

Report on “Equidistribution without stability for toric surfaces” by Jeffrey Diller and Roland Roeder

In this article, the authors consider certain toric rational maps $f : \mathbb{P}^2 \dashrightarrow \mathbb{P}^2$ that lack stable models. While there are various results in the literature discussing the dynamics of stable maps, equidistribution results in non-stable cases are rare. The authors investigate such problems for toric maps in this article. The article is nicely written, and the results are very interesting, therefore, I strongly recommend it for publication at *Commentarii Mathematici Helvetici*.

Here are my comments:

- Page 2, line 28: It might be useful to cite Theorem 5 in [Sto79] to imply that $\hat{\mathbb{T}}$ is compact.
- Theorem 1.3 seems to be a corollary of Theorem 1.5 in [BH].
- Page 9, line 7: Note that only *normal* toric surfaces/varieties are given by their fans.
- Page 9, line 23: Σ needs to be complete. See [CLS11] Chapter 8.
- Subsection 3.1. It might be reasonable to mention that similar ideas for the inverse limit were used in Section 5 in [FS97] to express that the direct limit of Chow groups in that setting coincides with McMullen polytope algebra.
- Page 26, it might be useful to mention that when $u = \log |f|$, where f is a Laurent polynomial, $\psi_u(v)$ coincides with the Ronkin function of f .
- Page 40, Proof of Proposition 7.14. Are you using the Weak Factorization Theorem to write $\pi_{\hat{X}} = \tau_k \circ \cdots \circ \tau_1$? I believe τ_i ’s need to contain blow-downs too. The statement should remain correct.

References

- [CLS11] David A. Cox, John B. Little, and Henry K. Schenck. *Toric varieties*, volume 124 of *Graduate Studies in Mathematics*. American Mathematical Society, Providence, RI, 2011.
- [FS97] William Fulton and Bernd Sturmfels. Intersection theory on toric varieties. *Topology*, 36(2):335–353, 1997.
- [Sto79] A. H. Stone. Inverse limits of compact spaces. *General Topology Appl.*, 10(2):203–211, 1979.