Assessed Coursework 2

- Available from 13:00, April 17th to 13:00, April 24th
- Please submit your work in PDF format directly on Blackboard
- This exam counts for %7.5 of your final course mark
- Feel free to discuss Q1 with each other, but I expect the solutions to Q2 to Q5 to be the outcome of your sole effort
- Please justify your answers in detail

Q1. (a) (**10 marks**) Find all the elements of $\mathrm{maxSpec}(\mathbb{C}[x])$ and $\mathrm{maxSpec}(\mathbb{C}[x, 1/x])$, respectively.

(b) (**10 marks**) Consider the isomorphism $\varphi : \mathbb{A}^1 \setminus \{0\} \longrightarrow \mathbb{A}^1 \setminus \{0\},\ a \longmapsto b = 1/a$, and the pullback map on the coordinate rings $\varphi^* : \mathbb{C}[x, 1/x] \longmapsto \mathbb{C}[y, 1/y]$. Compute $\varphi^*(1/x),\ \varphi^*(2x^2 + \frac{2x^3+4x}{x^5}),\ \varphi^*(2 - x)$.

Q2. Consider the affine algebraic hypersurface $V := \mathbb{V}(y - ux) \subseteq \mathbb{A}^3$.

(a) (**10 marks**) Prove that the projection $\mathbb{A}^3 \longrightarrow \mathbb{A}^2$, $(x, y, u) \longmapsto (x, u)$ restricts to an isomorphism between V and $\mathbb{A}^2$.

(b) (**10 marks**) Prove that the projection $\mathbb{A}^3 \longrightarrow \mathbb{A}^2$, $(x, y, u) \longmapsto (x, u)$ does not restrict to isomorphism between V and $\mathbb{A}^2$.

(c) (**10 marks**) Find $\mathcal{O}_V(D(u))$.

Q3. (**20 marks**) Prove that if V is an irreducible affine variety, then so is its projective closure $\overline{V}$.

Q4. (**10 marks**) What is the projective closure of the $\mathbb{V}(y - \sin(x))$ in $\mathbb{P}^2$? Would this contradict the Chow Lemma?

Q5. (**20 marks**) Consider the family of algebraic varieties, with parameter $t \in \mathbb{C}$, given by

$$V_t := \mathbb{V}(xy^2 - t) \subseteq \mathbb{A}^2.$$

Sketch the variety of V_0, V_1, and V_2 in $\mathbb{R}^2$. Which one of these varieties are smooth? Which one of these varieties are irreducible?