Questions on (NA) currents, equidistribution, ...

§1. Hybrid spaces

* Why do we care? It's a setting that brings together archimedean & non-archimedean

  1 → realizes the analogy in a genuine (topological) analytic space

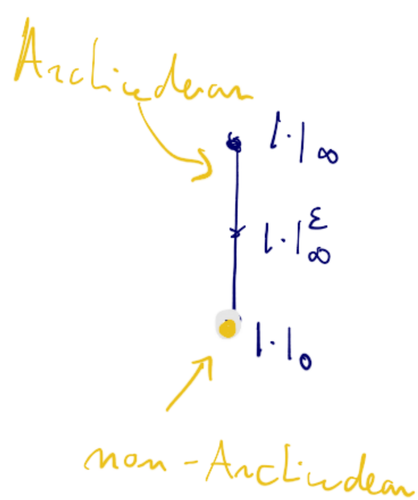
  2 → helpful to point out the right non-Archimedean object.

* definition

$$\mathbb{C}_{hyb} := (\mathbb{C}, |\cdot|_{hyb}) \qquad |c|_{hyb} := \max\{1, |c|\} \quad \forall c \in \mathbb{C}^\times$$

$$|0|_{hyb} = 0$$

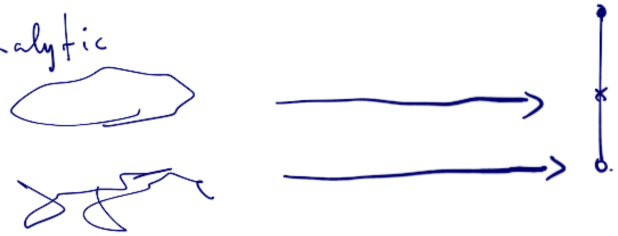
$$\mathcal{M}(\mathbb{C}_{hyb}) = \left\{ \text{mult seminorms } \mathbb{C} \longrightarrow \mathbb{R}_{\geq 0} \text{ bounded by } |\cdot|_{hyb} \right\}$$



$\forall \; X_{/\mathbb{C}}$ alg. variety

$$\rightsquigarrow \; X^{hyb}_{/\mathbb{C}} \longrightarrow \mathcal{M}(\mathbb{C}_{hyb})$$

$\mathbb{C}$-analytic



Berkovich space over $(\mathbb{C}, |\cdot|_0)$

<u>FACT</u>: we can use hybrid spaces to study families

$$\mathcal{X} \longrightarrow \mathbb{D}_{\mathbb{C}}(0,1)^+ = \{ t \in \mathbb{C} \, / \, |t| \leq 1 \}$$

$$\mathcal{X}_t \longleftarrow t$$

$$\mathbb{G}_m^{hyb} = \Big(\mathrm{Spec}\big(\underline{\mathbb{C}[t,t^{-1}]}\big)\Big)^{hyb} = \Big\{ |\cdot| : \mathbb{C}[t,t^{-1}] \longrightarrow \mathbb{R}_{\geq 0} \text{ bounded by } |\cdot|_{hyb} \text{ on } \mathbb{C} \Big\}$$

$\cup$

$$\mathbb{G}_m^{*} = \{ x \in \mathbb{G}_m^{hyb} \, / \, x(t) = \varepsilon \}$$

$x(t) = |t(x)|$

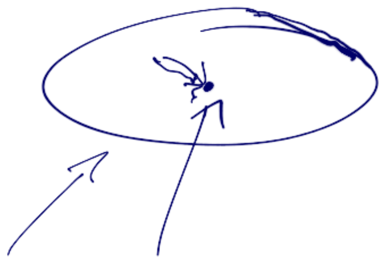
fix $0 < \varepsilon < 1$

$\mathbb{C}^\times$

$\mathbb{C}((t))$

$\mathcal{M}(\mathbb{C}_{hyb})$

$$\mathbb{C}^{\#}_{7m} \simeq$$



$\mathbb{C}((t))$

$$= \underbrace{\mathbb{D}(0,1)^{+} \setminus \{0\}}_{\text{punctured disk}} \cup \{pt\}$$

<u>Rmk:</u> consider $A_\varepsilon := \left\{ f = \sum_{n\in\mathbb{Z}} a_n T^n : \sum |a_n|_{hyb}\, \varepsilon^{n} < +\infty \right\}$

Prop (Poincaré)

$$\begin{tikzcd}
\mathbb{C}^{\#}_{7m} \cong \mathcal{M}(A_\varepsilon) \arrow[d, equal] & \simeq \{|T| = \varepsilon\} \subset \mathbb{A}^1_{\mathbb{C}_{hyb}} \\
\mathcal{C}_{hyb}(\varepsilon) & \\
\mathbb{P}^1_{\mathcal{C}_{hyb}(\varepsilon)} \circlearrowleft \arrow[u] &
\end{tikzcd}$$

Q. What about hybrid tropicalisations ?

Q. Toric models of hybrid spaces ?

Q. Convergence of currents in hybrid setting ?

Remark we do have convergence of measures in hybrid spaces, so we can try to describe convergence of currents in terms of convergence of measures (at least for integration currents).
One way to do this is to show that it is enough to check convergence after intersecting currents with generic affine spaces of complementary dimension.