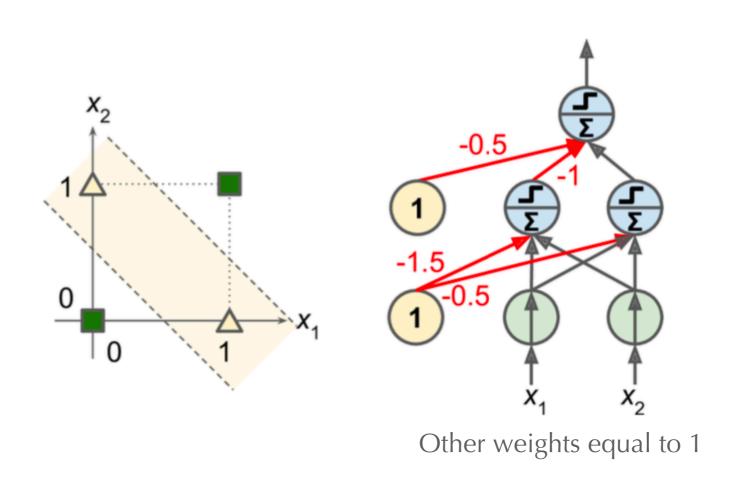


$$h(\mathbf{X}) = \phi(\mathbf{X}\mathbf{W} + \mathbf{b})$$

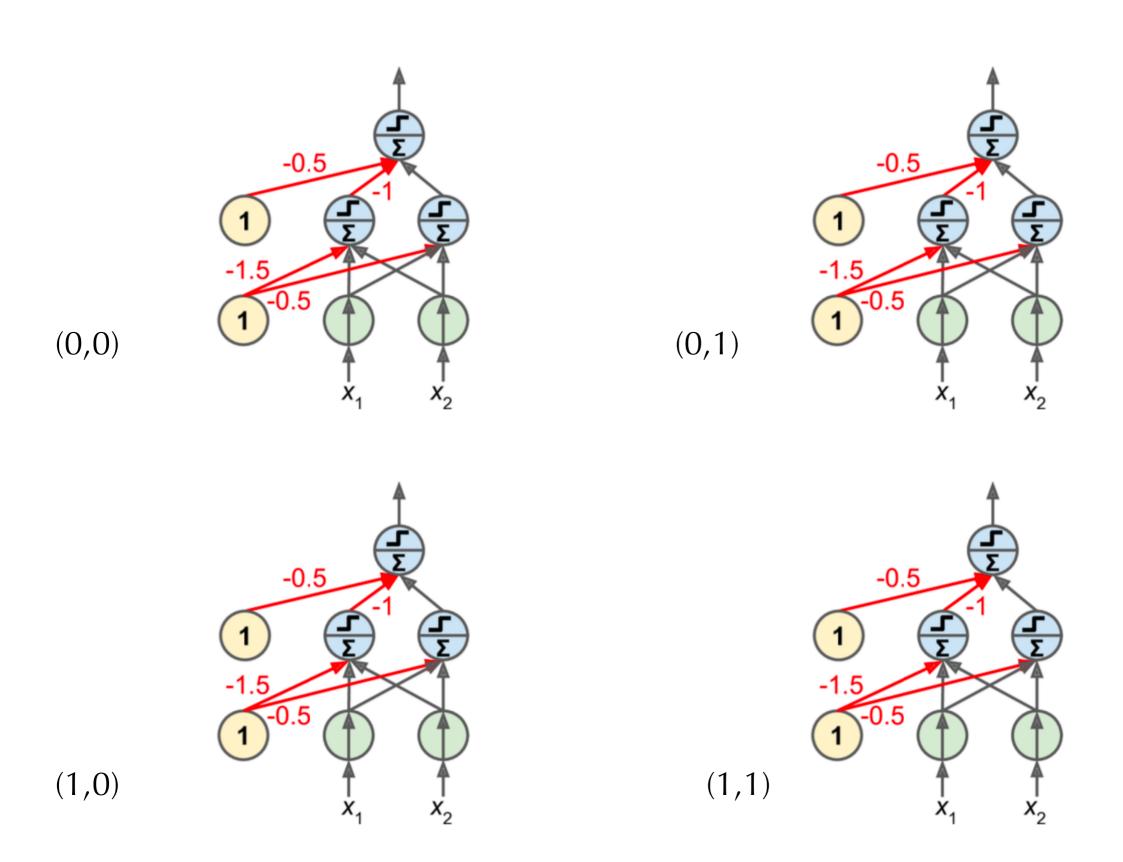
- Matrix of input features  $\mathbf{X} \in \mathbb{R}^{n \times d}$
- Matrix of connection weights  $\mathbf{W} \in \mathbb{R}^{d \times m}$
- Bias vector **b**: one bias term per artificial neuron
- ullet Activation or step function  $\phi$

# **Multilayer Perceptrons (MLPs)**

- Some of the limitations of Perceptrons can be eliminated by stacking multiple Perceptrons
- For example, the following network solves the Exclusive OR (XOR) problem

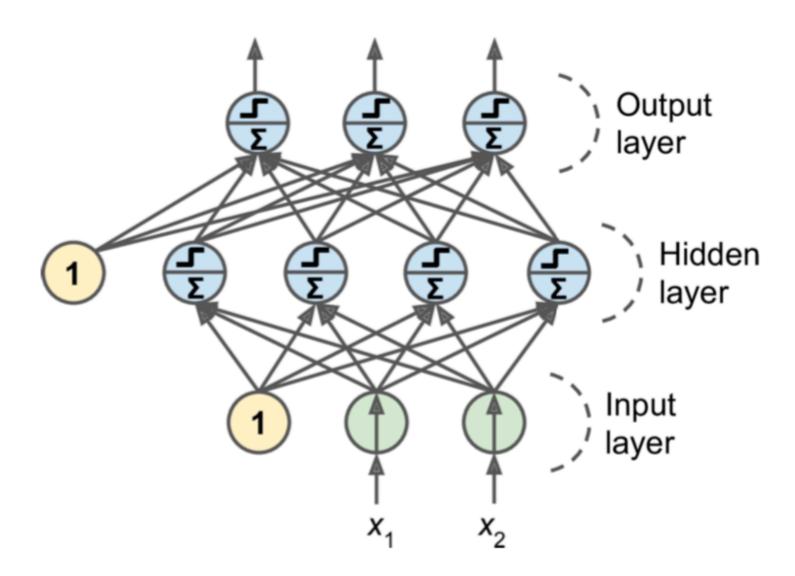


# **Solving XOR**



### **Architecture of Multilayer Perceptrons**

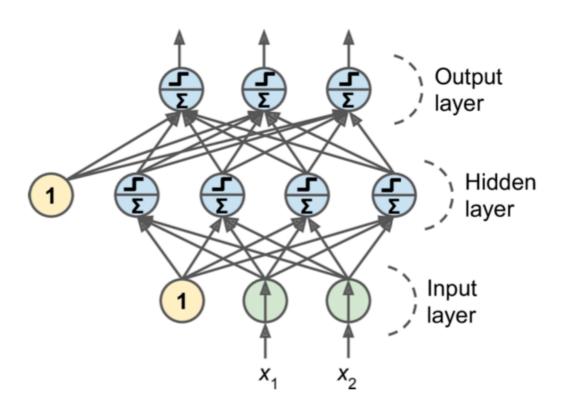
- MLP is composed of one input layer, one or more layers of TLUs (called hidden layers), and one final layer of TLUs (called output layer)
- Every layer except the output layer includes a bias neuron and is fully connected



• Input layer: 2 neurons, hidden layer: 4 neurons, and output layer: 3 neurons

# **Terminology**

 Features flow in one direction from the inputs to the outputs, so the architecture in the previous slide is an example of feedforward neural networks



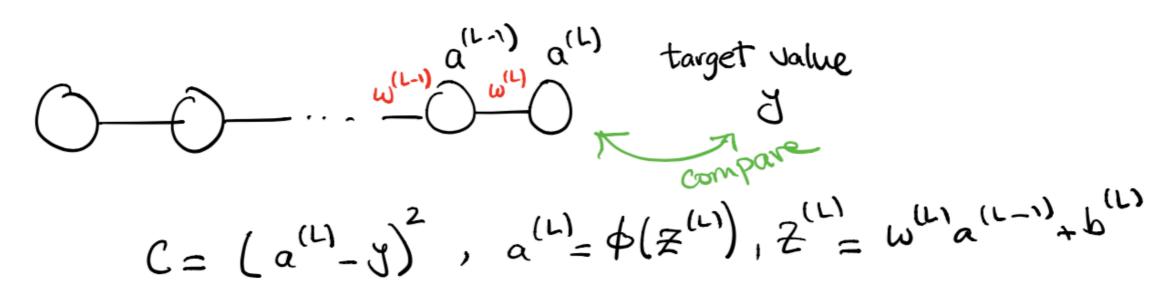
- When an ANN contains a deep stack of hidden layers (e.g., tens or hundreds of layers), it is called a deep neural network (DNN)
- Some people refer to networks with one or two hidden layers as Deep Learning

# **Training MLPs and backpropagation**

- We need to find out how each connection weight and bias term should be optimized in order to reduce the error
- We can use Gradient Descent and its variants if we compute the gradients
- General approach:
  - For each training instance, we first make a prediction (forward pass)
  - Measure the output error using a loss function
  - Go through each layer in reverse to measure the error contribution from each connection (reverse pass)
  - Update the connection weights to reduce the error
- We apply the chain rule in the backpropagation algorithm
- Note that we need to initialize connection weights randomly

#### **Example**

Consider a network with one neuron per layer



$$\frac{\partial C}{\partial w^{(L)}} = \frac{\partial C}{\partial a^{(L)}} \cdot \frac{\partial a^{(L)}}{\partial z^{(L)}} \cdot \frac{\partial Z^{(L)}}{\partial w^{(L)}}$$

$$= \frac{\partial C}{\partial a^{(L)}} \cdot \frac{\partial A^{(L)}}{\partial z^{(L)}} \cdot \frac{\partial Z^{(L)}}{\partial w^{(L)}}$$

$$= \frac{\partial C}{\partial a^{(L)}} \cdot \frac{\partial A^{(L)}}{\partial z^{(L)}} \cdot \frac{\partial Z^{(L)}}{\partial w^{(L)}}$$

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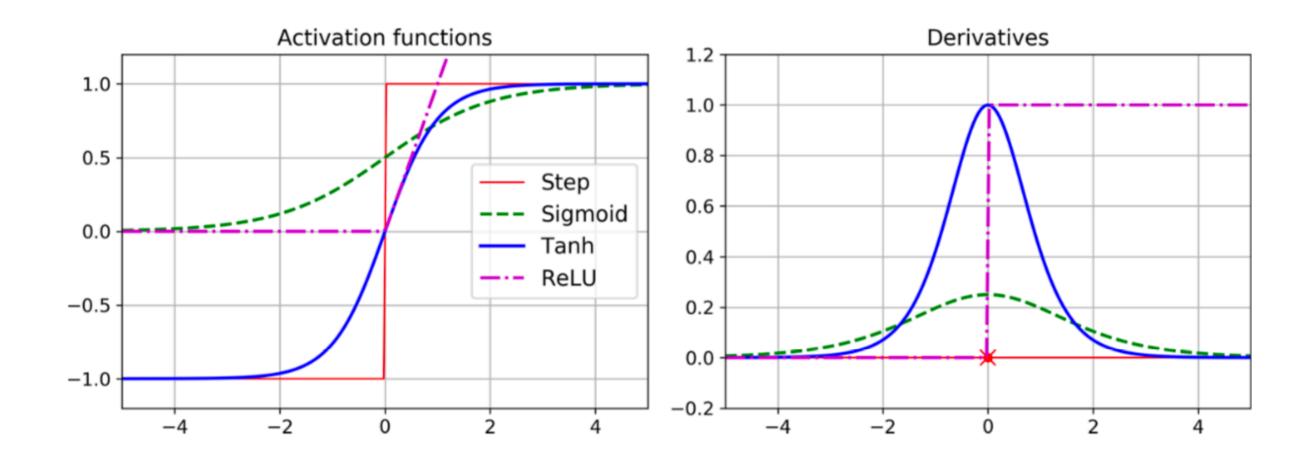
#### **Example**

$$\frac{\partial C}{\partial w^{(L)}} = \frac{\partial C}{\partial a^{(L)}} \cdot \frac{\partial a^{(L)}}{\partial z^{(L)}} \cdot \frac{\partial Z^{(L)}}{\partial w^{(L)}} \cdot \frac{\partial Z^{(L)}}{\partial z^{(L)}} \cdot \frac$$

#### **Activation functions**

- Replaced the step function with the logistic function  $\sigma(z) = \frac{1}{1 + \exp(-z)}$ 
  - Advantage: well-defined non-zero derivative everywhere
- Two other choices:
  - Hyperbolic tangent function  $tanh(z) = 2\sigma(2z) 1$ 
    - The output value ranges from -1 to 1
    - Each layer's output more or less centered around 0
  - Rectified Linear Unit function ReLU(z) = max(0,z)
    - Continuous but not differentiable at z = 0
    - The default activation function because it works very well

#### **Activation functions and their derivatives**



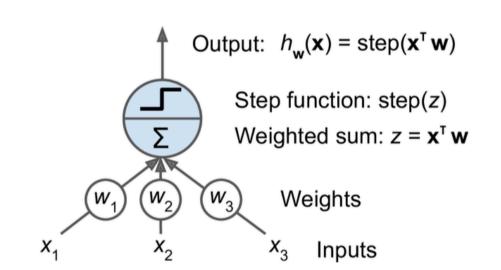
# Why activation functions?

- Why do we need activation functions in the first place?
  - Example: Consider two linear transformations

• 
$$f(x) = 2x + 3$$

• 
$$g(x) = 5x - 1$$

• 
$$f(g(x)) = 2(5x - 1) + 3 = 10x + 1$$



- If we don't have some form of non-linearity between layers, then the network
  is equivalent to a single layer network, incapable of solving complex problems
- On the other hand, a large enough deep neural network with non-linear activations can **theoretically** approximate any continuous function

# **Regression MLPs**

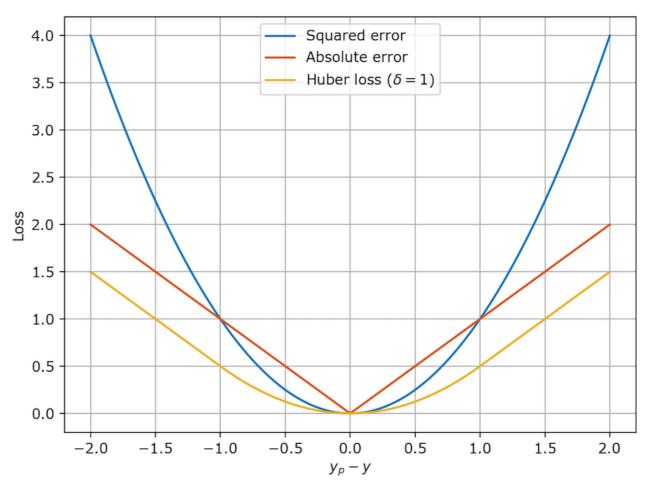
- For multivariate regression (i.e., predicting multiple values), you need one output neuron per output dimension
- We typically don't use any activation function for the output neurons so they are free to produce any range of values
  - If the output is always positive, you can use ReLU

If the output falls within a range of values, you can use logistic function and

scale the labels to the range 0 to 1



- Mean squared error (MSE)
- Mean absolute error (MAE)
- Huber loss



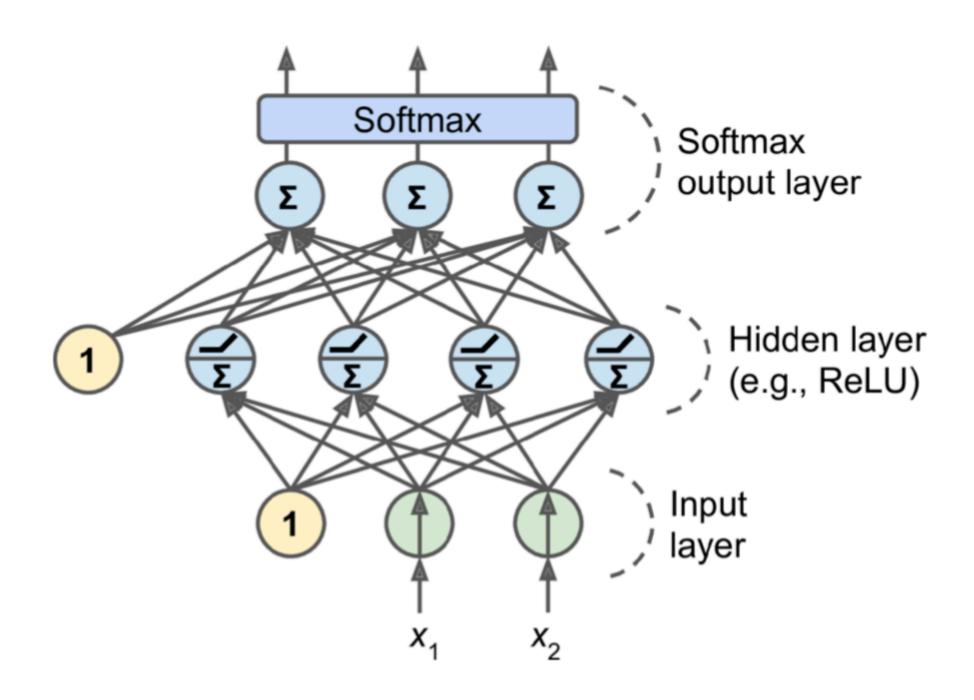
#### **Classification MLPs**

- For binary classification tasks, you just need a single output neuron with the logistic activation function
- If each instance belongs only to a single class, then we need one neuron per class with the softmax activation function
  - Example, classes 0 through 9 for digit image classification
- Loss function: we use the cross-entropy loss because the output layer is predicting probability distributions

$$-\sum_{c=1}^{C} y_c \log(p_c)$$

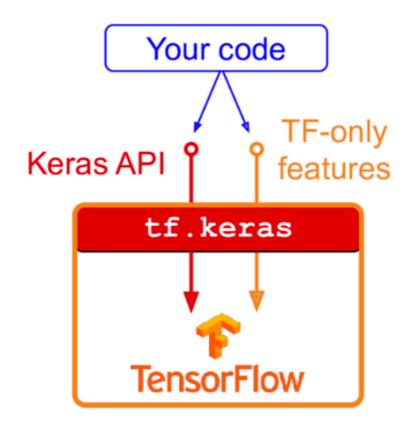
- $y_c$ : binary indicator (0 or 1)
- $p_c$ : predicted probabilities

#### A modern MLP architecture for classification



# **Implementing MLPs with Keras**

- Keras is a high-level deep learning API: <a href="https://keras.io/">https://keras.io/</a>
- TensorFlow comes bundled with its own Keras implementation tf.keras
  - It supports TensorFlow's Data API (load an preprocess data)



• Another popular library is PyTorch (quite similar to Keras and inspired by sklearn)

# **Installing TensorFlow**

• To test your installation, open a Python shell or Jupyter notebook

```
import tensorflow as tf

from tensorflow import keras

tf.__version__
'2.3.0'

keras.__version__
'2.4.0'
```

# Building an image classifier

- We work with the Fashion MNIST data set
  - 70,000 grayscale images of 28x28 pixels with 10 classes



(60000, 28, 28) [0 1 2 3 4 5 6 7 8 9]

print(X\_train\_full.shape, np.unique(y\_train\_full))

### **Loading data**

 When loading Fashion MNIST using Keras, the pixel intensities are represented as integers from 0 to 255

```
X_train_full.dtype
dtype('uint8')
```

 We scale the pixel intensities to the range 0-1 range and also generate a validation set

```
X_valid, X_train = X_train_full[:5000] / 255., X_train_full[5000:] / 255.
y_valid, y_train = y_train_full[:5000], y_train_full[5000:]
X_test = X_test / 255.

print(X_train.shape, X_train.dtype)

(55000, 28, 28) float64
```

List of class names

# Visualizing data

• Let's look at the first image in the training set and its label

```
plt.imshow(X_train[0], cmap="binary")
plt.axis('off')
plt.show()
```



```
class_names[y_train[0]]
'Coat'
```

#### Creating the model using the Sequential API

Classification MLP with two hidden layers

```
model = keras.models.Sequential()
model.add(keras.layers.Flatten(input_shape=[28, 28]))
model.add(keras.layers.Dense(300, activation="relu"))
model.add(keras.layers.Dense(100, activation="relu"))
model.add(keras.layers.Dense(100, activation="softmax"))
```

- "Flatten layer" converts each input image into a 1D array
  - No parameters because it only does preprocessing
  - You should specify the "input\_shape" because it is the first layer in the model
- "Dense" hidden layers with 300 and 100 neurons with ReLU activation functions
- "Dense" output layer with 10 neurons (one per class) with softmax

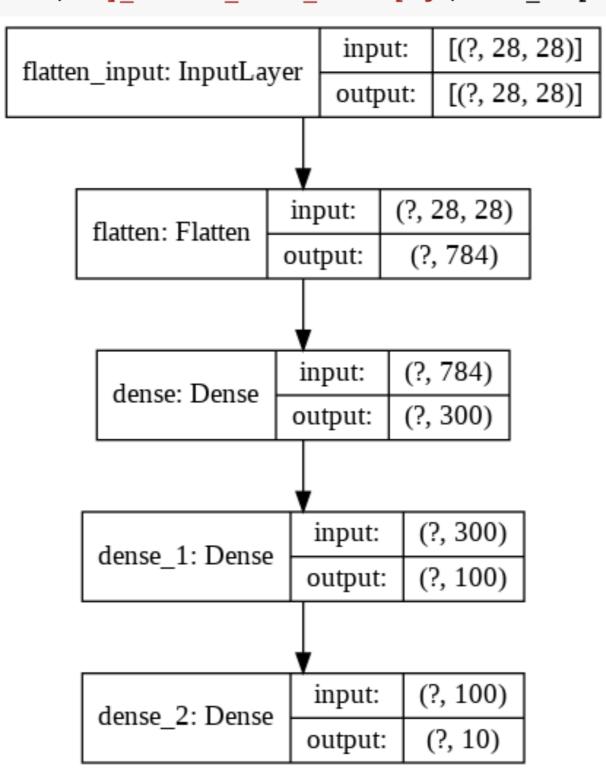
#### Easier way to define model

We can pass a list of layers when creating the sequential model

```
model = keras.models.Sequential([
    keras.layers.Flatten(input shape=[28, 28]),
    keras.layers.Dense(300, activation="relu"),
    keras.layers.Dense(100, activation="relu"),
    keras.layers.Dense(10, activation="softmax")
1)
model.summary()
Model: "sequential"
                                                         784 \times 300 + 300 = 785 \times 300 = 235500
                                                          Param #
                              Output Shape
Layer (type)
flatten (Flatten)
                              (None, 784)
                                                          235500 4
dense (Dense)
                              (None, 300)
dense 1 (Dense)
                                                          30100
                              (None, 100)
                              (None, 10)
                                                          1010
dense 2 (Dense)
Total params: 266,610
Trainable params: 266,610
Non-trainable params: 0
```

# Visualizing the model

keras.utils.plot\_model(model, "my\_fashion\_mnist\_model.png", show\_shapes=True)



# Accessing parameters for each layer

(784, 300) (300,)

```
hidden1 = model.layers[1]
weights, biases = hidden1.get weights()
                          Weights are initialized randomly and biases are initialized to zeros
weights
                                      https://keras.io/api/layers/initializers/
array([[ 0.02448617, -0.00877795, -0.02189048, ..., -0.02766046,
         0.03859074, -0.068893911,
       [0.00476504, -0.03105379, -0.0586676, ..., 0.00602964,
        -0.02763776, -0.04165364],
       [-0.06189284, -0.06901957, 0.07102345, ..., -0.04238207,
         0.07121518, -0.07331658],
       . . . ,
       [-0.03048757, 0.02155137, -0.05400612, ..., -0.00113463,
         0.00228987, 0.05581069],
       [0.07061854, -0.06960931, 0.07038955, ..., -0.00384101,
         0.00034875, 0.028784921,
       [-0.06022581, 0.01577859, -0.02585464, ..., -0.00527829,
         0.00272203, -0.06793761]], dtype=float32)
print(weights.shape, biases.shape)
```

### Compiling the model

- After a model is created, you must call its compile() method to specify the loss function and the optimizer to use
  - Additionally, specify a list of extra metrics to compute

- Why do we use "sparse" categorical cross entropy loss?
  - Labels are provided as integers
  - If labels are provided using one-hot representation, then we use categorical cross entropy loss

```
[0., 0., 0., 1., 0., 0., 0., 0., 0., 0.]
```

# Training and evaluating the model

• Keras will measure the loss and the extra metrics at the end of each epoch

```
loss: 0.7237 - accuracy: 0.7643 - val_loss: 0.5213 - val_accuracy: 0.8226
loss: 0.4842 - accuracy: 0.8316 - val_loss: 0.4349 - val_accuracy: 0.8528
loss: 0.4391 - accuracy: 0.8456 - val_loss: 0.5331 - val_accuracy: 0.7986
loss: 0.4123 - accuracy: 0.8565 - val_loss: 0.3916 - val_accuracy: 0.8654
loss: 0.3937 - accuracy: 0.8621 - val_loss: 0.3740 - val_accuracy: 0.8698
```

• If the performance on the training set is much better than on the validation set, the model is probably overfitting

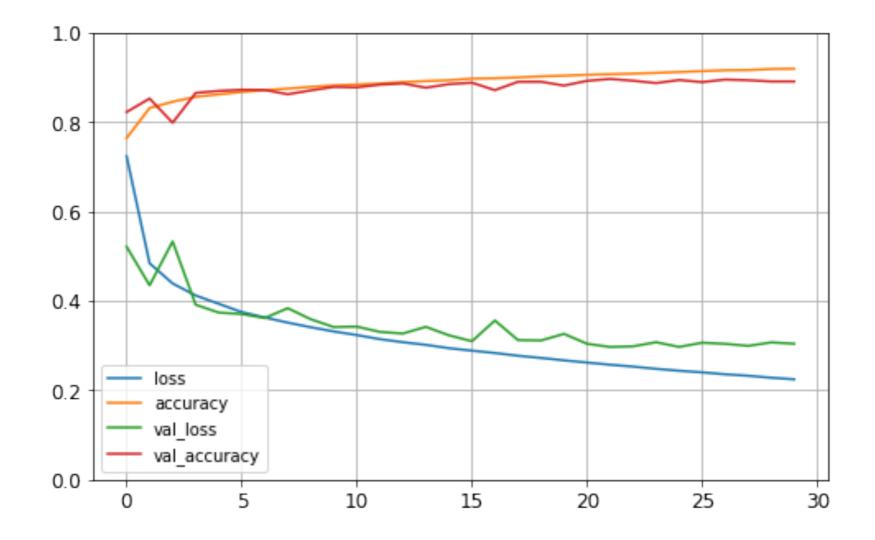
# **After training**

- The returned "history" object contains
  - Training parameters: history.params
  - List of epochs: history.epoch
  - Dictionary including the loss and extra metrics: history.history

# **Plotting learning curves**

```
import pandas as pd

pd.DataFrame(history.history).plot(figsize=(8, 5))
plt.grid(True)
plt.gca().set_ylim(0, 1)
plt.show()
```



# Compute the generalization error

 Once you are satisfied with your model's validation accuracy, you should evaluate it on the test set

#### Using the model to make predictions

- We can use the model's predict() method to make predictions on new instances
  - Probabilities

Classes

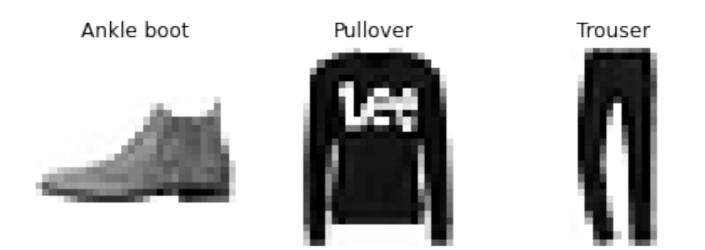
```
y_pred = model.predict_classes(X_new)
y_pred

WARNING:tensorflow:From <ipython-input-58-
Instructions for updating:
Please use instead:* `np.argmax(model.pred
array([9, 2, 1])</pre>
```

#### Visualization

• Let's look at these three predictions

```
plt.figure(figsize=(7.2, 2.4))
for index, image in enumerate(X_new):
    plt.subplot(1, 3, index + 1)
    plt.imshow(image, cmap="binary", interpolation="nearest")
    plt.axis('off')
    plt.title(class_names[y_test[index]], fontsize=12)
plt.subplots_adjust(wspace=0.2, hspace=0.5)
plt.show()
```



# Regression MLP using the sequential API

We use the California housing data set

```
from sklearn.datasets import fetch california housing
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
housing = fetch california housing()
X_train_full, X_test, y_train_full, y_test = train_test_split(housing.data,
                                              housing.target, random state=42)
X_train, X_valid, y_train, y_valid = train_test_split(X_train_full,
                                              y train full, random state=42)
scaler = StandardScaler()
X train = scaler.fit transform(X train)
X valid = scaler.transform(X valid)
X test = scaler.transform(X test)
```

```
print(X_train.shape, X_valid.shape, X_test.shape)
(11610, 8) (3870, 8) (5160, 8)
```

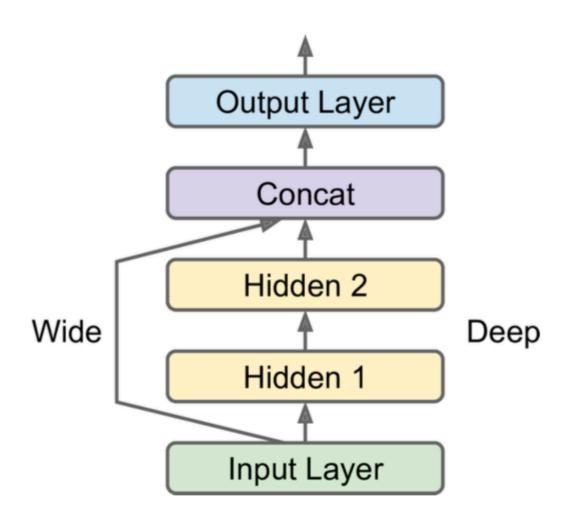
### **Training and evaluation**

• The output layer has a single neuron, one hidden layer, the loss function is MSE

```
model = keras.models.Sequential([
  keras.layers.Dense(30, activation="relu", input shape=X train.shape[1:]),
  keras.layers.Dense(1)
1)
model.compile(loss="mean squared error", optimizer=keras.optimizers.SGD(lr=1e-3))
history = model.fit(X train, y train, epochs=20, validation data=(X valid, y valid))
mse test = model.evaluate(X test, y test)
X \text{ new} = X \text{ test}[:3]
y pred = model.predict(X new)
Epoch 1/20
Epoch 2/20
Epoch 3/20
y pred
array([[0.38856652],
    [1.6792021],
    [3.1022797 ]], dtype=float32)
```

## **Building complex models using Keras**

- One example of non-sequential neural network is a "Wide & Deep" network
  - Connects the inputs directly to the output layer to learn both deep patterns (deep path) and simple rules (short path)



## **Building a complex model**

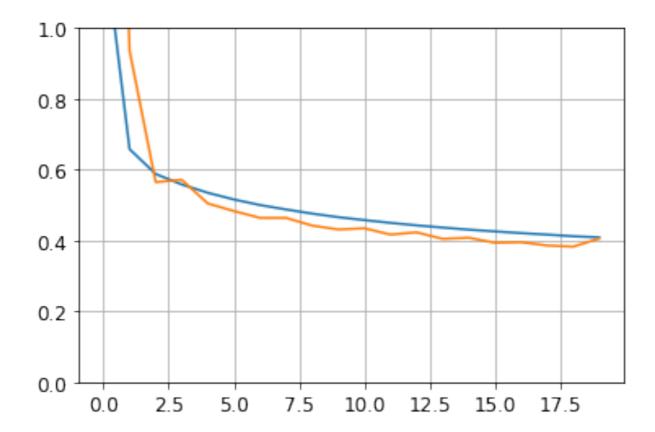
```
input = keras.layers.Input(shape=X train.shape[1:])
hidden1 = keras.layers.Dense(30, activation="relu")(input)
hidden2 = keras.layers.Dense(30, activation="relu")(hidden1)
concat = keras.layers.concatenate([input_, hidden2])
output = keras.layers.Dense(1)(concat)
model = keras.models.Model(inputs=[input], outputs=[output])
model.summary()
Model: "functional 1"
                                                     Param #
                                Output Shape
                                                                 Connected to
Layer (type)
input 1 (InputLayer)
                                [(None, 8)]
                                                     0
dense 5 (Dense)
                                (None, 30)
                                                     270
                                                                  input 1[0][0]
                                                                  dense_5[0][0]
dense 6 (Dense)
                                (None, 30)
                                                     930
concatenate (Concatenate)
                                (None, 38)
                                                     0
                                                                  input_1[0][0]
                                                                  dense_6[0][0]
                                                     39
                                (None, 1)
                                                                  concatenate[0][0]
dense 7 (Dense)
```

Total params: 1,239

Trainable params: 1,239 Non-trainable params: 0

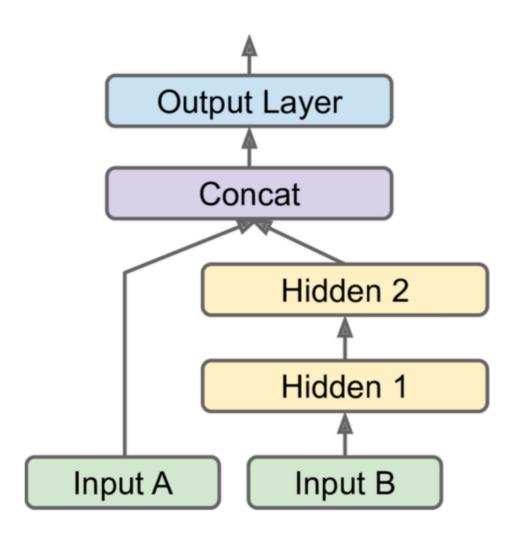
# **Training and evaluation**

• Similar to what we had before



#### More complex model

 Send a subset of the features through the wide path and a different subset through the deep path



One solution is to use "multiple inputs"

#### **Implementation**

```
input_A = keras.layers.Input(shape=[5], name="wide_input")
input_B = keras.layers.Input(shape=[6], name="deep_input")
hidden1 = keras.layers.Dense(30, activation="relu")(input_B)
hidden2 = keras.layers.Dense(30, activation="relu")(hidden1)
concat = keras.layers.concatenate([input_A, hidden2])
output = keras.layers.Dense(1, name="output")(concat)
model = keras.models.Model(inputs=[input_A, input_B], outputs=[output])
```

```
model.compile(loss="mse", optimizer=keras.optimizers.SGD(lr=1e-3))

X_train_A, X_train_B = X_train[:, :5], X_train[:, 2:]

X_valid_A, X_valid_B = X_valid[:, :5], X_valid[:, 2:]

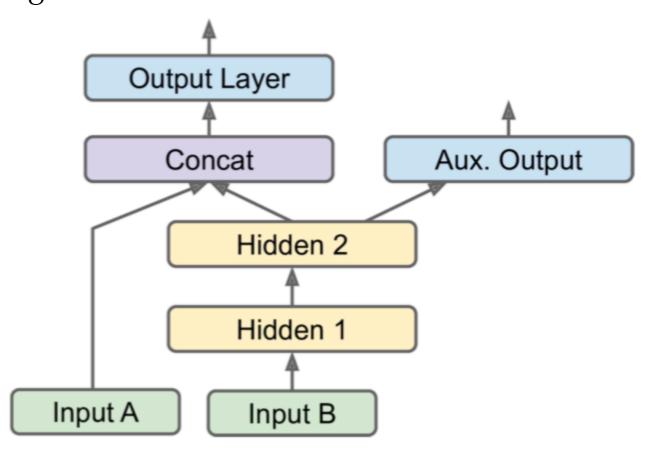
X_test_A, X_test_B = X_test[:, :5], X_test[:, 2:]

X_new_A, X_new_B = X_test_A[:3], X_test_B[:3]

history = model.fit((X_train_A, X_train_B), y_train, epochs=20, validation_data=((X_valid_A, X_valid_B), y_valid))
```

#### Case of multiple outputs

- There are many cases in which you want to have multiple outputs
  - Example 1: want to locate and classify main object in a picture (both regression and classification tasks)
  - Example 2: multi-task classification (e.g., classify facial expression and identify whether wearing glasses)
  - Example 3: some auxiliary outputs to ensure that the underlying network learns something useful



#### **Implementation**

Each output will need its own loss function and a weight parameter

Training

#### Saving and restoring a model

Now, let's see how we can save a trained Keras model

```
model = keras.models.Sequential([
    keras.layers.Dense(30, activation="relu", input_shape=[8]),
    keras.layers.Dense(30, activation="relu"),
    keras.layers.Dense(1)
])
```

```
model.compile(loss="mse", optimizer=keras.optimizers.SGD(lr=1e-3))
history = model.fit(X_train, y_train, epochs=10, validation_data=(X_valid, y_valid))
mse_test = model.evaluate(X_test, y_test)
```

• Keras uses HDF5 format to save both the model's architecture and the values of all the model parameters for every layer (i.e., connection weights and biases)

```
model.save("my_keras_model.h5")
```

We can also load the saved model

```
model_new = keras.models.load_model("my_keras_model.h5")
```

#### **Using Callbacks during training**

- When the training process takes a few hours, you should not only save your model at the end
  - Instead, save checkpoints at regular intervals to avoid losing everything

```
model = keras.models.Sequential([
    keras.layers.Dense(30, activation="relu", input_shape=[8]),
    keras.layers.Dense(30, activation="relu"),
    keras.layers.Dense(1)
])
```

## Fine-tuning neural network hyperparameters

- The flexibility of neural networks is also one of their main drawbacks
  - There are many hyperparameters
    - Number of layers (shallow vs deep models)
    - Number of neurons per layer
    - Activation functions
    - Weight initialization
    - Learning rate
    - Optimizer
    - Batch size

### **Exploring hyperparameter space**

- We can wrap our Keras models in objects that mimic regular Scikit-Learn regressors and then utilize GridSearchCV and RandomizedSearchCV
- Create a simple Sequential model for univariate regression

```
def build_model(n_hidden=1, n_neurons=30, learning_rate=3e-3, input_shape=[8]):
    model = keras.models.Sequential()
    model.add(keras.layers.InputLayer(input_shape=input_shape))
    for layer in range(n_hidden):
        model.add(keras.layers.Dense(n_neurons, activation="relu"))
    model.add(keras.layers.Dense(1))
    optimizer = keras.optimizers.SGD(lr=learning_rate)
    model.compile(loss="mse", optimizer=optimizer)
    return model
```

Create a KerasRegressor based on the above function

```
keras_reg = keras.wrappers.scikit_learn.KerasRegressor(build_model)
```

#### **Exploring hyperparameter space**

Training one model

Training different models

Finding the best estimator and its hyperparameters

Reading Assignment: Chapter 10 of Textbook