

## Assessed Coursework 2

- Available from 13:00, April 17th to 13:00, April 24th
- Please submit your work in PDF format directly on Blackboard
- This exam counts for %7.5 of your final course mark
- Feel free to discuss Q1 with each other, but I expect the solutions to Q2 to Q5 to be the outcome of your sole effort
- Please justify your answers in detail

Q1. (a) **(10 marks)** Find all the elements of  $\max\text{Spec}(\mathbb{C}[x])$  and  $\max\text{Spec}(\mathbb{C}[x, 1/x])$ , respectively.

(b) **(10 marks)** Consider the isomorphism  $\varphi : \mathbb{A}^1 \setminus \{0\} \longrightarrow \mathbb{A}^1 \setminus \{0\}$ ,  $a \longmapsto b = 1/a$ , and the pullback map on the coordinate rings  $\varphi^* : \mathbb{C}[x, 1/x] \longmapsto \mathbb{C}[y, 1/y]$ . Compute  $\varphi^*(1/x)$ ,  $\varphi^*(2x^2 + \frac{2x^3+4x}{x^5})$ ,  $\varphi^*(2-x)$ .

Q2. Consider the affine algebraic hypersurface  $V := \mathbb{V}(y - ux) \subseteq \mathbb{A}^3$ .

- (a) **(10 marks)** Prove that the projection  $\mathbb{A}^3 \longrightarrow \mathbb{A}^2$ ,  $(x, y, u) \longmapsto (x, u)$  restricts to an isomorphism between  $V$  and  $\mathbb{A}^2$ .
- (b) **(10 marks)** Prove that the projection  $\mathbb{A}^3 \longrightarrow \mathbb{A}^2$ ,  $(x, y, u) \longmapsto (x, y)$  does not restrict to isomorphism between  $V$  and  $\mathbb{A}^2$ .
- (c) **(10 marks)** Find  $\mathcal{O}_V(D(u))$ .

Q3. **(20 marks)** Prove that if  $V$  is an irreducible affine variety, then so is its projective closure  $\bar{V}$ .

Q4. **(10 marks)** What is the projective closure of the  $\mathbb{V}(y - \sin(x))$  in  $\mathbb{P}^2$ ? Would this contradict the Chow Lemma?

Q5. **(20 marks)** Consider the family of algebraic varieties, with parameter  $t \in \mathbb{C}$ , given by

$$V_t := \mathbb{V}(xy^2 - t) \subseteq \mathbb{A}^2.$$

Sketch the variety of  $V_0, V_1$ , and  $V_2$  in  $\mathbb{R}^2$ . Which one of these varieties are smooth? Which one of these varieties are irreducible?