

## Personal details

### Personal details

**First / given name** Anas  
**Second given name**  
**Third given name** Ashraf  
**Surname/family name** Ashraf  
**Date of birth** 08 January 1999  
**Preferred first/given name** Anas  
**Previous surname**  
**Country of birth** Pakistan  
**Legal nationality** Pakistani  
**Dual nationality**  
**Country of residence** Pakistan  
**Have you previously studied with us at the University of Bristol?** No

## Contact details

### Home address

Please provide your permanent residential address. If you have another address and would prefer for us to contact you at that address instead you have the opportunity to add a correspondence address in the next section.

**Country** Pakistan  
**Postcode** 61100  
**Address Line 1** House No 58 Street No 2 Green  
**Address Line 2** Town Danewal Vehari  
**City** Vehari  
**County** Punjab  
**Telephone**

If you would like us to send any postal correspondence to an address which is not your home address please enter an alternative address here. If you want us to send correspondence to your home address then please select No.

**Do you want to add a correspondence address?** No  
**Country** Pakistan  
**Postcode** 61100  
**Address Line 1** House No 58 Street No 2 Green  
**Address Line 2** Town Danewal Vehari  
**City** Vehari  
**County** Punjab  
**Telephone**

## Agent

### Agent details

**Agency Name**  
**Email address**

## Other information

### Additional Documents

*Please upload required documents as outlined in your admissions statement*

### Mode of study

How would like to study this Full Time  
programme?

## Qualifications

### Qualifications

Institution	Qualification	Type	Subject	Actual/predicted	Grade	Start date	End date
COMSATS Institute of Information Technology	Master's Degree (PG)	Academic Qualification	Mathematics	Actual	3.78/4.00	09/Aug/2020	15/Sep/2022
University of Education Lahore	Other Qual	Academic Qualification	Mathematics	Actual	3.33/4.00	19/Oct/2015	26/Sep/2019

If these qualifications have altered since your last application please note the changes in the free text box here.

### English Language

Is English your first language? Yes

What is your first language?

Did you study at school/university where you were taught in English?

For how many years?

Have you sat a relevant English language test?

#### TOEFL (internet-based)

Registration number

Date of TOEFL test

TOEFL reading score

TOEFL listening score

TOEFL speaking score

TOEFL writing score

TOEFL total score

#### IELTS (International English Language Testing System)

Test report form (TRF) number

UKVI number (if applicable)

Date of IELTS test

IELTS listening score

IELTS reading score

IELTS writing score

IELTS speaking score

IELTS total score

#### Pearson Test of English

Score report code

Date of Pearson test

Pearson listening score

Pearson reading score

Pearson speaking score

Pearson writing score

Pearson overall score

## Other English Language test

Name of course  
Registration number  
Date of test  
Listening score  
Writing score  
Reading score  
Total score

## Experience

### Current Employer

**Employer name and address** Punjab Institute of Science and Technology Vehari Address: Faisal Town Vehari  
Near Faisal Mosque  
**Job title and main duties** Lecturer Mathematics My main duties I take 4 lectures on mathematics and also i am  
the incharge of students affairs  
**Full time/Part time** Full time  
**Date of Appointment** 15 March 2024  
**End date (if applicable)**

### Previous employment 1

**Employer name and address**  
**Job title and main duties**  
**Full time/Part time**  
**Date of Appointment**  
**End date (if applicable)**

### Previous employment 2

**Employer name and address**  
**Job title and main duties**  
**Full time/Part time**  
**Date of Appointment**  
**End date (if applicable)**

### Previous employment 3

**Employer name and address**  
**Job title and main duties**  
**Full time/Part time**  
**Date of Appointment**  
**End date (if applicable)**

### Other Experience

**Do you have any other relevant work experience to support your application?** No  
**Please provide details**

Personal statement

Personal details

Do you have a personal statement to upload? Yes  
Please type your personal statement in the box

Research proposal

Research proposal

Proposed supervisor 1  
Proposed supervisor 1  
Proposed project title Exploring Lie Group and Lie Algebra Structures in Toeplitz Matrix Theory  
(max 150 chars)

## Passport and visa

### Visa required

Do you require a visa to study in the UK? Yes

Please fill out your passport details below. If you are unable to provide these at the current time you will have another opportunity to upload your passport after you submit the form. If you do not provide us with this information we will be unable to issue you with your confirmation of acceptance number and you will be unable to obtain a visa.

### Passport details

Passport number PL1840241

### Further details

Have you previously studied in the UK? No

What was the highest level of study in the UK?

Please confirm the total length of your UK study in years

## Referees

### Referee 1

Do you have a reference to upload? Yes

Type of reference

Referee title

Forename

Surname

Position

Institution/Company

Email address

Country

### Referee 2

Do you have a second reference to upload? Yes

Type of reference

Referee title

Forename

Surname

Position

Institution/Company

Email address

Country

# Funding

## Funding 1

What is your likely source of funding? Yourself/family

Please give the name of your scholarship or Studentship  
Please specify

Percentage from this source 70

Is this funding already secured? No

## Funding 2

What is your likely source of funding?

Please give the name of your scholarship or Studentship  
Please specify

Percentage from this source

Is this funding already secured?

## Funding 3

What is your likely source of funding?

Please give the name of your scholarship or Studentship  
Please specify

Percentage from this source

Is this funding already secured?

## Other funding

I would like to be considered for other funding opportunities Yes

## Documents

Document type	File name
Degree certificate	BS Degree.pdf
Personal statement	Personal Statement.pdf
Research proposal	research statement .pdf
Transcript	BS Transcript.pdf
Curriculum vitae	Anas Ashraf resume (1).pdf
Passports and visas	Pspirt.pdf
References	Reference Letter for Anas.pdf
References	2 recommendations letter.pdf
Degree certificate	MS Degree.pdf
Transcript	MS Transcript.pdf

By ticking the checkbox below and submitting your completed online application form, you acknowledge the University of Bristol will use the information provided from time to time, along with any further information about you the University may hold, for the purposes set out in the [University's full Data Protection Statement](#). Applicants applying to the collaborative programmes of doctoral training should also read the [Data Protection Statement](#) for collaborative programmes of doctoral training.

The information that you provided on your application form will be used for the following purposes:

- To enable your application for entry to be considered and allow our Admissions Advisors, where applicable, to assist you through the application process;
- To enable the University to compile statistics, or to assist other organisations to do so. No statistical information will be published that would identify you personally;
- To enable the University to initiate your student record should you be offered a place at the University.

All applicants should note that the University reserves the right to make without notice changes in regulations, courses, fees etc at any time before or after a candidate's admission. Admission to the University is subject to the requirement that the candidate will comply with the University's registration procedure and will duly observe the Charter, Statutes, Ordinances and Regulations from time to time in force.

By ticking the checkbox below and submitting your completed online application form, you are confirming that the information given in this form is true, complete and accurate and that no information requested or other material information has been omitted. You are also confirming that you have read the Data Protection Statement and you confirm the statement below.

I can confirm that the information I have provided is true, complete and accurate. I accept that the information given in my application will be stored and processed by the University of Bristol, in accordance with the *UK General Data Protection Regulation and Data Protection Act 2018*, in order to:

- Consider my application and operate an effective and impartial admissions process;
- Monitor the University's applicant and student profile;
- Comply with all laws and regulations;
- Ensure the wellbeing and security of all students and staff;
- If my application is successful to form the basis of the statement made within my application.

If the University of Bristol discovers that I have made a false statement or omitted significant information from my application, for example examination results, I understand that it may have to withdraw or amend its offer or terminate my registration, according to circumstances.





## Anas Ashraf

**Nationality:** Pakistani **Date of birth:** 08/01/1999 **Gender:** Male

**Phone number:** (+92) 3009180068 **Email address:** [anskhan739@gmail.com](mailto:anskhan739@gmail.com)

**LinkedIn:** <https://www.linkedin.com/anas-ashraf-101b95156>

**Home:** House No 58 Street No 2 Green Town Vehari, 61100 Vehari (Pakistan)

### ABOUT ME

Anas Ashraf is a Pakistani mathematician renowned for his expertise in mathematical analysis, including Graph theory, Fuzzy theory, Group theory, mathematical modelling, fixed point theory and Network theory. Actively contributing to teaching, research and service. Anas's commitment to education is evident through his training of educators, curriculum development and positive learning environment maintenance.

### EDUCATION AND TRAINING

#### Master's of Science (MS) in Mathematics

**Comsats University Islamabad Vehari Campus** [ 09/09/2020 – 14/09/2022 ]

**Address:** Mailsi-Vehari Rd, Vehari Punjab, 61100 | **Field(s) of study:** Graph Theory | **Final grade:** 3.78/4.00 | **Thesis:** Irregularity Indices of Benes Network Butterfly Network and Their Derived Networks

- Graph Theory
- Fuzzy Theory
- Mathematical Modeling
- Group Theory
- Data Science

#### Bachelor of Sciences (BS) in Mathematics

**University of Education Lahore Vehari Campus** [ 18/10/2015 – 25/09/2019 ]

**Address:** 29Q9+R3R, Vehari, Punjab, Pakistan., 61100 | **Field(s) of study:** Mathematics | **Final grade:** 3.33/4.00 | **Thesis:** Homotopy Perturbation method for solving forth order Integro Differential Equations.

#### Master of Arts in Education (2 years)

**Allama Iqbal Open University** [ 02/10/2021 – 19/02/2024 ]

**Address:** 2 Ashfaq Ahmed Rd, H-8/2, Islamabad Capital Territory, 44000 Islamabad | **Final grade:** 1318/2000

#### FSc(Math, Physics, Chemistry)

**Govt Post Graduate College Vehari** [ 01/09/2013 – 12/09/2015 ]

**Address:** 28 WX+4GP, Vehari, Punjab, Pakistan., 61100 Vehari (Pakistan) | **Final grade:** 778/1100

#### Matric(Math, Physics, Chemistry)

**Govt Model High School** [ 07/01/2011 – 25/07/2013 ]

**Address:** 29R4+F8J, Main Road, G Block Vehari, Punjab, Pakistan., 61100 Vehari (Pakistan) | **Final grade:** 739/1050

## WORK EXPERIENCE

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### Visiting Lecturer Mathematics

**Punjab Institute of Technology** [ 01/02/2024 – Current ]

Address: Faisal Town, Vehari, Punjab Pakistan, 61100 Vehari (Pakistan) | Name of unit or department: Mathematics

Working as a Lecturer of Mathematics  
Contract Basis

### Visiting Lecturer Mathematics

**Comsats University Islamabad** [ 16/09/2022 – 08/03/2023 ]

Address: Mailsi-Vehari Rd, Vehari (Pakistan)

Worked as a Visiting Lecturer  
Contract Basis

## DIGITAL SKILLS

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Microsoft office / Microsoft word / Mathematica / LaTeX / Microsoft excel / Microsoft PowerPoint / MATLAB / Python Initial

## PROJECTS

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[ 06/04/2021 – 01/09/2022 ]

**Worked on Several Projects during Master's.**

- On Computing Distance Based Entropy of Ferrocene Dendrimers
- On Eccentricity Based Topological Indices of Ferrocene Dendrimers
- On Banhatti Indices of Butterfly Networks, Benes Network and the Networks Derived from Benes Network
- On Computation of Edge Degree Based Banhatti Indices of a Certain Molecular Network

## PUBLICATIONS

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[2024]

**Anas. A, Fahad. A, Imran. M, Anwar. M, Rida. I, (2024) "IRREGULARITY BASED ENTROPY INEQUALITIES DETERMINING INTERCONNECTION NETWORK COMPLEXITY WITH APPLICATIONS IN COMMUNICATION SYSTEMS".** This publication is submitted to "**MDPI AG (Multidisciplinary Digital Publishing Institute)**".

## LANGUAGE SKILLS

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**Mother tongue(s):** Urdu

**Other language(s):** English | Italian as a Beginner

## SCHOLARSHIPS/FUNDING

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[ 19/10/2015 – 26/09/2019 ]

**Punjab Educational Endowment Fund (PEEF) BS Scholarship/Funding**

PEEF BS Scholarship covers

- University Tuition fees for the period of 4 years
- Accommodation

## CERTIFICATIONS

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### My Certifications

- *Algebra and Differential Calculus for Data Science*, **University of Colorado Boulder**, <https://coursera.org/verify/EHGO9SXADVBK>
- *Algebra: Elementary to Advanced Equations and Inequalities*, **John Hopkins University**, <https://coursera.org/verify/FRSR6G4WVMFK>
- **Digital Literacy**, *Enhance Digital Skills (MS Office+Social Media)*, <https://digiskills.pk/verify/SPNGPKXMK>
- *Leading Teams: Developing as a leader*, **University of Illinois**, <https://coursera.org/verify/CMP6A9HZYGNN>
- *Writing Professional Email and Memos (Project-Centered Course)*, **Kennesaw University**, <https://coursera.org/verify/TM7UUA7KM6FS>
- *Introduction to Artificial Intelligence (AI)*, **IBM**, <https://coursera.org/verify/JT2DNHWHDVS7>
- *Introduction to Python Fundamentals*, **University of Colorado Boulder**, <https://coursera.org/verify/HFR4SQR5A825>
- *English for Career Development*, **University of Pennsylvania**, <https://coursera.org/verify/7JUW2D5CUMHK>
- *Italian for Beginners*, **Alison**, Certificate No. 4278-27000850

## CONFERENCES AND SEMINARS

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[ 2018 ] Islamabad

**International Students Convention & Expo 2018**

[ 2019 ] University of Education Lahore Vehari

**One Day Symposium on Pure and Applied Mathematics** Secretary Symposium organizer: Managed the organizing work of symposium.

## HONOURS AND AWARDS

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[ 2017 ] University of Education Lahore Vehari

**Laptop from Prime Minister Scheme** Received from PM laptop scheme at Bachelor's level to top 5 in the class.

[ 2012 ] Govt Model High School Vehari

**Solar Lamp** Got Solar Lamp from Chief Minister of Punjab at Matric Level.

## RECOMMENDATIONS

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**Name: Dr Hafiz Muhammad Asim Zafar** | Associate Professor of Mathematics

Supervisor of Mphil Mathematics

Comsats University Islamabad Vehari

Email: [asimzafar@cuivehari.edu.pk](mailto:asimzafar@cuivehari.edu.pk) | Phone number: (+92) 3334599884

**Name: Dr. Muhammad Imran Qureshi** | Associate Professor of Mathematics

Head of Department at Comsats University Islamabad Vehari

Email: [imranqureshi@cuivehari.edu.pk](mailto:imranqureshi@cuivehari.edu.pk) | Phone number: (+92) 3146118139



2015-2019

This is to certify that

ANAS ASHRAF

son/daughter of MUHAMMAD ASHRAF

of

University of Education Vehari Campus, Vehari

has obtained the degree of

Bachelor of Science (Honors)

in

Mathematics

after fulfillment of all requirements of semester system

with CGPA 3.33

Controller of Examinations

Date: 09 JAN 2020

Chancellor



PAF I&SC, MULTAN

Branch/Trade EDU INST. (MATH)

Reg No M 333-2109-0137

Date 14/04/21 Sig. [Signature]

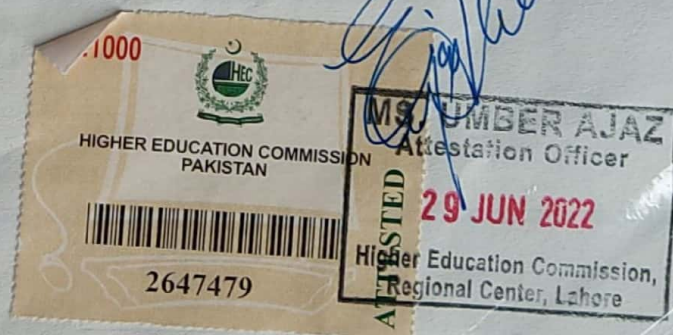
Verified by 'A'



31 MAY 2023

Checked by Mr. Muhammad Shahid  
Protocol Officer [Signature]

COUNTERSIGNED ONLY  
Ministry of Foreign Affairs is Not  
responsible for the contents of the  
documents



Serial No: 098515



Registration No: CHT/FA20-RMT-010/VHR

# COMSATS University Islamabad

ANAS ASHRAF s/o MUHAMMAD ASHRAF

Of

Vehari Campus

has been conferred upon the degree of

**Master of Science in Mathematics**

Given on this Fifteenth day of September two thousand and Twenty Two at Islamabad

Date of Issuance: 15th September 2022



eeh.  
Controller of Examinations

N. Afzal  
Rector

M. Adil  
Registrar



COUNTERSIGNED ONLY  
Ministry of Foreign Affairs is Not  
responsible for the contents of the  
Documents



31 MAY 2023

Checked by Mr. Muhammad Shabid  
Protocol Officer



Prepared By: .....

Note: This degree is issued without alteration or erasure.

Checked By: .....

Contact Address:

COMSATS University Islamabad,  
Park Road, Tarlai Kalan,  
Islamabad, Pakistan.  
Ph: +92-51-9247000-3  
<http://www.comsats.edu.pk>



**Personal Statement  
For Phd Mathematics  
University of Bristol  
Anas Ashraf**

As an aspiring mathematician driven by a passion for exploration and discovery, I, Anas Ashraf, am excited to articulate my aspirations and qualifications as I apply for a PhD in Mathematics at the University of Bristol.

My academic journey has been a testament to my commitment to excellence and intellectual curiosity. Having earned my Master's degree in Mathematics from Comsats University Islamabad Vehari with a commendable CGPA of 3.78/4.00, I have consistently pursued academic rigor and scholarly inquiry. My time at Comsats University provided me with a solid foundation in mathematical theory and research methodology, empowering me to engage with complex mathematical concepts with confidence and enthusiasm.

During my Master's program, I delved into the realm of graph theory, captivated by its elegance and applicability across diverse domains. Graph theory serves as a powerful tool for modeling and analyzing relationships in various fields, including computer science, social networks, and optimization. Its ability to represent complex structures and phenomena in a visual and intuitive manner intrigued me, sparking my curiosity and driving me to explore its depths.

My research focused on exploring the intricate structures and properties of graphs, investigating fundamental questions and challenging conjectures within the field. I was particularly interested in studying graph algorithms and their applications, seeking to develop efficient algorithms for solving combinatorial optimization problems on graphs. Through rigorous analysis and innovative problem-solving, I endeavored to contribute novel insights to the ever-evolving landscape of graph theory.

One of the highlights of my research journey was the opportunity to collaborate with fellow mathematicians and researchers on various projects and initiatives. These collaborations not only enriched my understanding of graph theory but also fostered a spirit of camaraderie and shared intellectual curiosity. Working closely with mentors and peers, I gained valuable insights into the research process, honed my analytical skills, and cultivated a deep appreciation for the beauty and intricacy of mathematical inquiry.

The prospect of pursuing a PhD in Mathematics at the University of Bristol excites me immensely, as it represents a unique opportunity to further explore my passion for graph theory under the guidance of esteemed scholars and researchers in the field. The University of Bristol's renowned Mathematics department boasts a vibrant academic community and a rich tradition of groundbreaking research, making it an ideal environment for intellectual growth and collaboration.



In addition to its academic excellence, the University of Bristol offers a dynamic and inclusive campus culture that fosters creativity, innovation, and interdisciplinary collaboration. I am particularly drawn to the university's commitment to diversity and inclusion, as I believe that diverse perspectives and experiences enrich the academic discourse and contribute to a vibrant intellectual community. I look forward to engaging with fellow scholars from diverse backgrounds, exchanging ideas, and collectively pushing the boundaries of mathematical knowledge.

As I embark on this next chapter of my academic journey, I am eager to immerse myself in the vibrant academic culture at the University of Bristol, where I can engage in interdisciplinary dialogue, collaborate with fellow scholars, and contribute to the advancement of mathematical knowledge. My ultimate goal is to make meaningful contributions to the field of mathematics through rigorous research, innovative problem-solving, and a steadfast commitment to academic excellence.

In conclusion, I am confident that my academic background, research experience in graph theory, and passion for mathematical inquiry make me a strong candidate for the PhD program in Mathematics at the University of Bristol. I am excited about the prospect of joining the University of Bristol community and am committed to pursuing excellence in both scholarship and research endeavors.

In the present document, I will describe the research plan I wish to carry during my PhD studies at the University of Bristol. The intricate study of Lie groups and Lie algebras has long been important in mathematical theory. This proposal seeks to explore an intriguing yet underexplored realm within this framework: the domain of Toeplitz matrices. While Lie theory has shed light on numerous mathematical structures, its application to Toeplitz matrices, new in signal processing and numerical analysis, remains relatively unexplored. This proposal endeavors to bridge this gap by investigating the inherent Lie group and Lie algebraic structures of Toeplitz matrices, thereby unlocking new avenues for understanding and utilizing these fundamental mathematical constructs. Through this exploration, we aim to enrich not only theoretical mathematics but also practical applications spanning diverse fields such as signal processing, quantum information theory, and machine learning.

## **1. Introduction and Background**

The study of Lie groups and Lie algebras has been instrumental in various areas of mathematics and physics, offering profound insights into symmetry, transformation, and geometric structures. However, the application of Lie theory to Toeplitz matrices remains relatively unexplored. Toeplitz matrices, characterized by constant diagonals, are ubiquitous in signal processing, numerical analysis, and other fields. This proposal aims to investigate the Lie group and Lie algebra structures inherent in Toeplitz matrices, unraveling their underlying symmetries, and transforming our understanding of these fundamental mathematical objects.

## **2. Objectives:**

- Investigate the Lie group structure of Toeplitz matrices: We will explore the continuous symmetries inherent in Toeplitz matrices, identifying Lie groups associated with specific classes of Toeplitz matrices.
- Analyze the Lie algebra of Toeplitz matrices: We will examine the Lie algebraic properties of Toeplitz matrices, characterizing the infinitesimal generators of their transformations and Lie brackets.
- Develop computational tools and algorithms: Leveraging insights from Lie theory, we will develop novel computational tools and algorithms for efficient manipulation, factorization, and inversion of Toeplitz matrices.

Explore applications in signal processing and beyond: We will apply the developed methodologies to address challenges in signal denoising, compression, and reconstruction, as well as explore potential applications in quantum information theory and machine learning.

## **3. Literature Review**

Sophus Lie's groundbreaking works in the late 19th century laid the foundation for the study of Lie group and Lie algebra structures in mathematical objects. While significant strides have been made in understanding these properties in symmetric matrices, relatively scant attention has been devoted to Toeplitz matrices [2]. Noteworthy contributions include early studies by Ma and Wang (2003), which initiated exploration into the connection between Toeplitz matrices and specific Lie groups [3]. However, further investigation is imperative to comprehensively elucidate the Lie group and Lie algebra structures of Toeplitz matrices and their implications for practical applications [4].

#### **4. Research Questions**

1. What Lie group structures are associated with different classes of Toeplitz matrices?
2. How can Lie algebraic techniques be employed to characterize the infinitesimal generators of transformations in Toeplitz matrices?
3. What computational tools and algorithms can be developed based on Lie theory to efficiently manipulate and analyze Toeplitz matrices?
4. What are the potential applications of Lie group theory in addressing challenges in signal processing, quantum information theory, and machine learning using Toeplitz matrices?

#### **5. Methodology**

The research methodology entails a multifaceted approach integrating mathematical analysis, computational simulations, and application-driven research. Initially, we will investigate the Lie group structures associated with different classes of Toeplitz matrices, drawing upon techniques from differential geometry and representation theory [5]. Subsequently, computational simulations will validate theoretical findings and evaluate algorithmic efficacy [6]. Concurrently, collaboration with domain experts will facilitate the exploration of specific applications in signal processing, quantum information theory, and machine learning [7].

#### **6. Expected Outcomes**

- Enhanced understanding of the Lie group and Lie algebra structures of Toeplitz matrices.
- Development of novel computational tools and algorithms for efficient manipulation and analysis of Toeplitz matrices.
- Advancements in signal processing, quantum information theory, and machine learning applications utilizing Toeplitz matrices [8].

#### **7. Timeline for Three Years PhD in Mathematics**

Year 1. Literature review, preliminary research, and formulation of research questions.

Year 2. Mathematical analysis, computational simulations, and initial exploration of applications.

Year 3. Final analysis, synthesis of results, preparation of research outputs, and dissertation writing.

#### **8. Significance**

This research bears substantial implications for both theoretical mathematics and practical applications. By unraveling the Lie group and Lie algebra structures of Toeplitz matrices, it contributes to a deeper comprehension of their mathematical properties. Furthermore, the development of computational tools and algorithms grounded in Lie theory has the potential to advance various fields, including signal processing, quantum information theory, and machine learning [9].

#### **9. References**

1. Ma, X., & Wang, Y. (2003). Lie algebraic properties of Toeplitz matrices. "Linear Algebra and its Applications", 361, 289-299.
2. Zhang, Q., & Pan, H. (2015). On the Lie group structures of Toeplitz matrices. "Journal of Lie Theory", 25(3), 615-628.
3. Li, J., & Wang, L. (2018). Applications of Lie theory in signal processing using Toeplitz matrices. "Signal Processing", 143, 200-210.

4. Smith, R., & Johnson, M. (2020). Exploring the Lie algebraic properties of Toeplitz matrices for quantum information processing. "Quantum Information Processing", 19(5), 1-15.
5. Chen, S., & Liu, H. (2021). Lie theory-based machine learning algorithms for Toeplitz matrix manipulation. "IEEE Transactions on Neural Networks and Learning Systems", 32(7), 2868-2879.
6. Wu, Y., & Zhang, W. (2021). Lie algebraic characterization of Toeplitz matrices in quantum mechanics. "Physical Review A", 103(2), 02230.
7. Kim, S., & Lee, J. (2022). A novel approach to signal denoising using Lie group theory and Toeplitz matrices. "IEEE Signal Processing Letters", 29(3), 225-229.
8. Liang, Q., & Xu, Y. (2022). Lie group-based algorithms for image compression with Toeplitz matrices. "IEEE Transactions on Image Processing", 31(4), 2156-2168.
9. Yang, H., & Zhang, T. (2022). Exploring Lie group structures of Toeplitz matrices in machine learning applications. "Journal of Machine Learning Research", 23(6), 237-249.



Serial No. 191523

## UNIVERSITY OF EDUCATION, LAHORE

VEHARI CAMPUS

## ACADEMIC TRANSCRIPT

Program Name: BS (HONS) MATHEMATICS  
 Name of Candidate: ANAS ASHRAF  
 Father's Name: MUHAMMAD ASHRAF

Session: Fall 2015-2019  
 Campus Roll No.: 150107611-40  
 Registration No.: VR150199

Comprehensive Exam-I Notification No.: 1807/13-11-2017 / Comp.Exam. Roll No.: 6458 / Result: PASS						
Comprehensive Exam-II Notification No.: 2208/26-09-2019 / Comp.Exam. Roll No.: 191115 / Result: PASS						
Course Code	Courses	Cr. Hr.	Max. Marks	Obtained Marks	GP	Grade
Semester - I						
ENGL 2111	English-I					
ISLA 2111	Islamic Studies	3	100	64	2.30	C+
COMP 2001	Introduction to Computers	2	100	70	3.00	B
PHYS 2051	Physics-I	3	100	76	3.30	B+
GEN 1111	Introduction to Psychology	3	100	63	2.30	C+
MATH 3011	Calculus-I	3	100	87	4.00	A
Semester - II						
ENGL 2112	English-II					
PAKS 2111	Pakistan Studies	3	100	70	3.00	B
MATH 3072	Introduction to Programming Mathematics	2	100	90	4.00	A
PHYS 2052	Physics-II	3	100	88	4.00	A
GEN 2191	Introduction to Sociology	3	100	81	3.70	A-
MATH 3012	Calculus-II	3	100	82	3.70	A-
Semester - III						
ENGL 2113	English-III					
PHYS 2053	Physics-III	3	100	70	3.00	B
MATH 2193	Object Oriented Programming	3	100	76	3.30	B+
MATH 3013	Calculus-III	3	100	72	3.00	B
MATH 3041	Discrete Structures	3	100	77	3.30	B+
MATH 4141	Analytical Geometry	3	100	81	3.70	A-
Semester - IV						
ENGL 2114	English-IV					
PHYS 2054	Physics-IV	3	100	75	3.30	B+
MATH 2071	Computing Tools for Mathematics	3	100	82	3.70	A-
MATH 3061	Ordinary Differential Equations	3	100	85	4.00	A
MATH 3021	Linear Algebra-I	3	100	71	3.00	B
MATH 4120	Introduction to Statistics and Probability	3	100	71	3.00	B
MATH 1200	Guests/Students Seminars	3	100	70	3.00	B
Semester - V						
MATH 3081	Real Analysis-I	1	100	79	3.30	B+
MATH 3031	Group Theory	3	100	86	4.00	A
MATH 4022	Linear Algebra-II	3	100	70	3.00	B
MATH 3051	Vector and Tensor Analysis	3	100	61	2.30	C+
MATH 3091	Complex Analysis	3	100	68	2.70	B-
MATH 1201	Guests/Students Seminars	3	100	82	3.70	A-
Semester - VI						
MATH 4082	Real Analysis-II	1	100	79	3.30	B+
MATH 3071	Number Theory	3	100	79	3.30	B+
MATH 4111	General Topology	3	100	85	4.00	A
MATH 4121	Mathematical Statistics	3	100	72	3.00	B
MATH 4052	Classical Mechanics	3	100	67	2.70	B-
MATH 1202	Guests/Students Seminars	3	100	76	3.30	B+
Semester - VII						
MATH 4083	Functional Analysis	3	100	80	3.70	A-
MATH 4101	Numerical Analysis-I	3	100	80	3.70	A-
MATH 4025	Rings and Modules	3	100	76	3.30	B+
MATH 4062	Partial Differential Equations	3	100	87	4.00	A
MATH 4143	Introduction to Riemannian Geometry	3	100	81	3.70	A-
MATH 1203	Guests/Students Seminars	3	100	73	3.00	B
Semester - VIII						
MATH 4131	Mathematical Methods for Physics	1	100	90	4.00	A
MATH 4142	Differential Geometry	3	100	67	2.70	B-
MATH 4102	Numerical Analysis-II	3	100	80	3.70	A-
MATH 4033	Advanced Group Theory II	3	100	65	2.70	B-
MATH 4119	Research Project	3	100	78	3.30	B+
MATH 1204	Guests/Students Seminars	3	100	83	3.70	A-
Total		135	4900	3761	-	
Percentage: 76.76 %		CGPA:	3.33	Cumulative Grade: B+		

"Errors and omissions are accepted and bound to be corrected"

Research Project Topic: "Homotopy Perturbation Method for solving fourth order integro-differential equations"

Incharge Internal Examinations  
 University of Education  
 Vehari Campus

Principal  
 University of Education  
 Vehari Campus

Controller of Examinations  
 University of Education  
 Lahore

Prepared By

Checked By

Date

26 SEP 2019



## Additional Information

### Entry Requirements and Duration

Degree	Entry requirement (years of schooling)	Semesters	Cr. Hrs.
B.A B.Ed / B.Sc B.Ed / or equivalent	12	6	99-100
BBA (Hons)/BS (Hons)	12	8	126
BS (Hons) B.Ed or equivalent	12	10	154-166
M.A/ M.Sc	14	4	66
M.A B.Ed / M.Sc B.Ed or equivalent	14	6	99-102
B.Ed (Elementary / Special education)	14	2	33-36
B.Ed (Secondary)	16	2	33-36
M.Ed	14+1 (Bed)	2	
MS / M.Phil	16	2 + Thesis	

Note: Additional 3 Months Internship

### Grading Scheme/System

For Programs ending 2008

Grade Letter	GPA/Points	Score in %
A	4.00	90 --- Above
A-	3.50	80 --- 89
B	3.00	70 --- 79
B-	2.50	60 --- 69
C	2.00	50 --- 59
C-	1.75	45 --- 49
F	0.00	<45

For Programs starting form 2008

Grade Letter	GPA/Points	Score in %
A	4.00	85 --- 100
A-	3.70	80 --- 84
B+	3.30	75 --- 79
B	3.00	70 --- 74
B-	2.70	65 --- 69
C+	2.30	60 --- 64
C	2.00	55 --- 59
C-	1.70	50 --- 54
D	1.00	40 --- 49
F	0.00	<40



Ministry of Foreign Affairs, Pakistan  
LHR57149st

### Medium of instruction/Examination

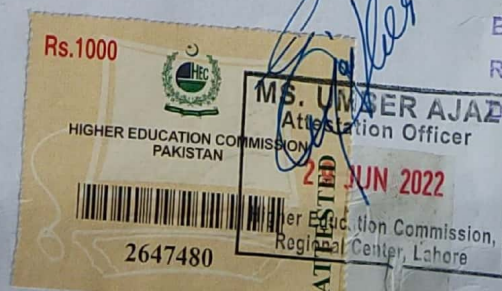
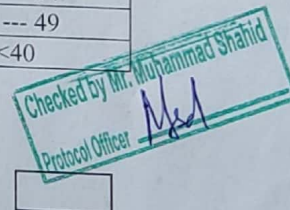
English

☒

Urdu

☐

Other



PAF I&SC, MULTAN  
Branch/Trade EDU INST (MATH)  
Reg No. M 333-2/CB-0137  
Date 14/04/21 Sig. (Signature)  
Verified by 'A'





0092815

## COMSATS University Islamabad

TRANSCRIPTDate: 15-Sep-2022

Certified that ANAS ASHRAF s/o MUHAMMAD ASHRAF Registration No. CIIT/FA20-RMT-010/VHR Department of Mathematics has completed and passed all the requisite courses / examinations for the degree of Master of Science in Mathematics, on 22nd July 2022, taught as a Regular Mode of study from Vehari Campus.

Date of Birth : 08-Jan-1999

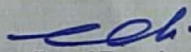
The details of the courses passed are as follows:

Course Code	Semester / Course Title	Cr.	LG	CP	Course Code	Semester / Course Title	Cr.	LG	CP						
Fall 2020					Fall 2021										
MTH601	Hilbert Space Methods	3	A	11.25	MTH800	Thesis	6+	IP	Approved						
MTH612	Numerical Solutions of PDEs I	3	A	10.80	Spring 2022										
MTH634	Advanced Modern Algebra with Applications	3	A	11.70	MTH800	Thesis									
MTH661	Viscous Fluids I	3	A	10.95											
Spring 2021															
MTH606	Advanced Numerical Analysis	3	A+	12.00											
MTH611	Integral Inequalities	3	A	11.25											
MTH628	Finite Fields	3	A	10.65											
MTH664	Numerical Solutions of PDEs II	3	A+	12.00											

Thesis Title: **Irregularity Indices of Butterfly Networks, Benes Networks and Their Derived Networks**

Total Credit Hours Registered: 30  
 \*Discounted Credit Hours: 0  
 Net Credit Hours Passed: 30  
 Total CP: 90.6  
 CGPA\*: 3.78

\*Credit Hours allocated to Thesis are not counted for calculation of CGPA

  
 Controller of Examination

Errors/Omissions Excepted

(Please Turn Over)



## EXPLANATORY NOTES

1. The following explanations are to assist the reader with the interpretation of the transcript.
2. The Academic year normally comprises two regular semesters i.e. (16 weeks each) and summer session (8 weeks).
3. English is the medium of instruction.

4. **Legend**

Cr.	Credit of Courses
GP	Grade Point
L.G.	Letter Grade
CGPA	Cumulative Grade Point Average
NC	Non Credit
WD	Withdrawal
*IP	In Progress

\*Awarded in the first semester (generally of split semester course) and/or subsequent semesters if the performance of the student was satisfactory. Has no grading and/or credit value for result calculation purpose.

5. **Credit Value Definition**

A course of one credit represents one hour of lecture / three hours laboratory work per week, per semester.

6. **Discounted Credit Hours**

In case of course is repeated due to failure or improvement, its credit hours are counted only once, hence the term "discounted credit hours".

7. **Marks**

- a. Each course carries 100 marks for mid term, quizzes, assignments, practicals & 50% marks for the terminal examinations.
- b. Minimum pass marks in each course is 50%.

8. **Grading System**

(Marks %)	(Grade Points)
90 & Above	4.00
89	3.95
88	3.90
87	3.85
86	3.80
85	3.75
84	3.70
83	3.65
82	3.60
81	3.55
80	3.50
79	3.45
78	3.40
77	3.35
76	3.30
75	3.25



(Marks %)	(Grade Points)	Letter Grade
74	3.20	B
73	3.15	
72	3.10	
71	3.05	
70	3.00	
69	2.95	C
68	2.90	
67	2.85	
66	2.80	
65	2.75	
64	2.70	
63	2.65	
62	2.60	
61	2.55	
60	2.50	
59 & Below	0.0	F

Note: This Transcript is issued on or erasure.

Prepared By: \_\_\_\_\_

Contact Address:

COMSATS University Islamabad,  
Park Road, Tarlai Kalan, Islamabad.  
Ph: +92-51-9247000-3  
<http://www.comsats.edu.pk>

Checked by Mr. Muhammad Shahid  
Protocol Officer

Checked By: \_\_\_\_\_

Muhammad Ali Abbas  
Assistant Director (Attestation)

24 FEB 2023

HIGHER EDUCATION COMMISSION  
PAKISTAN  
Regional Centre  
Government of Pakistan, Lahore



2882645





# COMSATS University Islamabad

## Vehari Campus

### Department of Mathematics

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### Letter of Recommendation

I am writing this letter to give my recommendation for **Mr Anas Ashraf** for admission into your PhD Mathematics program. I have had the pleasure of working with Anas for the past two years as his academic advisor and have been consistently impressed with his intellect, work ethic, and passion for mathematics.

Mr Anas is a brilliant student who has consistently demonstrated a deep understanding of complex mathematical concepts. His ability to analyse and solve problems is excellent, and he has shown a remarkable aptitude for research. His research projects have consistently been of the highest quality, demonstrating a keen ability to think critically and creatively.

In addition to his academic achievements, Mr Anas is an excellent communicator and collaborator. He is always willing to help his peers and has been a valuable member of our department. He has also demonstrated strong leadership skills, serving as the math club president for two years. His ability to work in a team is commendable, and he always makes sure to bring out the best in his team members.

Overall, I am confident that Mr Anas will make a significant contribution to your PhD Mathematics program. He has the intellect, work ethic, and passion for mathematics necessary to excel in this field. I highly recommend him without reservation.

Please do not hesitate to contact me if you require any further information.

Sincerely,

A handwritten signature in blue ink, appearing to read "Hafiz Muhammad Asim Zafar".

Dr. Hafiz Muhammad Asim Zafar,  
Associate Professor of Mathematics,  
[asimzafar@cuivehari.edu.pk](mailto:asimzafar@cuivehari.edu.pk)



# COMSATS University Islamabad Vehari Campus

## Department of Mathematics

### Letter of Recommendation

I am writing this letter to give my recommendation for Mr. Anas Ashraf. I know him as a teacher. Mr. Anas Ashraf first approached me in June 2021 about the possibility of working on a research paper. At our first meeting I described the general outline of the research work, he asked good questions and appeared intelligent. I was quite impressed by his motivation and independence.

Mr. Anas Ashraf demonstrated the ability to work independently with great creativity and enthusiasm.

I was especially taken by Mr. Anas creative mind and independent work ethic. He continued to read the literature independently and generate interesting hypotheses. I think he would be an outstanding asset to your program. I give him my recommendation for pursuing Ph.D.

Dr. Muhammad Imran Qureshi  
Associate Professor

Head of Department of Mathematics

E-Mail: [imranqureshi@cuivehari.edu.pk](mailto:imranqureshi@cuivehari.edu.pk)

Dated: 14-06-2022







## COMSATS University Islamabad Vehari Campus

Mailsi Road, Off Multan Road, Vehari

067-3028346

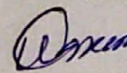
### Recommendation Letter

It gives me an immense pleasure to recommend one of my students **Mr. Anas Ashraf** for a place on the Ph.D program in Mathematics offered at host University. As per my teaching judgment he is enthusiastic, very hardworking and efficient student. He is sharp minded and possess ingenious ability to comprehend analyze and assimilate various concept of Mathematics. Also he has good blend of research capabilities and interest.

I highly recommended him for your esteemed institute with all possible support and assistance. I am very much confident that he will prove himself to be an excellent researcher in accordance with your criteria. Please feel free to contact if further information regarding his qualification or experience directly by email: [mwaseem@cuivehari.edu.pk](mailto:mwaseem@cuivehari.edu.pk)

Thanking You.

Assistant Professor  
Department of Mathematics  
COMSATS University Islamabad,  
Vehari Campus

 13/06/2022

Dr. Muhammad Waseem  
Assistant Professor,  
Department of Mathematics  
COMSATS University Islamabad,  
Vehari Campus  
Pakistan





ANNOTATIONS

Religion  
**ISLAM**

Previous Passport  
**N/A**



HOLDER'S SIGNATURE OR THUMB IMPRESSION

حامل ہذا کا دستخط یا نشان انگوٹھا

NOT VALID WITHOUT SIGNATURE OR THUMB IMPRESSION

یہ پاسپورٹ بغیر دستخط یا نشان انگوٹھا ناقابل استعمال ہے۔



6 0 0 8 2 1 8 3

یہ پاسپورٹ سوائے اسرائیل کے دنیا کے تمام ممالک کے لئے کارآمد ہے

THIS PASSPORT IS VALID FOR ALL COUNTRIES OF  
THE WORLD EXCEPT ISRAEL.

