Hints for CW 1

Q1

$\mathbb{C}^n$ with Euclidean topology is homeomorphic to $\mathbb{R}^{2n}$. In simpler terms, you can treat $\mathbb{C}^n$ as $\mathbb{R}^{2n}$ as far as Euclidean topology is concerned.

Q2 (b)

$\mathbb{V}(y^3 - x^2)$ is just an example of a closed affine algebraic variety. You can similarly prove that any closed affine algebraic variety in $\mathbb{A}^n$ is compact with the Zariski topology.

Q3 (d)

You have a variety which is given as the intersection of hypersurfaces. To understand it, solve the equations like you did in high school:

$$\begin{aligned} y^2 - x^2(x^2+1) &= 0, \\ y &= zx. \end{aligned}$$

and see what you get. To prove that different components are irreducible, you might use the idea from Q3(c) and Example 2.41 of the notes.

Q4

(a) Note that $V \neq V \cup \{a\}$.

(ii) Use Part (i) to write 1 as a linear combination of the generators with coefficients in $\mathbb{C}[x_1, \ldots, x_{n+1}]$.