Application from Sankei Sankei for Mathematics (PhD) to start in 01 September 2024

Personal details

Personal details

First / given name	Sankei
Second given name	Daniel
Third given name	Njoroge
Surname/family name	Sankei
Date of birth	14 February 1993
Preferred first/given name	Daniel
Previous surname	
Country of birth	Kenya
Legal nationality	Kenyan
Dual nationality	
Country of residence	Kenya
Have you previously studied with us at the University of Bristol?	No

Contact details

Home address

Please provide your permanent residential address. If you have another address and would prefer for us to contact you at that address instead you have the opportunity to add a correspondence address in the next section.

Country	Kenya
Postcode	60200
Address Line 1	972
Address Line 2	
City	Meru
County	Kenya
Telephone	

If you would like us to send any postal correspondence to an address which is not your home address please enter an alternative address here. If you want us to send correspondence to your home address then please select No.

Do you want to add a correspondence address?	No
Country	Kenya
Postcode	60200
Address Line 1	972
Address Line 2	
City	Meru
County	Kenya
Telephone	

Agent

Agent details

Agency Name	
Email address	

Other information

Additional Documents

Please upload required documents as outlined in your admissions statement

Mode of study

How would like to study this programme?	Full Time

Qualifications

Qualifications

Institution	Qualification	Type	Subject	Actual/predicted	Grade	Start date	End date
Meru University of Science and Technology	Master's Degree (PG)	Academic Qualification	Mathematics	Actual	pass	04/Jan/2022	27/Oct/2023
Meru University of Science and Technology	First degree BA/BSC etc	Academic Qualification		Actual	First Class Hon	10/Sep/2012	15/Dec/2017

If these qualifications have altered since your last application please note the changes in the free text box here.

English Language

Is English your first language? Yes

What is your first language?

Did you study at school/university where you were taught in English?

For how many years?

Have you sat a relevant English language test?

TOEFL (internet-based)

Registration number

Date of TOEFL test

TOEFL reading score

TOEFL listening score

TOEFL speaking score

TOEFL writing score

TOEFL total score

IELTS (International English Language Testing System)

Test report form (TRF) number

UKVI number (if applicable)

Date of IELTS test

IELTS listening score

IELTS reading score

IELTS writing score

IELTS speaking score

IELTS total score

Pearson Test of English

Score report code

Date of Pearson test

Pearson listening score

Pearson reading score

Pearson speaking score

Pearson writing score

Pearson overall score

Other English Language test

Name of course	
Registration number	
Date of test	
Listening score	
Writing score	
Reading score	
Total score	

Experience

Current Employer

Employer name and address	Meru University of Science and Technology, P.O Box 972-60200
Job title and main duties	Designation: Administrative Assistant Responsibility: Registration of candidates for assessments. Preparation of assessment schedules, formative assessment timetables, and assessment tools. Maintaining the portfolio of evidence
Full time/Part time	Full time
Date of Appointment	03 January 2022
End date (if applicable)	

Previous employment 1

Employer name and address	
Job title and main duties	
Full time/Part time	
Date of Appointment	
End date (if applicable)	

Previous employment 2

Employer name and address	
Job title and main duties	
Full time/Part time	
Date of Appointment	
End date (if applicable)	

Previous employment 3

Employer name and address	
Job title and main duties	
Full time/Part time	
Date of Appointment	
End date (if applicable)	

Other Experience

Do you have any other relevant work experience to support your application?	Yes
Please provide details	Part time teaching, conducting competency Based Education Training and Development of assessment tools

Personal statement

Personal details

Do you have a personal statement to upload?	Yes
Please type your personal statement in the box	

Research proposal

Research proposal

Proposed supervisor 1	Misha Rudnev
Proposed supervisor 1	Celine Maistret
Proposed project title (max 150 chars)	A NEW FORMULATION OF A SET OF ODD NUMBERS AND GENERATION OF TRIPLES OF ODD NUMBERS FOR APPLICATION IN PROVING WEAK GOLDBACH’S CONJECTURE

Passport and visa

Visa required

Do you require a visa to study in the UK?	Yes

Please fill out your passport details below. If you are unable to provide these at the current time you will have another opportunity to upload your passport after you submit the form. If you do not provide us with this information we will be unable to issue you with your confirmation of acceptance number and you will be unable to obtain a visa.

Passport details

Passport number	AK0635912

Further details

Have you previously studied in the UK?	No
What was the highest level of study in the UK?	
Please confirm the total length of your UK study in years	

Referees

Referee 1

Do you have a reference to upload?	Yes
Type of reference	
Referee title	
Forename	
Surname	
Position	
Institution/Company	
Email address	
Country	

Referee 2

Do you have a second reference to upload?	Yes
Type of reference	
Referee title	
Forename	
Surname	
Position	
Institution/Company	
Email address	
Country	

Funding

Funding 1

What is your likely source of funding?	University of Bristol scholarship
Please give the name of your scholarship or Studentship	
Please specify	
Percentage from this source	100
Is this funding already secured?	No

Funding 2

What is your likely source of funding?	
Please give the name of your scholarship or Studentship	
Please specify	
Percentage from this source	
Is this funding already secured?	

Funding 3

What is your likely source of funding?	
Please give the name of your scholarship or Studentship	
Please specify	
Percentage from this source	
Is this funding already secured?	

Other funding

I would like to be considered for other funding opportunities	Yes

Submission

Documents

Document type	File name
Personal statement	Personal Statement .pdf
References	Recommendation 1.pdf
References	Recommendation 2.pdf
References	Recommendation 2.pdf
Transcript	CERTIFICATE_AND_TRANSCRIPTS (1).pdf
Degree certificate	Msc Certificate.pdf
Transcript	Msc Result Slips.pdf
Research proposal	Research statement.docx
Passports and visas	Passport.pdf
Curriculum vitae	CV(1).pdf
Admissions documents (Miscellaneous)	Masters Thesis_Strong Goldbach Conjecture.pdf

By ticking the checkbox below and submitting your completed online application form, you acknowledge the University of Bristol will use the information provided from time to time, along with any further information about you the University may hold, for the purposes set out in the University's full Data Protection Statement. Applicants applying to the collaborative programmes of doctoral training should also read the Data Protection Statement for collaborative programmes of doctoral training.

The information that you provided on your application form will be used for the following purposes:

- To enable your application for entry to be considered and allow our Admissions Advisors, where applicable, to assist you through the application process;
- To enable the University to compile statistics, or to assist other organisations to do so. No statistical information will be published that would identify you personally;
- To enable the University to initiate your student record should you be offered a place at the University.

All applicants should note that the University reserves the right to make without notice changes in regulations, courses, fees etc at any time before or after a candidate's admission. Admission to the University is subject to the requirement that the candidate will comply with the University's registration procedure and will duly observe the Charter, Statutes, Ordinances and Regulations from time to time in force.

By ticking the checkbox below and submitting your completed online application form, you are confirming that the information given in this form is true, complete and accurate and that no information requested or other material information has been omitted. You are also confirming that you have read the Data Protection Statement and you confirm the statement below.

I can confirm that the information I have provided is true, complete and accurate. I accept that the information given in my application will be stored and processed by the University of Bristol, in accordance with the *UK General Data Protection Regulation and Data Protection Act 2018*, in order to:

- Consider my application and operate an effective and impartial admissions process;
- Monitor the University’s applicant and student profile;
- Comply with all laws and regulations;
- Ensure the wellbeing and security of all students and staff;
- If my application is successful to form the basis of the statement made within my application.

If the University of Bristol discovers that I have made a false statement or omitted signification information from my application, for example examination results, I understand that it may have to withdraw or amend its offer or terminate my registration, according to circumstances.

CURRICULUM VITAE

Personal Data

Postal Address: P.O. Box 2934-60200 Meru, Kenya
Telephone: +254 711584857
Date of Birth: 14-02-1993
Place of birth: Narok; Kenya
Marital Status: Single
Gender: Male
Email: dsankei@must.ac.ke

Professional Experience

Institution: Meru University of Science and Technology
Duration: Jan 2022 to-date
Designation: Administrative Assistant
Responsibility:

- Registration of candidates for assessments.
- Preparation of assessment schedules, formative assessment timetables, and assessment tools.
- Maintaining the portfolio of evidence and validation of the formative assessment tools as per the assessment requirements.
- Timetabling officer
- Offer Competency Based Education Training to trainers
- Teach weekend classes

Institution: Meru University of Science and Technology
Duration: Jan 2020 – Dec 2021
Designation: Tutor
Responsibilities:

I. Taught and examined candidates in the following Units:
   - Demonstrate numeracy skills
   - Apply Engineering Mathematics
   - Demonstrate digital literacy
   - Control ICT Security Threats
   - Mathematics
   - Introduction to computer and applications

II. Prepared Teaching and Examination timetables

Institution: Mituntu Girls Secondary School
Duration: March 2016 – Dec 2019
Designation: Mathematics and Chemistry Tutor
Responsibilities:

- Taught Mathematics and Chemistry
- Assisted and prepared students for projects for the Kenya Science and Engineering Fair schools' competitions
- Served as a coach for the football team

Institution: Brother Beausang Catholic Education Centre
Duration: Jan-Nov 2015
Designation: Volunteer Teacher
Responsibilities: Taught Mathematics and Chemistry

Academic Qualifications

University education

2022 -2023 Meru University of Science and Technology
Certification: Master of Science in Pure Mathematics
Thesis Title: " Formulation of a Set of Even Numbers and Generation of Pairs of Odd Numbers for Application in Proving Strong Goldbach's Conjecture,"

2012- 2017 Meru University of Science and Technology
Certification: Bachelor of Science in Mathematics and Computer Science (Pure Mathematics option)
Award: First Class Honours

Secondary school education

2008- 2011 Brother Beausang Catholic Education Centre
Certification: Kenya Certificate of Secondary Education (KCSE)
Award: B+

Primary school education

1998-2007 N/ENKARE District Education Board
Certification: Kenya certificate of Primary Education

Conference Presentations:

26th-28th June 2023 The 2nd International Conference on SDGs for Development
Theme: "Leveraging Interlinkages among the SDGs to Realize the 2030 Agenda Through Research and Innovation in the Post Covid Era''
Subtheme: Reimaging Pure and Applied Sciences for the Post Pandemic Future
Title of my presentation: A new formulation of a set of odd numbers
Award: Certificate of participation

Professional Training and Competency

1. Certificate in Competency Based Education Training(CBET) – 2020
2. Validation of Assessment Tools (TVET CDACC) -2021
3. Certificate of merit for participating in Kenya Science and Engineering Fair - 2017

Referees:

Dr. Loyford Njagi
Lecturer, Department of Mathematics
Meru University of Science and Technology
Tel: +254723101835

Dr. Simon Kubaison
Director, TVET
Meru University of Science and Technology
Tel: +254720637290

Peter Shenahan
Principal
Brother Beausang Catholic Education Centre
Tel: +254720782907

Martha Githinji
Principal
Mituntu Girls' High school
Tel: +254722839194

Certificate Number
5573

MERU UNIVERSITY OF SCIENCE & TECHNOLOGY

Foundation of Innovations

MERU UNIVERSITY OF SCIENCE & TECHNOLOGY

This is to certify that

Sankei Njoroge Daniel

having satisfied all the requirements for the award of the Degree of

Master of Science

IN

PURE MATHEMATICS

was admitted to the Degree at a congregation held at this University

on the **27th** *Day of* **October**
in the Year **2023**

[redacted]

Vice Chancellor

Deputy Vice-Chancellor
(Academic and Student Affairs)

* This certificate is issued without alteration

P.O Box 972-60200
Meru, Kenya
Passport No: AK0635912
dsankei@must.ac.ke
+254711584857

2nd January 2024

Admissions Committee
University of Bristol
Faculty of Mathematics
United Kingdom, BS8 1TW, Bristol, University Walk

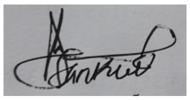
Dear Members of the Admissions Committee,

I am writing to express my sincere interest in applying for the PhD in Mathematics at the University of Bristol. With a profound passion for mathematics particularly in the realm of Number Theory and a dedicated academic background, I am keen to discover and contribute to the ongoing research in this field, and I believe that the University of Bristol offers the ideal environment for me to pursue my doctoral studies.
My academic journey in mathematics began during my undergraduate studies having pursued a Bachelor's degree in Mathematics and Computer Science (Pure Mathematics Option) and a Master's degree in Pure Mathematics, where I developed a fascination for the elegant and profound concepts in Pure Mathematics specializing in number theory. My master's thesis, titled " Formulation of a Set of Even Numbers and Generation of Pairs of Odd Numbers for Application in Proving Strong Goldbach's Conjecture," focused on introducing a new formulation of a set of even numbers that allows any even number to be partitioned into all pairs of odd numbers. These results were quite an advancement to the current solutions to the Strong Goldbach's Conjecture since, we were able to show that from this set of all pairs of odd numbers, there exist at least one pair of prime numbers satisfying this conjecture. With this formulation, we were able to extend the numerical verification of this conjecture to all even numbers not larger than 9×10^{18} from the current 4×10^{18}. Combining all these results, we were also able to provide a rigorous proof of the Strong Goldbach's Conjecture. This demonstrates my ability to engage deeply with complex mathematical concepts and to conduct independent research.

I have also taken part in examination of the developed modified Ohm's Law and the scrupulous analysis that ensued by Alex kimuya of Meru University of science and Technology (email: alexkimuya23@gmail.com) as well as presented a mathematical concept in an international conference.

Thank you for considering my application. I look forward to the opportunity to discuss my candidacy further in an interview.

Sincerely,



A NEW FORMULATION OF A SET OF ODD NUMBERS AND GENERATION OF TRIPLES OF ODD NUMBERS FOR APPLICATION IN PROVING WEAK GOLDBACH'S CONJECTURE

1. Introduction

In a letter to Euler on 7th of June 1742, Goldbach proposed two conjectures on the representation of integers as the sum of primes. The two conjectures maybe stated as follows:

i) Every even integer greater than 2 can be written as the sum of two primes (the strong Goldbach conjecture).

ii) Every odd integer greater than 7 can be written as the sum of three primes (the weak Goldbach conjecture).

It is strongly believed that ii) can be derived from i) [1]. Computationally, the conjectures have been shown to hold for very large values. In November of 2013, a paper was published by Thomás Oliveira e Silva, Siegfried Herzog, and Silvo Pardi which used advances in computational computing proving that the binary form of the Goldbach Conjecture is true up to 4×10^{18}[2]. The most recent computational approach is that of Daniel Sankei, Loyford Njagi and Josephine Mutembei who using a new formulation of a set of even numbers [3] and the fact that an even number of this form can be partitioned into all pairs of odd numbers [4] we were able to present a numerical verification of the strong goldbach conjecture for all even numbers not larger than 9×10^{18} [5]. The numerical verification of the Weak Goldbach's Conjecture on the other hand has been shown to be true up to $8.875 \cdot 10^{30}$[6].

While the two Goldbach conjectures remain unproven for almost 250 years despite considerable effort by many mathematicians throughout history, many mathematicians believe that they are true since a counter example is yet to be found. However, a formal proof of the conjectures remains an open problem in Number Theory [7]. Interesting to say is that in 1920, Yuan Wang, in the book, The Goldbach conjecture wrote that there is no method to attach this conjecture and the research is confined only to checking this conjecture by some numerical calculations or proposing some further conjectures. Further, quite notable also is that between March 20, 2000 and March 20, 2002, Faber offered a prize to anyone who proved Goldbach's conjecture, but the prize went unclaimed [8].

1.1 The Proposed proof of the Strong Goldbach's Conjecture by Daniel Sankei, Loyford Njagi and Josephine Mutembei

We have (My Master's supervisors and I) presented a manuscript to the Hindawi Journal of Mathematics for the peer-review process and publishing on a rigorous proof of the Strong Goldbach's Conjecture. The results in the paper shows that any two arbitrary sets of prime numbers when added together gives an even number. We also show that the set of all pairs of odd numbers obtained from partitioning an even number of the new formulation , contains two proper subsets of prime numbers such that in each of these proper subsets of prime numbers there exist at least one pair of prime numbers whose sum equals . Since the addition of any two arbitrary prime numbers gives an arbitrary even number and the converse has been shown to be true, then these results ensures that there exists at least a prime number in each of these two proper subsets of prime numbers such that the addition of the two prime numbers equalstherefore proving the Strong Goldbach's Conjecture.

2. Proposed PhD Research Work

2.1 Proposed Title of the PhD Thesis:

A NEW FORMULATION OF A SET OF ODD NUMBERS AND GENERATION OF TRIPLES OF ODD NUMBERS FOR APPLICATION IN EXTENDING AND PROVING WEAK GOLDBACH'S CONJECTURE

Remark 1

Since Christian Goldbach proposed the two problems on 7^{th} of June 1742, Mathematicians have dedicated their efforts to finding the solution to the Strong Goldbach's Conjecture as it is strongly believed that its solution implies the Weak Goldbach's Conjecture. This is to say that, if today a general proof of the Strong Goldbach's conjecture is to be accepted by the mathematical community, the Weak Goldbach's Conjecture would then become a corollary. The study proposes an independent solution to the weak Goldbach's Conjecture by formulating a new definition of a set of odd numbers.

2.2 Proposed New Formulation of a set of Odd numbers and its Application to Proving the Weak Goldbach's Conjecture

In mathematics, an odd number can be expressed as $2n+1$, where n is an integer and they are characterized by not being multiples of 2. The proposed new formulation of a set of odd numbers can be derived from this definition so as to allow any odd number say be written as a sum of three odd numbers. Since the statement of the Weak Goldbach's conjecture claims that Every odd integer greater than 7 can be written as the sum of three primes, then a new expression of the odd number as a sum of three odd numbers makes it possible to develop an algorithm that partitions it into a set containing all triples of odd numbers. Prime numbers greater than 2 are subsets of Odd numbers, It is further possible to show that from this set containing all the triples of odd numbers, there exist at least one triple of prime numbers whose sum is and therefore satisfying the Weak Goldbach's Conjecture.

Additionally, a computational algorithm can be developed using the algorithm for partitioning

into all triples of odd numbers so as to check whether we can extend the numerical verification of the Weak Goldbach's Conjecture from the current $8.875 \cdot 10^{30}$. Finally, since it is known that there are infinitely many prime numbers and the result of adding a triple of odd numbers is odd and the fact that it is possible to partition any odd number into a set containing all triples of odd numbers from which, we can obtain at least a triple of prime numbers, a rigorous proof of the Weak Goldbach's conjecture can be derived.

The Goldbach Conjecture has connections to computer science and cryptography. If proven true, it could have implications for algorithms related to number theory and factorization, which are crucial in areas like cryptography. The conjecture is related to the distribution of prime numbers, and the results proposed for the Weak Goldbach's Conjecture could lead to new insights into the patterns and structures of prime numbers thereby helping mathematicians find new method of attack to the Riemann Hypothesis Conjecture, Twin Prime Conjecture and Collatz Conjecture. The new formulation of a set of odd numbers could also help us extend the interpretation of the Strong Goldbach's Conjecture since, Christian Goldbach presented a relation where any even number can be written as a sum of two prime numbers but this relation is not entirely unique.

3. Why Bristol University?

Based on the results for REF 2021, the School of Mathematics at the University of Bristol was ranked 4th in the UK for Mathematical Sciences research based on independent analysis by Times higher Education. The REF results highlighted that 98% of the school's research is either 'world leading' or 'internationally excellent' in terms of **originality**, **significance** and rigour. This represents commitment, talent and expertise from all faculty members especially the supervisors.

The solution proposed here for the Weak Goldbach's Conjecture presents unique and original approach that no mathematician has explored since the proposed method of proving the Weak Goldbach's conjecture is independent of the methods used in attempting to prove the Strong Goldbach's Conjecture and these same results could have significant applications to other open problems in Number Theory. I am drawn to pursue a Ph.D. in mathematics at Bristol University for a scholarship award due to its academic excellence, research opportunities, and faculty expertise.

Receiving a doctoral scholarship would be a tremendous honor and a crucial step in realizing my aspirations in mathematical research. I am confident that my background, coupled with my enthusiasm for Number Theory, positions me as a strong candidate for this scholarship. I am excited about the prospect of joining the vibrant academic community at the University of Bristol and contributing to the ongoing advancements in pure mathematics.

4. Proposed supervisors

i) Celine Maistret
- Computational number theory and arithmetic geometry

ii) Misha Rudnev
- Number theory and combinatorics

5. References

1. Härdig, J. (2020). Goldbach's Conjecture.

2. e Silva, Tomás Oliveira, Siegfried Herzog, and Silvio Pardi. "Empirical Verification of the Even Goldbach Conjecture and Computation of Prime Gaps up to 4·1018." *Mathematics of Computation* (2014): 2033-2060
3. Sankei, D., Njagi, L., & Mutembei, J. (2023). A New Formulation of a Set of Even Numbers. *European Journal of Mathematics and Statistics*, 4(4), 93-97.
4. Sankei, D., Njagi, L., & Mutembei, J. (2023). Partitioning an even number of the new formulation into all pairs of odd numbers. *Journal of Mathematical Problems, Equations and Statistics*, 4(2), 35-37. https://doi.org/10.22271/math.2023.v4.i2a.104.
5. Daniel, S., Njagi, L., & Mutembei, J. (2023). A NUMERICAL VERIFICATION OF THE STRONG GOLDBACH CONJECTURE UP TO 9× 10^ 18. *GPH-International Journal of Mathematics*, 6(11), 28-37.
6. Helfgott, H. A., & Platt, D. J. (2013). Numerical verification of the ternary Goldbach conjecture up to 8.875· 1030. *Experimental Mathematics*, *22*(4), 406-409.
7. South, J. R. (2022). An algebraic approach to the goldbach conjecture. *arXiv preprint arXiv:2206.01179*.
8. Weisstein, E. W. (2002). Goldbach conjecture. *https://mathworld. wolfram. com/*.

Certificate Number
3460

Foundation of Innovations

MERU UNIVERSITY OF SCIENCE & TECHNOLOGY

This is to certify that

Sankei Daniel Njoroge

having satisfied all the requirements
for the award of the Degree of

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

FIRST CLASS HONOURS

was admitted to the Degree at a congregation
held at this University

on the **15th** *Day of* **December**
in the Year **2017**

No. 26578

MERU UNIVERSITY OF SCIENCE & TECHNOLOGY

TRANSCRIPT

Name of Student: SANKEI DANIEL NJOROGE
Registration No: MC593/0108/12
School: PURE AND APPLIED SCIENCES
Year of Admission: 2012
Programme: BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE
Year of Study: FIRST

UNIT CODE	UNIT DESCRIPTION	ACADEMIC HOURS	GRADE
SPH 2172	PHYSICS	45	A
SMA 2104	MATHEMATICS FOR SCIENCE	45	B
SMA 2101	CALCULUS I	45	A
SMA 2100	DISCRETE MATHEMATICS	45	A
ICS 2100	INTRODUCTION TO COMPUTER SYSTEMS	45	B
ICS 2102	INTRODUCTION TO COMPUTER PROGRAMMING	45	A
HRD 2101	COMMUNICATION SKILLS	45	C
SZL 2111	HIV/AIDS	45	PASS
SMA 2103	PROBABILITY AND STATISTICS I	45	A
SMA 2102	CALCULUS II	45	A
ICS 2101	COMPUTER ORGANIZATION	45	A
ICS 2105	DATA STRUCTURE AND ALGORITHMS	45	C
ICS 2104	OBJECT ORIENTED PROGRAMMING I	45	A
SCH 2110	CHEMISTRY	45	A
ICS 2200	ELECTRONICS	45	A
HRD 2102	DEVELOPMENT STUDIES AND SOCIAL ETHICS	45	B

RECOMMENDATION: PASS. PROCEED TO YEAR TWO OF STUDY

KEY TO GRADING SYSTEM

A=70% Above
B=60% & below 70%
C=50% & Below 60%
D=40% & Below 50%
E=Below 40%

Signed ______________________
Registrar (Academic, Research & Student Affairs)

Date ____________

MERU UNIVERSITY OF SCIENCE & TECHNOLOGY
REGISTRAR (ARS)
P.O. Box 972 60200 MERU

No: 26579

MERU UNIVERSITY OF SCIENCE & TECHNOLOGY

TRANSCRIPT

Name of Student:	SANKEI DANIEL NJOROGE	**Registration No:**	MC593/0108/12
School:	PURE AND APPLIED SCIENCES	**Year of Admission:**	2012
Programme:	BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE	**Year of Study:**	SECOND

UNIT CODE	UNIT DESCRIPTION	ACADEMIC HOURS	GRADE
SMA 2201	LINEAR ALGEBRA I	45	A
SMA 2230	PROBABILITY AND STATISTICS II	45	A
SMA 2200	CALCULUS III	45	A
SMA 2203	NUMBER THEORY	45	A
HRD 2103	GENERAL ECONOMICS	45	B
ICS 2203	INTERNET APPLICATION PROGRAMMING	45	B
ICS 2202	OPERATING SYSTEMS I	45	A
ICS 2201	OBJECT ORIENTED PROGRAMMING II	45	B
SMA 2220	VECTOR ANALYSIS	45	A
SMA 2231	PROBABILITY AND STATISTICS III	45	A
SMA 2221	CLASSICAL MECHANICS	45	A
SMA 2202	ALGEBRAIC STRUCTURES	45	A
ICS 2210	SYSTEMS ANALYSIS AND DESIGN	45	A
ICS 2206	DATA BASE SYSTEMS	45	A
HRD 2104	PRINCIPLES OF INDUSTRIAL MANAGEMENT	45	A

RECOMMENDATION: PASS. PROCEED TO YEAR THREE OF STUDY

KEY TO GRADING SYSTEM

A=70% Above
B=60% & below 70%
C=50% & Below 60%
D=40% & Below 50%
E=Below 40%

MERU UNIVERSITY OF SCIENCE & TECHNOLOGY
REGISTRAR (ARS)
09 MAY 2013
P.O. Box 972 60200 MERU

Signed ____________ **Date** ____________
Registrar (Academic, Research & Student Affairs)

No. 26580

MERU UNIVERSITY OF SCIENCE & TECHNOLOGY

TRANSCRIPT

Name of Student:	SANKEI DANIEL NJOROGE	**Registration No:**	MC593/0108/12
School:	PURE AND APPLIED SCIENCES	**Year of Admission:**	2012
Programme:	BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE	**Year of Study:**	THIRD

UNIT CODE	UNIT DESCRIPTION	ACADEMIC HOURS	GRADE
ICS 2207	SCIENTIFIC COMPUTING	45	A
ICS 2208	OPERATING SYSTEM II	45	A
HRD 2115	ACCOUNTS AND FINANCE	45	A
SMA 2301	REAL ANALYSIS I	45	A
SMA 2305	COMPLEX ANALYSIS I	45	A
SMA 2321	NUMERICAL ANALYSIS I	45	A
SMA 2304	ORDINARY DIFFERENTIAL EQUATION I	45	A
ICS 2305	SYSTEMS PROGRAMMING	45	A
ICS 2306	COMPUTER NETWORKS	45	A
ICS 2311	COMPUTER GRAPHICS	45	B
SMA 2343	OPERATIONS RESEARCH	45	A
SMA 2302	REAL ANALYSIS II	45	A
SMA 2303	GROUP THEORY I	45	A
SMA 2307	RING THEORY	45	A

RECOMMENDATION: PASS. PROCEED TO YEAR FOUR OF STUDY

KEY TO GRADING SYSTEM

A=70% Above
B=60% & below 70%
C=50% & Below 60%
D=40% & Below 50%
E=Below 40%

Signed ______________________
Registrar (Academic, Research & Student Affairs)

MERU UNIVERSITY OF SCIENCE & TECHNOLOGY
REGISTRAR (ARS)
P. O. Box 972 60200 MERU

Date ______________________

No. 26581

MERU UNIVERSITY OF SCIENCE & TECHNOLOGY

TRANSCRIPT

Name of Student: SANKEI DANIEL NJOROGE
Registration No: MC593/0108/12
School: PURE AND APPLIED SCIENCES
Year of Admission: 2012
Programme: BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE
Year of Study: FOURTH

UNIT CODE	UNIT DESCRIPTION	ACADEMIC HOURS	GRADE
HRD 2401	ENTREPRENEURSHIP SKILLS	45	B
SMA 2400	PARTIAL DIFFERENTIAL EQUATIONS I	45	A
ICS 2303	MULTIMEDIA SYSTEMS	45	B
ICS 2302	SOFTWARE ENGINEERING	45	A
SMA 2401	TOPOLOGY I	45	A
SMA 2306	LINEAR ALGEBRA II	45	A
SMA 2407	FUNCTIONAL ANALYSIS	45	A
SMA 2404	FIELD THEORY	45	A
ICS 2408	SELECTED/ADVANCED TOPICS IN COMPUTER SCIENCE	45	A
ICS 2308	ARTIFICIAL INTELLIGENCE	45	B
ICS 2307	SIMULATION AND MODELLING	45	A
SMA 2405	COMPLEX ANALYSIS II	45	A
SMA 2402	TOPOLOGY II	45	A

RECOMMENDATION: PASS. AWARDED BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE FIRST CLASS HONOURS

KEY TO GRADING SYSTEM

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B=60% & below 70%
C=50% & Below 60%
D=40% & Below 50%
E=Below 40%

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RESULT SLIP

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PROGRAMME: MASTER OF SCIENCE IN PURE MATHEMATICS

YEAR OF STUDY: ONE **SEMESTER:** ONE **DATE OF ADMISSION:** JANUARY 2022

UNIT CODE	UNIT TITLE	UNIT(S)	GRADE	REMARKS
SMA 5001	Measure and Integration	1	B	PASS
SMA 5002	Functional Analysis I	1	A	PASS
SMA 5003	Group Theory I	1	A	PASS
SMA 5004	General Topology I	1	B	PASS
SMA 5006	Complex Analysis III	1	A	PASS

Key to the Grading System:

A = 70% and Above
B = 60% and Below 70%
C = 50% and Below 60%
D = 40% and Below 50%
E = Below 40%

NB: 1 unit consists of 45 lecture hours or equivalent: (3 Practical hours or 2 Tutorial hours are equivalent to one lecture hour).

SIGNED.......................................DATE 2/11/2022.......................................
COD – Mathematics

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Dean – School of Pure and Applied Sciences

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RESULT SLIP

REGISTRATION NUMBER: SC405/201114/20

NAME: DANIEL SANKEI

PROGRAMME: MASTER OF SCIENCE IN PURE MATHEMATICS

YEAR OF STUDY: ONE **SEMESTER:** TWO **DATE OF ADMISSION:** JANUARY 2022

UNIT CODE	UNIT TITLE	UNIT(S)	GRADE	REMARKS
SMA 5005	Graph Theory I	1	B	PASS
SMA 5007	Functional Analysis II	1	A	PASS
SMA 5008	Group Theory II	1	B	PASS
SMA 5015	Coding Theory	1	C	PASS
SMA 6006	Operator Theory I	1	A	PASS
SMA 6010	Introduction to Mathematical Cryptography	1	B	PASS

Key to the Grading System:

A = 70% and Above
B = 60% and Below 70%
C = 50% and Below 60%
D = 40% and Below 50%
E = Below 40%

NB: 1 unit consists of 45 lecture hours or equivalent: (3 Practical hours or 2 Tutorial hours are equivalent to one lecture hour).

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COD – Mathematics

SIGNED............................DATE 06/06/23
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FORMULATION OF A SET OF EVEN NUMBERS AND GENERATION OF PAIRS OF ODD NUMBERS FOR APPLICATION IN PROVING STRONG GOLDBACH'S CONJECTURE

SANKEI DANIEL NJOROGE

A Thesis Submitted in Partial Fulfilment of Requirements for Conferment of the Degree of Master of Science in Pure Mathematics of Meru University of Science and Technology

2023

Declaration

This thesis is my original work and has not been presented for a degree in any other institution.

Signature Date: ………6/10/2023………………

SANKEI DANIEL NJOROGE

SC405/201114/20

This thesis has been submitted with our approval as University Supervisor(s).

Signature Date: …….6/10/2023…………..

Dr. Loyford Njagi (PhD.)

Meru University of Science and Technology

Signature … ………………………… Date: ……6/10/2023………..

Dr. Josephine Mutembei (PhD.)

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Lastly, Special thanks to Meru University of Science and Technology for providing a serene environment for high-quality learning. I express immense gratitude to my family, colleagues, friends, and everyone who made my studies at Meru University of Science and Technology worthy in different ways.

Dedication

This work is dedicated to Glory Kendi, my father Raphael Tajeu Sankei, my mother Agnes Senteu and my brother Kelvin Saitoti for their earnest efforts towards my education.

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Notations

We shall make use of the following notations:

GRH Generalized Riemann Hypothesis

P The set of all primes

N Positive natural number

E The set of all even numbers

O The set of all odd numbers

d For any two $p_i, p_j \in P$, $d \in (p_j - p_i)^n$

BGC Binary Goldbach conjecture

Operational Definitions

Definition 1 An even number is an integer that can be divided by two and remain an integer. It is an integer of the form $n = 2k$, where k is also an integer (Even and Odd Numbers (Definition, Properties, and Examples), n.d.).

Definition 2 An odd number is an integer of the form $n = 2k \pm 1$, where k is an integer, or Odd numbers are numbers that cannot be divided by 2 evenly (Even and Odd Numbers (Definition, Properties, and Examples), n.d.).

Definition 3 A number $p > 1$ is called a prime number (or simply a prime) if it has only two divisors (namely p and 1). A natural number n > 1 which is not prime, will be called composite (Prime and Composite Numbers - Definition, Examples, List and Table, n.d.).

Definition 4 A Conjecture is a supposition that is consistent with known data but has neither been proved or disproved (Definition of Conjecture, n.d.).

Definition 5 (Goldbach partition)

The pair of two prime numbers, p, and q, where p + q = n and n being an even integer, is called a Goldbach partition of n (Wikipedia Contributors, 2021).

Abstract

The Strong Goldbach's conjecture also known as the Binary Goldbach conjecture (BGC) is one of the oldest and best-known unsolved problems in Number Theory and all of mathematics. It states that every even integer greater than 2 can be expressed as the sum of two primes. A Goldbach number is a positive even integer that can be expressed as the sum of two odd primes. Since 4 is the only number greater than 2 that requires the even prime 2 to be written as the sum of two primes, another form of the statement of the Strong Goldbach's Conjecture is that all even integers greater than 4 are Goldbach numbers. The BGC has set a persistent challenge to the exploration of the foundations of mathematics in general and Number Theory in particular as it remains unproven for almost 250 years despite considerable efforts by mathematicians throughout history. The best-known result so far is that of Chen proving that every sufficiently large even integer N can be written as the sum of a prime and the product of at most two prime numbers. The known algorithms for attempting to prove or verify the BGC on a given interval $[a , b]$ consist of finding two sets of primes p_i and p_j such that $p_i + p_j$ cover all the even numbers in the interval $[a , b]$. The traditional representation of an even number is $2k$ for $k \in \mathbb{N}$ and this formulation has not provided mathematicians with a direct pathway to easily obtain all Goldbach partitions for any even number of this form. This study introduces a new formulation of a set of even numbers as an integer E of the form $E_{ij} = n_i + n_j + (n_j - n_i)^n$ for all $n \in \mathbb{N}$. The proof that this new formulation holds $\forall n < \infty$ is provided. This new definition will have two consequences: (1) using the new formulation of a set of even numbers, it has been proved that any even number say E_{ij} can be partitioned into all pairs of all odd numbers whose sum is E_{ij}, and (2) from these set of pairs of odd numbers, it has been shown that there exists at least one Goldbach partition for E_{ij}. Finally, a rigorous proof of the Strong Goldbach's conjecture is provided.The study further recommends the exploration of the method of partitioning any even number into all pairs of odd numbers so as to find a new method of attacks to theTwin Prime Conjecture and the Weak Goldbach Conjecture.

CHAPTER ONE: INTRODUCTION

1.1 Background

One of the challenging questions in Number Theory yet to be answered is Goldbach's conjecture. The conjecture is named after Christian Goldbach, an 18th-century mathematician from Prussia, who first proposed it in a letter to the famous mathematician Leonhard Euler in 1742 (Lemmermeyer and Mattmu¨ller, 2015). On 7th of June 1742, the German mathematician Christian Goldbach wrote a letter to Leonhard Euler (letter XLIII) in which he proposed the following conjectures (Härdig, 2020): Every even integer can be written as the sum of two primes (the Strong Goldbach Conjecture) but he could not prove it (Bournas, 2013). In the letter, Goldbach stated that he believed that every even number greater than 2 could be expressed as the sum of two primes, and he challenged Euler to prove or disprove the conjecture (Shi et al., 2019). Since then, many mathematicians have attempted to prove or disprove the conjecture. Goldbach's original conjecture was formulated in such a way that, every even integer greater than 2 can be expressed as the sum of two prime numbers (William-West, 2016). For example, 4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, 10 = 3 + 7 = 5 + 5, and so on. He then proposed a second conjecture on the margin of his letter: Every odd integer greater than 7 can be written as the sum of three primes (the weak Goldbach conjecture) (Martelli, 2013). A Goldbach number is a positive even integer that can be expressed as the sum of two odd primes (Mehendale, 2020) and since 4 is the only even number greater than 2 that requires the even prime 2 in order to be written as the sum of two primes, another form of the statement of Goldbach's conjecture is that all even integers greater than 4 are Goldbach numbers (South, 2022).

Despite centuries of effort by mathematicians to prove or disprove the conjecture, it remains unsolved to this day (Aouessare *et al.,* 2016). However, there has been a lot of progress towards it. The strong Goldbach conjecture implies the conjecture that all odd numbers greater than 7 are the sum of three odd primes, which is known today as the weak Goldbach conjecture (Vega, 2018). In the 20th century, the development of computers allowed mathematicians to check the conjecture for very large even numbers, and they found that the conjecture holds true for billions of even integers (Oliveira *et al.,* 2014). Various techniques have been used to attack the problem, including analytic number theory, algebraic number theory, and sieve methods, but so far none of them have led to a complete solution. Despite its unsolved status, the Goldbach conjecture has become famous for its simplicity and accessibility, and for the fact that it remains unsolved despite centuries of effort by some of the greatest mathematicians in history (Wang, 2022).

There have been many partial results and related conjectures, but no one has been able to prove the Goldbach Conjecture in its full generality. It remains one of the most tantalizing and intriguing open problems in mathematics. The conjecture has inspired a great deal of research in Number Theory and has also led to the development of new techniques and methods in the field (Wang, 2022).

The “strong” conjecture has also been shown to hold up through 4×10^{18} (Oliveira *et al.,* 2014) but remains unproven for almost 250 years despite considerable effort by many mathematicians throughout history (O'Regan, 2022). In his famous speech at the 2nd international congress of mathematics held in Paris in 1900, Hilbert proposed 23 problems for mathematicians in the 20th century, and the Strong Goldbach conjecture was part of his 8th problem (Matiyasevich, 1996). In 1912, the Strong Goldbach Conjecture was regarded as one of the four famous unsolved problems in the theory of prime numbers proposed by Landau in his speech at the 5th international congress held in Cambridge (Maynard, 2022). Furthermore, at the mathematical society of Copenhagen in 1921, Hardy pronounced that the strong Goldbach conjecture was probably as difficult as any of the unsolved problems in mathematics (Härdig, 2020).

This problem appeared for the first time in a letter from Goldbach to Euler in the year 1742 (Härdig, 2020) and since its publication in 1984, it is still unsolved despite the best efforts by eminent mathematicians (Al-Ameen & Muhi, 2022) and the closest related results are that: (i) there exists an integer S such that every integer is the sum of at most S primes, and (ii) every sufficiently large even integer may be written as the sum of a prime number and of the product of at most two prime numbers (Barca, 2022). Other key milestones towards the proof of Goldbach conjecture in the past 278 years and beyond include: 1) The Euclid's *Fundamental Theorem of Arithmetic* (FTA) that revealed there is always a unique prime factorization for any integer (Dawson & Dawson, 2015; Härdig, 2020) The Legendre's *Sieve of Eratosthenes* (1808) that provided a foundation for modern sieve theories (Wang, 2022).

On the other hand, this conjecture has also been numerically verified up to 4×10^{11} (Richstein, 2001). Yuan Wang (1920) in the book, The Goldbach conjecture wrote that there is no method to attach this conjecture and the research is confined only to checking this conjecture by some numerical calculations or proposing some further conjecture on this conjecture. The majority of mathematicians believe that Goldbach's conjecture is true, especially in statistical considerations based on the probabilistic distribution of prime numbers (Bado,2018). The larger the number, the more manners available to represent it as a sum of two or more. Despite the lack of proof, the Goldbach conjecture has been influential in the development of Number Theory and has inspired many other important results. While the Goldbach conjecture has not been proved, it has been verified empirically for very large even numbers, and many mathematicians believe that it is true for all even integers. However, a formal proof of the conjecture remains an open problem in mathematics (South, 2022).

This study has formulated a new set of even numbers that has made it easier to generate the set of all pairs of odd numbers whose sum is a given even number and shown that there exists at least one pair of prime in these set of pairs of odd numbers whose sum is the given even number and provided a rigorous proof to the Strong Goldabach conjecture.

1.2 Statement of the Problem

A Goldbach number is a positive even integer that can be expressed as the sum of two odd primes (Mehendale, 2020). Mathematicians have put great efforts into trying to prove the strong Goldbach's conjecture to date (Richstein, 2001; Wang, 2022; Mutafchiev, 2014; Son ,2019) among others. Despite the considerably great determinations put in by various mathematicians in an attempt to prove this conjecture, it still remains unsolved (O'Regan, 2022; Watanabe, 2018). While the conjecture has been tested for all even numbers up to 4×10^{18} (Oliveira *et al.,* 2014) and no counterexamples have been found, a general proof is still lacking. The Goldbach Conjecture is therefore an active area of research in number theory, and mathematicians continue to search for a proof or a counterexample. The conjecture has also inspired the development of new mathematical techniques and ideas associated with the distributions of prime numbers.

This study aims at providing a new formulation of a set of even numbers. This new formulation is proven to hold for all natural numbers. The results obtained have been shown to have a great impact in partitioning any even number of the form introduced, into all pairs of odd numbers. It is finally shown that in this set of pairs of odd numbers there exists at least one Goldbach partition proving the Strong Goldbach Conjecture.

1.3 Objectives of the Study

1.3.1 General objective

To use the new formulation of a set of even numbers to prove the Strong Goldbach conjecture.

1.3.2 Specific objectives

i. To introduce a new formulation of a set of even numbers.
ii. To partition any even number into all pairs of odd numbers.
iii. To extend the numerical verification of the Strong Goldbach Conjecture
iv. To obtain Goldbach partitions and prove the strong Goldbach's conjecture.

1.4 Significance of the Study

The breakthrough on the study of Goldbach's conjecture is clearly inseparable from the great achievements on analytic Number Theory in the 19th century, in particular, the theory of chebyshev, Dirichlet, Riemann, Hadamard, de la Vallee Poussin and Von Mangoldt on the distribution of prime numbers which is the prerequisite of the present research. Additionally, the GRH is one of the most important unsolved problems in mathematics and if solved, it would help mathematicians understand the distribution of prime numbers much better. In fact, if GHR was proved, the ternary Goldbach conjecture would be a corollary. This study has introduced a new formulation of a set of even numbers that has made it easy to partition a given even number into all pairs of odd numbers and extended the verification of Goldbach's Conjecture up to 9×10^{18}. A rigorous proof of the strong Goldbach's conjecture has also been formulated. Further, the new formulation of a set of even numbers has been shown to provide a new method of attack to the Weak Goldbach Conjecture and the Twin Prime Conjecture.

CHAPTER TWO: LITERATURE REVIEW

2.1 Introduction

This section contains work that has been carried out by researchers on areas relating to our research.

The first great achievements on the study of Goldbach problem were obtained in 1920s. using their ‘‘circle method’’ British mathematicians Hardy and Littlewood proved in 1923 that every sufficiently large odd number is the sum of three odd primes and almost all even integer are sum of two primes if the Grand Riemann Hypothesis is assumed to be true (Wang, 2022). In 1937, Vinogradov using the circle method and his ingenious method on the estimation of exponential sum with prime variable, was able to remove the dependence on the Grand Riemann Hypothesis, thereby giving unconditional proofs of the above two conclusions of hardy and Littlewood (Wang, 2022).

Using methods developed by Vinogradov, over the next two years Nikolai Chudakov proved that all but a finite number of even integers can be written as the sum of two primes (Chudakov, 1938; van der Corput, 1937; Estermann, 1938). In 1939, K. Borozdin (Tao, 2011) was the first to find an upper bound, $3^{14348907}$, for describing Vinogradov’s "sufficiently large odd number."

Norwegian mathematician Brun established in 1919 by the sieve method that every large even integer is a sum of two numbers each having at most nine prime factors. After a series of important improvements to Brun’s method and his result, in 1966, Chen Jing established that every large even integer is the sum of a prime and the product of at most two primes (Wang,2022).

In 1938, Estermann by combining the ideas of Hardy and Littlewood with Vinogradow’s discovery proved that almost all even numbers are the sums of two primes whereas Schnirelman introduced an approach to Goldbach’s conjecture in the additive theory of prime numbers whereby it can be shown that every integer larger than unity can be written as the sum of a bounded number of prime numbers (Wang,2022).

In 1951, Linnik proved under the assumption of the GRH, and later in 1953 unconditionally, that every sufficiently large even integer can be written as the sum of two primes and K powers of 2, where K is an unspecified but absolute constant. The first explicit value for K, K = 770, was proved in 1998 by J. Y. Liu(Han and Liu, 2024).

In 1964,Mok-kong shen attempted to check Goldbach's conjecture on a digital computer and the validity of the conjecture was verified up to 33×10^6 and further calculations up to 10^8 was made by light et al. Further, Jorg (2000) and Matti (1993) verified the binary Goldbach conjecture up to 4×10^{14} (Bado, 2019). In 1977, Vaughan proved that every natural number is the sum of at most twenty-six prime numbers, and thus every natural number greater than unity is the sum of at most twenty-seven (Wang,2022).

In 1930, Lev Schnirelmann made progress on the strong Goldbach conjecture by showing that every even integer greater than 2 can be written as the sum of at most twenty primes, a result that has been improved several times. Schnirelman (1939) also proved that every even number can be written as the sum of not more than 300,000 primes (Weisstein, 2002), which seems a rather far cry from proof for two primes. Additionally, Pogorzelski (1977) claimed to have proven the Goldbach conjecture, but his proof is not generally accepted (Weisstein, 2002).

According to Hardy (1999, p. 19), "It is comparatively easy to make clever guesses; indeed, there are theorems, like 'Goldbach's Theorem,' which have never been proved and which any fool could have guessed. Faber and Faber offered a \$1,000,000 prize to anyone who proved Goldbach's conjecture between March 20, 2000 and March 20, 2002, but the prize went unclaimed and the conjecture remains open.

In a recent 2012 preprint that was to appear in Mathematics of Computation, Terence Tao (Tao, 2011) improved Kaniecki's result by proving that all odd integers are the sum of at most five primes without the Riemann hypothesis. Also in 2012, Tom´as Olivera e Silva, (Oliveira, 2014) computationally verified that the strong conjecture is true up to 4×10^{18}.

In 2017, Marshall, proved elementary that every even number is the sum of two prime numbers and in 2019, Son, N. K, proved that, for every even natural number $N > 10$, it can always be expressed as the sum of two primes. Although the conjecture has been extensively tested for even numbers up to very large values, no general proof has been found.

In 2019, Bado, gave a rigorous proof on Goldbach's conjecture based on a formulation that every even integer has a primo-radius. This proof is mainly based on the application of Chebotarev-Artin's theorem, Mertens' formula and the Principle exclusion-inclusion of Moivre.

In 2022, Stephen H. Jarvis, used a mathematical analysis of zero-dimensionality in deriving the natural numbers, offering a solution to Goldbach's conjecture and the Riemann hypothesis.

In 2022, Ameen and Muhi, proved that the sum of any two odd prime numbers is an even number and thereby the sequence of all even numbers > 4 can be expressed as the sum of two odd primes and infinitum (Goldbach's strong conjecture). While also in 2022, Yingxu Wang (Wang, 2022) presented a formal proof of the Goldbach conjecture based on a discovery of mirror primes and their recursive properties.

While there have been several advancements and partial results related to the Goldbach Conjecture, the Conjecture has been tested extensively for even numbers up to incredibly large values, and no counterexamples have been found. This empirical verification provides strong evidence for the conjecture's truth but does not constitute a proof (Li, 2023). Some mathematicians have derived conditional results related to the Goldbach Conjecture using advanced techniques and assumptions from other areas of mathematics. For example, in 2013, Harald Helfgott proved a conditional result known as the "Ternary Goldbach Conjecture," which states that every odd integer greater than 5 can be expressed as the sum of three prime numbers. Other researchers have made progress on related problems that shed light on the Goldbach Conjecture. For example, the Hardy-Littlewood circle method has been employed to make advancements in understanding the behavior of prime numbers and the additive properties of prime constellations whereas with the advent of powerful computers and sophisticated algorithms, researchers have conducted extensive computational searches for counterexamples to the Goldbach Conjecture. These searches have significantly expanded the range of verified cases but have not provided a conclusive proof. The Conjecture therefore remains unsolved primarily because no one has been able to provide a general proof that applies to all even integers greater than 2 (Li, 2023).

The statement of the Strong Goldbach Conjecture sounds fairly simple in its nature and it still draws the attention of mathematicians even to this date, more than 250 years after it was proposed. Worth mentioning is that the solution to this conjecture has been geared towards checking its validity up to a finite integer (Li, 2019; Martelli, 2013; O'Regan, 2022; da Costa, 2022). Other researchers have endeavored to provide a prove of this conjecture (Mehendale, 2020; Barca, 2022; Wang, 2022; Patel *et al.,* 2020), but none of these results have been accepted as a generalization solution to the strong Goldbach's conjecture and it remains open for further research (Wikipedia Contributors, 2021). It is important to note that it requires the knowledge of simple arithmetic to confirm Goldbach's conjecture with the first few even numbers, but as the even numbers grow bigger, the number of ways to represent them as a sum of two primes becomes practically difficult. The greatest challenge faced in trying to prove this conjecture is that the set of even and prime numbers are infinite (Infinitely Many Primes | Brilliant Math & Science Wiki, 2020), and therefore, as the value of the even number increases, advanced options such as the use of powerful computers are used to verify this conjecture.

The expected mode of solving this Conjecture appears to be that a given even number say $E_{1,2}$ of the form $2k$, where k is any natural number is partitioned into two primes P_1 and P_2. Using this definition of an even number, researchers have explored various approaches to proving the Strong Goldbach Conjecture under different assumptions whereas others have confirmed the validity of this conjecture to very large numbers. The conjecture therefore remains an open problem to be explored. This study presents a different approach to the solution of this conjecture that have been shown to more efficient in giving a better relationship between a given even number and a pair of primes. The study introduces a new formulation of a set of even numbers that has made it easier obtain all Goldbach partitions for a given even number of the new formulation. It has been shown how, starting with the sum of any two primes P_1 and P_2, and the addition of the difference between these two primes $d_{1,2}$ to the sum $P_1 + P_2$ yields another even number say $E_{3,4}$ that can be partitioned into all pairs of odd numbers whose sum is $E_{3,4}$. Further, it has been shown that it is always possible to obtain all Goldbach partitions of $E_{3,4}$ from these sets of pairs of odd numbers. Finally, based on these results, a rigorous proof of the Strong Goldbach's conjecture is provided.

CHAPTER THREE: A NEW FORMULATION OF A SET OF EVEN NUMBERS

3.0 Introduction

The chapter introduces four sections focusing on the conventional understanding of a set of even number whereas section two deals with the new formulation of even numbers for n_1 and $n_2 \in E$ for $\forall$ $k \in \mathbb{N}$. In section three it is proven that the new formulation holds for n_1 and $n_2 \in O$ for k $\in \mathbb{N}$, and since prime numbers greater than 2 are subsets of odd numbers, the final section of this chapter shows that new formulation of a set of the even numbers holds for all prime numbers. These results therefore give a better relation between a given even number and a pair of primes.

3.1 Basic known Mathematical Concepts on Even numbers

Even numbers are important in many areas of mathematics and computer science, and are often used in algorithms and data structures (Lehman et al., 2017). They play an important role in many mathematical concepts, such as algebra, number theory, and geometry. They are also commonly used in everyday life, such as when dividing objects into equal parts (Rocha, 2019). The set of even numbers is denoted by the symbol "E", and is a subset of the set of integers. Note that the set of even numbers includes both positive and negative integers, as well as zero. They have many interesting properties, such as the fact that any even number can be expressed as the sum of two prime numbers (known as Goldbach's conjecture) (Shi *et al.,* 2019). An easy way to identify even numbers is to check the last digit of the number (Zazkis, 1999). If the last digit belongs to the set {0, 2, 4, 6, 8}, then the number is even. For example, 132, 4620,164, and 8888 are all even numbers because their last digits are members of this set.

In general, any even number can be represented as $2n$, $\forall n \in \mathbb{N}$. Properties of even numbers can be helpful in solving mathematical problems and understanding patterns in numbers. Some interesting properties that even numbers possess includes:

Premise (1): Sum of two even numbers is even (Even Numbers and Odd Numbers - Definition, Properties, Examples, 2022).

Premise (2): Sum of two odd numbers is even (Lin and Tai, 2016).

The following theorem proves premise (2). The concepts discussed in this theorem will be used later in Chapter 4 to show the approach used to partition an even number into two all pairs of odd numbers.

Theorem 3.1.1

The sum of two odd numbers is an even number.

Proof (CASE 1)

A number is odd if its rightmost digit belongs to the set {1, 3, 5, 7, 9} while a number is even if its rightmost digit belongs to the set {0, 2, 4, 6, 8}. To find the rightmost digit of the sum of two numbers, you only have to add the rightmost digits of the two numbers and take the rightmost digit of that. For example, consider the numbers 1345 and 629. The rightmost digits are 5 and 9. Adding these gives us 14, whose rightmost digit is 4. So, we expect the rightmost digit of 1345 + 629 to be 4. And it is: 1345 + 629 = 1974. This tells us that in order to verify that the sum of any two odd numbers is an even number, we just have to check whether 4 is in the sum of any two odd digits on the right. It might not seem like a big deal at first, but this actually helps a lot. We have gone from talking about all the odd numbers (infinity of them) to talking about just five digits. We just checked this criterion for 5 and 9. We have to go through every case so that we're sure it always works:

$$
\begin{array}{cccc}
1+1=2 & 3+1=4 & \dots & 9+1=10 \\
1+3=4 & 3+3=6 & \dots & 9+3=12 \\
1+5=6 & 3+5=8 & \dots & 9+5=14 \\
1+7=8 & 3+7=10 & \dots & 9+7=16
\end{array}
$$

$$1 + 9 = 10 \qquad 3 + 9 = 12 \qquad \dots \quad 9 + 9 = 18$$

In every single one of these cases the rightmost digit is even (Algebraic Proof of Arithmetic Results, n.d).

Proof (CASE 2)

A number is odd if it can be written as 2x + 1, where x is some integer. A number is even if it can be written as 2x, where x is some integer. To start, pick any two odd numbers. We can write them as 2n + 1 and 2m + 1. The sum of these two odd numbers is (2n + 1) + (2m + 1). This can be simplified to 2n + 2m + 2 and further simplified to 2(n + m + 1). The number 2(n + m + 1) is even because n + m + 1 is an integer. Therefore, the sum of the two odd numbers is even (Prove that the sum of two odd numbers is even, n.d).

Premise (3): For all even numbers, when adding an even number and an odd number the sum will always be odd (Lin & Tsai, 2016).

Theorem 3.1.2

The sum of any even number and any odd number is always odd.

Proof

Let the even number be $2n$ and the odd number be $2m + 1$. Adding gives: $2n + (2m + 1) = 2n + 2m + 1 = 2(n + m) + 1$. Thus, the expression can be written as one more than a multiple of 2, so it is odd (Algebraic Proof of Arithmetic Results, n.d).

$Premise$ (4): Every integer is either even or odd (Show that every positive integer is either even or odd, n.d).

There is a unique relationship between even and prime numbers, which is that the only even prime number is 2. Given that 2 is both even and prime and that all the other even numbers > 2

are divisible by 2 and therefore has at least two positive integer divisors (1 and 2). This means that no even number greater than 2 can be prime.

3.2 Discussion of Results

Euclid defined an even number as "an integer which is divisible into two equal parts''. He also provided a method for generating even numbers using the formula $2n$ where n is any integer, and the representation of even numbers became more standardized. This study explores this definition of even numbers $(2n)$ and partition it into two even partitions say $2n = 2n_1 + 2n_2$. Further, partitions of each of the even partitions into two such that $2n_1 = k_1 + k_2$ and $2n_2 = k_2 - k_1, \forall k_1, k_2 \in \mathbb{N}$ where $\mathbb{N}$ is the set of all natural numbers. Therefore, a new formulation of a set of even numbers is introduced as an integer, E, of the form $E = (k_1 + k_2) + (k_2 - k_1)^n$ $\forall k_1, k_2, n \in \mathbb{N}$ and $(n \geq 1) \in \mathbb{N}$.

This new formulation of even numbers has been shown to hold for all natural numbers. It is proven that this new formulation of a set of even numbers, $E = (k_1 + k_2) + (k_2 - k_1)^n$ $\forall k_1, k_2, n \in \mathbb{N}$ holds in three cases: Case (1) $\forall\ k_1, k_2 \in E$, Case (2) $\forall\ k_1, k_2 \in O$, and Case (3) $\forall\ k_1, k_2 \in P$, where $n \in \mathbb{N}$ and P the set of all prime numbers. The following theorem introduces the general formulation of the new representation of a set of even numbers.

Theorem 3.2.1

Let n_1 and $n_2 \in \mathbb{N}$, then $(n_1+n_2)+(n_2-n_1)^n$ is even for all $n_2 > n_1$ and $(n \geq 1) \in \mathbb{N}$, where $\mathbb{N}$ is the set of all natural numbers. (3.2.1.1)

Remark 1 (Theorem 3.2.1)

Since n_1 and $n_2 \in \mathbb{N}$ then it follows that n_1 and $n_2 \in \mathbb{N}$ can either be odd or even. Expression (3.2.1) has been proven for even values of n_1 and $n_2 \in \mathbb{N}$ and finite values of $n = 1,2,$ and generalized for all even values of n_1 and n_2.

CASE 1

This section introduces Corollary 3.2.1 and proves the new formulation of even numbers for finite values of $n = 1,$ and 2 and generalize the proof for all values of n.

Corollary 3.2.1

Let n_1 and $n_2 \in E$, $\forall$ k$\in \mathbb{N}$ and $n=1$, then $(n_1+n_2)+(n_2-n_1)^1$ is even. (3.2.1.2)

Proof

Since n_1 and n_2 are even, then from the definition of even numbers they can be written as $n_1 = 2k_1, \forall k_1 \in \mathbb{N}$ and $n_2 = 2k_2, \forall k_2 \in \mathbb{N}$. Substituting $n_1 = 2k_1$ and $n_2 = 2k_2$ in expression (3.2.1.2) yields $(2k_1+2k_2)+(2k_2-2k_1)^1$. Since $n_2 > n_1$ then it implies that $k_2 > k_1 \Rightarrow$ $k_2 - k_1 > 0$.

$\Rightarrow 2(k_1+k_2)+2(k_2-k_1)^1$ and from the definition of even numbers, both $2(k_1+k_2)$ and $2(k_2-k_1)$ are even hence $2(k_1+k_2)+2(k_2-k_1)^1$ is even.

In order to illustrate Corollary 3.2.1, we give the following:

Example 3.2.1

Let $n_1 = 5686548$, $n_2 = 8976992$ and $n=1$,then

$(n_1+n_2)+(n_2-n_1)^1 = (5686548+8976992)+(8976992-5686548)^1$

$= 14{,}663{,}540 + 3{,}290{,}444 = 17{,}953{,}984 \in 2k,$

$\forall k \in \mathbb{N}$

Corollary 3.2.2

Let n_1 and $n_2 \in E$ for $\forall$ k$\in \mathbb{N}$ and $n = 2$, then $(n_1 + n_2) + (n_2 - n_1)^2$ is even. (3.2.2.1)

Proof

Substituting $n_1 = 2k_1, \forall k_1 \in \mathbb{N}$ and $n_2 = 2k_2, \forall k_2 \in \mathbb{N}$ in expression (3.2.2.1) yields $(2k_1 + 2k_2) + (2k_2 - 2k_1)^2 \Rightarrow (2k_1 + 2k_2) + (2k_2 - 2k_1)^2 \Rightarrow 2(k_1 + k_2) + (4k_2^{\ 2} - 8k_2k_1 + 4k_1^{\ 2}) \Rightarrow$ $2(k_1 + k_2) + 4(k_2^{\ 2} - 2k_2k_1 + k_1^{\ 2})$

$\Rightarrow 2(k_1 + k_2) + 4(k_2 - k_1)^2 \Rightarrow 2(k_1 + k_2) + 2^2(k_2 - k_1)^2$

$\Rightarrow 2(k_1 + k_2)$ is even and $2^2(k_2 - k_1)^2 = 2(2(k_2 - k_1)^2)$, for $k_2 - k_1 > 0 \Rightarrow (k_2 - k_1)^2 \in \mathbb{N}$ and $2(k_2 - k_1)^2$ is even from the definition of even numbers. This further implies that $2(2(k_2 - k_1)^2)$ is also even and finally, this shows that $(2k_1 + 2k_2) + (2k_2 - 2k_1)^2$ is even.

In order to illustrate Corollary 3.2.2, we give the following example:

Example 3.2.2

Let $n_1 = 678953356$, $n_2 = 678957854$ and $n = 2$,then

$$(n_1 + n_2) + (n_2 - n_1)^2 = (678953356 + 678957854) +$$
$$(678957854 - 678953356)^2 = 1{,}357{,}911{,}210 + 20{,}232{,}004$$
$$= 1{,}378{,}143{,}214 \in 2k,$$
$$\forall k \in \mathbb{N}$$

Theorem 3.2.2

let n_1 and $n_2 \in E$,then $(n_1 + n_2) + (n_2 - n_1)^n$ is even for all $n_2 > n_1$ and $(n \geq 1) \in \mathbb{N}$. (3.2.2.1)

Proof

From expressions (3.2.1.2) and (3.2.2.1) it is clear that $(n_1 + n_2)$ in theorem (3.2.2) is even , and we are left to prove that $(n_2 - n_1)^n$ is also even.

$\Rightarrow (2k_2 - 2k_1)^n \Rightarrow 2^n(k_2 - k_1)^n = 2(2^{n-1}(k_2 - k_1)^n)$,where $(k_2 - k_1)^n \in \mathbb{N}$

$\Rightarrow (2^{n-1}(k_2 - k_1)^n) \in \mathbb{N} \Rightarrow 2(2^{n-1}(k_2 - k_1)^n)$ is even from the definition of even numbers. This further implies that $(2k_1 + 2k_2) + (2k_2 - 2k_1)^n$ is even.

In order to illustrate Theorem 3.2.2, we give the following example:

Example 3.2.3

Let $n_1 = 4$, $n_2 = 12$ and $n = 10$,then

$$(n_1 + n_2) + (n_2 - n_1)^{10} = (4 + 12) + (12 - 4)^{10} = 16 + 1{,}073{,}741{,}824 = 1{,}073{,}741{,}840 \in 2k, \forall k \in \mathbb{N}$$

CASE 2

This section introduces Corollary 3.2.3 and proves the new representation of even numbers for odd values of $n_1 and\ n_2$ using values of $n = 1, 2,$ and then generalize the proof for all values of n.

Corollary 3.2.3

Let $n_1$and $n_2 \in O$ $\forall$ k$\in \mathbb{N}$ and $n = 1$, then $(n_1 + n_2) + (n_2 - n_1)^1$ is even. (3.2.3.1)

Proof

Since n_1 and n_2 are both odd, from the definition of odd numbers, they can be written as $n_1 = 2k_1 + 1, \forall k_1 \in \mathbb{N}$ and $n_2 = 2k_2 + 1, \forall k_2 \in \mathbb{N}$. Substituting $n_1 = 2k_1 + 1$ and $n_2 = 2k_2 + 1$ in expression (3.2.3.1) yields $(2k_1 + 1 + 2k_2 + 1) + (2k_2 + 1 - 2k_1 - 1)^1$. Since $n_2 > n_1$ then it implies that $k_2 > k_1 \Rightarrow k_2 - k_1 > 0$.

$\Rightarrow 2(k_1 + k_2 + 1) + 2(k_2 - k_1)^1$and from the definition of even numbers, both $2(k_1 + k_2 + 1)$ for $(k_1 + k_2 + 1) \in \mathbb{N}$ and $2(k_2 - k_1)$ are even, hence $2(k_1 + k_2 + 1) + 2(k_2 - k_1)^1$ is even.

In order to illustrate Corollary 3.2.3, we give the following example:

Example 3.2.4

Let $n_1 = 1455674459$, $n_2 = 3578166737$ and $n = 1$,then

$$(n_1 + n_2) + (n_2 - n_1)^1 = (1455674459 + 3578166737) + (3578166737 - 1455674459)^1 =$$
$$5{,}033{,}841{,}196 + 2{,}122{,}492{,}278 = 7{,}156{,}333{,}474 \in 2k,$$
$$\forall k \in \mathbb{N}$$

Corollary 3.2.4

Let n_1 and $n_2 \in O$ for $\forall$ k$\in \mathbb{N}$ and $n = 2$ then $(n_1 + n_2) + (n_2 - n_1)^2$ is even. (3.2.4.1)

Proof

Substituting $n_1 = 2k_1 + 1, \forall k_1 \in \mathbb{N}$ and $n_2 = 2k_2 + 1, \forall k_2 \in \mathbb{N}$ in expression (3.2.4.1) yields

$(2k_1 + 2k_2 + 2) + (2k_2 - 2k_1)^2$

$\Rightarrow 2(k_1 + k_2 + 1) + 2^2(k_2 - k_1)^2 \Rightarrow 2(k_1 + k_2 + 1) + 4(k_2{}^2 - 2k_2k_1 + k_1{}^2) \Rightarrow$

$2(k_1 + k_2 + 1) + 2(2(k_2{}^2 - 2k_2k_1 + k_1{}^2))$

From the definition of an even number, we have that $2(k_1 + k_2 + 1)$ and $2(2(k_2^2 - 2k_2k_1 + k_1^2)$ are both even proving expression (3.2.4.1).

In order to illustrate Corollary 3.2.4, we give the following example:

Example 3.2.5

Let $n_1 = 501247$, $n_2 = 501259$ and $n = 2$,then

$(n_1 + n_2) + (n_2 - n_1)^2 = (501{,}247 + 501{,}259) + (501{,}259 - 501{,}247)^2$
$= 1{,}002{,}506 + 144 = 1{,}002{,}650 \in 2k,$

$\forall k \in \mathbb{N}$

Theorem 3.2.3

let n_1 and $n_2 \in O$,then $(n_1 + n_2) + (n_2 - n_1)^n$ is even for all $n_2 > n_1$ and $(n \geq 1) \in \mathbb{N}$. (3.2.3.1)

Proof

From expressions (3.2.3.1) in corollary 3.2.3 and (3.2.4.1) it is clear that $(n_1 + n_2)$ in expression (3.2.3.1) in theorem 3.2.3 is even. we are therefore left to prove that $(n_2 - n_1)^n$ is also even.

$(2k_2 - 2k_1)^n \Rightarrow$, $(2k_2 - 2k_1)^n \Rightarrow 2^n(k_2 - k_1)^n = 2(2^{n-1}(k_2 - k_1)^n)$ where $(k_2 - k_1)^n \in \mathbb{N}$

$\Rightarrow (2^{n-1}(k_2 - k_1)^n) \in \mathbb{N} \Rightarrow 2[(2^{n-1}(k_2 - k_1)^n]$ is even. This further implies that $(2k_2 - 2k_1)^n$ is even.

In order to illustrate Theorem 3.2.3, we give the following example:

Example 3.2.6

Let $n_1 = 5$, $n_2 = 7$ and $n = 20$,then

$$(n_1 + n_2) + (n_2 - n_1)^{20} = (5+7) + (7-5)^{20} = 12 + 1{,}048{,}576 = 1{,}048{,}588 \in 2k,\ \forall k \in \mathbb{N}$$

CASE 3

This section gives a brief overview of prime numbers and focus on the new formulation of even numbers from prime numbers.

3.3 Brief Overview of Prime Numbers

Prime numbers are important in mathematics and computer science because they are used in cryptography. Cryptography and prime numbers have a significant and fascinating relationship. Prime numbers play a crucial role in modern cryptographic algorithms, providing the foundation for secure communication and data protection (Kościelny *et al.,* 2013). Odd numbers, on the other hand, are more commonly used in basic arithmetic and algebra. Further discussion on odd numbers and their relationship with even numbers will be done in Chapter 4.

Prime numbers greater than 2 are subsets of odd numbers and therefore the results obtained in theorem 3.2.3 covers the proof for both composite odd numbers and prime numbers. The theorem therefore proves expression (3.3.1.1).

Corollary 3.3.1

Let p_1 and $p_2 \in P$,then $(P_1 + P_2) + (P_2 - P_1)^n$ is even $\forall n \in \mathbb{N}$ and $p_2 > p_1$ (3.3.1.1)

This expression will have remarkable application in generating all pairs of odd numbers associated with a given even number of form in expression (3.3.1.1). These results will be used to investigate the possibility of obtaining at least one Goldbach partition from these pairs of odd numbers.

In order to illustrate Corollary 3.3.1, we give the following example:

Example 3.3.1

Let $n_1 = 11$, $n_2 = 13$ and $n = 2$,then
$(n_1 + n_2) + (n_2 - n_1)^2 = (11 + 13) + (13 - 11)^2 = 24 + 4 = 28 \in 2k, \ \forall k \in \mathbb{N}$

CHAPTER FOUR: GENERATION OF PAIRS OF ODD NUMBERS AND THE PROOF OF STRONG GOLDBACH'S CONJECTURE

4.0 Introduction

The chapter is divided into three sections. Section one gives a brief introduction of odd numbers whereas section two deals with the generation of all pairs of odd numbers from partitioning any given even number into all pairs of odd numbers. The study also presents an algorithm and the general proof of partitioning any even number into all pairs of odd numbers. In section three, it is shown that there exist at least one Goldbach partition in these pairs of odd numbers generated and the Strong Goldbach Conjecture is proven. Finally, the study shows that the algorithm used in partitioning any even number into all pairs of odd numbers could have ground breaking solutions to the Weak Goldbach Conjecture and Twin Prime conjecture.

4.1 Basic Known Mathematical Concepts on Odd Numbers

In Mathematics, odd numbers are a type of integers, which are whole numbers that can be positive or negative. An odd number is any integer that is not divisible by 2, meaning that it has a remainder of 1 when divided by 2. They are important in mathematics and can be used in many different ways. For instance, odd numbers can be used in basic arithmetic to perform operations like addition, subtraction, multiplication, and division. They can also be used in algebra to solve equations and manipulate expressions (What Are Odd Numbers? Definition, Examples, Properties, All Odd Numbers, n.d.).

Odd numbers can be added or subtracted just like any other number. When odd numbers are added or subtracted together, the result is always an even number. However, when an even number is added or subtracted from an odd number, the result is always an odd number (What Are Odd Numbers? Definition, Examples, Properties, All Odd Numbers, n.d.).

Odd numbers have several interesting properties in mathematics, including their role in prime numbers and Number Theory like its contribution to the study of divisibility, prime numbers, arithmetic operations (Odd Numbers ⋆ Definition, Properties, List, Examples, n.d.), congruence relations, recurrence relations, and solving Diophantine equations. Their properties and relationships form the basis for many number-theoretic investigations. They also play a significant role in geometry, where they are often used to construct regular polygons and other geometric figures (Hage-Hassan, 2023). While the role of odd numbers in geometry may not be as prominent as in number theory, they are still relevant in constructing regular polygons, determining symmetry, creating tessellations, forming geometric progressions, and counting diagonals. These applications demonstrate the interplay between numbers and geometric shapes.

The study of odd numbers and their properties has led to many important mathematical discoveries, including the famous Goldbach Conjectures. Although Odd numbers do not directly play a role in the statement of the Goldbach Conjecture, as the conjecture is only concerned with even numbers (Härdig, 2020), the odd numbers can be indirectly related to the conjecture through the use of parity.

Parity refers to whether a number is odd or even (Parity (Mathematics), 2022). Every even number can be expressed as the sum of two odd numbers (for example, 8 = 3 + 5) (Lin et al., 2016). Similarly, every odd number can be expressed as the sum of an even number and an odd number (for example, 7 = 4 + 3) (Algebraic Proof Of Arithmetic Results, n.d.). Using parity and the fact that every even number can be expressed as the sum of two prime numbers (according to the Strong Goldbach Conjecture), we can make some conjectures about odd numbers. For example, it is conjectured that every odd number greater than 5 can be expressed as the sum of three prime numbers (for example, 7 = 2 + 2 + 3). This is known as the Weak Goldbach Conjecture for odd numbers (Mateos, 2012).

4.2 Partitioning Any Even Number into All Pairs of Odd Numbers

4.2.1 Introduction

The statement of the Strong Goldbach Conjecture in its simple form gives a relationship between a given even number and two primes as $2n = p_i + p_j, \forall p_i, p_j \in P, and, n \in \mathbb{N}$. However, the traditional definition of an even number as $2n$ does not directly bring out its relation to a pair of prime. The new formulation of a set of even numbers formulated on the other hand as $2n = (p_1 + p_2) + (p_2 - p_1)^n = p_i + p_j$ (according to Goldbach Conjecture, any even number greater than 4 can be partitioned into two pairs of primes) gives a direct relation between a given even number and two primes. This definition therefore makes it possible to generate an algorithm that allows any even number be partitioned into all pairs of odd numbers (primes numbers greater than 2 are subsets of odd numbers). The study presents the following algorithm as a general approach to partitioning any even number into all pairs of odd numbers.

It has been shown in chapter 3 that the new formulation of a set of even numbers $(n_1 + n_2) + (n_2 - n_1)^n$ will always be even for all $n_2 > n_1$ and $(n \geq 1) \in \mathbb{N}$, where $\mathbb{N}$ is the set of all natural numbers. From this formulation, one is able to obtain an expression of the formulation of a new set of even numbers , $(P_1 + P_2) + (P_2 - P_1)^n$ from two prime numbers p_1 and $p_2 \in P$. The study further proposes following algorithm that partitions any even number into all pairs of odd numbers:

Step 1 : Let P_1 and $P_2 \in P$,then $(P_1 + P_2) + (P_2 - P_1)^n$ is even, $\forall n \in \mathbb{N}$, and $p_2 > p_1$.

Step 2: Let d be even and belong to the half-open interval$[1, [\frac{1}{2}((p_1 + p_2) + (p_2 - p_1)^n]]$.

Step 3: Let $z_i, and, y_i \in 1 \leq O \leq \frac{1}{2}((P_1 + P_2) + (P_2 - P_1)^n)$ for $d = p_2 - p_1$ since $p_2 > p_1$ and $i \in 1 \leq O \leq (\frac{1}{2}(p_1 + p_2) + (p_2 - p_1)^n)$.

Remark 2:

The set $y_i \in O$ since the difference between an even number and an odd number is odd.

With p_1, p_2, d and z_i, we partition $(p_1 + p_2) + (p_2 - p_1)^n$ as follows:

Partition 1 : $(p_1 + p_2) + (p_2 - p_1)^n - (d + z_1) = y_1$

Partition 2 : $(p_1 + p_2) + (p_2 - p_1)^n - (d + z_3) = y_3$

Partition 3: $(p_1 + p_2) + (p_2 - p_1)^n - (d + z_5) = y_5$

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Partition i: Partition i: $((p_1 + p_2) + (p_2 - p_1)^n)$ -
$(d + z_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)}) = y_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)}$

The set of pairs $(d + z_1, y_1), (d + z_3, y_3), (d + z_5, y_5), \ldots,$

$(d + z_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)}, y_{((\frac{1}{2}((p_1+p_2)+(p_2-p_1)^n)-1)})$ of odd numbers are all partitions of the even number $(p_1 + p_2) + (p_2 - p_1)^n$.

4.2.2 Illustration of the algorithm of partitioning an even number into all pairs of odd numbers for n = 1,2,3.

Example 4.2.2.1

Step 1 : Let $p_1 = 3$, $p_2 = 5$ and $n = 1$,then

$(p_1 + p_2) + (p_2 - p_1)^1 = (3+5) + (5-3)^1 = 8 + 2 = 10$ is even.

Step 2 : obtain $d = p_2 - p_1, d = 5 - 3 = 2 > 0$

Step 3 : Take odd numbers in the range $1 \le O \le \frac{1}{2}(10) \to (1,3,5)$

Then, using $d = 2$ and the set of odd numbers in step 3,we partition 10 as follows:

I. $10 - (2+1) = 7$

II. $10 - (2+3) = 5$

III. $10 - (2+5) = 3$

The partitions of 10 are therefore: $\big((2+1),7\big), \big((2+3),5\big), ((2+5),3) \quad \Rightarrow$

$(3,7), (5,5), (7,3)$. It should be noted that the three pairs generated here are all pairs of odd numbers. Therefore, the algorithm has partitioned 10 into 3 pairs of odd numbers.

Example 4.2.2.2

Step 1 : Let $p_1 = 13$, $p_2 = 23$ and $n = 1$,then

$(p_1 + p_2) + (p_2 - p_1)^1 = (13+23) + (23-13)^1 = 36 + 10 = 46$ is even.

Step 2 : we take , $d = 23 - 13 = 10 > 0$

Step 3 : Take odd numbers in the range $1 \le O \le \frac{1}{2}(46) \to (1,3,5,7,9,11,13,15,17,19,21,23)$

Then, using $d = 10$ and the set of odd numbers in step 3, we partition 46 as follows:

I. $46-(10+1)=35$ VII. $46-(10+13)=23$

II. $46-(10+3)=33$ VIII. $46-(10+15)=21$

III. $46-(10+5)=31$ IX. $46-(10+17)=19$

IV. $46-(10+7)=29$ X. $46-(10+19)=17$

V. $46-(10+9)=27$ XI. $46-(10+21)=15$

VI. $46-(10+11)=25$ XII. $46-(10+23)=13$

The partitions of 46 are therefore: $\big((10+1),35\big),\big((10+3),33\big),\big((10+5),31\big),\big((10+7),29\big),\big((10+9),27\big),\big((10+11),25\big),\big((10+13),23\big),\big((10+15),21\big),\big((10+17),19\big),\big((10+19),17\big),\big((10+21),15\big),((10+23),13) \Rightarrow$

$(11,35),(13,33),(15,31),(17,29),(19,27),(21,25),(23,23),(25,21),(27,19),(29,17),(31,15),$

$(33,13)$. Therefore, the algorithm has partitioned 46 into 12 pairs of odd numbers.

The algorithm has partitioned 10 and 46 into all pairs of odd numbers for $n=1$ and $d=2$ and $d=10$ respectively. The same algorithm can be used to partition 10 and 46 into the same pairs of odd numbers for $n=1$ with any multiples of d belonging to the half-open interval$[1,\ [\tfrac{1}{2}(\ (p_1+p_2)+(p_2-p_1)^n]]$. The following theorem gives a general approach to partitioning an even number into pairs of odd numbers for $n=1$ and any multiples of d belonging to the half-open interval$[1,\ [\tfrac{1}{2}(\ (p_1+p_2)+(p_2-p_1)^n]]$.

Theorem 4.2.2.1

Let p_1 and $p_2 \in P$, where P is the set of all prime numbers and let d be the difference between p_1 and p_2 such that $d>0$ and Let $z_i \in 1 \le O \le \tfrac{1}{2}(\ {}^{(p_1+p_2)+(p_2-p_1)^n})$ be the set of odd numbers for $d=p_2-p_1$ since $p_2>p_1$ and $i \in 1 \le \mathbb{N} \le (\tfrac{1}{2}(p_1+p_2)+(p_2-p_1)^n)$ then any multiple of d in the range $1 < d \leq \frac{1}{2}((p_1 + p_2) + (p_2 - p_1)^n)$ can be used to generate the same set of all pairs of odd numbers whose sum is $(p_1 + p_2) + (p_2 - p_1)^n$.

Remark 3 (for Theorem 4.2.2.1)

For $p_1 = 3$, $p_2 = 5$ and $n = 1$, then $(p_1 + p_2) + (p_2 - p_1)^1 = (3+5) + (5-3)^1 = 8 + 2 = 10$, then the set of even numbers (multiples of d) in the range $d \in 1 \leq E \leq \frac{1}{2}(\ (p_1 + p_2) + (p_2 - p_1)^n)$ are: $2, and\ 4$. Theorem 4.2.2.1 confirms that any of these even numbers (multiples of d) will generate the same pairs of odd partitions of 10.

Example 4.2.2.3

Step 1 : Let $p_1 = 3$, $p_2 = 5$ and $n = 1$, then $(p_1 + p_2) + (p_2 - p_1)^1 = (3+5) + (5-3)^1 = 8 + 2 = 10$ is even.

Step 2 : let $d = 4$

Step 3 : Take odd numbers in the range $1 \leq O \leq \frac{1}{2}(\ 10\ ) \rightarrow (1,3,5)$

Then we partition 10 as follows for $d = 4$:

I. $10 - (4+1) = 5$

II. $10 - (4+3) = 3$

III. $10 - (4+5) = 1$

The partitions of 10 are therefore: $\big((4+1), 5\big), \big((4+3), 3\big), ((4+5), 1) \quad \Rightarrow$ $(5,5), (7,3), (9,1)$. It should be noted that the three partitions of 10 generated here are all pairs of odd numbers and the two pairs $(5,5)\ and\ (7,3)$ are the same pairs of odd numbers generated when $d = 2$.

Similar approach can be used to partition 46 into pairs of odd numbers using any multiple of d in the range $d \in 1 < E \leq \frac{1}{2}(46) \quad = \{2,4,6,8,10,12,14,16,18,20, and, 22\}$. We illustrate using examples that for $d = 22, 20$ and 8 (any other multiple of d belonging to this set generates similar results) the algorithm partitions 46 into the same set of pairs of odd numbers as the set of odd numbers generated using $d = 10$ or the algorithm will generate a subset of pairs of odd numbers belonging to the set of odd numbers generated using $d = 10$.

Example 4.2.2.4

Step 1: Let $E = 46$ and $n = 1$

Step 2 : $d = 22$

Step 3 : Take odd numbers in the range $1 \leq O \leq \frac{1}{2}(46) \rightarrow (1,3,5,7,9,11,13,15,17,19,21,23)$

Then we partition 46 as follows:

I.	$46-(22+1)=23$	VII.	$46-(22+13)=11$
II.	$46-(22+3)=21$	VIII.	$46-(22+15)=9$
III.	$46-(22+5)=19$	IX.	$46-(22+17)=7$
IV.	$46-(22+7)=17$	X.	$46-(22+19)=5$
V.	$46-(22+9)=15$	XI.	$46-(22+21)=3$
VI.	$46-(22+11)=13$	XII.	$46-(22+23)=1$

The partitions of 46 are therefore: $\big((22+1),23\big),\big((22+3),21\big),\big((22+5),19\big),\big((22+7),17\big),\big((22+9),15\big),\big((22+11),13\big),\big((22+13),11\big),\big((22+15),9\big),\big((22+17),7\big),\big((22+19),5\big),\big((22+21),3\big),((22+23),1) \quad \Rightarrow$

$(23{,}23), (25{,}21), (27{,}19), (29{,}17), (31{,}15), (33{,}13), (35{,}11), (37{,}9), (39{,}7), (41{,}5), (43{,}3)$

and (45,1)

Which are all pairs of odd numbers. Therefore, using $d = 22$ the algorithm has partitioned 46 into a set containing the same 12 pairs of odd numbers as $d = 10$.

Example 4.2.2.5

Step 1 : Let $E = 46$ and $n = 1$

Step 2 : $d = 20$

Step 3 : Take odd numbers in the range $1 \le O \le \frac{1}{2}(46) \rightarrow (1,3,5,7,9,11,13,15,17,19,21,23)$

Then we partition 46 as follows:

I. $46 - (20 + 1) = 25$

II. $46 - (20 + 3) = 23$

III. $46 - (20 + 5) = 21$

IV. $46 - (20 + 7) = 19$

V. $46 - (20 + 9) = 17$

VI. $46 - (20 + 11) = 15$

VII. $46 - (20 + 13) = 13$

VIII. $46 - (20 + 15) = 11$

IX. $46 - (20 + 17) = 9$

X. $46 - (20 + 19) = 7$

XI. $46 - (20 + 21) = 5$

XII. $46 - (20 + 23) = 3$

The partitions of 46 are therefore: $\big((20 + 1), 25\big), \big((20 + 3), 23\big), \big((20 + 5), 21\big), \big((20 + 7), 19\big), \big((20 + 9), 17\big), \big((20 + 11), 15\big), \big((20 + 13), 13\big), \big((20 + 15), 11\big), \big((20 + 17), 9\big), \big((20 + 19), 7\big), \big((20 + 21), 5\big), ((20 + 23), 3) \quad \Rightarrow$

$(21,25), (23,23), (25,21), (31,15), (27,19), (29,17), (31,15), (33,13), (35,11), (37,9), (39,7), (41,5) and\ (43,3)$

Which are all pairs of odd numbers. Therefore, using $d = 20$ the algorithm has partitioned 46 into the set containing the same 12 pairs of odd numbers as $d = 10\ and\ 22$.

Example 4.2.2.6

Step 1 : Let $E = 46$ and $n = 1$

Step 2 : $d = 8$

Step 3 : Take odd numbers in the range $1 \leq O \leq \frac{1}{2}(46) \rightarrow (1,3,5,7,9,11,13,15,17,19,21,23)$

Then we partition 46 as follows:

I.	$46-(8+1)=37$	VII.	$46-(8+13)=25$
II.	$46-(8+3)=35$	VIII.	$46-(8+15)=23$
III.	$46-(8+5)=33$	IX.	$46-(8+17)=21$
IV.	$46-(8+7)=31$	X.	$46-(8+19)=19$
V.	$46-(8+9)=29$	XI.	$46-(8+21)=17$
VI.	$46-(8+11)=27$	XII.	$46-(8+23)=15$

The partitions of 46 are therefore: $\big((8+1),37\big),\big((8+3),35\big),\big((8+5),33\big),\big((8+7),31\big),\big((8+9),29\big),\big((8+11),27\big),\big((8+13),25\big),\big((8+15),23\big),\big((8+17),21\big),\big((8+19),19\big),\big((8+21),17\big),((8+23),15) \quad \Rightarrow$

$(9,37),(11,35),(13,33),(15,31),(17,29),(19,27),(21,25),(23,23),(25,21),(27,19),(29,17)$, $(41,5)$ *and* $(31,15)$. Which are all pairs of odd numbers. Therefore, using $d = 8$ the algorithm has partitioned 46 into the set containing the same 12 pairs of odd numbers as $d = 10, 20$ *and* 22.

Therefore, using $d = 2,4,6,12,14,16$ *or* 18 , the algorithm partitions 46 into the same 12 pairs of odd numbers as $d = 10, 20$ *and* 22.

In general, any multiple of d in the range $1 \leq d \leq \frac{1}{2}((p_1 + p_2) + (p_2 - p_1)^n)$ can be used to generate the same set of all pairs of odd numbers whose sum is $(p_1 + p_2) + (p_2 - p_1)^n$. The same algorithm can be used to partition a given even number of the form $(p_1 + p_2) + (p_2 - p_1)^n$ using any multiple of d in the range $\frac{1}{2}((p_1 + p_2) + (p_2 - p_1)^n) < d < (p_1 + p_2) + (p_2 - p_1)^n$ into pairs of odd numbers belong to the a set or a subset of pairs of odd numbers generated using any multiple of d in the range $1 \leq d \leq \frac{1}{2}((p_1 + p_2) + (p_2 - p_1)^n)$. The study introduces the following theorem 4.2.2.2 that summarizes the partitioning of an even number into pairs of odd numbers using multiples of d greater than half the given even number.

Theorem 4.2.2.2

Let p_1 $and\, p_2 \in P$, where P is the set of all primes and let d be the difference between p_1 $and\, p_2$ such that $d > 0$ and Let $z_i \in 1 \leq O \leq \frac{1}{2}((p_1 + p_2) + (p_2 - p_1)^n)$ be the set of odd numbers. For $d = p_2 - p_1 > 0$ since $p_2 > p_1$ and $i \in 1 \leq \mathbb{N} \leq (\frac{1}{2}(p_1 + p_2) + (p_2 - p_1)^n)$, then any multiple of d in the range $\frac{1}{2}((p_1 + p_2) + (p_2 - p_1)^n) < d < (p_1 + p_2) + (p_2 - p_1)^n$ can be used to generate the same set of odd numbers or a subset of pairs of odd numbers whose sum is $(p_1 + p_2) + (p_2 - p_1)^n$. It should be noted that for this set of values of d greater than half the even number, as the value of d gets closer to $(p_1 + p_2) + (p_2 - p_1)^n$,the set of pairs of odd numbers generated reduces significantly.

N/B. The multiples of d in the range $\frac{1}{2}((p_1 + p_2) + (p_2 - p_1)^n) < d < (p_1 + p_2) + (p_2 - p_1)^n$ are :$24,26,28,30,32,34,36,38,40,42$ and 44. The study chooses to use $d = 24,30,40$ and 44 to verify theorem 4.2.2.2.

Example 4.2.2.7

Step 1: Let $E = 46$ and $n = 1$

Step 2: $d = 24$

Step 3: Take odd numbers in the range $1 \le O \le \frac{1}{2}(46) \to (1,3,5,7,9,11,13,15,17,19,21,23)$

Then we partition 46 as follows:

I.	$46-(24+1)=21$	VII.	$46-(24+13)=9$
II.	$46-(24+3)=19$	VIII.	$46-(24+15)=7$
III.	$46-(24+5)=17$	IX.	$46-(24+17)=5$
IV.	$46-(24+7)=15$	X.	$46-(24+19)=3$
V.	$46-(24+9)=13$	XI.	$46-(24+21)=1$
VI.	$46-(24+11)=11$	XII.	$46-(24+23)=-1$

The partitions of 46 are therefore: $\big((24+1),21\big),\big((24+3),19\big),\big((24+5),17\big),\big((24+7),15\big),\big((24+9),13\big),\big((24+11),11\big),\big((24+13),9\big),\big((24+15),7\big),\big((24+17),5\big),\big((24+19),3\big),\big((24+21),1\big),((24+23),-1) \quad \Rightarrow$

$(25{,}21),(27{,}19),(29{,}17),(31{,}15),(33{,}13),(35{,}11),(37{,}9),(39{,}7),(41{,}5),(43{,}3),(45{,}1).$

Which are all pairs of odd numbers. Therefore, using $d = 24$ the algorithm has partitioned 46 into a subset of 11 pairs of odd numbers belonging to the set pairs of odd numbers generated by $d = 8, 10, 20\ or\ 22$.

Notice that we don't consider the pair , $\big((24+23),-1\big) = (47,-1)$ to be part of the partition of 46 because of the $-1(\,which\ is\ not\ positive)$ and the fact that 47 is larger than 46.

Example 4.2.2.8

Step 1: Let $E = 46$ and $n = 1$

Step 2: $d = 30$

Step 3: Take odd numbers in the range $1 \le O \le \frac{1}{2}(46) \to (1,3,5,7,9,11,13,15,17,19,21,23)$

Then we partition 46 as follows:

I.	$46-(30+1)=15$	VII.	$46-(30+13)=3$
II.	$46-(30+3)=13$	VIII.	$46-(30+15)=1$
III.	$46-(30+5)=11$	IX.	$46-(30+17)=-1$
IV.	$46-(30+7)=9$	X.	$46-(30+19)=-3$
V.	$46-(30+9)=7$	XI.	$46-(30+21)=-5$
VI.	$46-(30+11)=5$	XII.	$46-(30+23)=-7$

The positive partitions of 46 are therefore : $\big((30+1),15\big),\big((30+3),13\big),\big((30+5),11\big),\big((30+7),9\big),\big((30+9),7\big),\big((30+11),5\big),\big((30+13),3\big),\big((30+15),1\big) \Rightarrow$

$(31,15),(33,13),(35,11),(37,9),(39,7),(41,5),(43,3)\ and\ (45,1)$. Which are all positive pairs of odd numbers. Therefore, using $d = 30$, the algorithm has partitioned 46 into a subset of 8 positive pairs of odd numbers belonging to the set generated by $d = 8, 10, 20\ or\ 22$.

Example 4.2.2.9

Step 1 : Let $E = 46$ and $n = 1$

Step 2 : $d = 40$

Step 3 : Take odd numbers in the range $1 \le O \le \frac{1}{2}(46) \to (1,3,5,7,9,11,13,15,17,19,21,23)$

Then we partition 46 as follows:

I. $46-(40+1)=5$

II. $46-(40+3)=3$

III. $46-(40+5)=1$

IV. $46-(40+7)=-1$

V. $46-(40+9)=-3$

VI. $46-(40+11)=-5$

VII. $46-(40+13)=-7$

VIII. $46-(40+15)=-9$

IX. $46-(40+17)=-11$

X. $46-(40+19)=-13$

XI. $46-(40+21)=-15$

XII. $46-(40+23)=-17$

The positive partitions of 46 are therefore : $\big((40+1),5\big),\big((40+3),3\big),\big((40+5),11\big) \Rightarrow (41,5),(43,3)\ and\ (45,1)$. Which are all positive pairs of odd numbers. Therefore, using $d = 40$ the algorithm has partitioned 46 into a subset of 3 positive pairs of odd numbers belonging to the set of pairs of odd numbers generated by $d = 8, 10, 20\ or\ 22$.

Example 4.2.2.10

Step 1 : Let $E = 46$ and $n = 1$

Step 2 : $d = 44$

Step 3 : Take odd numbers in the range $1 \leq O \leq \frac{1}{2}(46) \rightarrow (1,3,5,7,9,11,13,15,17,19,21,23)$

Then we partition 46 as follows:

I. $46-(44+1)=1$

II. $46-(44+3)=-1$

III. $46-(44+5)=-3$

IV. $46-(44+7)=-5$

V. $46-(44+9)=-7$

VI. $46-(44+11)=-9$

VII. $46-(44+13)=-11$

VIII. $46-(44+15)=-13$

IX. $46-(44+17)=-15$

X. $46-(44+19)=-17$

XI. $46-(44+21)=-19$

XII. $46-(44+23)=-21$

$\Rightarrow$The positive partition of 46 is therefore : $\big((44+1),1\big) \Rightarrow (45,1)$.

It is noted here that as the value of d approaches $(p_1 + p_2) + (p_2 - p_1)^n$, the pairs of odd numbers whose sum is the given even numbers reduces significantly. In fact for the value of $d = 44$, only one positive pair of odd number is obtained.

4.3 Partitioning Any Even Number into All Pairs of Odd Numbers For $1 < n < \infty$.

The solution to the Strong Goldbach Conjecture lies in finding a method that will take an even number and partition it into at least a pair of primes. Since all prime numbers greater than 2 are subsets of odd numbers, another approach to the Goldbach problem, could be finding another line of attack that allows an even number to be partitioned into pairs of odd numbers.

The sum of two odd numbers will always be even (Prove that the sum of two odd numbers is even, n.d).This statement in its simple form gives a direct relationship between the sum of two odd numbers with a given even number. It confirms that any even number can always be partitioned into two odd numbers. This result however does not guarantee partitioning the even number into all pairs of odd numbers and this section of the thesis will provide a general proof of partitioning any even number of the form $(p_1 + p_2) + (p_2 - p_1)^n$ into all pairs of odd numbers. These results will have impressive application into the solution of the Strong Goldbach conjecture since we will show that in this set of pairs of odd numbers there exist at least one pair of prime. The following theorem provides a general proof of partitioning any even number of the form $(p_1 + p_2) + (p_2 - p_1)^n$ into all pairs of odd numbers. This theorem will have many applications into the solution of the Goldbach Conjectures and the Twin Prime Conjecture.

Theorem 4.3.1

Let p_1 $and\, p_2 \in P$, where P is the set of all primes, and let d be the difference between p_1 $and\, p_2$ such that $d > 0$ and Let $z_i \in 1 \leq O \leq \frac{1}{2}(\,(p_1 + p_2) + (p_2 - p_1)^n)$ be the set of odd numbers for $d = p_2 - p_1 > 0$ since $p_2 > p_1$ and $i \in 1 \leq \mathbb{N} \leq (\frac{1}{2}(p_1 + p_2) + (p_2 - p_1)^n)$, then any multiple of d in the range $1 < d \le (\frac{1}{2}((p_1 + p_2) + (p_2 - p_1)^n))$ can be used to partition $(p_1 + p_2) + (p_2 - p_1)^n$ into all pairs of odd numbers.

Proof

Let $p_1 \; and \; p_2 \in P$, where P is the set of all primes and let d be the difference between $p_1 \; and \; p_2$ such that $d > 0$. Then we show that $(p_1 + p_2) + (p_2 - p_1)^n$ can be partitioned into all pairs of odd numbers. Since $p_1 \; and \; p_2 \in P$ are odd numbers, they can be represented as $p_1 = 2k_1 + 1$ and $p_2 = 2k_2 + 1 \Rightarrow (p_1 + p_2) + (p_2 - p_1)^n$ can be partitioned as follows for all $z_i, and, y_i \in 1 \le O \le [\frac{1}{2}((p_1 + p_2) + (p_2 - p_1)^n)]$ and $i \in 1 \le \mathbb{N} \le (\frac{1}{2}(p_1 + p_2) + (p_2 - p_1)^n)$:

For Partition 1 : $((P_1 + P_2) + (P_2 - P_1)^n)$-$(d + z_1) = y_1$ in section 4.2.1, We need to prove that for the Even number $(P_1 + P_2) + (P_2 - P_1)^n$, both $d + z_1$ and y_1 are odd numbers. Since $(p_1 + p_2) + (p_2 - p_1)^n$ and d are both even and z_1 is odd, then it follows that partition 1 can be expressed as : $((P_1 + P_2) + (P_2 - P_1)^n - (d + z_1) \Rightarrow$

$$[((2k_1 + 1) + (2k_2 + 1) + (2k_3)^n)] - (2k_3 + 2k_4 + 1) \quad , \forall k_1, k_2, k_3, k_4 \in N$$

$$\Rightarrow ((2(k_1 + k_2 + (2^{n-1} k_3{}^n) \;\; + 1)) \;\; - (2k_3 + 2k_4 + 1)$$

Where, $p_1 = 2k_1 + 1, p_2 = 2k_2 + 1, d = 2k_3, and, z_1 = 2k_4 + 1$.

This further implies that $2(k_1 + k_2 + (2^{n-1} k_3^n) + 1) \in 2n$ and $(2k_3 + 2k_4 + 1) \in 2n_1 + 1$. We are to show that $[2(k_1 + k_2 + (2^{n-1} k_3^n) + 1)] - [(2k_3 + 2k_4 + 1)]$ belongs to the set of odd numbers. An odd number subtracted from an even number gives an odd number since, let $2n_1 = 2(k_1 + k_2 + (2^{n-1} k_3^n) + 1)$ and $2n_2 + 1 = 2k_3 + 2k_4 + 1$, then it follows that $2n_1 - (2n_2 + 1) = 2n_1 - 2n_2 - 1 \Rightarrow 2(n_1 - n_2) \in 2n$ and therefore $2n_3 - 1 \in 2n + 1$ for $n_3 = n_1 - n_2$.

This proves that $((P_1+P_2)+(P_2-P_1)^n)$-$(d+z_1)=y_1\in 2n+1$ and hence, the even number $(P_1+P_2)+(P_2-P_1)^n$ has been partitioned into a pair of odd numbers $d+z_1$ and y_1.

The same argument can be used to show that the even number $(P_1+P_2)+(P_2-P_1)^n$ can been partitioned into pairs of odd numbers for all the other partitions 2,3,4, …, and i in the algorithm we have presented for partitioning an even number into all pairs of odd numbers in section 4.2.1.These results proves that any even number of the form $(P_1+P_2)+(P_2-P_1)^n$ can be partitioned into all pairs of odd numbers.Further, the results show that in these pairs of odd numbers, there exists at least one Goldbach partition.

4.3.1 Verification of the proof for values of $n = 2, 3$ and 4.

For $n = 2$:

Step 1 : Let $p_1 = 3$, $p_2 = 5$ and $n = 2$,then

$(p_1+p_2)+(p_2-p_1)^2 = (3+5)+(5-3)^2 = 8+4 = 12$

is even.

Step 2 : $d = p_2 - p_1 > 0$, $d = 5-3 = 2 > 0$

Step 3 : Take odd numbers in the range $1 \le O \le \frac{1}{2}(12) \to (1,3,5)$

Then we partition 12 as follows:

I. $12-(2+1)=9$

II. $12-(2+3)=7$

III. $12-(2+5)=5$

The partitions of 12 are therefore: $((2+1),9),((2+3),7),((2+5),5) \Rightarrow$ $(3,9),(5,7),and\ (7,5)$, which are all pairs of odd numbers. Therefore, the algorithm has partitioned 12 into 3 pairs of odd numbers. The other multiples of d in the range $1 < d \leq \frac{1}{2}(12)$ $\Rightarrow$ 2,4,6 has been shown to yield similar results.

For $n = 3$:

Step 1 : Let $p_1 = 3$, $p_2 = 5$ and $n = 3$,then

$(p_1 + p_2) + (p_2 - p_1)^3 = (3+5) + (5-3)^3 = 8 + 8 = 16$ is even.

Step 2 : $d = p_2 - p_1 > 0$, $d = 5 - 3 = 2 > 0$

Step 3 : Take odd numbers in the range $1 \leq O \leq \frac{1}{2}(16) \rightarrow (1,3,5,7)$

Then we partition 16 as follows:

I. $16 - (2+1) = 13$

II. $16 - (2+3) = 11$

III. $16 - (2+5) = 9$

IV. $16 - (2+7) = 7$

The partitions of 16 are therefore : $((2+1),13),((2+3),11),((2+5),9),((2+7),7) \Rightarrow$ $(3,13),(5,11),(7,9)$, and $(9,7)$, which are all pairs of odd numbers. Therefore, the algorithm has partitioned 16 into 4 pairs of odd numbers. The other multiples of d in the range $1 < d \leq \frac{1}{2}(16) \Rightarrow$ 2,4,6,8 has been shown to yield similar results.

For $n = 4$:

Step 1 : Let $p_1 = 3$, $p_2 = 5$ and $n = 4$,then

$(p_1 + p_2) + (p_2 - p_1)^4 = (3+5) + (5-3)^4 = 8 + 16 = 24$ is even.

Step 2 : $d = p_2 - p_1 > 0$, $d = 5 - 3 = 2 > 0$

Step 3 : Take odd numbers in the range $1 \le O \le \frac{1}{2}(24) \to (1,3,5,7,9,11)$

Then we partition 24 as follows:

IV.	$24-(2+1)=21$	IV.	$24-(2+7)=15$
V.	$24-(2+3)=19$	V.	$24-(2+9)=13$
VI.	$24-(2+5)=17$	VI.	$24-(2+11)=11$

The partitions of 24 are therefore: $((2+1),21),((2+3),19),((2+5),17),((2+7),15),((2+9),13),((2+11),11) \Rightarrow (3,21),(5,19),(7,17),(9,15),(11,13),(13,11)$, which are all pairs of odd numbers. Therefore, the algorithm has partitioned 24 into 6 pairs of odd numbers. The other multiples of d in the range $1 \le d \le \frac{1}{2}(24) \Rightarrow 2,4,6,8,10,12$, has been shown to yield similar results. For example for $d = 12$:

Step 1 : Let $p_1 = 3$, $p_2 = 5$,and $n = 4$,then

$(p_1 + p_2) + (p_2 - p_1)^4 = (3+5) + (5-3)^4 = 8 + 16 = 24$ is even.

Step 2 : $d = 12$

Step 3 : Take odd numbers in the range $1 \le O \le \frac{1}{2}(24) \to (1,3,5,7,9,11)$

Then we partition 24 as follows:

I.	$24-(12+1)=11$	IV.	$24-(12+7)=5$
II.	$24-(12+3)=9$	V.	$24-(12+9)=3$
III.	$24-(12+5)=7$	VI.	$24-(12+11)=1$

The partitions of 24 are therefore : $\left((12+1),11\right),\left((12+3),9\right),\left((12+5),7\right),\left((12+7),5\right),\left((12+9),3\right),((12+11),1) \quad \Rightarrow$

$(13,11),(15,9),(17,7),(19,5),(21,3),(23,1)$.Which are all pairs of odd numbers. Therefore, the algorithm has partitioned 24 into 6 pairs of odd numbers as the ones generated by $d = 2$.

The algorithm for partitioning any even number of the form $(P_1+P_2)+(P_2-P_1)^n$ into all pairs of odd numbers works from two given prime numbers p_1 and p_2 that allows us form an even number that we are able to partition. It should be noted that this algorithm is not restricted to formulating the even number by starting with any two given prime numbers p_1 and p_2 in order to form the even number $(P_1+P_2)+(P_2-P_1)^n$. In fact given an even number $2n$, the algorithm allows us to obtain the multiples of d, such that $1 \le d \le \frac{1}{2}(2n)$ and the set of odd numbers $1 \le O \le \frac{1}{2}(2n)$ that we can use to partition $2n$ into all pairs of odd numbers. The following Corollary 4.3.1.1 gives a statement of partitioning an even number of the form $2n$ into all pairs of odd numbers.

Corollary 4.3.1.1

Let $2n$ be any even number and Let $z_i, y_i \in 1 \le O \le \frac{1}{2}(2n)$ be the set of odd numbers in such a way that $d \in 1 \le E \le \frac{1}{2}(2n)$, then $2n$ can be partitioned into a set of all pairs of odd numbers such that $2n-(d+z_i)=y_i \in O$.

4.3.2 Verification of the Corollary 6 for values of $2n = 500$ *and* $10,000$

For $\mathbf{2n = 500}$:

Step 1 : Let $2n = 500$

Step 2 : Then $d \in 1 < E \leq \frac{1}{2}(2n)$,say $d = \frac{1}{2}(500) = 250$

Step 3 : Take odd numbers in the range $1 \leq O \leq \frac{1}{2}(500)$, we obtain the set

(1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 125, 127, 129, 131, 133, 135, 137, 139, 141, 143, 145, 147, 149, 151, 153, 155, 157, 159, 161, 163, 165, 167, 169, 171, 173, 175, 177, 179, 181, 183, 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 231, 233, 235, 237, 239, 241, 243, 245, 247, 249)

It should be noted that, using the algorithm proposed here when $d = 250$, each of these odd numbers will partition 500 into a pair of odd number. In this example, we choose the set of odd numbers: $13, 37, 103, 127$ *and* 229 to partition 500 as follows:

I. $500 - (250 + 21) = 229$

II. $500 - (250 + 237) = 13$

III. $500 - (250 + 213) = 37$

IV. $500 - (250 + 147) = 103$

V. $500 - (250 + 123) = 127$

We obtain the following partitions of 500: $\big((250 + 21), 229\big), \big((250 + 237), 13\big), \big((250 + 213), 37\big), \big((250 + 147), 103\big), \big((250 + 123), 127\big) \quad \Rightarrow$

$(271,229), (487,13), (463,37), (397,103), (373,127)$. Which are all pairs of odd numbers.

For 2n = 10,000:

Step 1 : Let $2n = 10{,}000$

Step 2 : Then $d \in 1 < E \leq \frac{1}{2}(2n)$,say $d = \frac{1}{2}(10000) = 5000$

Step 3 : Some of the odd numbers belonging to the set $1 \leq O \leq \frac{1}{2}(10{,}000)$ are:

(…1987, 1993, 1997, 1999, 2003, 2011, 2017, 2027, 2029, 2039, 2053, 2063, 2069, 2081, 2083, 2087, 2089, 2099, 2111, 2113, 2129, 2131, 2137, 2141, 2143, 2153, 2161, 2179, 2203, 2207, 2213, 2221, 2237, 2239, 2243, 2251, 2267, 2269, 2273, 2281, 2287, 2293, 2297, 2309, 2311, 2333, 2339, 2341, 2347, 2351, 2357, 2371, 2377, 2381, 2383, 2389, 2393, 2399….7699, 7703, 7717, 7723, 7727, 7741, 7753, 7757, 7759, 7789, 7793, 7817, 7823, 7829, 7841, 7853, 7867, 7873, 7877, 7879, 7883, 7901, 7907, 7919…)

For $d = 5000$, each of these odd numbers will partition 10000 into pairs of odd numbers. In this example, we choose the set of odd numbers: $2{,}559, 2{,}919, 2{,}793\ and\ 2{,}727$ to partition $10{,}000$ as follows:

I. $10{,}000 - (5000 + 2{,}559) = 2{,}441$

II. $10{,}000 - (5000 + 2{,}919) = 2{,}081$

III. $10{,}000 - (5000 + 2{,}793) = 2{,}207$

IV. $10{,}000 - (5000 + 2{,}727) = 2{,}273$

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Some of the partitions obtained for 10,000 are: $\big((5{,}000 + 2{,}559), 2{,}441\big), \big((5000 + 2{,}919), 2{,}081\big), \big((5{,}000 + 2{,}793), 2{,}207\big), \big((5{,}000 + 2{,}727), 2{,}273\big) \Rightarrow (7559{,}2441), (7919{,}2081), (7793{,}2207), (7727{,}2273)$, which are all pairs of odd numbers.

4.3.3 Verification of partitioning any even number into all pairs of odd numbers for larger values of n up to 9×10^{18}

In order to be able to verify that the algorithm developed in Appendix 1 for partitioning any even number into all pairs of odd numbers works for larger numbers too, the interface containing an even number as an input and the partitions of the same even numbers as the output is displayed here for selected even numbers used as examples in this study. To illustrate this, the study seeks to show a few partitions of the even numbers selected as $2n = 1000000, 9989748, 100000000, 1000000000,$ and 9000000000000000000 as the pairs of odd numbers generated are too many. The interfaces in the figures below clearly shows how a given even number is partitioned into all pairs of odd numbers.

Remark 4: (software used in producing the following figures)

The algorithm in appendix 2 has been developed in Java using IDE (Integrated Development Environment) and NetBeans. When the program executes, it is able to display an interface where one is able to input an even number and the program displays all the partitions associated with the given even number. We have therefore taken the pictures of the output in the display to get the following figures:

For $2n = \mathbf{1,000,000}$, some partitions of this even number includes:

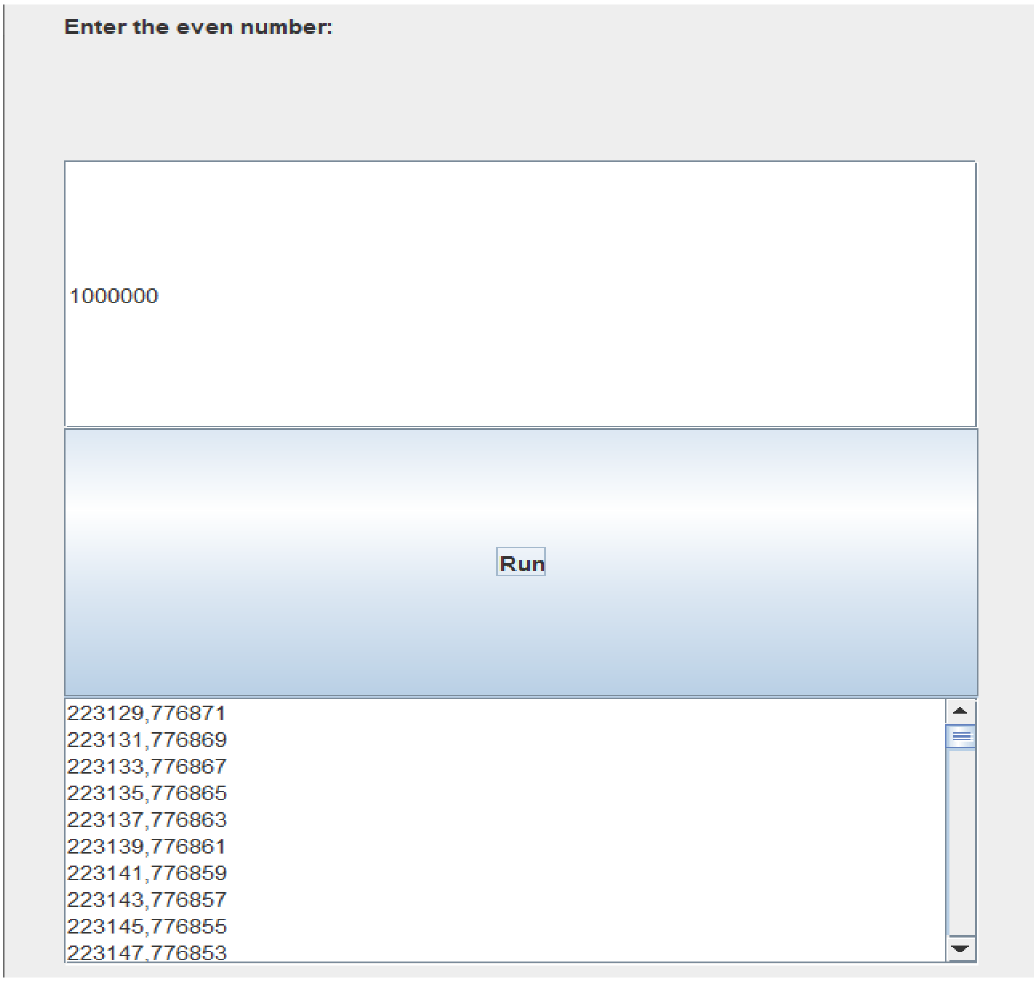

Figure 1 Partition the even number 2n=1,000,000 into pairs of odd numbers.

For $2n = \mathbf{9,989,748}$, some partitions of this even number includes:

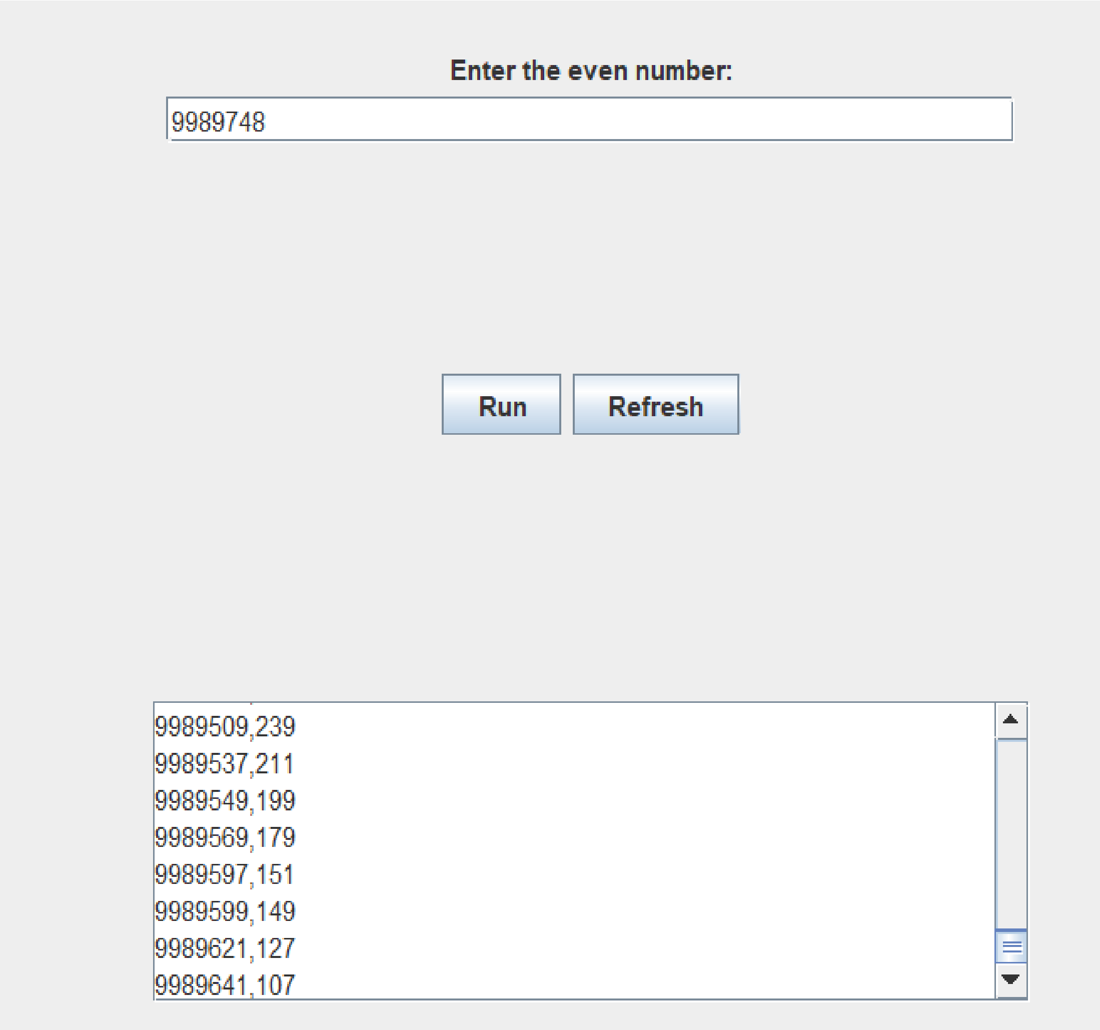

Figure 2 Partitions 9,989,748 into all pairs of odd numbers displaying a subset of the set of all partitions of odd numbers.

For $2n$ =**100,000,000** some partitions of this even number includes:

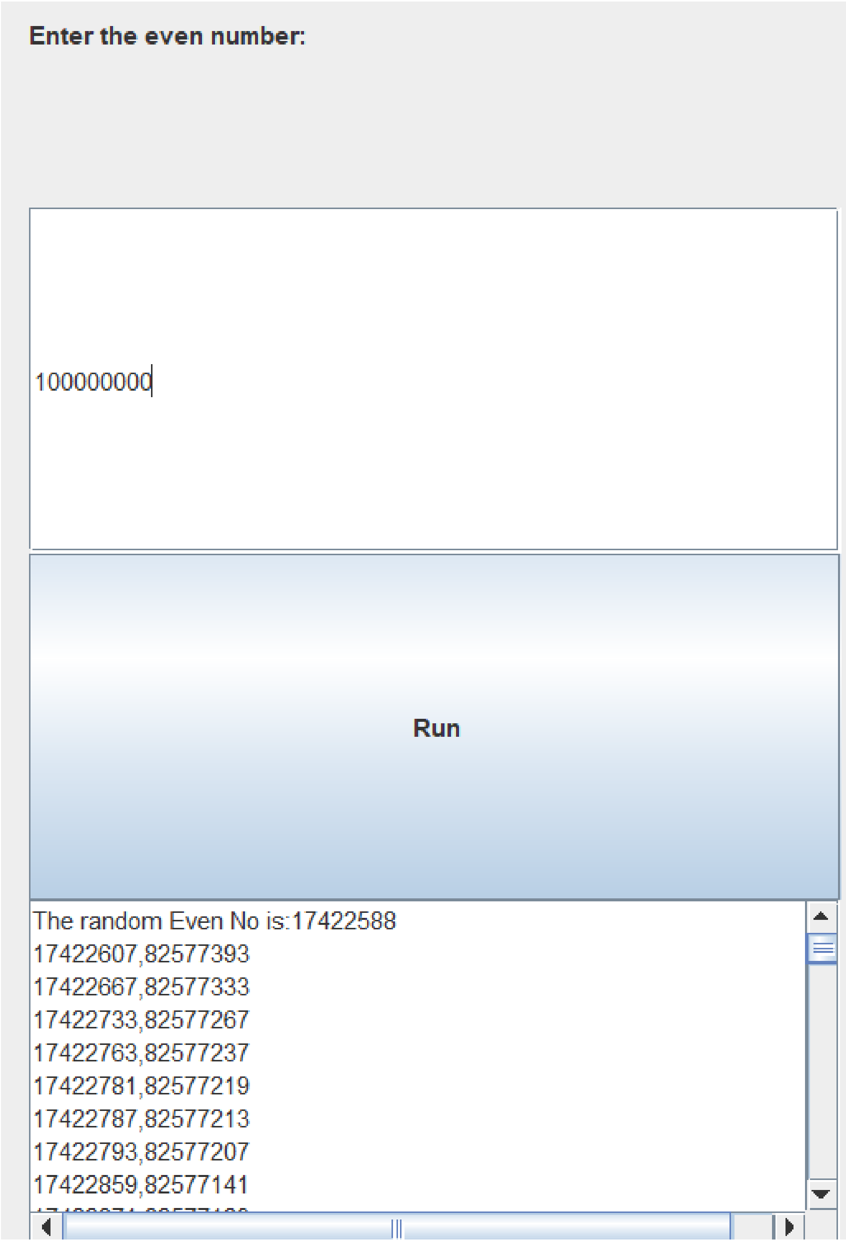

Figure 3 Partitions 100000000 into all pairs of odd numbers displaying a subset of the set of all partitions of odd numbers.

The program partitions 100000000 into all pairs of odd numbers but since the partitions are too many, we choose to display a subset of the set of all partitions of odd numbers.

For $2n = \mathbf{1,000,000,000}$, some partitions of this even number includes:

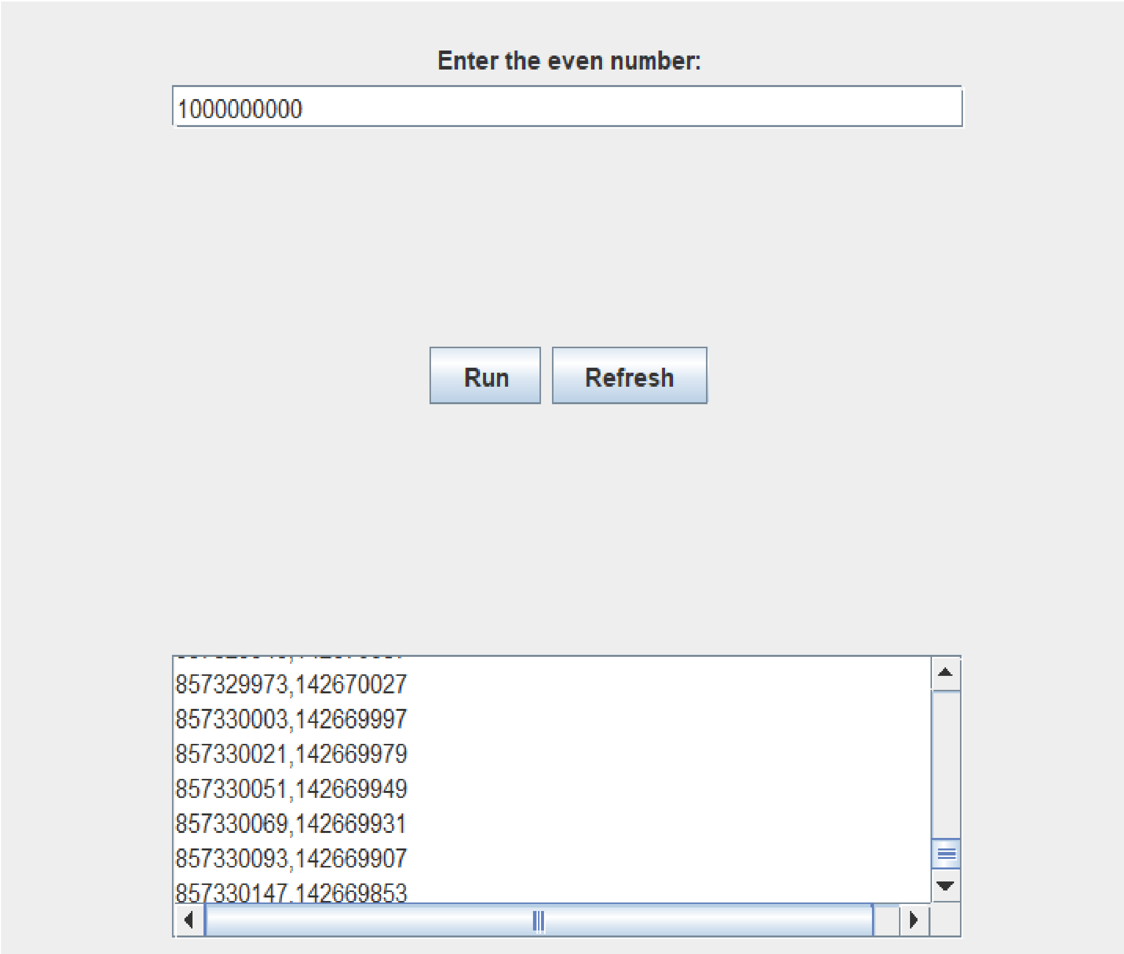

Figure 4 Partitions 1,000,000,000 into all pairs of odd numbers displaying a subset of the set of all partitions of odd numbers.

For $2n = \mathbf{1,000,000,000,000,000,000}$ some partitions of this even number includes:

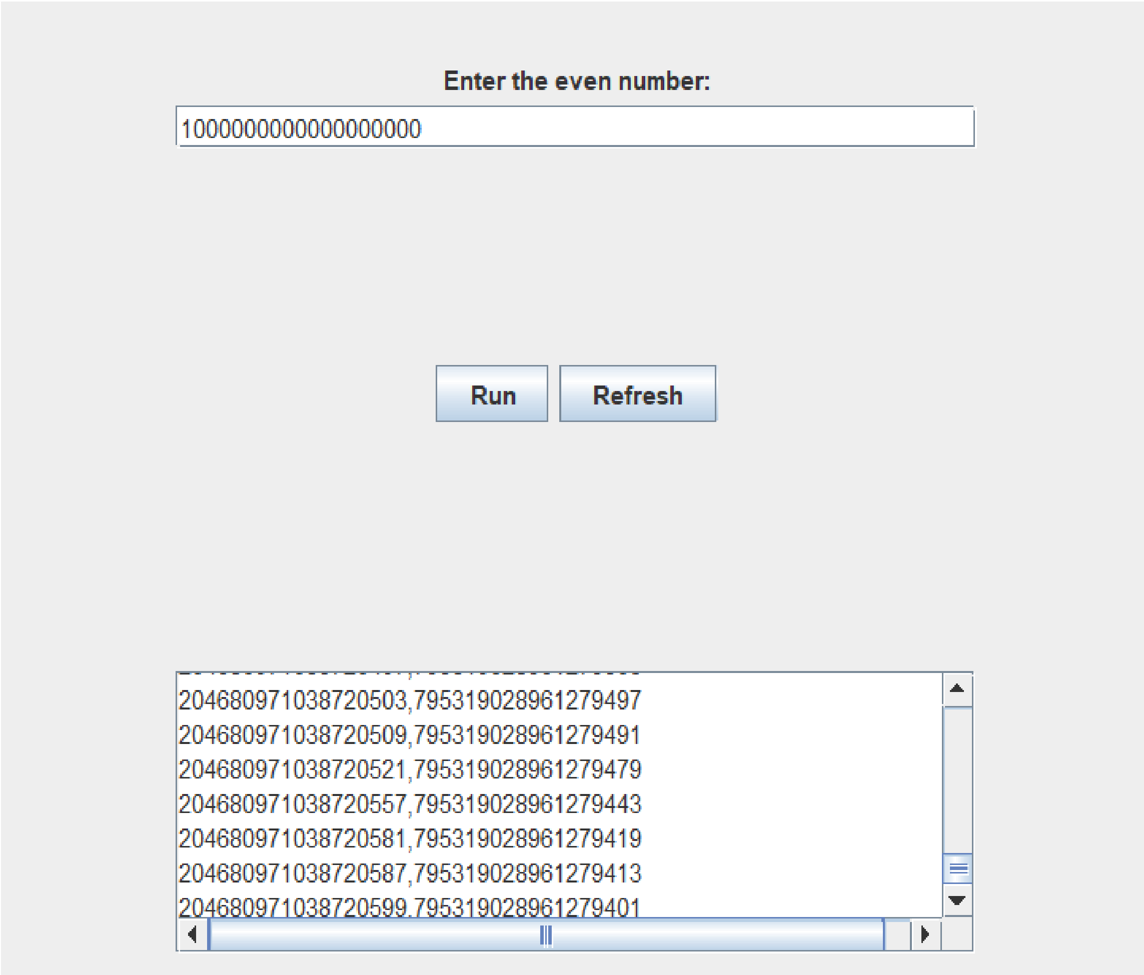

Figure 5 Partitions 1,000,000,000,000,000,000. into all pairs of odd numbers displaying a subset of the set of all partitions of odd numbers.

For $2n = \mathbf{9,000,000,000,000,000,000}$ some partitions of this even number includes:

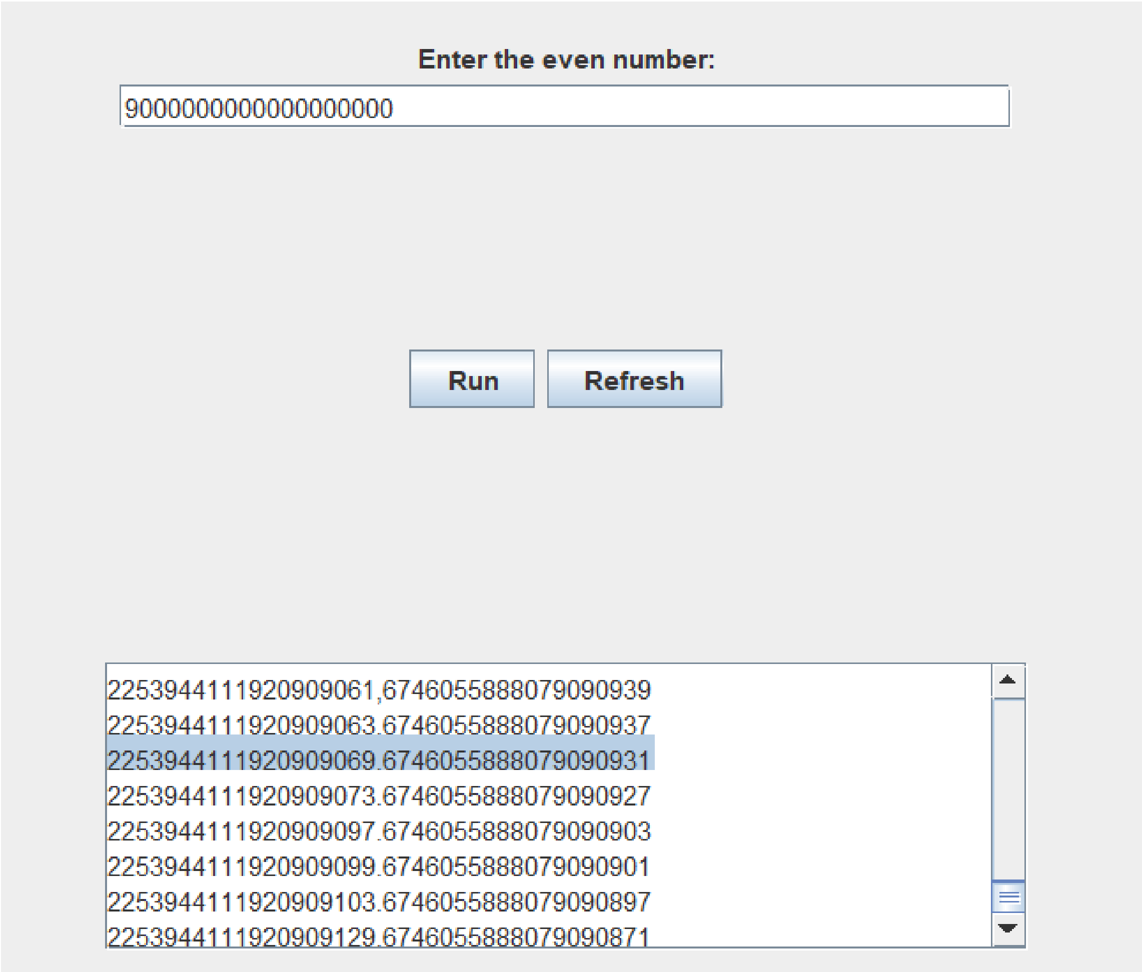

Figure 6 Odd pairs of partitions of 9,000,000,000,000,000,000.

4.4 Extending the Verification Up To 9×10^{18} And Proof of Strong Goldbach's Conjecture

4.4.1 Extending the Verification of the Strong Goldbach Conjecture up to 9×10^{18}

The Goldbach Conjecture is a famous unsolved problem in number theory. It states that every even integer greater than 2 can be expressed as the sum of two prime numbers. While the conjecture has been tested extensively for even numbers up to very large values, no counterexamples have been found, and it holds true for all tested cases. However, a rigorous proof for the Goldbach Conjecture is yet to be discovered, and it remains an open problem in mathematics (Marshall, 2017).

The Goldbach Conjecture specifically deals with even numbers, so it does not make any statements about odd numbers. Therefore, it does not provide any direct information about the properties of odd numbers or their relationship to prime numbers. In fact, odd numbers cannot be expressed as the sum of two prime numbers (unless one of the primes is 2), as the sum of two odd numbers is always even. Therefore, the Strong Goldbach Conjecture does not apply to odd numbers.

On the other hand, the Goldbach Conjecture is indeed closely related to prime numbers. It suggests that every even integer greater than 2 can be expressed as the sum of two prime numbers. In other words, it proposes that for any even number n (where $n > 2$), there exist prime numbers p and q such that $p + q = n$. Since the conjecture involves the decomposition of even numbers into the sum of two primes, prime numbers play a fundamental role. The conjecture asserts that even numbers can be expressed as the sum of two primes (Marshall, 2017), it emphasizes the significance of prime numbers in understanding the properties and structure of even integers.

It is worth noting that while the Goldbach Conjecture has been tested extensively and holds true for a vast range of even numbers, despite the lack of proof, mathematicians widely believe the conjecture to be true based on the empirical evidence (Weber et al., 2014). There are several different ways mathematicians have tried to solve the conjecture, ranging from statistical and probabilistic approaches to analytic number theory (Härdig, 2020). Using computing approach, mathematicians have extended the verification of Goldbach Conjectures to larger numbers. This study presents Figure 6 as a proof that the binary Goldbach conjecture is true up to 9×10^{18}.

These results are quite an advancement of the results published by Thomás Oliveira e Silva which also used advances in computational computing proving that the binary form of the Goldbach Conjecture is true up to 4×10^{18} (Oliveira et al., 2014).

The following table summarizes the partitioning of selected even numbers into all pairs of odd numbers up to 9×10^{18} while showing the Goldbach partition for the even number.

Table 1 Summarizes the partitioning of selected even numbers into all pairs of odd numbers and finding the distinct Goldbach partitions

S/ no	Even number : $(p_1+p_2)+(p_2-p_1)^n$	Pairs of odd numbers whose sum is $(p_1+p_2)+(p_2-p_1)^n$	Distinct Goldbach partitions whose sum is $(p_1+p_2)+(p_2-p_1)^n$
1	10	(3,7), (5,5), (7,3)	… (5,5), (7,3)…
2	24	(3,21), (5,19), (7,17), (9,15), (11,13), (13,11)	… (5,19), (7,17), (11,13)..
3	46	(11,35), (13,33), (15,31), (17,29), (19,27), (21,25), (23,23), (25,21), (27,19), (29,17), (31,15), (33,13)	… (17,29), (23,23)…
4	500	…, (271,229), (487,13), (463,37), (397,103) (373,127), …	…, (271,229), (487,13), (463,37), (397,103) (373,127),…
5	10,000	…, (7559,2441), (7919,2081), (7793,2207), (7727,2273),…	... (7559,2441), (7919,2081), , (7793,2207), (7727,2273), ...
6	10^6	…, (999809,191), (999827,173),(999863,137) (999893,107), (999899,101),(999917,83),…	…, (999917,83), (999863,137), (999809,191), …

7	9,989,748	…, (9989537,211), (9989549,199), (9989569,179), (9989597,151),(9989599,149) (9989621,127), …	…, (9989689,59), (9989677,71), (9989569,179), …
8	10^8	…, (99999821,179), (99999827,173), (99999833,167), (99999941,59), (9999994753), …	**…,** (17422763,82577237), (99999941,59),(99999827,173) , (99999821,179), **…**
9	10^9	…, (857330003,142669997), (857330021,142669979), (857330051,142669949), (857330069,142669931) (857330093,142669907), …	…, (857330021,142669979), …
10	10^{18}	…, (204680971038720509,795319028961279 491) (204680971038720521,795319028961279 479) (204680971038720557,795319028961279 443) (204680971038720581,795319028961279 419), …	…, (204680971038720557,795319 028961279443), …
11	9×10^{18}	… (2253944111920909063,67460558880790 90937)	

(2253944111920909069,6746055888079090931), (2253944111920909073,6746055888079090927), (2253944111920909097,6746055888079090903), (2253944111920909099,6746055888079090901), (2253944111920909103,6746055888079090897)…	… (2253944111920909069,6746055888079090931) …

The Strong Goldbach Conjecture, which states that every even integer greater than 2 can be expressed as the sum of two prime numbers, has been extensively verified for all integers up to 4×10^{18} (Marshall, 2017). However, despite the significant computational effort and the absence of counterexamples mathematicians are nowhere near to providing a general proof. This study has extended the verification of the Strong Goldbach Conjecture for all even numbers up to 9×10^{18} as in Figure 6.

4.4.2 A Rigorous Proof of the Strong Goldbach Conjecture

4.4.2.1 Methodology of the proof

Proving the strong Goldbach conjecture involves a systematic and rigorous approach that combines various mathematical techniques. The proof presented here combines results obtained in Chapter 3 of a new formulation of a set of even numbers and the fact that it has been proven that any even number of this new formulation can be partitioned into all pairs of odd numbers.

The study further uses the fact that from the following theorem , it is clear that there exist infinitely many prime numbers.

Theorem 4.4.2.1.1

There are infinitely many prime numbers (Hirschhorn, 2002).

Proof

Given the list of prime numbers $p_1, p_2, \dots, p_n$, the number $N = (p_1 * p_2 * p_3 * \cdots * p_n) + 1$ must contain a prime factor not among the primes used in its construction. To see this, notice p_1 does not divide N since it leaves a remainder of 1 (or alternatively N/p_1 is clearly not an integer). Similarly, the other $pi\,'s$ does not divide N. We therefore conclude that any finite list of primes is not complete, and therefore there must be infinitely many primes (Goldston, 2007).

From **Theorem 4.3.1**, it is clear that if we let $(p_1 + p_2) + (p_2 - p_1)^n$ be any even number and let $z_i, y_i \in 1 \le O \le \frac{1}{2}(\,p_1 + p_2) + (p_2 - p_1)^n\,)$ be the set of odd numbers such that $d \in 1 \le E \le \frac{1}{2}(p_1 + p_2 + (p_2 - p_1)^n)$ and $i \in 1 \le O \le \frac{1}{2}(\,p_1 + p_2) + (p_2 - p_1)^n\,)$, then $(p_1 + p_2) + (p_2 - p_1)^n$ is partitioned into two sets of odd numbers $\{d + z_i\}$ and $\{y_i\}$ such that ($p_1 + p_2) + (p_2 - p_1)^n - (d + z_i) = y_i$.

4.4.2.2 Construction of the proof

Let's consider two finite sets of prime numbers $A = \{p_1, p_2, \dots, p_n\}$ and $B = \{p_1, p_2, \dots, p_m\}$. According to Theorem 4.4.2.1.1, there are infinitely many prime numbers, and therefore it is expected that for set A, if p_n is the large prime number in the set, there is another larger prime number than pn say $N = (p_1 * p_2 * p_3 \cdots * p_n) + 1$ and $M = (p_1 * p_2 * p_3 * \cdots * p_m) + 1$ is a larger prime number than p_m from set B. From Theorem 3.1.1 in Chapter 3, the sum of any two odd numbers is an even number. Since prime numbers greater than 2 are subsets of odd numbers, the addition of elements of set A to elements of set B will also give an even number. For the sake of illustration, let us consider adding two real sets of prime numbers ≥ 3 (although 2 is a prime number, we exempt it from each of this sets since the addition of 2 to any prime number results to an odd number and not an even number). Therefore, the elements in set A for example will begin from: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97,..., pn, and $(p1 * p2 * p3 *\cdots* pn) + 1$. We therefore create table 2 to illustrate the results of adding elements of set A and set B and show the result (A+B) gives an even number as follows:

Table 2 Strong Goldbach Conjecture table (excluding 2)

Set A	Set B	Sum (A + B)	Cont′ Set A	Cont′ Set B	Cont′ Sum (A + B)
3	3	6	7	13	20
3	5	8	7	17	24
3	7	10	7	19	26
3	11	14	7	23	30
3	13	16	11	11	22
3	17	20	11	13	24
3	19	22	11	17	28
3	23	26	11	19	30
5	5	10	11	23	34
5	7	12	13	13	26
5	11	16	13	17	30
5	13	18	13	19	32
5	17	22	13	23	36
5	19	24	17	17	34
5	23	28	17	19	36
7	7	14	17	23	40
7	11	18	19	19	38
			.	.	.
			.	.	.
			.	.	.
			pn	pm	$2n_{nm}$
			$(p_1 * p_2 * p_3 * \cdots * p_n) + 1$	$(p_1 * p_2 * p_3 * \cdots * p_m) + 1$	$2n_{(pn1,pm1)}$

In order to be able to prove this conjecture, we start by interpreting the results in Table 2. It is straightforward that the even numbers are formed by adding two prime numbers from set A and

Set B. If the process of adding the elements of set A and set B continues, eventually one would get to a very large pair of prime numbers ($((p_1 * p_2 * p_3 * \cdots * p_n) + 1),((p_1 * p_2 * p_3 * \cdots * p_m) + 1))$ whose sum is a positive integer say $2n_{(pn1,pm1)}$,that is also an even number. It is important to note that this larger pair of prime is chosen arbitrarily, and therefore the result $2n_{(pn1,pm1)}$ could be any arbitrary even number. In other words, these results imply that any arbitrary even number can be written as a sum of two prime numbers. For that reason, the results in Table 2 can be used to provide a rigorous proof of the Strong Goldbach conjecture using the results obtained using the fact that any even number of the form $(p_1 + p_2) + (p_2 - p_1)^n$ can be partitioned into all pairs of odd numbers and from this set of pairs of odd numbers, there exists at least one pair of prime.

4.4.2.3 The Proof of the Strong Goldbach Conjecture

Combining the results in section 4.4.2.2, the study proposes to present the following rigorous proof for the Strong Goldbach conjecture.

Theorem 4.4.2.3.1 (Strong Goldbach Conjecture)

Every even integer greater than two is the sum of two prime numbers.

Proof

Using a new formulation of set of even numbers, $(p_1 + p_2) + (p_2 - p_1)^n$, it has been shown that it is possible to extend the results in Table 2 so as to allow the given even number to be partitioned into all pairs of odd numbers as in Example 4.2.2.2 in Section 4.2.2. From theorem 4.3.1, it is seen that using a new formulation of a set of even numbers as $(p_1 + p_2) + (p_2 - p_1)^n$ and the algorithm presented in this study for partitioning an even number, one is always able to partition the given even number into all pairs of odd numbers for all $z_i, and, y_i \in 1 \leq O \leq \frac{1}{2}(p_1 + p_2 + (p_2 - p_1)^n))$, $d = p_2 - p_1$, and

$i, j, k \in 1 \leq O \leq (\frac{1}{2}(p_1 + p_2) + (p_2 - p_1)^n)$. The results also show that partitioning any even number of this form, generates two sets of odd numbers $\{d + z_i\}$ and $\{y_i\}$ such that $(p_1 + p_2) + (p_2 - p_1)^n = \{d + z_i\} + \{y_i\}$ for all pairs of odd numbers of the elements in each partition.

Further, it is clearly seen from Example Example 4.2.2.2 in Section 4.2.2 that, from the two sets of odd numbers $\{d + z_i\}$ and $\{y_i\}$,there exist two proper subsets of prime numbers say $\{d + z_k\} \subset \{d + z_i\}$ and $\{y_k\} \subset \{y_i\}$ in such a way that each proper subset contains at least one prime number say $(d + z_j) \in \{d + z_k\} \subset \{d + z_i\}$ and $(y_j) \in \{y_k\} \subset \{y_i\}$ such that $(p_1 + p_2) + (p_2 - p_1)^n = (d + z_j) + y_j$. The results discussed in table 2 ensures that the addition of any two arbitrarily chosen prime numbers will always give an even number. It also confirms that any arbitrary even number, however large it is, can always be written as a sum of two prime numbers. These results therefore guarantees that the two sets of odd numbers $\{d + z_i\}$ and $\{y_i\}$ obtained from partitioning any arbitrarily chosen even number $(p_1 + p_2) + (p_2 - p_1)^n$ contains two proper subsets of prime numbers $\{d + z_k\} \subset \{d + z_i\}$ and $\{y_k\} \subset \{y_i\}$, and that each proper subsets of prime numbers must each contain at least one prime number say $(d + z_j) \in \{d + z_k\} \subset \{d + z_i\}$ and $(y_j) \in \{y_k\} \subset \{y_i\}$ such that $(p_1 + p_2) + (p_2 - p_1)^n = (d + z_j) + y_j$. The results therefore prove the Strong Goldbach conjecture restated as every even integer greater than 6 can be expressed as the sum of two prime numbers. Since it is known that $4 = 2 + 2$ and $6 = 3 + 3$, the proof presented here is considered a generalization proof of the Strong Goldbach conjecture.

4.5 Application of the Generation of Pairs of Odd Numbers on the Twin Prime Conjecture and Weak Goldbach Conjecture

4.5.1 A Brief Overview of the Twin Prime Conjectures

The twin prime conjecture is an open problem in number theory which asserts that there are infinitely many pairs of twin primes (De Corte, 2015). Formally, the conjecture states that there exist infinitely many pairs of primes $(p, p + 2)$ such that both p and $p + 2$ are prime numbers (Nazardonyavi, 2012). The Twin Prime Conjecture has not been proven or disproven (Mothebe, 2019), and it remains one of the most famous unsolved problems in number theory. However, mathematicians have made progress toward understanding this conjecture. The conjecture has been around for centuries and has been studied by many famous mathematicians, including Euler and Hardy-Littlewood (Weisstein, 2002).

4.5.1.1 The application of the generation of pairs of odd numbers on twin prime conjecture.

The Twin prime conjecture ascertains that there exist infinitely many pairs of prime that differ by 2. The algorithm proposed here for generating the pairs of odd numbers can be used when $d = 2$ to generate the list of all odd numbers less than a given even number that differs by 2. And we show that in this list of odd numbers there exist at least one pair of prime that differ by 2.

In Example 4.2.2.1 in Section 4.2.2, the results obtained from partitioning 10 are : $\big((2 + 1), 7\big), \big((2 + 3), 5\big), ((2 + 5), 3)$ and from this partitions, the list of distinct set of odd numbers obtained from the least to the largest is : $(3,5,7)$, and the two pairs of primes that differ by 2 are: [$(3,5)$ and $(5,7)$].

In Section 4.3.2, partitioning the even number 24 for $n = 4$, We were able to obtain the partitions of 24 as:$\big((2+1),21\big),\big((2+3),19\big),\big((2+5),17\big),\big((2+7),15\big),\big((2+9),13\big),((2+11),11) \Rightarrow$ $(3,21),(5,19),(7,17),(9,15),(11,13),(13,11)$. we can list the distinct set of odd numbers obtained from the least to the largest as: (3,5,7,9,11,13,15,17,19). From the list we obtain the following pairs of primes that differ by 2: $[(3,5),(5,7),(11,13), and\ (17,19)]$.

In Example 4.2.2.2 in Section 4.2.2, The positive partition of 46 are : $\big((2+1),43\big),\big((2+3),41\big),\big((2+5),39\big),\big((2+7),37\big)\big((2+9),35\big),\big((2+11),33\big),\big((2+13),31\big),\big((2+15),29\big),\big((2+17),27\big),\big((2+19),25\big),\big((2+21),23\big) and\ ((2+23),21)$. List of distinct set of odd numbers are: (3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37,39,41,43). From the list we obtain the following pairs of primes that differ by 2: $[(3,5),(5,7),(11,13),(17,19),(29,31)\ and\ (41,43)]$.

From these results, one is able to see that as the even number $(p_1 + p_2) + (p_2 - p_1)^n$ grows bigger, the list of odd numbers in the range $1 \le O \le \frac{1}{2}(\,(p_1 + p_2) + (p_2 - p_1)^n\,)$ increases. This subsequently, increases the numbers of pairs of primes that differ by 2. This research further proposes that the algorithm proposed here that generates all pairs of odd number for the even number $(p_1 + p_2) + (p_2 - p_1)^n$ could be explored to provide a solution to the twin prime conjecture since a given even number can always be partitioned into all pairs of odd numbers.

4.5.2 Application of the partitioning of an even number to all pairs of odd numbers to the solution to the Weak Goldbach Conjecture.

4.5.2.1 A Brief Overview of the Weak Goldbach Conjectures

The Weak Goldbach Conjecture, also known as the Odd Goldbach Conjecture, states that every odd integer greater than 5 can be expressed as the sum of three prime numbers. The conjecture is named after the German mathematician Christian Goldbach, who first proposed it in a letter to Swiss mathematician Leonhard Euler in 1742 (Wang, 2002). Goldbach's conjecture gained significant attention and has been extensively studied by mathematicians over the centuries, but it remains an unsolved problem in number theory.

While the conjecture has been tested extensively for numerous cases and found to hold true, a rigorous proof for all odd integers greater than 5 is yet to be discovered. The difficulty of proving the conjecture lies in the intricate nature of prime numbers and their distribution (Helfgott et al., 2013).

The relationship between the two conjectures is that a proof of the Strong Goldbach Conjecture would automatically imply the truth of the Weak Goldbach Conjecture. This is because if every even number can be expressed as the sum of two primes, it follows that every odd number greater than 5 can be expressed as the sum of three primes (by adding 3, which is a prime, to any even number expressed as the sum of two primes). The Weak Goldbach Conjecture is a specific case of the Strong Goldbach Conjecture, and a proof of the Strong Goldbach Conjecture would establish the truth of the Weak Goldbach Conjecture. However, as of now, both conjectures remain unsolved mathematical problems (Collins, 2020).

The new definition of a set of even numbers as $(p_1 + p_2) + (p_2 - p_1)^n$ has proved to have impressive application in partitioning an even number into all pairs of odd numbers. In these set of pairs of pairs odd numbers, we have shown that there exits at least a pairs of prime proving the strong Goldbach conjecture. This new definition of even numbers could be extended to produce a new formulation of a set of odd number. This new definition of odd numbers is expected to provide a different method of an attack to the weak Golbach conjecture independent from the approaches used to solving the strong Goldbach conjecture. The following section gives a brief outline of the extension of a new formulation of a set of even numbers.

4.5.2.2 A New formulation of a Set of Odd Numbers formed from extending the definition of a New formulation of a Set of Even Number defined as $E = (n_1 + n_2) + (n_2 - n_1)^n$

It has been shown an odd number is that which differs from an even number by a unit. The conventional definition of an odd number is $2n + 1, for\ all\ n \in \mathbb{N}$. An exploration of this definition of odd numbers as $(2n + 1)$ is done and extended from the new definition of even numbers $E = (n_1 + n_2) + (n_2 - n_1)^n$ to provide a new definition of odd numbers. This new representation of odd numbers could be arrived at using the fact the sum of any three odd numbers will always be odd. Note that the standard definition of an odd number as $2n + 1$ has not provided mathematicians with an easy approach to solve the weak Goldbach conjecture independently from the results obtain in solving the Strong Goldbach Conjecture. It has also not provided a direct relationship between an odd number and prime numbers and as such this conjecture remains an open area of research in number theory (Collins, 2020). It is also believed that a solution to the Strong Goldbach conjecture will automatically prove the weak conjecture. Therefore, there is need to formulate a new representation of odd numbers that provides an independent approach to the solution to the Weak Goldbach conjecture.

CHAPTER FIVE: CONCLUSIONS, RECOMMENDATIONS AND PUBLICATION

5.1 Conclusions

In chapter three, we have presented a new formulation of a set of even numbers of the form $(n_1+n_2)+(n_2-n_1)^n$ in theorem 3.2.1 This new definition of even numbers has been proved to hold for all natural number n in Theorem 3.2.2, Theorem 3.2.3 and in Corollary 3.3.1. It has also proved to give a better relationship between an even number and the corresponding prime numbers whose sum is the given even number of the form $E=(n_1+n_2)+(n_2-n_1)^n$.

In chapter four, using the new definition of even numbers, it was shown that any even number can be partitioned into all pairs of odd numbers in Section 4.2.1 and Section 4.3 and from these pairs of odd numbers, there exist at least one pair of prime. Using the algorithm presented in this study of partitioning any even number into all pairs of odd numbers, it was possible to extend and show that the Strong Goldbach Conjecture holds for all even numbers up to 9×10^{18}(see Table 1). Using the results in table 2, it was clear that the set of odd numbers obtained from partitioning an even number of the new form $E=(p_1+p_2)+(p_2-p_1)^n$, contains two prime numbers in each of these proper subsets of prime numbers contained in this set of odd numbers. It has also been shown that the addition of any two arbitrary prime numbers gives an even number and these results ensure that there exists at least a prime number in each of these two proper subsets of prime numbers contained in the set of odd numbers such that the addition of the two prime numbers equals $(p_1+p_2)+(p_2-p_1)^n$. Based on these results, the Strong Goldbach conjecture is considered to be proved.

5.2 Recommendations

The study recommends that researchers explore the algorithm proposed in Section 4.2.1 and Section 4.3 for partitioning any even number into all pairs of odd numbers to find a new method of attack to the solutions to the Weak Goldbach Conjecture and the Twin Prime Conjecture.

5.3 Publication

Sankei, D., Njagi,L. & Mutembei, J. (2023). A New Formulation of a Set of Even Numbers. *European Journal of Mathematics and Statistics, 4* (4), 78–82. https://doi.org/10.24018/ejmath.2023.4.4.249

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APPENDICES

Appendix 1: Publication

A New Formulation of a Set of Even Numbers

Daniel Sankei, Loyford Njagi, and Josephine Mutembei

***Abstract* — We present a new definition of an Even number as an integer, *E*, of the form**
$E\ n\ n = + + -(\ _{1\ 2})\ (n\ n_{2\ 1})^{n}$**,** □$n\ n\ n\ N_{1\ 2},\ ,$ □ **. We have shown the new set representation of even numbers holds for all natural numbers. This new representation opens up new doors to the solution of the Strong Goldbach Conjecture. The proofs obtained here will have impressive application in partitioning a given even number into all pairs of odd numbers.**

Keywords — **Even numbers, Goldbach Conjecture, Natural numbers, Odd numbers, Prime numbers.**

I. INTRODUCTION

Even numbers are important in many areas of mathematics and computer science and are often used in algorithms and data structures [1], they play an important role in many mathematical concepts, such as algebra, number theory, and geometry. They are also commonly used in everyday life, such as when dividing objects into equal parts [2]. The set of even numbers is denoted by the symbol "E", and is a subset of the set of integers. Note that the set of even numbers includes both positive and negative integers, as well as zero. They have many interesting properties, such as the fact that any even number can be expressed as the sum of two prime numbers (known as the Strong Goldbach's conjecture) [3]. An easy way to identify even numbers is to check if the last digit belongs to the set [0, 2, 4, 6, 8] [4]. For example, 132, 4620,164 and 8888 are all even numbers because their last digit belongs to this set. In general, any even number can be represented as $2n$, where n is an integer.

II. BASIC MATHEMATICAL CONCEPTS

We introduce important known mathematical concepts that cover the basic aspects of numbers. We will use this understanding, to discuss and prove the new representation of even numbers. Properties of even numbers can be helpful in solving mathematical problems and understanding patterns in numbers. Some interesting properties that even numbers possess include:

Statement (1): Any even number added to another even number will always give an even number [5]. *Statement (2):* For all odds, when adding an odd and an odd number the sum will always be even [6].

The following *Theorem A* proves *statement (2):* It will help us understand the proof that will be made for the new set representation of even numbers when the natural numbers are odd.

A. Theorem

The sum of two odd numbers is an even number.

Proof (CASE 1)

A number is odd if its rightmost digit belongs to the set [1, 3, 5, 7, 9] while a number is even if its rightmost digit belongs to the set [0, 2, 4, 6, 8]. To find the rightmost digit of the sum of two numbers, you only have to add the rightmost digits of the two numbers and take the rightmost digit of the sum. For example, consider the numbers 1345 and 629. The rightmost digits are 5 and 9. Adding these gives us 14, whose rightmost digit is 4. So, we expect the rightmost digit of 1345 + 629 to be 4. And it is 1345 + 629 = 1974. This tells us that in order to verify that the sum of any two odd numbers is an even number, we just have to check whether the sum of any two odd digits has an even digit on the right. We have gone from talking about all the odd numbers (infinity of them) to talking about just five digits. We just checked this criterion for 5 and 9. We have to go through every case so that we are sure it always works: 1+1=2   3+1=4   …   9+1=10

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1+3=4	3+3=6	…	9+3=12
1+5=6	3+5=8	…	9+5=14
1+7=8	3+7=10	…	9+7=16
1+9=10	3+9=12	…	9+9=18

In every single one of these cases, the rightmost digit is even [7].

Proof (CASE 2)

A number is odd if it can be written as 2x + 1, where x is some integer (The word integer means whole number, either positive, negative, or zero). A number is even if it can be written as 2x, where x is some integer. To start, pick any two odd numbers. We can write them as 2n + 1 and 2m + 1. The sum of these two odd numbers is (2n + 1) + (2m + 1). This can be simplified to 2n + 2m + 2 and further simplified to 2(n + m + 1). The number 2(n + m + 1) is even because n + m + 1 is an integer. Therefore, the sum of the two odd numbers is even [7].
Statement (3): when adding an even number and an odd number the sum will always be odd [6]. We introduce *Theorem B* to prove this statement.

B. Theorem

The sum of any even number and any odd number is odd.

Proof

Let the even number be 2n and the odd number be 2m + 1. Adding gives 2n + (2m + 1) = 2n + 2m + 1 = 2 (n + m) +1. Thus, the expression can be written as one more than a multiple of 2, so it is odd [9]. *Statement (4):* Every integer is either even or odd (i.e., there are no other possibilities).

C. Relationship Between Even and Prime Numbers

There is a unique relationship between even and prime numbers: the only even prime number is 2. All other even numbers are divisible by 2 and therefore not prime. This relationship arises because a prime number is defined as a positive integer greater than 1 that has no positive integer divisors other than 1 and itself. However, any even number greater than 2 is divisible by 2 and therefore has at least two positive integer divisors (1 and 2). This means that no even number greater than 2 can be prime.

III. FORMULATION OF A SET OF EVEN NUMBERS Euclid defined an even number as "a number which is divisible into two equal parts''. He also provided a method for generating even numbers using the formula $2n$, where n is any integer, and the representation of even numbers became more standardized. We explore this definition of even numbers ($2n$) and partition it into two even partitions say $2n=2n_1+2n_2$. Further, we partition each of the even partitions into two such that $2n_1 = k_1 + k_2$ and $2n_2 = k_2 - k_1, \forall k_1, k_2 \in N$, where N is the set of all natural numbers. We, therefore, wish to define a new representation of even numbers E as, $E = (k_1+k_2)+(k_2-k_1)^n \; \forall k_1, k_2, n \in N$.
We introduce this new representation of even numbers and show that the new formulation will always give an even number. We prove this new representation, $E = (k_1+k_2)+(k_2-k_1)^n \; \forall k_1, k_2, n \in N$ is even in three cases: (1) when $k_1, k_2 \in 2n$, (2) when $k_1, k_2 \in 2n+1$, and (3) when $k_1, k_2 \in p$, where n is any natural number and P the set of all prime numbers. We introduce expression (1) which will be the general representation of the new formulation of sets of even numbers.

A. The New Representation of Even Numbers

Let n_1 and $n_2 \in N$, then $(n_1 + n_2) + (n_2 - n_1)^n$ is even for all $n_2 \geq n_1$ and $(n \geq 1) \in N$ (1) where N is the set of all natural numbers Remark (for Expression (1)).

Since n_1 and n_2 □N then n_1 and n_2 □N can either be odd or even. Expression (1) will first be proved for even values of n_1 and n_2 □N and finite values of $n = 1,2$ and then generalized for all even values of n_1 and n_2.

CASE 1

This section will introduce *Corollary C and D* and provide proof for the new formulation of even numbers for finite values of $n = 1$ and 2 and generalize the proof for all values of n.

B. Corollary

Let n_1 and n_2 □$2k$,□ k□N and n=1, then $(n\, n_1 + + -_2)\,(n\, n_{2\ 1})^1$ is even. (2)

Proof

Since n_1 and n_2 are even, then from the definition of even numbers they can be written as $n_1 = 2k\ k\ N_1$,□ □$_1$ and $n_2 = 2k_2$,□ □$k_2 N$. Substituting $n_1 = 2k_1$and vin expression (2) yields

$(2k_1 + 2k_2) + (2k_2 - 2\ )k_1{}^1$. Since n_2 □$n_1$then it implies that k_2 □k_1 □ $k\ k_2 - {}_1$ □ 0 .

□ $2(k\ k_1 + +_2)\ 2(k\ k_2 - {}_1)^1$ and from the definition of even numbers, both $2(k\ k_1 + {}_2)$ and $2(k\ k_2 - {}_1)$ are even hence

$2(k\ k_1 + +_2)\ 2(k\ k_2 - {}_1)^1$ is even.

C. Corollary

Let n_1 and n_2 □$2k$,□ k□N and n=2, then $(n\, n_1 + + -_2)\ (n\, n_2\ \ {}_1)^2$ is even. (3)

Proof

Substituting $n_1 = 2k\ k\ N_1$,□ □$_1$ and $n_2 = 2k_2$,□ □$k_2 N$ in expression (3) yields $(2k_1 + 2\ )\ (2k_2 + k_2 - 2\ )k_1{}^2$

□ $(2k_1 + 2\ )\ (2k_2 + k_2 - 2\ )k_1{}^2$ □ $2(k\ k_1 + +_2)\ (4k_2^2 - 8k\ k_{2\,1} + 4k_1^2)$ □ $2(k\ k_1 + +_2)\ 4(k_2^2 - 2k\ k\ k_{2\,1} + {}_1^2)$

□ $2(k\ k_1 + +_2)\ 4(k\ k_2 - {}_1)^2$ □ $2(k\ k_1 + +_2)\ 2\ (^2 k\ k_2 - {}_1)^2$

□ $2(k\ k_1 + {}_2)$ is even and $2\ (^2 k\ k_2 - {}_1)^2 = 2(2(k\ k_2 - {}_1)\ )^2$, for $k\ k_2 - {}_1$ □ 0 □ $(k\ k_2 - {}_1)^2$ □N and $2(k\ k_2 - {}_1)^2$ is even from the definition of even numbers. This further implies that $2(2(k\ k_2 - {}_1)\ )^2$ is also even and finally, this shows that $(2k_1 + 2\ )\ (2k_2 + k_2 - 2\ )k_1{}^2$ is even.

D. Theorem let n_1 and n_2 □$2k$,then $(n\, n_1 + + -_2)\ (n\, n_{2\,1})^n$ is even for all n_2 □n_1 and $(n$□ □$1)\ N$ (4)

Proof

From expressions (2) and (3) it is clear that $(n\, n_1 + {}_2)$ in expression (4) is even , and we are left to prove that $(n\, n_2 - {}_1)^n$ is also even.

□ $(2k_2 - 2\ )k_1{}^n$ □ $2\ (^n k\ k_2 - {}_1)^n = 2(2\ (^{n-1} k\ k_2 - {}_1)\ )^n$, where $(k\ k_2 - {}_1)^n$ □N

□ $(2\ (^{n-1} k\ k_2 - {}_1)\ )^n$ □N □ $2(2\ (^{n-1} k\ k_2 - {}_1)\ )^n$ is even from the definition of even numbers. This further implies that

$(2k_1 + 2\ )\ (2k_2 + k_2 - 2\ )k_1{}^n$ is even.

CASE 2

This section will introduce *Corollary F and G* and provide proofs for the new representation of even numbers for odd values of $n_1 and\ n_2$ using values of $n = 1$ and 2, and then generalize the proof for all values of n.

E. Corollary

Let n_1 and $n_2 \square +2k\ 1 \square$ k$\square N$ and n=1, then $(n\ n_1 + + -_2)\ (n\ n_2\ \ _1)^1$ is even. (5)

Proof

Since n_1 and n_2 are both odd, from the definition of odd numbers, they can be written as $n_1 = +2k_1\ 1, \square \square k\ N_1$ and $n_2 = 2k_2 + 1, \square \square k_2 N$. Substituting $n_1 = +2k_1\ 1$ and $n_2 = 2k_2 + 1$in expression (5)

yields $(2k_1 + +1\ 2k_2 + +1)\ (2k_2 + - -1\ 2k_1\ 1)^1$. S$^{\text{ince}}$ $n_2 \square n_1$then it implies that $k_2 \square k_1 \square$ $k\ k_2 - {}_1 \square 0$.

$2(k_1 + k_2 + 1)$ for

$\square\ 2(k\ k_1 + + +_2\ 1)\ 2(k\ k_2 - {}_1)^1$ and from the definition of even numbers, both $(k\ k_1 + +_2$ $1)\square N$ and $2(k\ k_2 - {}_1)$ are even, hence $2(k\ k_1 + + +_2\ 1)\ 2(k\ k_2 - {}_1)^1$ is even.

F. Corollary

Let n_1 and $n_2 \square +2k\ 1$for $\square$ k$\square N$ and n=2, then $(n\ n_1 + + -_2)\ (n\ n_2\ \ _1)^2$ is even (6)

Proof

Substituting $n_1 = +2k_1\ 1, \square \square k\ N_1$ and $n_2 = 2k_2 + 1, \square \square k_2 N$ in expression (6) yields

$(2k_1 + + +2k_2\ \ 2)\ \ (2k_2 - 2\ )k_1{}^2$

$\square\ 2(k\ k_1 + + +_2\ 1)\ 2\ ({}^2 k\ k_2 - {}_1)^2 \square\ 2(k\ k_1 + + +_2\ 1)\ 4(k_2^2 - 2k\ k\ k_{2\,1} + {}_1{}^2) \square$

$2(k\ k_1 + + +_2\ 1)\ 2(2(k_2^2 - 2k\ k\ k_{2\,1} + {}_1{}^2)) \square$ From the definition of even numbers, we have that $2(k\ k_1 + +_2\ 1)$ and $2(2(k_2^2 - 2k\ k\ k_{2\,1} + {}_1{}^2))$ are both even proving expression (6).

G. Theorem

Let n_1 and $n_2 \square +2k\ 1$, then $(n\ n_1 + + -_2)\ (n\ n_2\ \ _1)^n$ is even for all $n_2 \square n_1$ and $(n\square \square 1)\ N$ (7)

Proof

From expressions (5) and (6) it is clear that $(n\ n_1 + {}_2)$ in equation (7) is even. we are therefore left to prove that $(n\ n_2 - {}_1)^n$ is also even.

$(2k_2 - 2\ )k_1{}^n \square\ 2\ ({}^n k\ k_2 - {}_1)^n = 2(2\ ({}^{n-1} k\ k_2 - {}_1)\ )^n$, where $(k\ k_2 - {}_1)^n \square N$

$\square\ (2\ ({}^{n-1} k\ k_2 - {}_1)\ )^n \square N \square\ 2[(2\ ({}^{n-1} k\ k_2 - {}_1)\ )]^n$ is even. This further implies that $(2k_2 - 2\ )k_1{}^n$ is even.

CASE 3

This section will give a brief overview of Prime numbers and introduce corollary *A*.

IV. Brief Overview of Prime Numbers

Every prime number is an odd number, except for 2, which is the only even prime number. However, not every odd number is a prime number. For example, 9 is an odd number, but it is not a prime number because it can be divided by 3.

The study of prime numbers and related problems such as the Goldbach Conjecture is an active area of research in mathematics and has important applications in Cryptography, Coding Theory, and Computer Science. Prime numbers are important in mathematics and computer science because they are used in cryptography, which is the study of codes and ciphers [8]. Odd numbers, on the other hand, are more commonly used in basic arithmetic and algebra.

A. Corollary

Prime numbers greater than 2 are subsets of odd numbers and therefore the results obtained in theorem *H* covers both composite odd numbers and primes. The theorem therefore proves expression (8).

Let $p_1$and $p_2 \square P$,then $(p\ p_1 + + -_2)\ (p_2 p_1)^n$ is even $\square \square n\ N$ and $p_2 \square p_1$ (8)

This expression will have remarkable application in generating all pairs of odd numbers associated with a given even number of form in expression (8). These results will be used to investigate the possibility of obtaining at least one Goldbach partition from these pairs of odd numbers.

V. CONCLUSION

The new formulation of sets of even numbers has been proved in three cases: Case 1 asserts that the new representation of even numbers holds for all even natural numbers. For Case 2, this formulation of even numbers will always be even for all odd natural numbers. Since prime numbers greater than 2 are subsets of odd numbers, Case 3 gives an expression of the formulation of even numbers for all prime natural numbers. This new formulation of the form in expression (8) can be used to investigate the possibility of partitioning a given even number into all pairs of odd numbers. We expect the results obtained here to have an impact to finding the solution to the Strong Goldbach Conjecture.

Appendix 2: The Algorithm of Partitioning Any Even number into all pairs of Odd numbers.

The following algorithm was developed in Java and partitions any even number into all pairs of odd number.

```
import java.util.*;

import javax.swing.*;

import javax.swing.Timer;

import java.awt.event.*;

import java.awt.*;

import java.io.*;

import javax.imageio.*;

public class EGuiImproved {

   Timer clock;

   long primeOne, i, randomEvenNo, primeTwo, evenNumber, halfEvenNo;

JFrame frame = new JFrame();

JPanel panel = new JPanel(), subPanel1 = new JPanel(), subPanel2 = new JPanel(),
buttonPanel = new JPanel();

JTextField enterEven = new JTextField (40);

JButton button = new JButton ("Run"), clear = new JButton ("Refresh");

JScrollPane sPane = new JScrollPane();

JTextArea result= new JTextArea(8, 40);

JLabel label= new JLabel ("Enter the even number:");

public EGuiImproved(){

   GUI();

}

public void GUI() {

frame.add(panel, BorderLayout.CENTER);

panel.add(subPanel1);

subPanel1.add(label);

subPanel1.add(enterEven);
```

```
panel.add(buttonPanel);
buttonPanel.add(button);
buttonPanel.add(clear);
panel.add(subPanel2);
subPanel2.add(sPane);
sPane.setViewportView(result);
result.setEditable(false);
panel.setBorder (BorderFactory.createEmptyBorder (30, 30, 10, 30)); panel.setLayout(new GridLayout(0, 1));
frame.setDefaultCloseOperation (JFrame.EXIT_ON_CLOSE);
frame.setTitle("ODDS & PRIMES FROM EVEN: DANIEL SANKEI");
frame.setSize(600,500);
frame.setLocation(200,60);
frame.setVisible(true);
clear.addActionListener(new ActionListener () { public void actionPerformed(ActionEvent e){
frame.dispose();
new EGuiImproved();
}});
button.addActionListener(new ActionListener () { public void actionPerformed(ActionEvent e){
clock = new Timer (1000, this);
String evenTotal = enterEven.getText();
evenNumber = Long.parseLong(evenTotal);
//evenNumber = Integer.valueOf(enterEven.getText());
if (evenNumber%2 == 0){
halfEvenNo = (evenNumber)/2;
Random randomEven = new Random();
randomEvenNo = randomEven.nextLong(halfEvenNo);
while(randomEvenNo % 2!=0 || randomEvenNo == 0){

randomEvenNo = randomEven.nextLong(halfEvenNo);
```

```
}
result.append("The random Even No is:" + randomEvenNo + "\n");
for (i = 0; i < evenNumber; i++){
if (i%2 != 0){
primeOne = randomEvenNo + i;
primeTwo = evenNumber - primeOne;
if((primeOne<evenNumber && primeTwo>1) &&
        (primeOne%3 !=0 && primeTwo%3!=0) &&
        (primeOne%5 !=0 && primeTwo%5!=0) &&
        (primeOne%7 !=0 && primeTwo%7!=0) &&
        (primeOne%11!=0 && primeTwo%11 !=0) &&
        (primeOne%13 !=0 && primeTwo%13!=0) &&
         (primeOne%17 !=0 && primeTwo%17!=0) &&
        (primeOne%19 !=0 && primeTwo%19!=0) &&
        (primeOne%23 !=0 && primeTwo%23 !=0) &&
        (primeOne%29 !=0 && primeTwo%29 !=0) &&
        (primeOne%31 !=0 && primeTwo%31 !=0) &&
        (primeOne%37 !=0 && primeTwo%37 !=0) &&
        (primeOne%41!=0 && primeTwo%41 !=0) &&
        (primeOne%43 !=0 && primeTwo%43 !=0) &&
        (primeOne%47!=0 && primeTwo%47!=0) &&
        (primeOne%49 !=0 && primeTwo%49!= 0)
)
{result.append(primeOne + "," + primeTwo + "\n");
}
}}



}else {
if(evenNumber%2 != 0){
```

```
JOptionPane.showMessageDialog(null, evenTotal + " is not an even number." , "NOT EVEN NUMBER", JOptionPane.INFORMATION_MESSAGE);}

}

}});

}

   public static void main(String[] args) {

      new EGuiImproved();

   }

}
```

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2nd January 2024

Admissions Office,
University of Bristol
United Kingdom, BS8 1TW, Bristol, University Walk

RE: RECOMMENDATION FOR MR. SANKEI DANIEL NJOROGE

I am pleased to recommend Mr Daniel Sankei who is applying for the PhD scholarship in Mathematics at University of Bristol. I have known him for the last four years during his undergraduate studies and for the two years I taught and supervised him when he was undertaking his Master's programme in Pure Mathematics. Mr. Sankei is a hard working scholar, honest and a person of integrity. He was the class representative during his undergraduate studies and demonstrated good leadership skills.

His overall academic achievements have been exemplary and this is depicted from the outstanding academic certificates and transcripts he possesses. As an academician, he has demonstrated commitment, enthusiasm and a keen interest in research specifically Number Theory. He has significantly contributed in the research field by publishing three quality research papers on the Strong Goldbach's Conjecture in international peer reviewed journals and one paper on a rigorous proof of the Strong Goldbach's Conjecture has been submitted in a peer reviewed journal for consideration for publication. With the research Mr. Sankei has conducted at the postgraduate level, I am confident that he is capable of conducting independent research going forward.

He has also participated in a research conference during his Masters course, and currently he is the Administrative Assistant in one of our schools at the University showing capacity to high multitasking skills. As an innovator, the opportunity to pursue a PhD degree would be very valuable to him and to the general research community in Mathematics.

I am confident that his dedication, intellectual prowess, and passion for number theory make him an outstanding candidate for your Ph.D. program. Kindly consider his application for this scholarship. Please do not hesitate to reach out to me using the above contact information at any time for additional information about this individual that you may find necessary throughout this process.

Yours faithfully,

....

Dr. Josephine Mutembei (PhD)

Pure Mathematics Program Advisor

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1st January 2024

Admissions Committee
University of Bristol-UK

Dear Members of the Admissions Committee,

RE: REFERENCE FOR MR. DANIEL SANKEI

I am writing to support Mr. Daniel Sankei's application for a PhD programme in Mathematics at the University of Bristol. I taught Mr. Daniel during his undergraduate and Master's studies, and I was one of his supervisors when he was undertaking his research in the field of Number Theory.

Mr. Sankei has a good base and grounding that is necessary for postgraduate studies and research in Pure Mathematics. He has strong analytical and computing skills which are essential in this demanding field of Mathematics. Moreover, his communication and interpersonal skills are exceptional. He is able to engage with peers, contribute to group projects, and collaborate with faculty members and have shown good leadership qualities.

Mr. Sankei is young, energetic, dynamic and hardworking and I can guarantee that he will achieve more in this chosen field of Mathematics. He was exceptional during his undergraduate studies and he will impress you with his thinking. This is clearly depicted from the academic results he possesses. His Postgraduate research work at master's level has made a remarkable contribution in Number Theory that could bring a new paradigm shift to how mathematicians now understand the Strong Goldbach Conjecture.

He has published three papers so far in internationally recognized peer-reviewed journals on the formulation of a set of even numbers, partitioning an even number of a new formulation into all pairs of odd numbers and using these results, he managed to extend the numerical verification of the Strong Goldbach conjecture to a very large even number hitting a new scope. He has also submitted another manuscript in a peer-reviewed journal on a rigorous proof of the Strong Goldbach Conjecture. This candidate has the ability to formulate and address research questions in Number Theory and therefore he is capable of conducting independent research.

I therefore fully support his application for the PhD programme in Mathematics and please feel free to contact me at any time for any further information about this gentleman that you might deem necessary during this process.

Yours sincerely,

Dr. Loyford Njagi

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I therefore fully support his application for the PhD programme in Mathematics and please feel free to contact me at any time for any further information about this gentleman that you might deem necessary during this process.

Yours sincerely,

Dr. Loyford Njagi

DESCRIPTION / *MAELEZO / DESCRIPTION*

Bearer / *Mwenye Pasi / Titulaire*

DANIEL NJOROGE SANKEI

Place of Residence / *Mahali aishipo / Lieu de Résidence*

TIGANIA WEST, KIRINDINE

Height / *Urefu / Hauteur* **5'11"** *ft/in* **1.8** *m/cm*

Colour of Eyes / *Rangi ya Macho / Couleur des Yeux*

DARK BROWN

Special Peculiarities / *Alama yoyote isiyo ya kawaida / Particularités Spéciales*

NOT APPLICABLE

Signature of Holder / *Sahihi ya Mwenye Pasi / Signature du Titulaire*

Signature of Passport Officer / *Sahihi ya Afisa wa Pasi / Signature de l'Agent des Passeports*

REPUBLIC OF KENYA / *JAMHURI YA KENYA / REPUBLIQUE DE KENYA*

PASSPORT/*PASI/PASSEPORT*

Type/*Aina/Type*: P

Country Code/*Nambari ya Nchi/Code du Pays*: KEN

Passport No./*Nambari ya Pasi/N° de Passeport*: AK0635912

Surname/*Jina la Ukoo/Nom*: SANKEI

Given Names/*Majina Aliyopewa/Prénoms*: DANIEL NJOROGE

Nationality/*Uraia/Nationalité*: KENYAN

Date of Birth/*Tarehe ya Kuzaliwa/Date de Naissance*: 14 FEB 1993

Personal No./*Nambari ya Kibinafsi/N° Personnel*: 2512467

Sex/*Jinsia/Sexe*: M

Place of Birth/*Mahali pa Kuzaliwa/Lieu de Naissance*: NAROK, KEN

Date of Issue/*Tarehe ya Kutolewa/Date de Délivrance*: 29 NOV 2019

Issuing Authority/*Mamlaka ya kutoa Pasi/Autorité*: GOVERNMENT OF KENYA

Date of Expiry/*Tarehe ya Mwisho/Date d'Expiration*: 28 NOV 2029

Holder's Signature/*Sahihi ya Mwenye Pasi/ Signature du Titulaire*

P<KENSANKEI<<DANIEL<NJOROGE<<<<<<<<<<<<<<<<<<

AK06359124KEN9302143M2911283330586915<<<<<<18