

Algebraic Geometry, Lecture 1

Farhad Babaee

University of Bristol

Schedule // Assessments // Office Hours
Schedule // Assessments // Office Hours item options
Schedule // Assessments // Office Hours Lectures and Problem Classes
Mondays 15:00 - 17:00 (Fry G.16) Fridays 10:00 - 11:00 (Fry G.16)
Assessment
Problem Classes Presentations (50) Final exam (50)
Master's students: Final written exam (80)
PhD/research students: Oral exam or presentation.
Office Hours (My office Fry 2.13)
Wednesdays 15:30 - 16:30

Admin stuff

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 - Assessed Homework 1, 22.5%, title: Affine Varieties, dates: Feb 04, noon — Feb 11, noon. , Individual upload
 - Assessed Homework 2, 22.5%, title: General varieties, dates: March 04, noon — March 11 noon.

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 - Problem Classes Presentations (5%) along the course, presenting solutions, individual/group presentation
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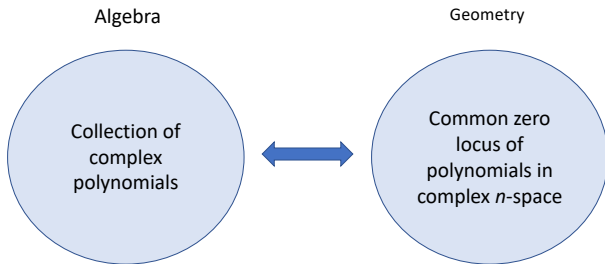
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What is the (Complex) Algebraic Geometry?



The goal of our course

- Describe basic objects in algebraic geometry
- Describe dimension, degree, smoothness, etc. in both algebraic and geometric settings
- In toric varieties read off a lot of info from combinatorial data

Some examples

Example

The zero locus of the polynomial $x^2 + y^2 + 1$ is empty in \mathbb{R}^2 but non-empty in \mathbb{C}^2 .

Example (Fermat's last theorem)

The zero locus of $x^n + y^n + z^n$ is empty in \mathbb{Q}^3 , for integer $n \geq 3$.

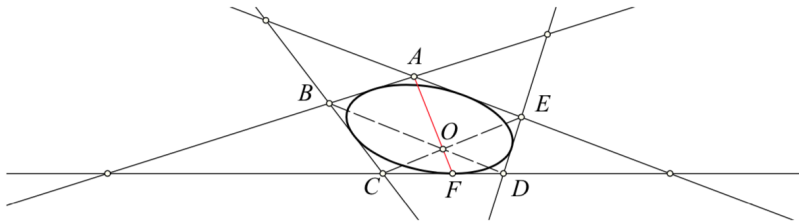
Remark

Algebraic geometry can be done in any field, but for simplicity and intuition, we mostly deal with complex numbers in this course.

Some history



Babylonians (2000-1500BC) seemed to know how to solve $ax^2 + bx = c$.
in \mathbb{R} . They also knew Pythagoras Theorem!



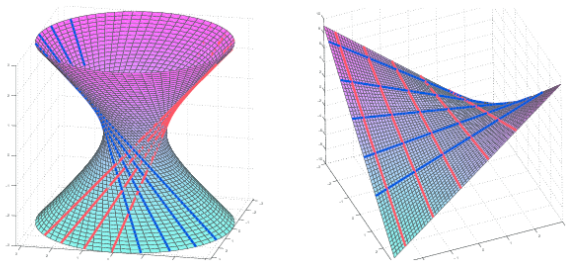
Appolonius (262-190 BC) seemed to know that a non-degenerate plane conic is determined by 5 tangent lines.

Mid-nineteenth century



Bernhard Riemann (1828-1866) showed that compact Riemann surfaces can be described as zero sets of a polynomial function. We later state a generalisation of this statement which is called the Chow Theorem.

Mid-nineteenth century



Any quadratic surface (zero set of a degree 2 polynomial in 3 variables) can be covered by lines.

Any cubic surface contains exactly 27 lines!

Turn of the century

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Assume that C_1 and C_2 are two curves of degree d_1 and d_2 in the complex projective space \mathbb{P}^2 . Then, the number of intersection points, counting the multiplicities, is $d_1 d_2$.

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An idea for the proof.

- Proof for $d_1 = 1$, $d_2 = 2$.
- By moving the curves we can assume the intersections take place in $\mathbb{C}^2 \subseteq \mathbb{P}^2$.
- The theorem is true if the defining functions of C_1 and C_2 are of the form $f(x, y) = a_1 y - b_1 x$, and $g(x, y) = (a'_1 y - b'_1 x)(a'_2 x - b'_2 y)$.
- Intuitively, the number of intersection points, taking into account the multiplicities, do not change if we perturb the curves.



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- **Not complete!**

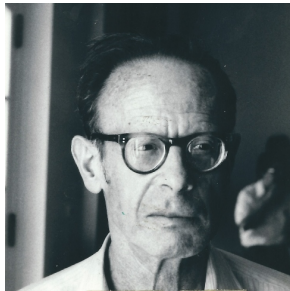


Beginning of 20th century



David Hilbert (1862-1943) and Emmy Noether (1882-1935) set the algebraic foundations for solid algebraic geometry.

20th century



Oscar Zariski 1899-1986, and André Weil 1906-1998 with many others revived the topic and developed it.



Alexander Grothendieck (1928-2014 - Fields Medal 1966) aided by Artin, Mumford (Fields 1974) and many others, introduced Scheme Theory and lifted Algebraic Geometry to a “dizzying heights of abstraction”. This abstraction made algebraic geometry more natural, general, and often simplified.

What do we study in this course?

- Basic foundations and many nice constructions/theorems:
 - Affine, Projective, and Quasi-Projective Algebraic Varieties
 - Degree
 - Gluing
 - Some flavours of Scheme Theory
 - Smoothness and resolution of singularities
 - Some toric geometry

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- Please register your attendance!