Remarks and Responses

Remark 1: The topology of the space of currents $D^p(X)$ (see e.g., Prop. 1.2, 2.6) is defined as follows: a sequence converges to a current if it converges weakly and the sequence of $*$-norm is bounded. That is, you can write each element of the sequence as the difference of two positive closed currents of bounded masses. This point is important.

Response 1: Fixed.

Remark 2: It seems that when we say that the Hodge group $H^{p,q}$ is the Dolbeault group, people in algebraic geometry don't like it. For them, these groups are only isomorphic to Dolbeault groups.

Response 2: I see. I've changed the equality on Page 3 into an isomorphism.

Remark 3: Corollary 2.9: Is it a consequence of 2.8?

Response 3: Yes, because locally we have an isomorphism when we don't intersect the exceptional divisors.

Remark 4: Lemma 5.6: I guess you assume that the first intersection is transversal. Did you prove that the intersection is independent of the choice of H_i? It seems to me that the inequality is actually an equality, as currents with continuous superpotentials have no mass on divisors.

Response 4: Ah you're right! I've clarified the assumption about transversality, however, even in the transversal case

$$\overline{\mathscr{T}}_{\mathrm{aff}(\sigma)} \leq \overline{\mathscr{T}_{H_1} \wedge \cdots \wedge \mathscr{T}}_{H_{n-p}},$$

and the choice of H_i matters, but we only need the inequalities here. It is discussed later that the 'angles' between H_i give rise to multiplicities.

Regarding the second comment, we prove a more general case later. At this stage, I only needed the inequality to establish continuity of the superpotential, deferring equality until we address the general case in Proposition 5.9.

Remark 5: The proof of Lemma 5.7 seems correct. However, should you clarify the choice of $\widehat{\Sigma}$, which depends on Σ? The notion of compatibility between $\widehat{C}$ and $\widehat{\Sigma}$ seems slightly different from Definition 5.1.

Response 5: I've added that $\widehat{\Sigma}$ is a refinement of Σ satisfying the required compatibility.

Remark 6: In Definition 3.2: Did you mix up τ and σ?

Response 6: Yes, you're correct. Fixed it. With the new numbering, this is Definition 3.2.

Remark 7: For Definition 3.3, by "top dimension," do you mean the "expected top dimension"? The intersection may have a dimension lower than expected. The notation $\sigma_1 \cap \sigma_2$ is a bit confusing. Maybe we shouldn't specify σ_1, σ_2 explicitly here, as they are not unique? Also, what do you mean by a "generic" v? I only see that v is in some open set, which could potentially be small. Am I missing something?

Response 7: I agree it's a bit confusing, but it's a standard definition. The condition $\dim(\sigma_1 + \sigma_2) = n$ fixes everything. Indeed, there is a lemma justifying why it makes sense. Basically, you're correct: we perturb C_1 by selecting a vector v in some suitable open subset of $\mathbb{R}^n$ (typically $\mathbb{R}^n \setminus \{C_1 \cup C_2\}$), ensuring the intersection becomes transversal. We then show that the intersection is continuous and independent of this choice of v. This procedure is analogous to SP-convergence. This is the content of Theorem 3.5.

If you prefer, we can just define the stable intersection as the limit of

$$C_1 \cap \epsilon v + C_2$$

as $\epsilon \to 0$ and mention as a theorem in tropical geometry that the definition doesn't depend on the generic choice of v.