BÉZOUT INEQUALITY AND APPLICATIONS

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Abstract.

1. Introduction

Let X be a complex manifold of dimension k. Let $Z_1, \ldots, Z_n$ be analytic subsets of X of dimension $q_1, \ldots, q_n$ respectively. When $q_1 + \cdots + q_n < k$, the inetersection $Z_1 \cap \ldots \cap Z_n$ is expected to be empty. Otherwise, it is expected to have dimension $q_1 + \cdots + q_n - k$ when it is non-empty. More precisely, $Z_1 \cap \ldots \cap Z_n$ is an analytic subset of X whose irreducible components has a dimension at least $q_1 + \cdots + q_n - k$. When this dimension is strictly larger than $q_1 + \cdots + q_n - k$ we refer to the "dimension excess" phenomenon. We will define to each component Y of $Z_1 \cap \ldots \cap Z_n$ a multiplicity m_Y which is a positive integer.

Here is our main result where the degree $\|Y\|$ of an analytic subset of pure dimension of X is defined by

$$\|Y\| := \int_Y \omega^{\dim Y}.$$

Theorem 1.1 (Bézout inequality)**.** *Let X be a compact Kähler manifold of dimension k and ω be a Kähler form on X. Let $Z_1, \ldots, Z_n$ be analytic subsets of X of dimension $q_1, \ldots, q_n$ respectively. Let q be an integer. Denote by $Y_1, \ldots, Y_l$ the irreducible components of dimension q of $Z_1 \cap \ldots \cap Z_n$ and $m_1, \ldots, m_l$ their multiplicities. Then there is a constant $A > 0$ depending only on (X, ω) such that*

$$\sum_{i=1}^{l} m_i \|Y_i\| \leq A \|Z_1\| \ldots \|Z_n\|.$$

This is an application of the density theory for currents.

Applying this result to the intersection of $\Gamma_n \cap \pi_1^{-1}(Z)$ and Δ where Γ_n is the graph of f^n and Δ is the diagonal of $X \times X$.

Corollary 1.2. *Let X be a compact Kähler manifold of dimension k and ω be a Kähler form on X. Let f be a dominant meromorphic correspondence from X to X. Let P_n be the set of isolated periodic points of period n of f where we count points with multiplicities. Then there is a constant A depending only on (X, ω) such that if Z is any analytic subset of pure dimension q of X we have*

$$\#(P_n \cap Z) \leq A \|Z\| \max_{0 \leq p \leq q} d_{n,p}, \qquad \textit{where} \quad d_{n,p} := \int_X (f^n)^*(\omega^p) \wedge \omega^{k-p}.$$

Recall that the limit

$$d_p := \lim_{n \to \infty} (d_{n,p})^{1/n}$$

exists and is called the dynamical degree of order p of f. It follows that the RHS of the inequality in the corollary can be bounded by a constant times $(\max d_p + \epsilon)^n$ for each $\epsilon > 0$.

The important fact in the corollary is that the estimate is uniform on Z. The case of endomorphism of $\mathbb{P}^k$ without multiplicities was obtained in [DY25] and has been used to get the exponential rate of equidistribution for periodic points. In this case, we have $d_{n,p} = d^{pn}$, where d is the algebraic degree of the map.

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Corollary 1.3. *Growth of the indeterminacy sets of f^n ? Other applications ? How to get a good estimate for periodic points of a Henon map on a curve ?*

References

[DY25] Tien-Cuong Dinh and Jit Wu Yap. *Unpublished note*, 2025.

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