Application from Jake Masters for Mathematics (PhD) to start in 01 September 2025

Personal details

Personal details

First / given name	Jake
Second given name	Aaron
Third given name	
Surname/family name	Masters
Date of birth	27 January 2003
Preferred first/given name	Jake
Previous surname	
Country of birth	United Kingdom
Legal nationality	British National
Dual nationality	
Country of residence	United Kingdom
Have you previously studied with us at the University of Bristol?	No

Contact details

Home address

Please provide your permanent residential address. If you have another address and would prefer for us to contact you at that address instead you have the opportunity to add a correspondence address in the next section.

Country	United Kingdom
Postcode	LS16 8FS
Address Line 1	2 Woodsley Drive
Address Line 2	
City	Leeds
County	West Yorkshire
Telephone	07432520390

If you would like us to send any postal correspondence to an address which is not your home address please enter an alternative address here. If you want us to send correspondence to your home address then please select No.

Do you want to add a correspondence address?	No
Country	United Kingdom
Postcode	LS16 8FS
Address Line 1	2 Woodsley Drive
Address Line 2	
City	Leeds
County	West Yorkshire
Telephone	07432520390

Agent

Agent details

Agency Name	
Email address	

Other information

Additional Documents

Please upload required documents as outlined in your admissions statement

Mode of study

How would like to study this programme?	Full Time

Qualifications

Qualifications

Institution	Qualification	Type	Subject	Actual/predicted	Grade	Start date	End date
University of Oxford, UK	Master's Degree (PG)	Academic Qualification		Predicted	Distinction (Fi	17/Oct/2021	15/Jul/2025

If these qualifications have altered since your last application please note the changes in the free text box here.

English Language

Is English your first language? Yes

What is your first language?

Did you study at school/university where you were taught in English?

For how many years?

Have you sat a relevant English language test?

TOEFL (internet-based)

Registration number

Date of TOEFL test

TOEFL reading score

TOEFL listening score

TOEFL speaking score

TOEFL writing score

TOEFL total score

IELTS (International English Language Testing System)

Test report form (TRF) number

UKVI number (if applicable)

Date of IELTS test

IELTS listening score

IELTS reading score

IELTS writing score

IELTS speaking score

IELTS total score

Pearson Test of English

Score report code

Date of Pearson test

Pearson listening score

Pearson reading score

Pearson speaking score

Pearson writing score

Pearson overall score

Other English Language test

Name of course

Registration number

Date of test

Listening score

Writing score

Reading score

Total score

Experience

Current Employer

Employer name and address

Job title and main duties

Full time/Part time

Date of Appointment

End date (if applicable)

Previous employment 1

Employer name and address

Job title and main duties

Full time/Part time

Date of Appointment

End date (if applicable)

Previous employment 2

Employer name and address

Job title and main duties

Full time/Part time

Date of Appointment

End date (if applicable)

Previous employment 3

Employer name and address

Job title and main duties

Full time/Part time

Date of Appointment

End date (if applicable)

Other Experience

Do you have any other relevant work experience to support your application? No

Please provide details

Personal statement

Personal details

Do you have a personal statement to upload?	Yes
Please type your personal statement in the box	

Research proposal

Research proposal

Proposed supervisor 1	Philip Welch
Proposed supervisor 1	
Proposed project title (max 150 chars)	Inner Models of Set Theory

Passport and visa

Visa required

Do you require a visa to study in the UK? No

Please fill out your passport details below. If you are unable to provide these at the current time you will have another opportunity to upload your passport after you submit the form. If you do not provide us with this information we will be unable to issue you with your confirmation of acceptance number and you will be unable to obtain a visa.

Passport details

Passport number

Further details

Have you previously studied in the UK?

What was the highest level of study in the UK?

Please confirm the total length of your UK study in years

Referees

Referee 1

Do you have a reference to upload? No

Type of reference Academic

Referee title Professor

Forename Stuart

Surname White

Position Professor of Mathematics

Institution/Company University of Oxford

Email address stuart.white@maths.ox.ac.uk

Country United Kingdom

Referee 2

Do you have a second reference to upload? No

Type of reference Academic

Referee title Professor

Forename Bartek

Surname Klin

Position Associate Professor of Computer Science

Institution/Company University of Oxford

Email address bartek.klin@cs.ox.ac.uk

Country United Kingdom

Funding

Funding 1

What is your likely source of funding?	University of Bristol scholarship
Please give the name of your scholarship or Studentship	
Please specify	
Percentage from this source	100
Is this funding already secured?	No

Funding 2

What is your likely source of funding?	Engineering and Physical Sciences Research Council
Please give the name of your scholarship or Studentship	
Please specify	
Percentage from this source	100
Is this funding already secured?	No

Funding 3

What is your likely source of funding?	Other
Please give the name of your scholarship or Studentship	
Please specify	I would like to be considered for the Heilbronn Doctoral Partnership.
Percentage from this source	100
Is this funding already secured?	No

Other funding

I would like to be considered for other funding opportunities	Yes

Submission

Documents

Document type	File name
Personal statement	Bristol.pdf
Research proposal	BristolProposal.pdf
Admissions documents (Miscellaneous)	projectSample.pdf
Transcript	Transcript.pdf
Curriculum vitae	resume28.11.24Anon.pdf

By ticking the checkbox below and submitting your completed online application form, you acknowledge the University of Bristol will use the information provided from time to time, along with any further information about you the University may hold, for the purposes set out in the University's full Data Protection Statement. Applicants applying to the collaborative programmes of doctoral training should also read the Data Protection Statement for collaborative programmes of doctoral training.

The information that you provided on your application form will be used for the following purposes:

- To enable your application for entry to be considered and allow our Admissions Advisors, where applicable, to assist you through the application process;
- To enable the University to compile statistics, or to assist other organisations to do so. No statistical information will be published that would identify you personally;
- To enable the University to initiate your student record should you be offered a place at the University.

All applicants should note that the University reserves the right to make without notice changes in regulations, courses, fees etc at any time before or after a candidate's admission. Admission to the University is subject to the requirement that the candidate will comply with the University's registration procedure and will duly observe the Charter, Statutes, Ordinances and Regulations from time to time in force.

By ticking the checkbox below and submitting your completed online application form, you are confirming that the information given in this form is true, complete and accurate and that no information requested or other material information has been omitted. You are also confirming that you have read the Data Protection Statement and you confirm the statement below.

I can confirm that the information I have provided is true, complete and accurate. I accept that the information given in my application will be stored and processed by the University of Bristol, in accordance with the *UK General Data Protection Regulation and Data Protection Act 2018*, in order to:

- Consider my application and operate an effective and impartial admissions process;
- Monitor the University's applicant and student profile;
- Comply with all laws and regulations;
- Ensure the wellbeing and security of all students and staff;
- If my application is successful to form the basis of the statement made within my application.

If the University of Bristol discovers that I have made a false statement or omitted signification information from my application, for example examination results, I understand that it may have to withdraw or amend its offer or terminate my registration, according to circumstances.

EDUCATION

Oxford University
October 2021 – July 2025
Mathematics and Computer Science
Oxford, UK

- Year 1: Distinction (First Equivalent)
- Year 2: First
- Modules in: Real and Complex Analysis; Linear Algebra; Probability; Groups; Algorithms; Models of Computation; Integration; Rings; Topology; Lambda Calculus; Principles of Programming Languages
- Year 3: First
- Modules in: Logic; Set Theory; Functional Analysis; Information Theory; Logic and Proof; Machine Learning; Quantum Information

The Grammar School at Leeds
September 2007 – July 2021
A Levels
Leeds, UK

- 4 A* (Maths, Further Maths, Computer Science, Physics)

PROGRAMMING LANGUAGES

- Python: 7 years
- Haskell: 4 years
- C++: 4 years
- Scala: 3 years
- Git: 2 year
- LaTeX: 2 year
- C: 1 year

EXPERIENCE

Mathematics Intern
June-August 2024
HMG

- Worked in a team to produce reports on two areas of research
- Allocated time between both projects ensuring that both were successful
- Developed programs that formed the basis of research on one of the projects
- Updated problem supporters regularly on my progress, ensuring that the projects were achieving their goals

Oxford Admissions Test Marker
November 2024
Oxford University

- Marked the Oxford Admissions test for Mathematics and Computer Science applicants
- Communicated with the team of markers to ensure consistency across 3300 papers

TALKS

STUK14
October 2024
Cambridge University Gave a talk on orbit-finite sets and their relation to Set Theory and Computer Science

PROJECTS

Automated Quadratic Homogeneous Diophantine Equation solver Python

- A program to efficiently solve equations of the form $ax^2 + by^2 = K$ where $a, b, K \in Z$ are given, for integer solutions for x and y

IFS visualiser Scala

- Allows for visualisation of an IFS attractor and animation of how the attractor changes upon changes to the system

Tamagotchi Clone Python

- Led a team in designing and implementing a Tamagotchi Clone
- Created and distributed black-box specifications for every component of the project

IoT Visualiser Python

- Group project for Cisco
- Worked in a team to design and implement a visualiser for large numbers of IoT devices to assist IT staff
- Produced a data synthesiser that generates realistic data for trialing the visualiser

ACHIEVEMENTS

Casberd Scholarship
October 2022
Achieved a distinction in Preliminary Examinations
Dr Matthew Nicholls

British Informatics Olympiad Finalist
2021
One of a handful of people to be invited to the second round of the BIO

Starting my undergraduate, I had some notions that Formal Logic, Set Theories, and other foundational topics existed, and piqued my curiosity. Over the last few years, this curiosity motivated me to take courses on formal Calculi, logic, and Set Theory, which feels like pulling back the curtain of Mathematics and seeing the underlying structures and similarities across all aspects of Mathematics and Computer Science. The way that these topics seem to lie behind many significant results, such as Topology being behind Genericity in Lambda Calculus and analysis being highly dependent on which version of Set Theory we work in, is very alluring; this cemented my interest in the foundations of Mathematics and Computer Science. Reading survey papers on techniques beyond my courses, like forcing for Set-Theory and recursion Theory; reverse Set-Theory; alternatives to Set-Theory e.g. Type Theory; Modal Logic; and the generalised reals, encouraged me to attend the 'Set Theory in the UK' event recently, where I saw presentation on research into 'Cardinal Characteristics', 'Codes for Ramsey Positive Sets', and 'End extensions of models of KP'. Meeting Andrew Brooke-Taylor and other people in the set theory community was both enjoyable and encouraging. I'm excited to continue in this direction by undertaking a PhD in Mathematics.

I learned of this course during STUK14, where people mentioned Bristol's strong history with set theory. To this extent I would love to work with Philip Welch, as he is one of the only set theorists in the UK working on inner models.

I've heard plenty of good things about Bristol; the city is said to be a great place to live and the university is said to be a great place to work. From these descriptions, Bristol would be perfect location to be during my PhD.

During the last few years, I have tutored multiple students for their Mathematics and Computer Science A-Levels, as well as for university admissions tests. I really enjoyed this teaching and found it rewarding to watch my students' abilities grow and see them succeed. Additionally, it was fun to come up with problems to challenge them, and guide them through how to come up with solutions.

Recently, I was asked to mark the MAT, the Oxford admissions test for Mathematics and Computer Science. For this, I worked in a team, and we communicated to ensure marking was consistent across the thousands of papers.

Last summer, I took part in an internship at a UK government department, where I was responsible for research on two projects. I worked in teams for these projects, and I experienced explaining my work to the other team members and how to collaborate on a complex project. I performed some independent research and learnt how to manage my time to ensure that the important areas would be completed in the short time that we were given. Producing talks and presenting them to a variety of audiences (from academic to technical), was a part of the internship, as well as writing a report and adapting it to comments.

At the moment, I'm extremely enjoying working on my master's project on computation over almost-finite sets with Bartek Klin, involving understanding the implementation of a specialised language, using ideas from Category theory, and presenting programs in an elegant manner so that transformations over them produce more general programs.

The aim of inner-model theory is to generalise Gödel's constructible universe and analyse the consequences of these models. Having spoken to Philip Welch, I would like to work on this by seeing how properties of these models interact with different notions of computation.

I will use the first year to develop my understanding of the problems so that I can start working on them by the end of the first year. My proposed timeline is as follows:

Term 1: I will learn the prerequisite theory. I will use the standard literature e.g. Jech's Set Theory to gain an understanding of the subject and begin to read literature and recent papers on inner-model theory and trans-finite machines.

Term 2: I will continue to read the relevant literature and gain an understanding of the open questions. At this point, I expect to have an idea of relevant theorems or constructions.

Term 3: I will continue to read the literature and discuss my ideas with the relevant researchers. At this point, I may start to work on my ideas.

During the second and third years, I will be able to work on my proposed problems and simultaneously type-set my work. This should allow me to avoid having to dedicate large periods of time at the end to writing up, hence will be a more efficient use of the time.

ACADEMIC TRANSCRIPT

Personal Information

Student:	Jake Aaron Masters
University Reference:	1424769
Qualification Sought:	Master of Mathematics and Computer Science
Start Date:	10 October 2021
HESA Reference:	2111565067613
FHEQ Level:	Masters
Expected end date:	30 June 2025

Programme Information

Teaching institution:	University of Oxford	Awarding Institution:	University of Oxford
College:	St John's College	Mode of Attendance:	Full-time
Programme of Study:	Master of Mathematics and Computer Science	Language of Instruction:	English

Award Information

The student has yet to complete the programme of study shown above

Assessment Information (Academic Year, Assessment Name, Result Mark/Grade)

Qualifying examinations

2021/22	Continuous Mathematics and Probability	96
2021/22	Functional Programming, and Design and Analysis of Algorithms	78
2021/22	Imperative Programming	91
2021/22	Mathematics I	88
2021/22	Mathematics II	93
2021/22	Practical Work (Year 1)	Distinction

Final Degree examinations

2022/23	A0 Linear Algebra	85
2022/23	A2 Metric Spaces and Complex Analysis	83
2022/23	A4 Integration	91
2022/23	A5 Topology	85
2022/23	Algorithms and Data Structures	79
2022/23	Group Design Practical	Distinction
2022/23	Lambda Calculus and Types	88
2022/23	Models of Computation	93
2022/23	Practical Work (Year 2)	Distinction
2022/23	Principles of Programming Languages	89
2023/24	Functional Analysis I	91
2023/24	Functional Analysis II	90
2023/24	Information Theory	95
2023/24	Logic	89
2023/24	Logic and Proof	75
2023/24	Machine Learning	65
2023/24	Practical Work (Year 3)	Distinction
2023/24	Quantum Information	91

Registrar

Transcript issued on 05 July 2024

Personal Information

Student:	Jake Aaron Masters
University Reference:	1424769
Qualification Sought:	Master of Mathematics and Computer Science
Start Date:	10 October 2021
HESA Reference:	2111565067613
FHEQ Level:	Masters
Expected end date:	30 June 2025

2023/24	Set Theory	81

End of Transcript

ONLY VALID AS AN ELECTRONIC DOCUMENT ONLY VALID AS AN

Registrar

Transcript Issued on 05 July 2024

University of Oxford
scan to verify
scan to verify
SmartVerify+

About the University of Oxford
The University of Oxford is an independent self-governing university. It is the oldest university in the English-speaking world and has been in continuous existence for some nine centuries. It is an international leader in learning, teaching and research. As a collegiate institution, it comprises the central university and 39 colleges and 6 permanent private halls

University of Oxford Transcripts
The transcript should not be released to another person, organisation or institution except to officials internal to your own organisation or institution who have a reasonable business use for the information. Release to other parties requires the written consent of the student. The following information is provided to aid in the evaluation of this student's academic record. Further explanation or detailed information can be obtained by contacting Degree Conferrals via the email address edocuments.support@admin.ox.ac.uk.
Under University regulations, Boards of Examiners may, where appropriate, take account of information additional to the profile of marks listed overleaf in deciding the final degree classification awarded to any student.
The explanatory text on the transcript is subject to change until such time that the programme of study is completed.

Academic Credit
The University does not routinely apply credit weightings to its programmes and its courses are not generally taught on a modular basis. We take each year of full-time undergraduate study to equal 120 UK credits and 180 UK credits for Masters-level postgraduate study according to the Higher Education Credit Framework for England. In relation to the European Credit Transfer Scheme (ECTS), this is equivalent to 60 credits for undergraduate study and 90 credits for Masters-level postgraduate study.

Framework for Higher Education Qualifications (FHEQ levels)

8 (Doctoral)	Doctoral Degrees (e.g. DPhil, DClinPsych)
7 (Masters)	Master's Degrees (including Integrated Master's Degrees)
	Postgraduate Diplomas & Certificates
6 (Honours)	Bachelor's Degrees with Honours
	Bachelor's Degrees
	Professional Graduate Certificate in Education
5 (Intermediate)	Undergraduate Diplomas
4 (Cert)	Undergraduate Certificates
	Certificate of Higher Education

Mark Scales
All marks included on a final academic transcript have been ratified by the Registrar. Examiners are required to express final agreed marks on all formally assessed work according to the following marking scales:

Foundation Year Programmes (Cert HE)

70-100	Distinction
60-69	Merit
40-59	Pass
0-39	Fail

Undergraduate Programmes

	Model 1	**Model 2**
70-100	First Class	Distinction
60-69	Upper Second Class	Pass
50-59	Lower Second Class	Pass
40-49	Third Class	Pass
30-39	Pass	Fail
0-29	Fail	Fail

Model 1 will be used for all final assessments. Model 2 will be used for all qualifying assessments unless the explanatory text overleaf states otherwise.

Postgraduate Taught Programmes
For students who started their courses **before** October 2018.

Model 1	**Model 2**	
70-100	70-100	Distinction
50-69	60-69	Pass
0-49	0-59	Fail

For students who started their courses **from** October 2018.

Model 1	**Model 2**	
70-100	70-100	Distinction
N/A	65-69	Merit
50-69	50-64	Pass
0-49	0-49	Fail

Model 2 will be used for all Award Programmes unless the explanatory text overleaf states otherwise.

Transcript Terminology
Results Not Moderated (On-Course Transcripts Only): Indicates a mark that may be subject to moderation in the process of concluding the final outcome of an examination comprising more than one part and taken over more than one year.
Declared to have deserved: the exam board considered the candidate was absent from part of the examination for good cause and declared them to deserve the Award.

Programme Information
The relevant *Examination Regulations* for the programme are available at: https://examregs.admin.ox.ac.uk/

Authentication
This academic transcript can be authenticated by scanning the QR code which is visible in the main section of the document. Further information on authentication may be obtained by contacting Degree Conferrals on the email address edocuments.support@admin.ox.ac.uk

The following is an extract from my ongoing master's project. The purpose of this document is to provide evidence of independent research.

In this document, I show that under certain conditions on the pure sets and the group and cardinal κ in the pure sets, a finite-support permutation model of some model of ZFA is also a finite-support permutation model of some model of $ZFA + |\mathbb{A}| = \kappa$ (Theorem 5.4, Lemma 5.9, and Corollary 5.8).
This has the consequence that all 'Basic Fraenkel Models' (the finite-support permutation model induced by $Sym(\mathbb{A})$) with the same pure sets, are isomorphic given that the pure universe satisfies choice (Example 5.10).

The new results included here are:

- Theorem 5.4
- Corollary 5.8
- Lemma 5.9
- Example 5.10

5 Cardinal Transfer

5.1 Motivation

When talking about computation in permutation models of ZFA, we often assume that the atoms are countable.

This motivates theorems that say that permutation models arising from models of ZFA with uncountable atoms, may also arise from models of $ZFA + \mathbb{A} = \aleph_0$.

We will make use of the following theorems:

Theorem 5.1 (Eric J. Hall [5]). *Let $\mathcal{N} \subseteq \mathcal{M}$ be transitive models of ZFA with the same kernel and same set of atoms, and with $\mathcal{M} \models AC$ then ...*
$\mathcal{N}$ is a permutation model of $\mathcal{M}$ $\iff$ $\mathcal{M}$ is a generic extension of $\mathcal{N}$ by some almost homogeneous notion of forcing.

Lemma 5.2 (Andreas Blass [9]). *Let $\mathcal{N}$ be a model of ZFA with generic extension $\mathcal{N}[\Gamma]$. Then $S \in \mathcal{N}[\Gamma]$ is a pure set iff there is a pure name $\dot{S}$ such that $S = Val_\Gamma\left(\dot{S}\right)$.*

Theorem 5.3 (Brunner [3]). *Let $\mathcal{M}$ be a model of $ZFA + AC^{pure}$ with permutation model $\mathcal{N}$. Then $\mathcal{N}$ is a finite-support permutation model of $\mathcal{M}$ iff $\mathcal{N} \models \mathbb{A}^{<\omega}$ is χ (a witness to SVC).*

5.2 Transfer Downwards

Theorem 5.4. *Let $\langle \mathcal{M}, \mathbb{A}, \in \rangle$ be a model of $ZFA + AC^{pure}$*
Let $G \leqslant Sym(\mathbb{A})$
Let $\mathcal{N} = PM(\mathcal{M}, G, \mathcal{F}_{fin})$
Let κ be a cardinal in the pure sets of $\mathcal{M}$
If there exists $t_\alpha \subseteq \mathbb{A}^\alpha$ for $\alpha \in \kappa$ such that ...

1. *For all* $\alpha \in \kappa$, t_α *is a* G*-orbit and each* $f \in t_\alpha$ *is injective.*

2. *For all* $\alpha, \beta \in \kappa$, *if* $\alpha \leq \beta$ *then* t_α *are the restrictions of* t_β *to the domain* α.

3. *Let* $\mathbb{P} := \bigcup_{\alpha\in\kappa, f\in t_\alpha} \mathcal{P}_{fin}(f)$. *For* $f, g \in \mathbb{P}$, *there is* $\sigma \in Aut^{\mathcal{N}}(\mathbb{P}, \supseteq)$ *and* $h \in \mathbb{P}$ *such that* $\sigma(f), g \subseteq h$. *(*$(\mathbb{P}, \supseteq)$ *is almost-homogeneous in* $\mathcal{N}$*)*

4. *For all* $f \in \mathbb{P}$ *and* $a \in \mathbb{A}$, *there is* $g \in \mathbb{P}$ *such that* $f \subseteq g$ *and* $a \in ran(g)$.

Then there is a model of ZFAC+"$|\mathbb{A}| = \kappa$*" with* $\mathcal{N}$ *being a finite-support permutation model.*

Proof.

Claim 5.5. $\mathbb{P} \in \mathcal{N}$

Proof. The elements of $\mathbb{P}$ are all hereditarily finite sets, so $\mathbb{P} \subseteq \mathcal{N}$. $\mathbb{P}$ is invariant under G by construction, so $\mathbb{P} \in \mathcal{N}$. □

Claim 5.6. Forcing with $(\mathbb{P}, \supseteq)$ adds no pure sets.

Proof. Let $\dot{S}$ be a pure name and $p \in \mathbb{P}$.
Sps $p \Vdash \dot{S} \subset \check{X}$ for some $X \in \mathcal{N}^{pure}$
Without loss of generality, p supports $\dot{S}$ as we can strengthen p by 4.
Let $x \in X$
Sps there are $\mathbb{P} \ni q_0, q_1 \supseteq p$ such that

- $q_o \Vdash \check{x} \in \dot{S}$

- $q_1 \Vdash \check{x} \notin \dot{S}$

Let $\alpha = max(dom(q_0), dom(q_1))$.
By 2, there are $q_0', q_1' \in t_\alpha$ such that $q_0 \subseteq q_0'$ and $q_1 \subseteq q_1'$.
By 1, there is $\sigma \in G$ such that $\sigma(q_0') = q_1'$ so $\sigma(q_0), q_1 \subseteq q_1'$.
σ fixes p (as p is common to q_0 and q_1), so fixes S and the pure sets x, X so...

- $\sigma(q_0) \Vdash \check{x} \in \dot{S}$
- $q_1 \Vdash \check{x} \notin \dot{S}$

So $\sigma(q_0)$ and q_1 are incomparable.

This is a contradiction so every extension of p agrees on the elements of S, so $S = \left\{x \in X | p \Vdash \check{x} \in \check{X}\right\} \in \mathcal{N}$.

So by Lemma 5.2, the generic extension of $\mathcal{N}$ given by $(\mathbb{P}, \supseteq)$ has the same pure sets as $\mathcal{N}$. $\square$

Claim 5.7. Forcing with $(\mathbb{P}, \supseteq)$ adds a bijection $f : \kappa \to \mathbb{A}$.

Proof. Let $\dot{f} := \{\langle \check{q}, p\rangle | p \in \mathbb{P} \text{ and } p = \{q\}\}$.

$\varnothing \Vdash \dot{f}$ is a bijection $\check{\kappa} \to \check{\mathbb{A}}$ as...

Let $\Gamma \subseteq \mathbb{P}$ be generic and $f := Val_\Gamma\left(\dot{f}\right)$.

$f \subseteq \kappa \times \mathbb{A}$ by definition of $\dot{f}$.

For all $\alpha \in \kappa$, let $D = \{p \in \mathbb{P} | \alpha \in dom(p)\}$.

D is dense as for $h \in \mathbb{P}$, there exists $\beta \in \kappa$ and $h' \in t_\beta$ such that $h \subseteq h'$.

By 2, there is $h'' \in t_{\alpha^+ \cup \beta}$ with $h' \subseteq h''$.

Now, $h \subseteq h'' \restriction_{dom(h) \cup \{\alpha\}} \in D$

So there is $p \in D \cap \Gamma$ so $\langle \alpha, p(\alpha)\rangle \in f$.

So $dom(f) = \kappa$.

For all $a \in \mathbb{A}$, let D $= \{p \in \mathbb{P} | a \in ran(p)\}$.

D is dense as for $h \in \mathbb{P}$, by 4, there is $h' \in \mathbb{P}$ such that $h \subseteq h'$ and $h' \in D$.

So there is $p \in D \cap \Gamma$ so $\langle p^{-1}(a), a\rangle \in f$.

So $dom(f) = \kappa$ and $ran(f) = \mathbb{A}$.

Suppose that $p \in \mathbb{P}$ extends both $\{\langle x, a\rangle\} \in \Gamma$ and $\{\langle y, b\rangle\} \in \Gamma$,

p is an injection so ($x = y$ and $a = b$) or ($x \neq y$ and $a \neq b$).

All elements of Γ are comparable, so f is an injection.

So $f : \kappa \to \mathbb{A}$ is a bijection. $\square$

Now for all $\Gamma \subseteq \mathbb{P}$ generic, $\mathcal{N}[\Gamma]$ contains the same pure sets as $\mathcal{N}$, is an almost-homogeneous extension of $\mathcal{N}$, and contains a bijection $\kappa \to \mathbb{A}$ (so also satisfies AC).

So by Theorem 5.1, $\mathcal{N}$ is a permutation model of $\mathcal{N}[\Gamma]$.

$\mathcal{N}$ is a finite-support model of $\mathcal{M}$, so $\mathcal{N} \models \mathbb{A}^{<\omega}$ is χ by Theorem 5.3.

So $\mathcal{N}$ is a finite-support model of $\mathcal{N}[\Gamma]$ by Theorem 5.3. $\square$

There is now the question as to how we find automorphisms as required in condition 3.

The obvious constructions are to find automorphisms on $\mathbb{P}$ by...

1. Finding permutations on $\mathbb{A}$ as in Lemma 5.9.

2. Finding permutations on κ as in Corollary 5.8 and inspired by the proof of Theorem 5.1.

Corollary 5.8. *Let $\langle \mathcal{M}, \mathbb{A}, \in \rangle$ be a model of $ZFA + AC^{pure}$*

Let $G \leqslant Sym(\mathbb{A})$

Let $\mathcal{N} = PM(\mathcal{M}, G, \mathcal{F}_{fin})$

Let κ be a cardinal in the pure sets of $\mathcal{M}$

If there exists a subset $\mathbb{B} \subseteq \mathbb{A}$ such that the following holds:

1. $|\mathbb{B}| = \kappa$

2. *For $m \in \mathbb{N}$, $\mathbf{a}$, $\mathbf{b} \in \mathbb{B}^m$ and is $\sigma' \in G$ such that $\sigma'(\mathbf{a}) = \mathbf{b}$ then there is $\sigma \in G_{\mathbb{B}}$ such that $\sigma(\mathbf{a}) = \mathbf{b}$*

3. *For all $\mathcal{S} \subseteq_{fin} \mathbb{B}$ and $\mathcal{S}' \subseteq_{fin} \mathbb{A}$, there is $\sigma \in G$ fixing the elements of $\mathcal{S}$ and with $\sigma(\mathcal{S}') \subseteq \mathbb{B}$*

Then $\mathcal{N}$ is a finite-support permutation model in some model of ZFA with the same pure sets as $\mathcal{M}$ but where $|\mathbb{A}| = \kappa$

Proof. Let $j : \kappa \to \mathbb{B}$ be a bijection.

For $\beta \in \kappa$, let $t_\beta := G \cdot (j \restriction_\beta)$.

So conditions 1 and 2 in Theorem 5.4 hold.

Let $a \in \mathbb{A}$, $f \in \mathbb{P}$, and let $S := dom\,(f)$.

Write $f = \pi\,(j \restriction_S)$ for some $\pi \in G$.

By condition 3, there is $\sigma \in G$ fixing the elements of $j\,(S)$ and sending $\pi^{-1}\,(a)$ into $\mathbb{B}$.

Let $f' := \pi\left(\sigma^{-1}\left(j \restriction_{S \cup \{\sigma(j^{-1}(\pi^{-1}(a)))\}}\right)\right) \in \mathbb{P}$.

$f \subseteq f' \in \mathbb{P}$ and $a \in ran\,(f')$.

So condition 4 of Theorem 5.4 holds.

For $\pi \in G_{\mathbb{B}}$, let $\pi_\kappa = j^{-1} \circ \pi \circ j$.

Let $G_\kappa = \{\pi_\kappa | \pi \in G\}$.

$G_\kappa \in \mathcal{N}$ as they are all pure sets.

G_κ acts on $\mathbb{P}$ by $\pi\,(p) = p \circ \pi^{-1}$ as ...

For $\pi \circ j \restriction_S \in \mathbb{P}$, for some $\sigma \in G_{\mathbb{B}}, S \subseteq_{fin} \mathbb{B}$...

$\sigma \circ j \restriction_S \circ \pi_\kappa^{-1} = \sigma \circ j \restriction_S \circ j^{-1} \circ \pi^{-1} \circ j = (\sigma \circ \pi^{-1}) \circ j \restriction_{(j^{-1} \circ \pi \circ j)(S)} \in \mathbb{P}$

The elements of G_κ are automorphisms of $(\mathbb{P}, \supseteq)$ as if $f \subseteq g$ and π acts on $dom\,(g)$, then $f \circ \pi \subseteq g \circ \pi$.

Now take $f, g \in \mathbb{P}$,

Let $\pi \in G$ such that $\pi\,(ran\,(f) \cup ran\,(g)) \subseteq \mathbb{B}$, by 3.

Write $f = \sigma \circ j \restriction_S$ for $\sigma \in G$ and $S \subseteq_{fin} \mathbb{B}$.

$\pi \circ \sigma$ keeps $j\,(S)$ inside $\mathbb{B}$ so, by 2, there is $\pi' \in G_{\mathbb{B}}$ such that $\pi' \restriction_{j(S)} = \pi \circ \sigma \restriction_{j(S)}$.

$\pi'_\kappa \in G_\kappa$ and ...

$\pi'_\kappa\,(f) = \sigma \circ j \circ j^{-1} \circ \pi'^{-1} \circ j \restriction_{(j^{-1} \circ \pi' \circ j)(S)} = \pi \circ j \restriction_{S'}$ for some $S' \subseteq_{fin} \kappa$.

Similarly, find $\pi''_\kappa \in G_\kappa$ with $\pi''_\kappa\,(g) = \pi \circ j \restriction_{S''}$ for some $S'' \subseteq_{fin} \kappa$.

So $\left(\pi''^{-1}\pi'\right)_\kappa (f), g \subseteq \pi \circ j \restriction_{S' \cap S''} \in \mathbb{P}$

So condition 3 of Theorem 5.4 holds.

So the conclusion of Theorem 5.4 holds.

□

Lemma 5.9. *Suppose we are in the case of Theorem 5.4 with conditions 1, 2, and 4, then condition 3 follows from* $\overline{G \cap \mathcal{N}} \supseteq G$*, where the closure is in the topology of point-wise convergence in* $\mathcal{M}$*.*

Proof. Let $p_0, p_1 \in \mathbb{P}$.
So for some $\alpha, \beta \in \kappa$, $S_0 \subseteq_{fin} \alpha, S_1 \subseteq_{fin} \beta$ and $f_0 \in t_\alpha, f_1 \in t_\beta$, $p_0 = f_0 \restriction_{S_0}$ and $p_1 = f_1 \restriction_{S_1}$.
By condition 2, we may assume that $\alpha = \beta$.
By condition 1, there is $\sigma \in G$ such that $\sigma(f_0) = f_1$.
As $\overline{G \cap \mathcal{N}} \supseteq G$, there is $\sigma' \in G \cap \mathcal{N}$ such that $\sigma' \restriction_{S_0 \cup S_1} = \sigma \restriction_{S_0 \cup S_1}$.
σ' is an automorphism of $(\mathbb{P}, \supseteq)$.
$\sigma'(p_0), p_1 \subseteq f_1 \restriction_{S_0 \cup S_1} \in \mathbb{P}$.
So condition 3 holds. □

Example 5.10 (Basic Fraenkel Model)**.** For a fixed pure universe satisfying choice, all the Basic Fraenkel Models of models of models of ZFA with the specfified pure universe are isomorphic.

Proof. Let $\mathcal{V}$ be a model of ZFC.
Construct a class, $\mathcal{N}^*$ with ω representing $\mathbb{A}$, modeling $ZFAC + |A| = \aleph_0$ inside $\mathcal{V}$.
By Mostowski's collapse lemma for ZFA, if $\mathcal{N}$ is a model of ZFA with pure sets $\mathcal{V}$ and with a bijection $f : \omega \to \mathbb{A}$, then $\mathcal{N} \cong \mathcal{N}^*$.
So all models of $ZFA + |A| = \aleph_0$ with the same pure universe $\mathcal{V}$ are isomorphic.
So it suffices to show that all 'Basic Fraenkel Models' are the 'Basic Fraenkel Model' of some model of $ZFA + |A| = \aleph_0$.

Let $\mathcal{M}$ be a model of ZFA with pure sets $\mathcal{V}$.

The basic Fraenkel Model is the finite-support permutation model $PM\left(\mathcal{M}, Sym\left(\mathbb{A}\right), \mathcal{F}_{fin}\right)$.

Should the conditions of Lemma 5.9 hold for $\kappa = \aleph_0$, then we are done as every permutation model of $\mathcal{M}'$ contains the basic Fraenkel Model of $\mathcal{M}'$ as a permutation model and the only permutation model inside a basic Fraenkel Model is itself.

For $n \in \aleph_0$, let $t_n := \{\mathbf{a} \in \mathbb{A}^n | \mathbf{a} \text{ is injective}\}$.

t_n is clearly closed under the action of G, and for $p_0, p_1 \in t_n, \ldots$

$S := ran\left(p_0\right) \cup ran\left(p_1\right)$ is finite so there is a finite bijection $\sigma : S \to S, \sigma \in \mathcal{N}$ such that $\sigma\left(p_0\right) = p_1$.

So $\sigma' = \sigma \cup id_{\mathbb{A} \setminus S} \in G$.

So condition 1 is satisfied.

Condition 2 is obvious from the definition.

Let $f \in \mathbb{P}$, write $f = f' \restriction_S$ for some $n \in \aleph_0, f' \in t_n$, and $S \subseteq n$.

Let $a \in \mathbb{A}$, if $a \in ran\left(f'\right)$ then let $f'' = f' \in \mathbb{P}$.

Otherwise, let $f'' = f' \cup \{\langle n, a\rangle\} \in \mathbb{P}$.

Either way, $f \subseteq f''$ and condition 4 is satisfied.

Now for $\sigma \in G$ and $S \subseteq_{fin} \mathbb{A}$, there is finite bijection $\pi : S \cup \sigma\left(S\right) \to S \cup \sigma\left(S\right) \in \mathcal{N}$ agreeing with σ on S.

Let $\sigma' := \pi \cup id_{\mathbb{A} \setminus \left(S \cup \sigma\left(S\right)\right)} \in \mathcal{N} \cap G$, so the alternative to condition 3 is satisfied.

$\square$

Question 5.11. Is there some 'simple' statement on the theory of the structure inducing G implying these conditions?

Question 5.12. What is the group that induces $\mathcal{N}$ in the extension?

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Oxford, December 1, 2024

REFERENCE LETTER

This is to recommend **Mr. Jake Masters** for a PhD position in Mathematics at the University of Bristol.

I am an Associate Professor in Computer Science at the University of Oxford. I am the supervisor of Jake's MSc project, which he started working on over the summer of 2024 and is expected to complete in June 2025. In this capacity, I meet Jake weekly to discuss his progress and offer guidance.

The original goal of Jake's project was to determine whether a few widely known string-searching algorithms (such as Knuth-Morris-Pratt, Aho-Corasick, Boyer-Moore, etc.) generalise to the setting of orbit-finite sets with atoms, also known as ZFA, or as nominal sets. Essentially, such a generalisation amounts to replacing various finite structures with "orbit-finite" ones, i.e., infinite but finite up to bijective renaming of certain atomic data values. In this context, the generalisation amounts to string searching over an infinite alphabet, where the pattern to be found is defined by a first-order formula rather than as a list of specific letters. Jake's task was to study whether certain well-known algorithms terminate, determine their complexity, and perhaps implement them in a domain-specific language that has been developed for programming in orbit-finite sets.

This is a typical scope and scale of an MSc project in Oxford, normally enough to get a very high grade. However, it soon became clear that the project is far too modest for Jake's ability and drive. By the end of October he had essentially done everything that I originally planned for him to do. Then he immediately embarked on a far more ambitious project, aimed at understanding set-theoretic properties of the universe of sets with atoms, with a particular focus on uncountable atom universes and their countable sub-universes. For this, he familiarised himself with literature on set theory: permutation models, forcing, etc. His initial results are already quite interesting, and they are likely to reach publication-level quality and scale before Jake's submission of his MSc dissertation.

I think very highly of Jake. Both his mathematical ability and his computational intuitions are very strong, his knowledge of logic and set theory is remarkable, and he attacks research-grade problems with confidence and enthusiasm. I have no doubt that he would thrive in any graduate program. I recommend him without hesitation.

Best regards,

Bartek Klin

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UNIVERSITY OF OXFORD

The postgraduate admissions committee

30th November 2024

Dear members of the selection committee,

Re: Mr Jake Masters

It is a pleasure to write in support of Jake Masters' application for a place in your Phd programme. I've known Jake since October 2021, as one of the Mathematics Tutors at St John's College. Jake is in the fourth and final year of his integrated Bachelors and Masters programme in Mathematics and Computer Science at St John's College at the University of Oxford.

Jake has performed exceptionally in exams. The exam system in Oxford is designed so that exams are scaled after they are taken so that 70 represents a 'first class performance' (roughly speaking an A in a North American system) and marks above 80 are rare. Although the score is out of 100, they should not be viewed as percentages. Jake scored 88.9 in his first year, and 86.2 and 86.3 in his second and third year respectively, placing him 3rd in an exceptionally strong cohort of Mathematics and Computer Science undergraduate students across Oxford University. His performance also ranked ahead of all but one student on the Computer Science degree. This exceptional set of results demonstrates Jake's depth of mastery of all his courses, together with the ability to apply his knowledge creatively to solve problems under time pressure. Highlights in his most recent set of exams include 95 in quantum information (second out of 47 students across the Mathematical Sciences taking the paper), 91 in functional analysis 1 (1st out of 10 maths and computer science students taking the paper), and 90 in functional analysis (3rd out of 45 students across the Mathematical Sciences taking the paper).

In the first two years teaching in Oxford is through small college tutorials, in which students meet with their tutor for an hour in pairs every fortnight to discuss their work. Right from the outset Jake consistently produced excellent work for tutorials, invariably giving complete solutions to all problems, including challenging bonus questions. I've taught Jake in around 16 hours of tutorials, covering linear algebra, analysis and measure theory. At the beginning of his degree, Jake's answers, particularly in analysis, were perhaps a bit over fussy as he worked out exactly which were the salient details to include, but he took on feedback well and his work subsequently has been excellently written, combining a careful attention to detail with good judgement about what prior facts should be assumed (but clearly stated).

Jake is a lively participant in tutorials who is happy to explain his ideas and engage in mathematical discussion with others. He picks up on new concepts and ideas incredibly quickly, asking salient questions and is a pleasure to teach. Jake's interest in logic and mathematical foundations dates has been ever present. In our very first analysis tutorial I remember us discussing issues around the axiom of choice, and it's very nice to see how his interests in foundations have continued to deepen as his studies have continued.

Jake's care and attention to detail, led me to recently nominate him to join the team of DPhil students marking the Oxford Mathematics Admissions test for applicants to our undergraduate degrees in Mathematics and Computer Science. It is very rare for a final year Masters student to undertake this role, but I had no hesitation in nominating Jake, and my understanding is that he performed this work excellently.

Jake has good presentational skills; he gave a well-researched and presented seminar on 'Fractals and Chaos' to our College second year mathematics seminars, and I'm looking forward to the seminar he's volunteered next week on 'Sets with Atoms: Computation and Permutation Models' to our termly

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undergraduate seminar series.

I am confident Jake a will be an excellent graduate student working on any topic in the foundations of Mathematics or Computer Science. I recommend him for extremely tcompetitive programmes worldwide in exceptionally strong terms.

Sincerely,

Stuart White
Professor of Mathematics
Tutorial Fellow at St John's College