

Assignment 12

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2 2018210031

3 Assignment 12

3.1 Non-Linear Least Square Fitting with regularization

Github Link: <https://github.com/farhan-93/assignment12.git>

Import required libraries

```
In [5]: import numpy as np
        import matplotlib.pyplot as plt
        import scipy as sp
        from scipy import signal
        plt.rcParams.update({'figure.max_open_warning': 0})
```

Below function will generate the data.

```
In [10]: def make_data():

    num = 1001
    std = 5

    def fun (x):
        #f = np.sin (x) * (1 / (1 + np.exp (-x)))
        f = np.abs (x) * np.sin (x)
        return f

    n = np.random.rand (num)
    nn = n - np.mean (n)
    x = np.linspace (-10,10, num)
    y1 = fun (x)
    y2 = y1 + nn * std

    plt.figure(0)
```

```

plt.title("Clean Data")
plt.plot (x, y1, 'b.')
plt.plot(x,y1,'b')

plt.figure(1)
plt.title("Noisy Data")
plt.plot (x, y2, 'k.')
plt.plot(x,y2,'k')

plt.figure(3)
plt.title("Clean and Noisy Data")
plt.plot (x, y2, 'k.' )
plt.plot(x,y2,'k',label="Noisy")
plt.plot (x, y1, 'b.')
plt.plot(x,y1,'b',label="Clean")
plt.legend()
plt.show ()

return x, y2, y1

```

Below function will calculate the best nonlinear curve line for above random generated data.
And calculate the approximation function

$$\hat{f}(x) = \theta_0 + \theta_2 x + \dots + \theta_{p-1} x^{p-1}$$

Least square fitting formula with regularization:

$$E(\cdot) = \|Ay\|_2^2 + \|\cdot\|_2^2$$

```
In [7]: ##### d=p-1
def least_squares(x, y,d,lamb):
```

```

n = len(x)
##### Create the matrix A (polynomial matrix)
a=np.c_[np.ones(n)]
for z in range(1,d):
    a = np.c_[a,x**z]
##### convert Feature matrix into sparse matrix
a=sp.sparse.csr_matrix(a)
##### MAKE IDENTITY MATRIX
I=sp.sparse.identity(d)
##### Convert identity sparse matrix into compressed sparse row matrix
I=sp.sparse.csr_matrix(I)
##### create zero matrix
zero=sp.sparse.csc_matrix((1,d))
##### Create block matrix A for regularized least square fitting
A=sp.sparse.bmat([[a.T,np.sqrt(lamb)*I]]).toarray()

```

```

##### Implement pseudo inverse method for least square problem
Ai=A.dot(A.T)
Ain=sp.sparse.csc_matrix(Ai)
Ainn=sp.sparse.linalg.inv(Ain)
B=sp.sparse.bmat([[y,zero]]).toarray()

right=A.dot(B.T)

u=Ainn.dot(right)

##### u is the model parameter.
u=np.asarray(u)
##### calculating curve or line by calculating approximation function d
curve=0.0
#####
for k in range(0,d):
    curve= curve +(u[k]*x**k)

return u,curve

```

d is the degree of polynomial. We calculate the 15 degree polynomials below. It can be change and can be any integer.

```

In [11]: ##### generate data
        x, y, y_clean = make_data()
        # d is the degree of plynomial
        count=1
        count1=0
        ##### for loop that control the polynomial degree from 8 to 17
        for i in range(8,17):

            error=[]

            ##### for loop that control the lambda value from 2**-10 to 2**10
            for j in range(-10,10):
                ##### Calculate the theta values
                lamb=2**j
                ##### call the least square function that function returns model parameters
                theta,curve= least_squares(x, y,i,lamb)
                ##### calculating the error
                residual =np.linalg.norm(y-curve)

                ##### print Least square error fitting on noisy data
                plt.figure(count)
                plt.title("Least Square Approximation at polynomial degree= "+str(i)+" Lambd"
                plt.scatter(x, y, marker=".")
                ##### cakculate the polynomial fit.

```

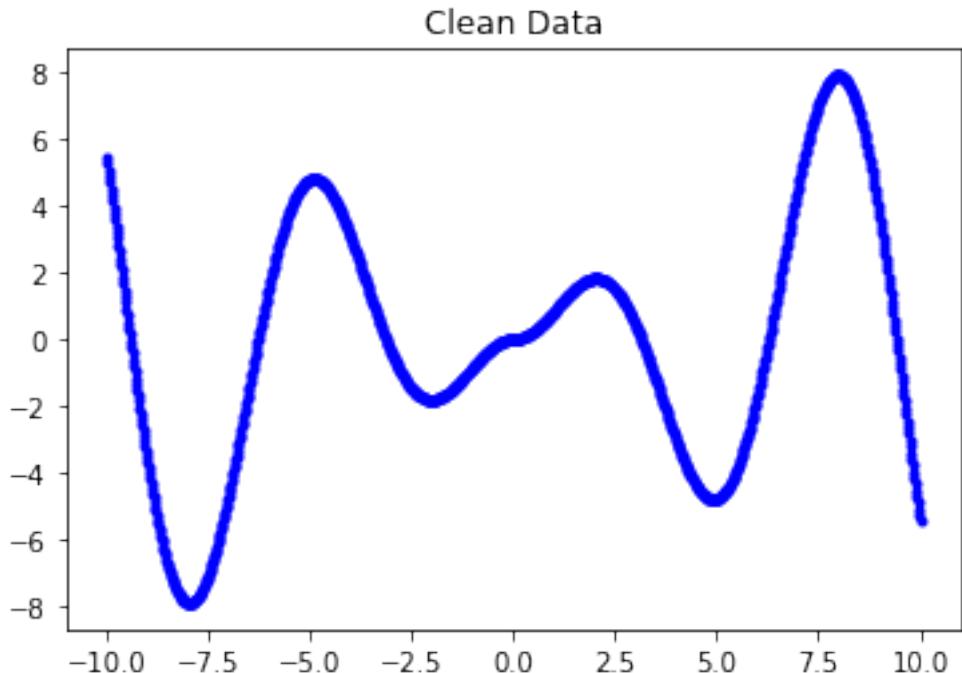
```

plt.plot(x, curve, 'r', label="Least square fitting")
plt.legend()
error=np.append(error,residual)
count+=1
count1+=1

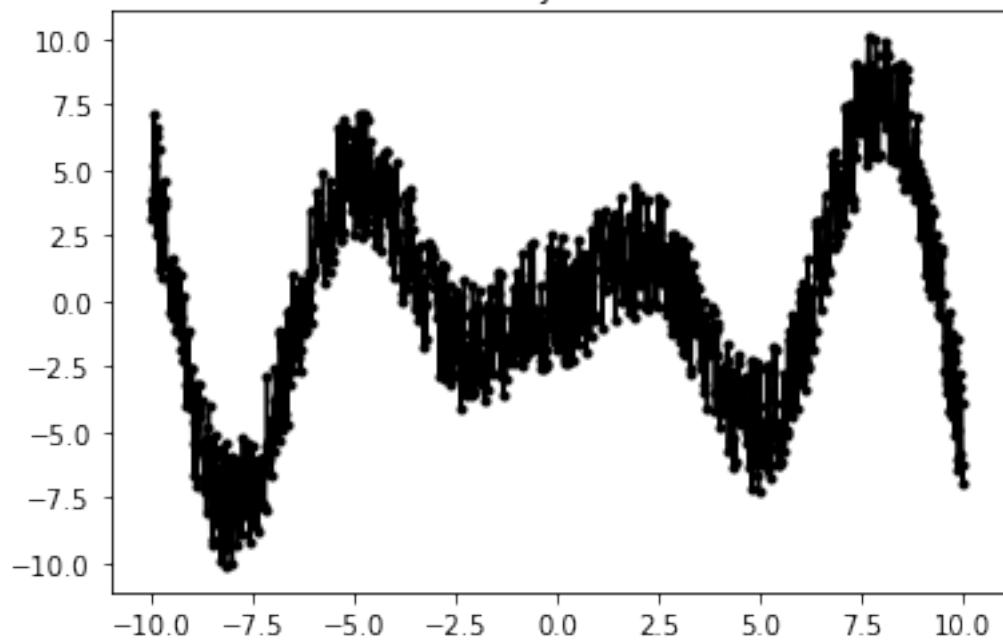
##### plot the Error/energy graph
plt.figure(500)
plt.title("Graph: Error")
plt.xlabel("Lambda index")
plt.ylabel("Error")
plt.plot(error,"-o")

plt.show()
print("-----")

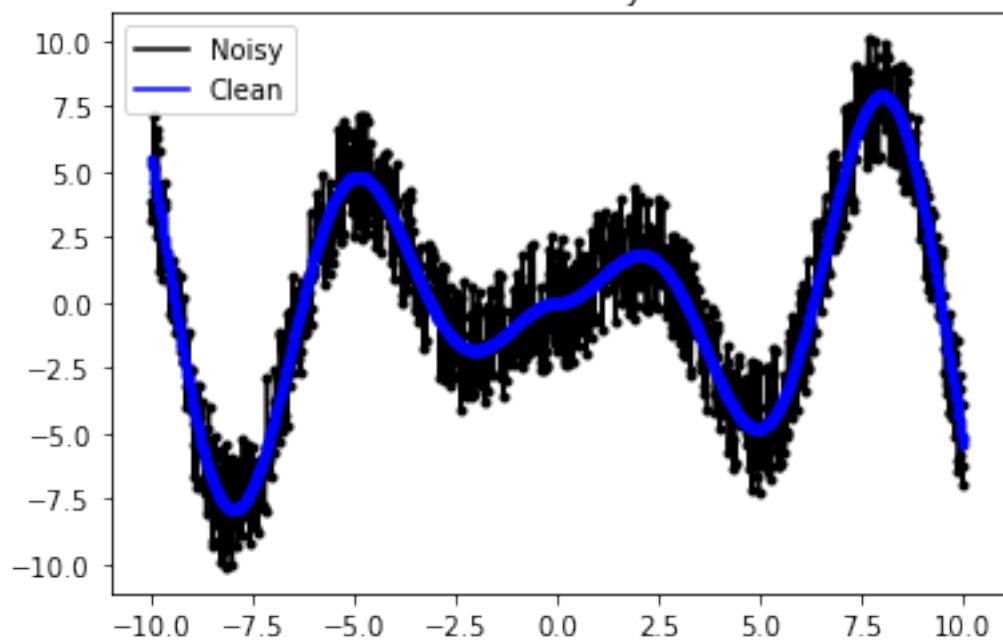
```



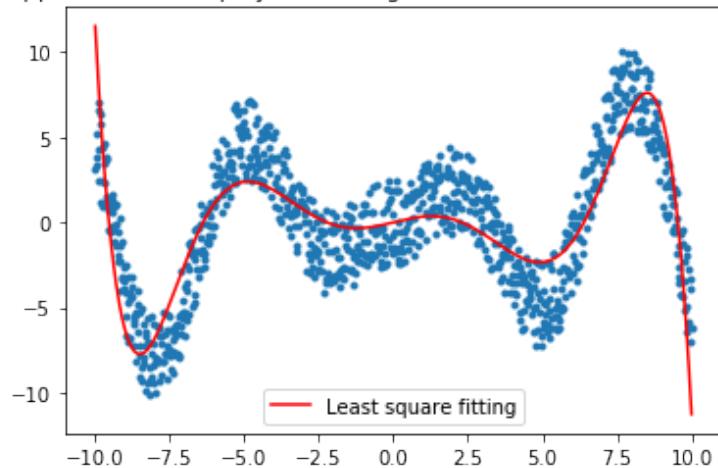
Noisy Data



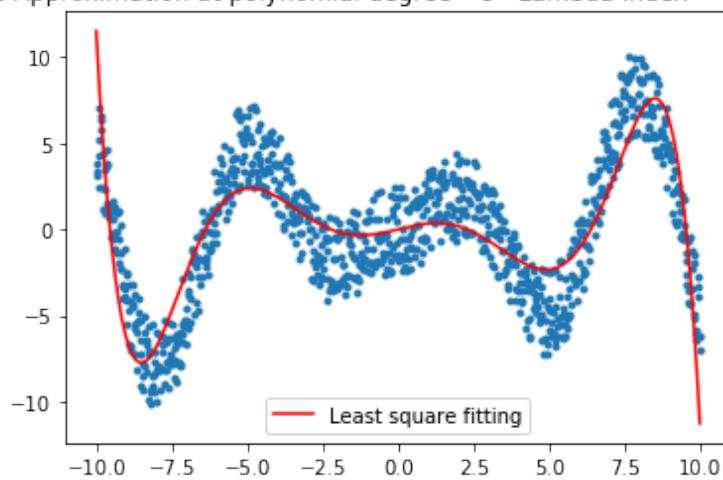
Clean and Noisy Data



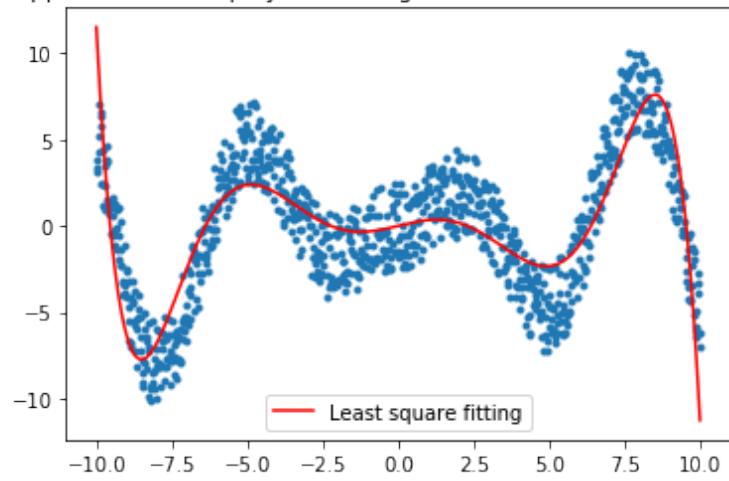
Least Square Approximation at polynomial degree= 8 Lambda index = 0 (Lambda: 2**-10)



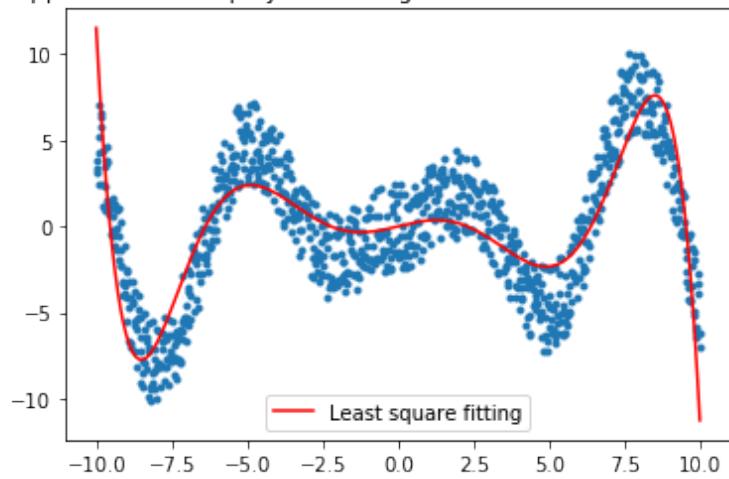
Least Square Approximation at polynomial degree= 8 Lambda index = 1 (Lambda: 2**-9)



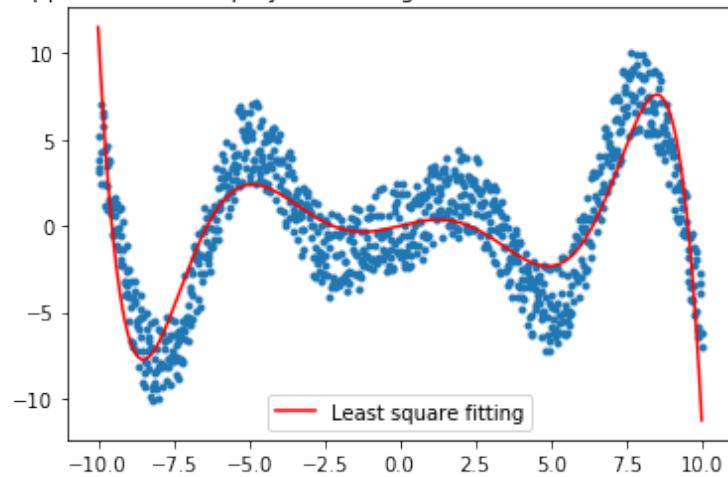
Least Square Approximation at polynomial degree= 8 Lambda index = 2 (Lambda: 2**-8)



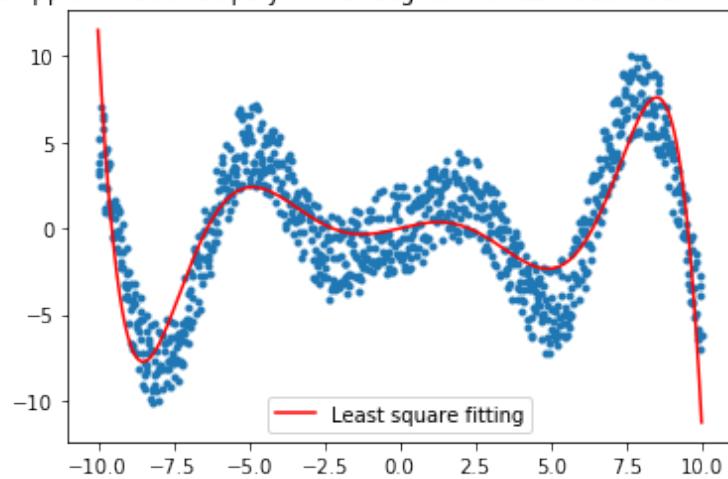
Least Square Approximation at polynomial degree= 8 Lambda index = 3 (Lambda: 2**-7)



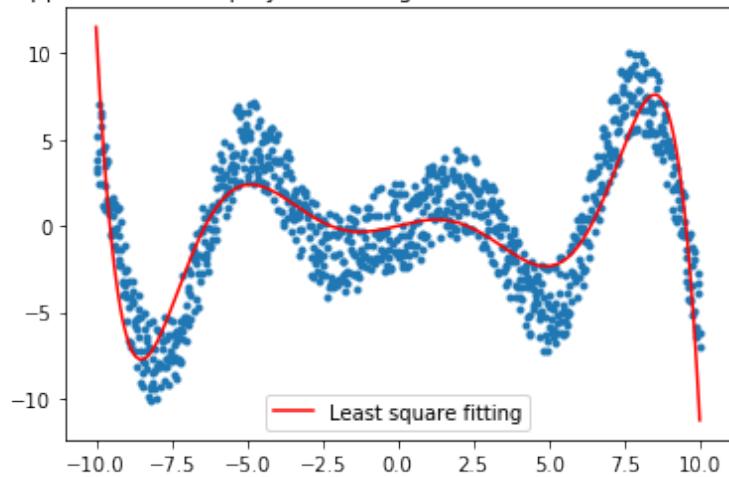
Least Square Approximation at polynomial degree= 8 Lambda index = 4 (Lambda: 2**-6)



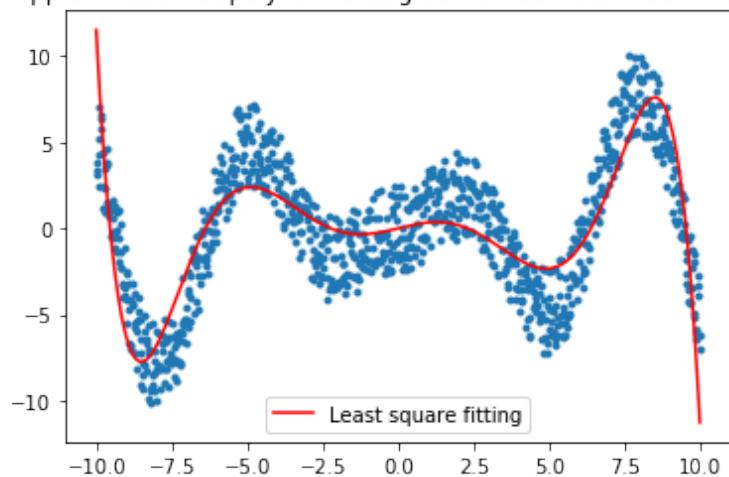
Least Square Approximation at polynomial degree= 8 Lambda index = 5 (Lambda: 2**-5)



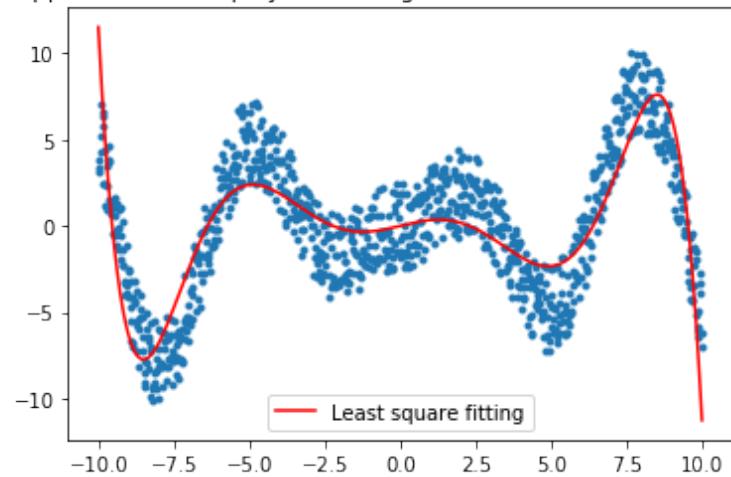
Least Square Approximation at polynomial degree= 8 Lambda index = 6 (Lambda: 2**-4)



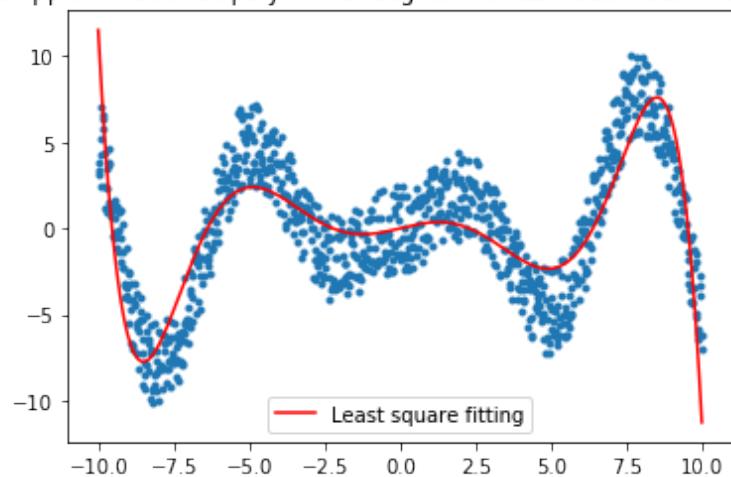
Least Square Approximation at polynomial degree= 8 Lambda index = 7 (Lambda: 2**-3)



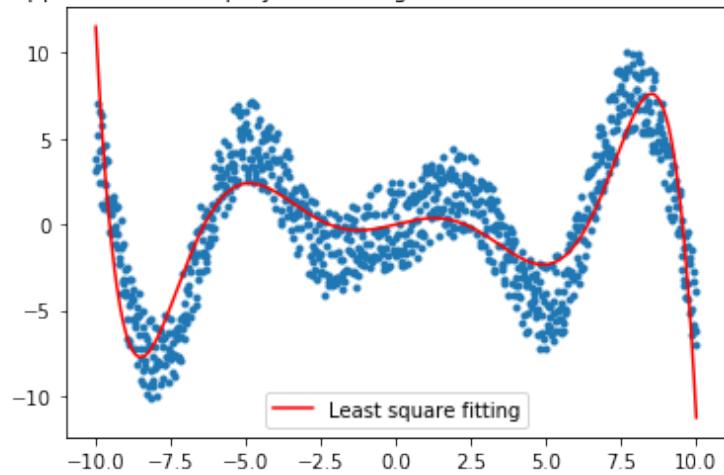
Least Square Approximation at polynomial degree= 8 Lambda index = 8 (Lambda: 2**-2)



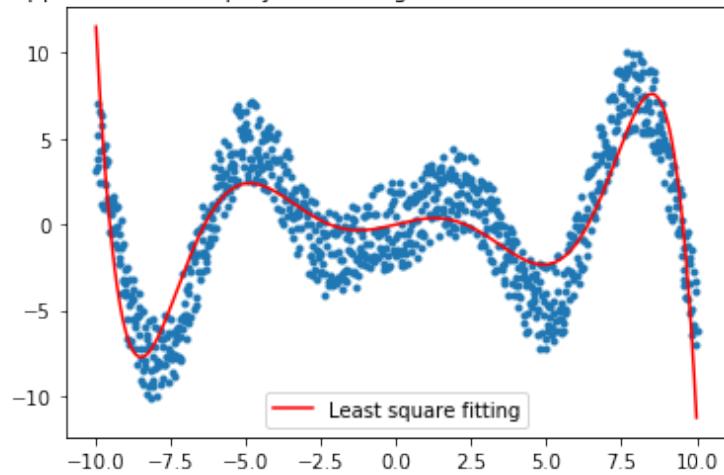
Least Square Approximation at polynomial degree= 8 Lambda index = 9 (Lambda: 2**-1)



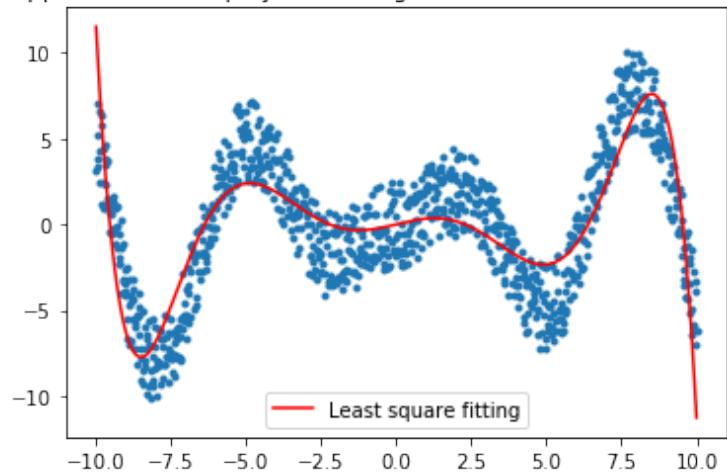
Least Square Approximation at polynomial degree= 8 Lambda index = 10 (Lambda: 2**0)



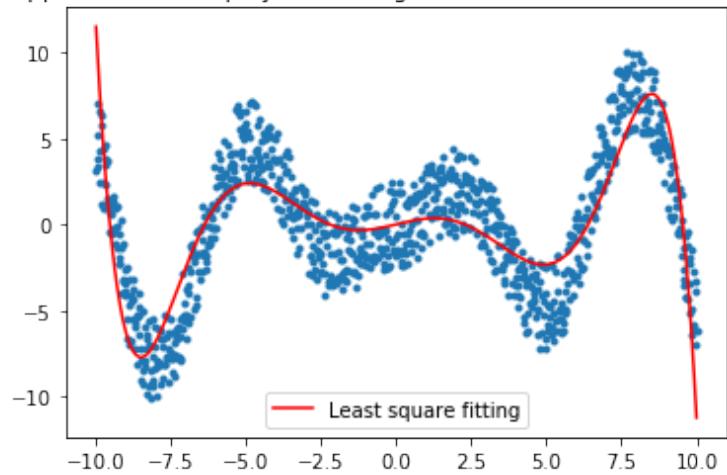
Least Square Approximation at polynomial degree= 8 Lambda index = 11 (Lambda: 2**1)



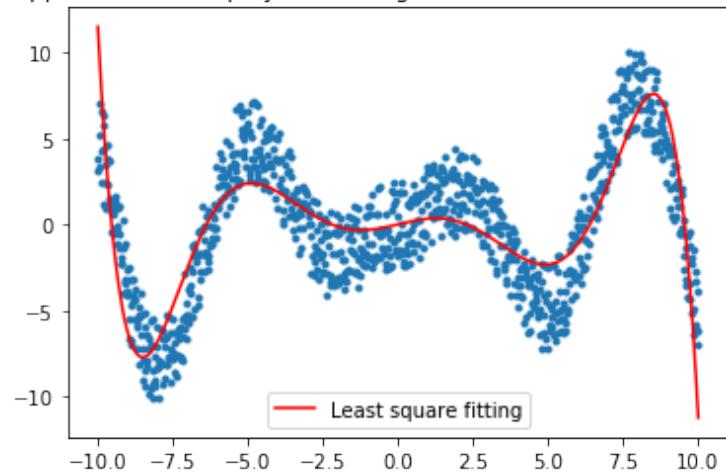
Least Square Approximation at polynomial degree= 8 Lambda index = 12 (Lambda: 2**2)



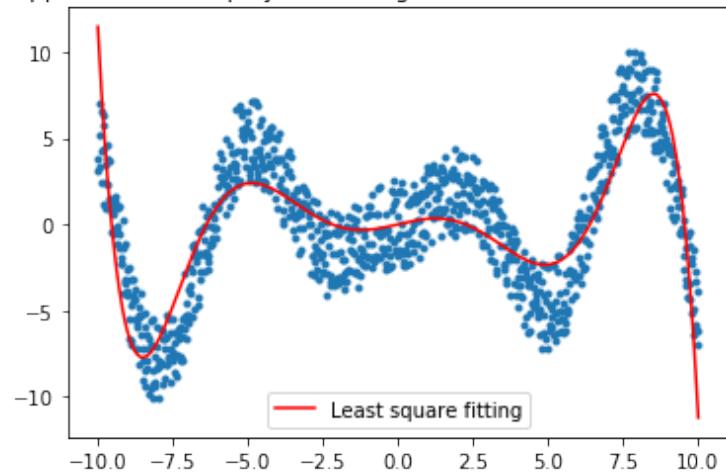
Least Square Approximation at polynomial degree= 8 Lambda index = 13 (Lambda: 2**3)



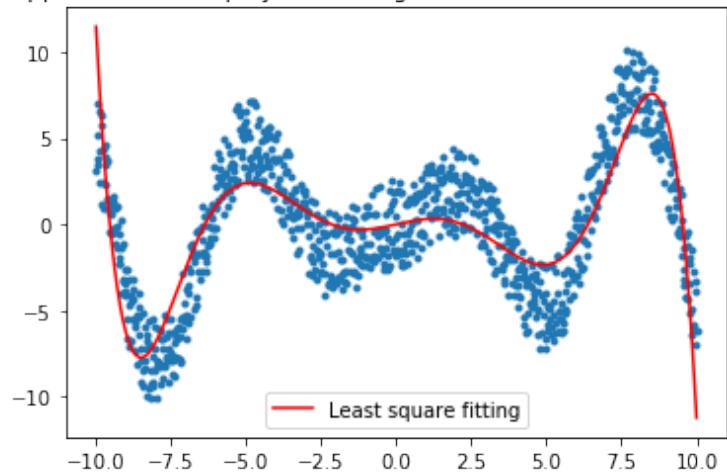
Least Square Approximation at polynomial degree= 8 Lambda index = 14 (Lambda: 2**4)



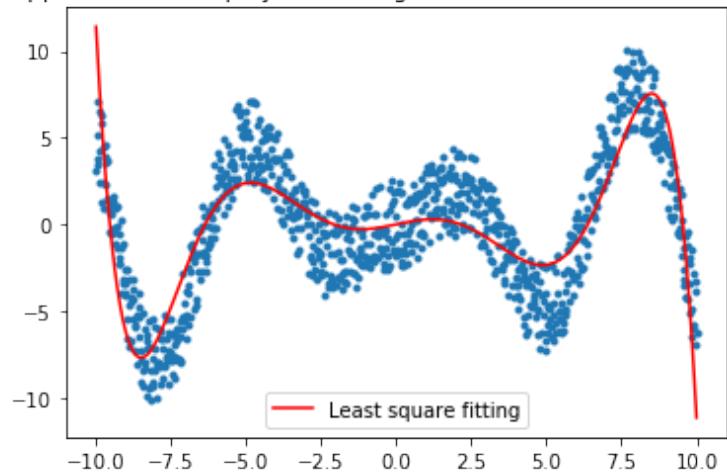
Least Square Approximation at polynomial degree= 8 Lambda index = 15 (Lambda: 2**5)



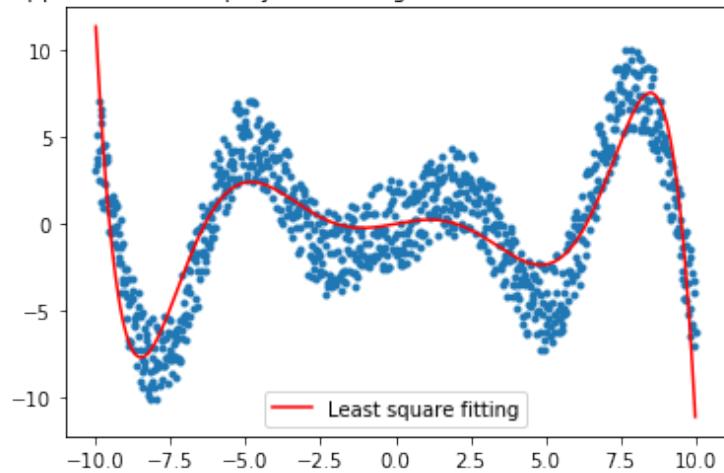
Least Square Approximation at polynomial degree= 8 Lambda index = 16 (Lambda: 2**6)



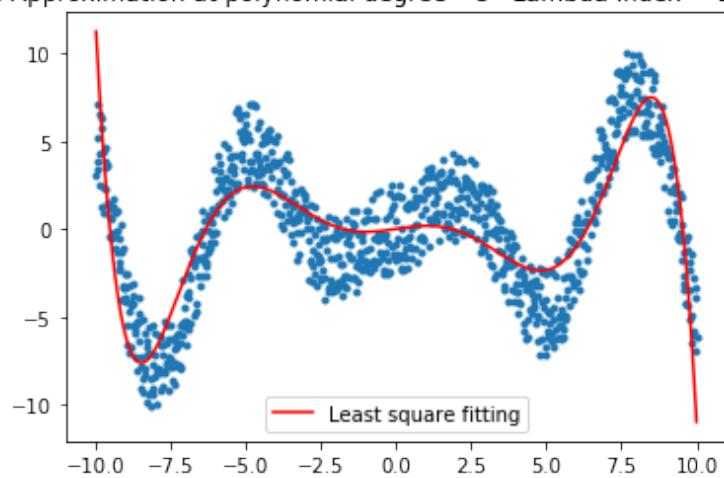
Least Square Approximation at polynomial degree= 8 Lambda index = 17 (Lambda: 2**7)



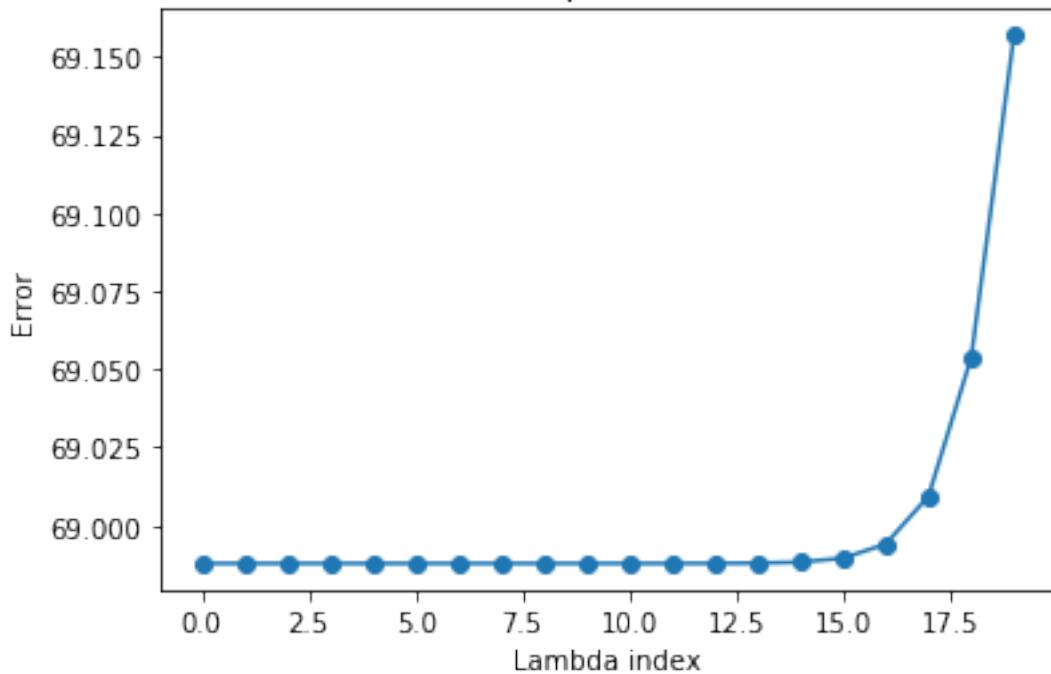
Least Square Approximation at polynomial degree= 8 Lambda index = 18 (Lambda: 2**8)



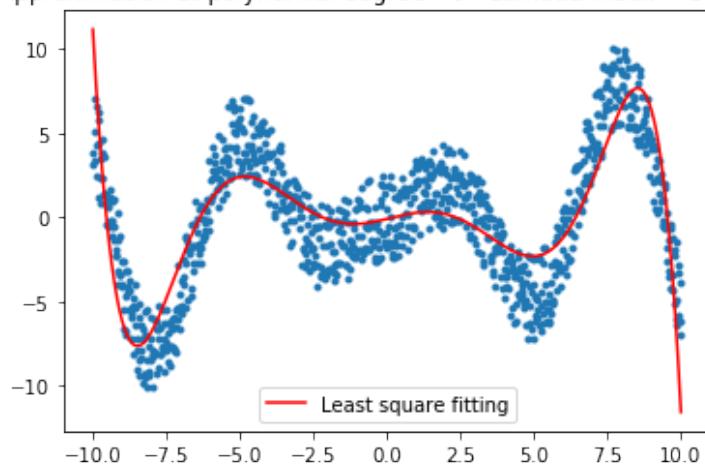
Least Square Approximation at polynomial degree= 8 Lambda index = 19 (Lambda: 2**9)



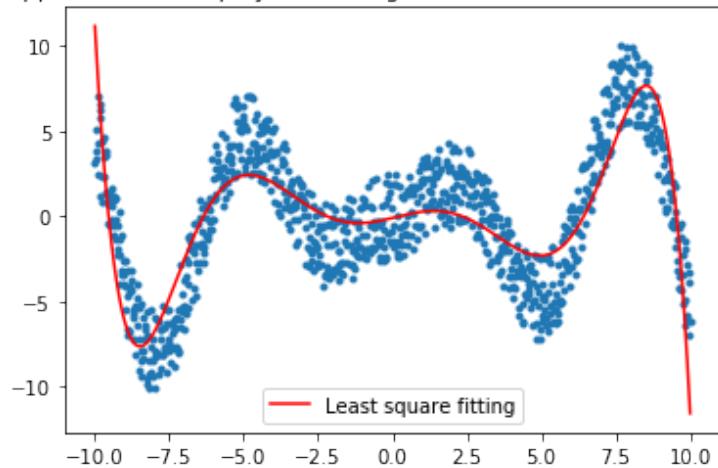
Graph: Error



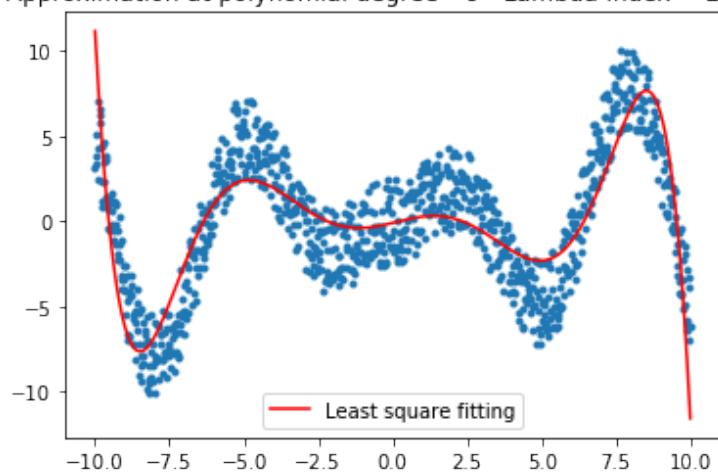
Least Square Approximation at polynomial degree= 9 Lambda index = 20 (Lambda: 2**-10)



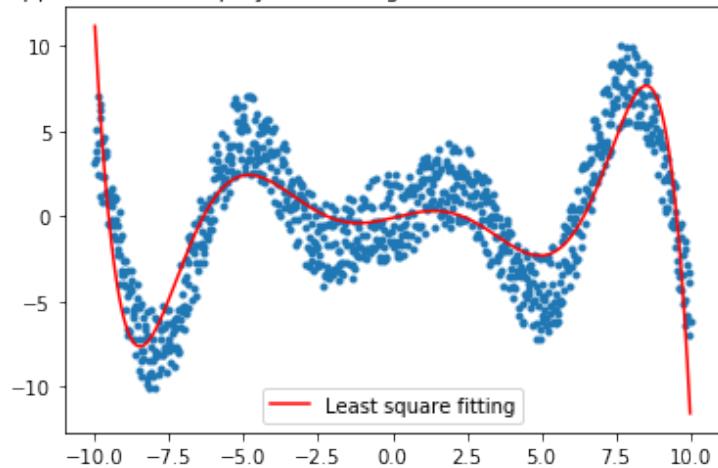
Least Square Approximation at polynomial degree= 9 Lambda index = 21 (Lambda: 2^{**-9})



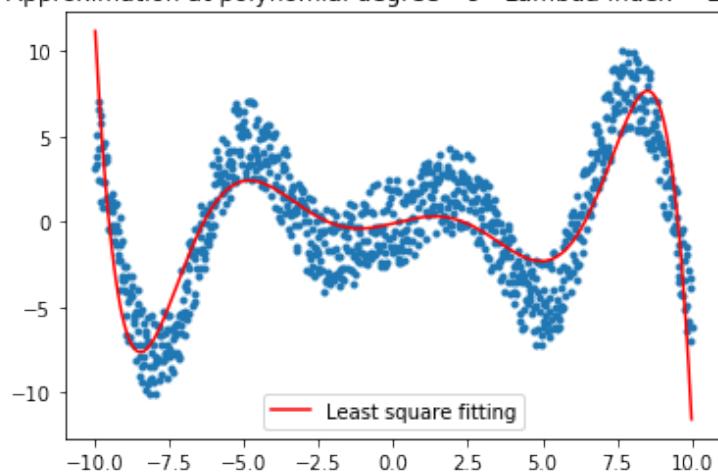
Least Square Approximation at polynomial degree= 9 Lambda index = 22 (Lambda: 2^{**-8})



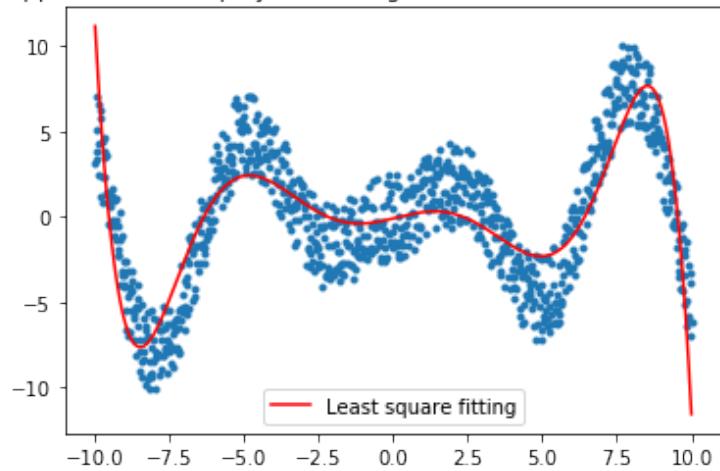
Least Square Approximation at polynomial degree= 9 Lambda index = 23 (Lambda: 2**-7)



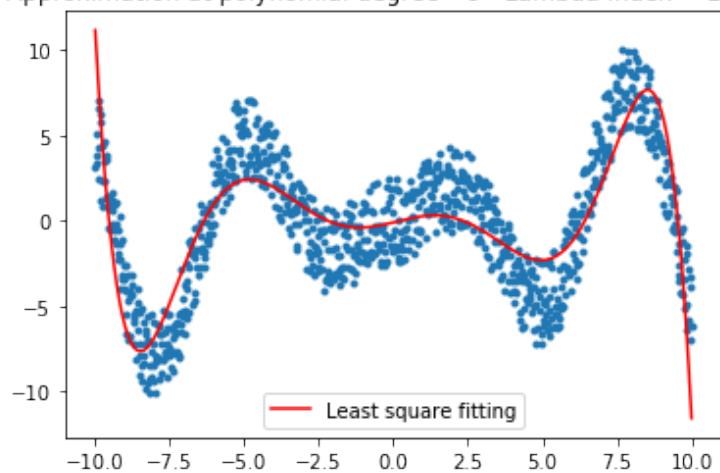
Least Square Approximation at polynomial degree= 9 Lambda index = 24 (Lambda: 2**-6)



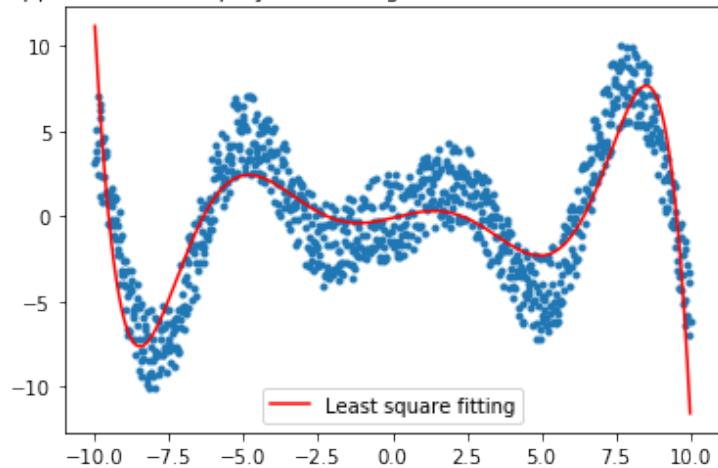
Least Square Approximation at polynomial degree= 9 Lambda index = 25 (Lambda: 2**-5)



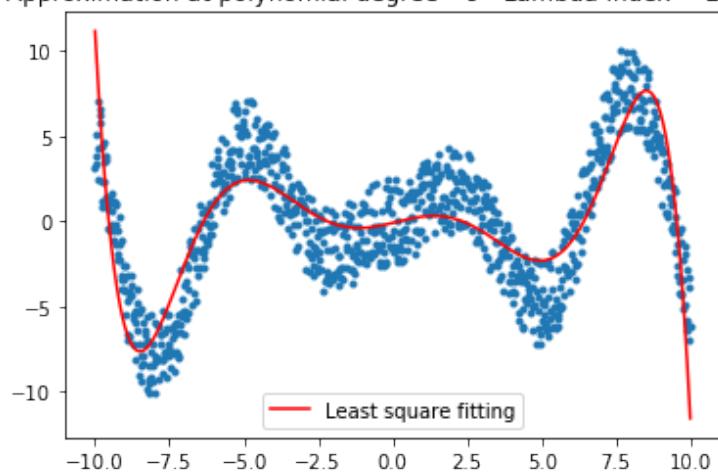
Least Square Approximation at polynomial degree= 9 Lambda index = 26 (Lambda: 2**-4)



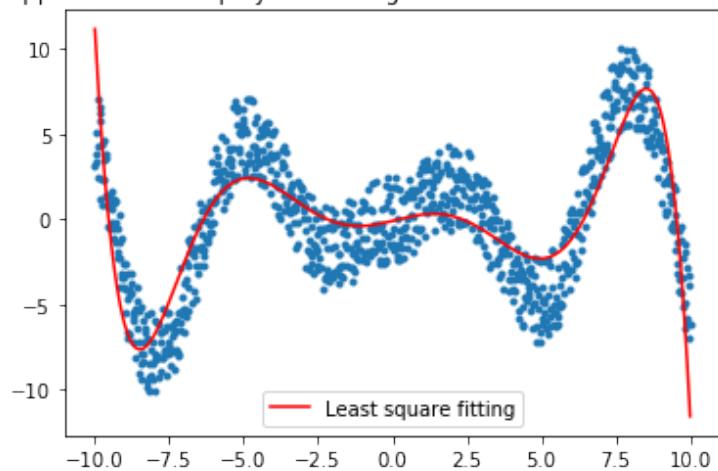
Least Square Approximation at polynomial degree= 9 Lambda index = 27 (Lambda: 2**-3)



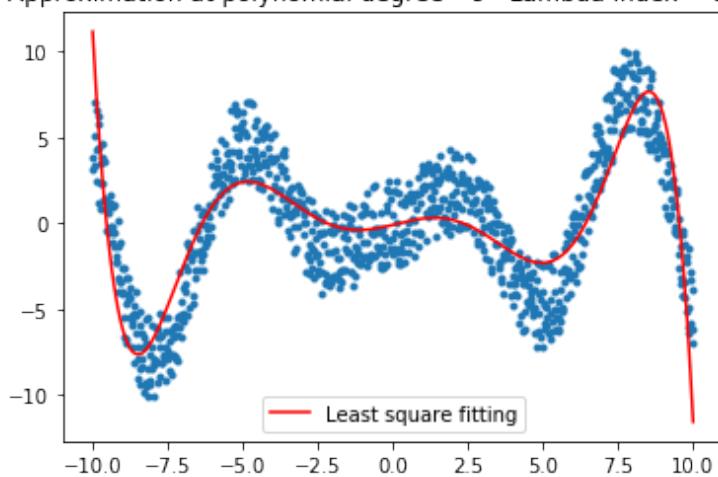
Least Square Approximation at polynomial degree= 9 Lambda index = 28 (Lambda: 2**-2)



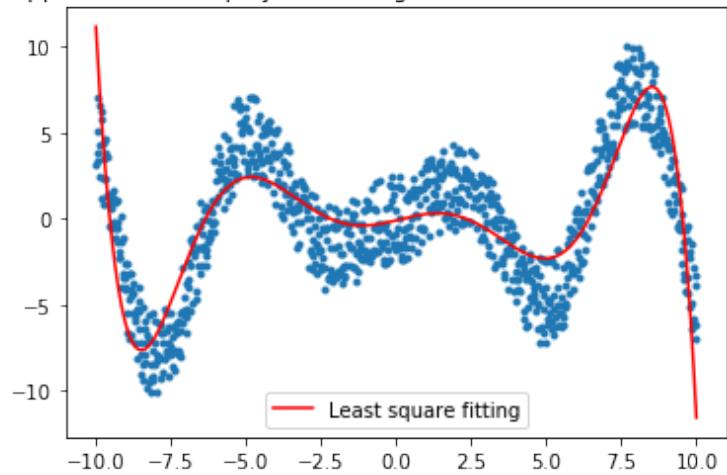
Least Square Approximation at polynomial degree= 9 Lambda index = 29 (Lambda: 2**-1)



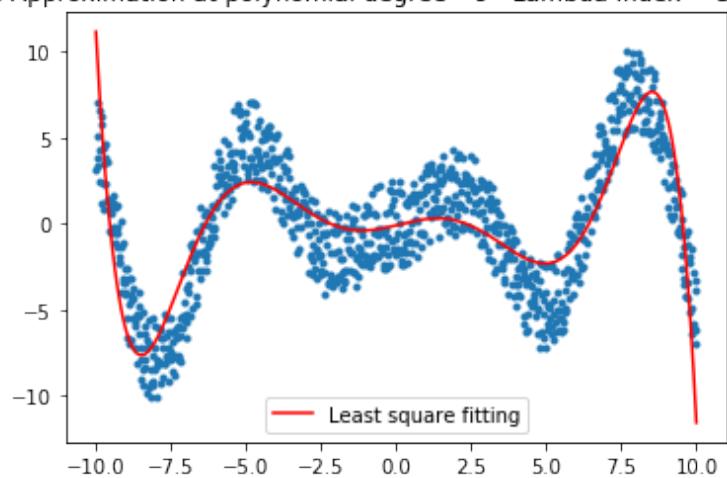
Least Square Approximation at polynomial degree= 9 Lambda index = 30 (Lambda: 2**0)



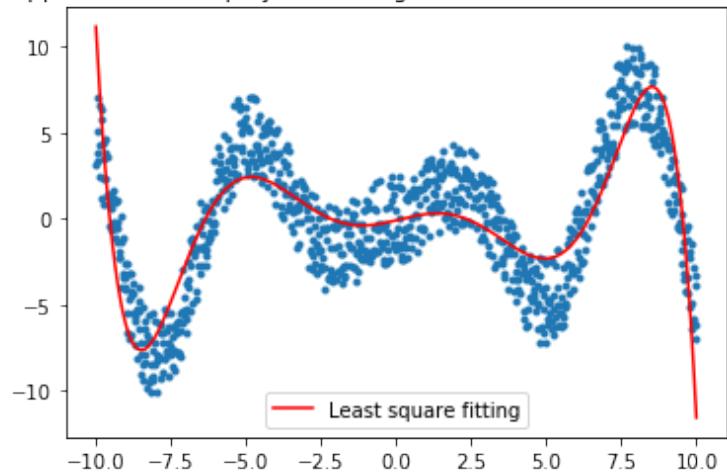
Least Square Approximation at polynomial degree= 9 Lambda index = 31 (Lambda: 2**1)



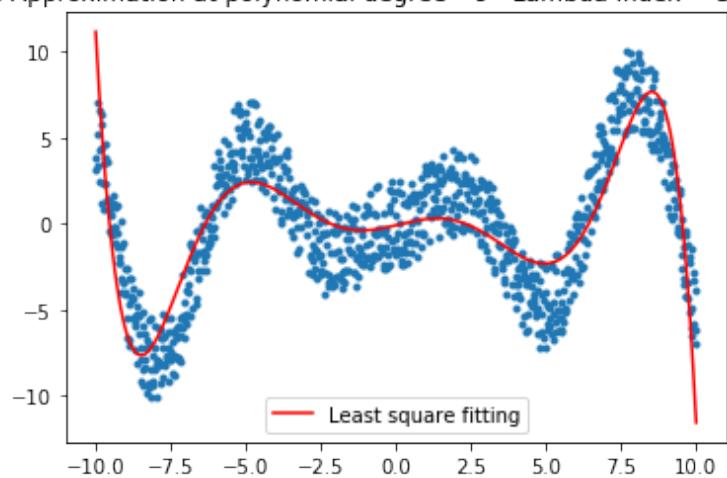
Least Square Approximation at polynomial degree= 9 Lambda index = 32 (Lambda: 2**2)



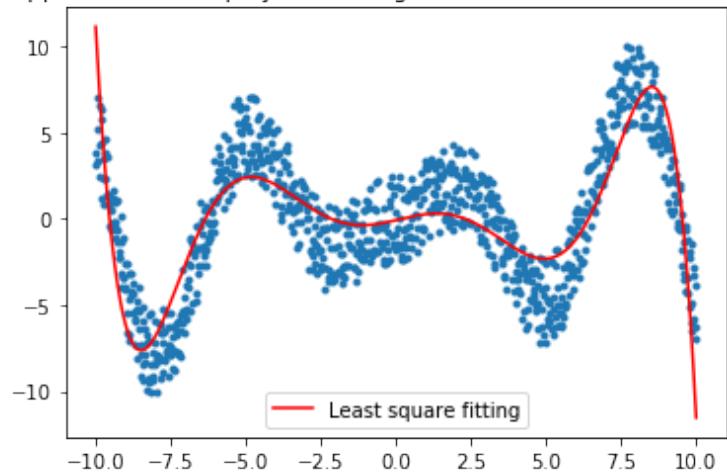
Least Square Approximation at polynomial degree= 9 Lambda index = 33 (Lambda: 2**3)



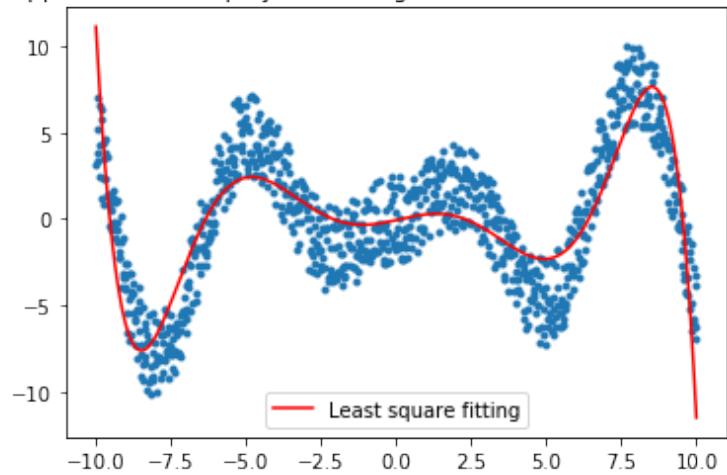
Least Square Approximation at polynomial degree= 9 Lambda index = 34 (Lambda: 2**4)



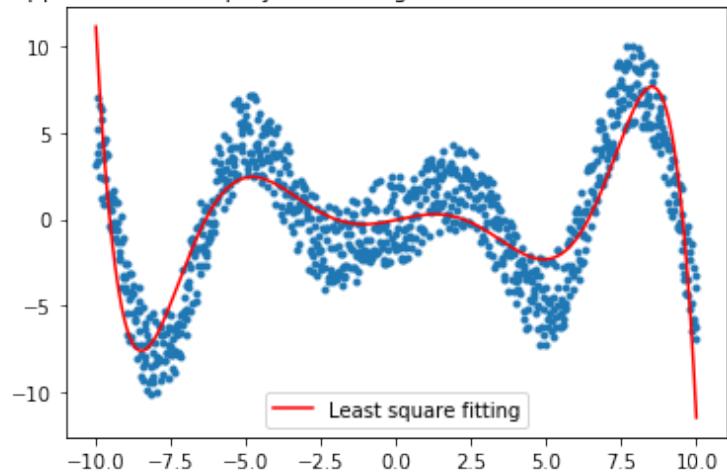
Least Square Approximation at polynomial degree= 9 Lambda index = 35 (Lambda: 2**5)



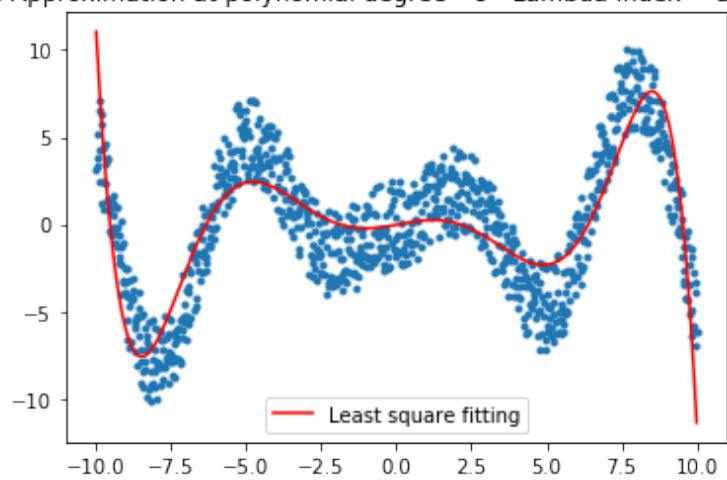
Least Square Approximation at polynomial degree= 9 Lambda index = 36 (Lambda: 2**6)



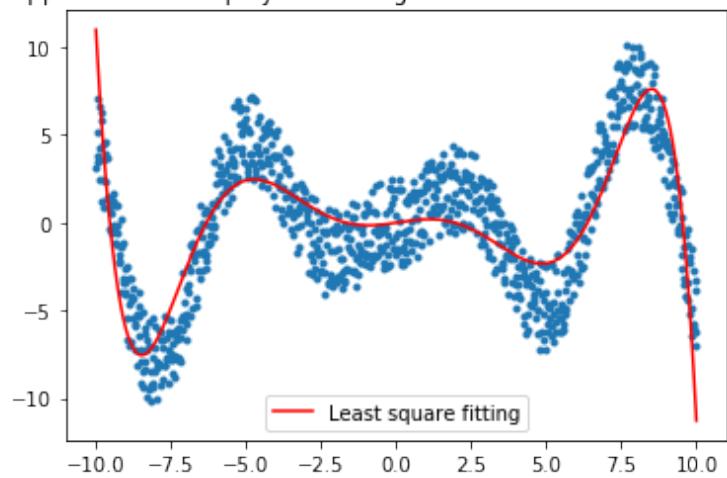
Least Square Approximation at polynomial degree= 9 Lambda index = 37 (Lambda: 2**7)



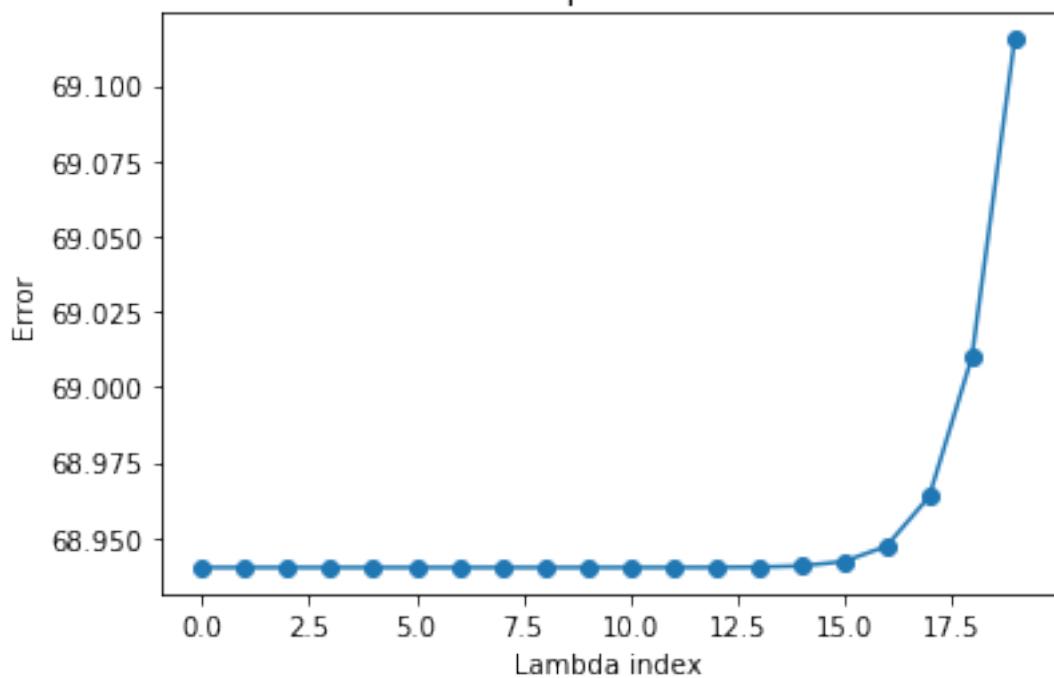
Least Square Approximation at polynomial degree= 9 Lambda index = 38 (Lambda: 2**8)



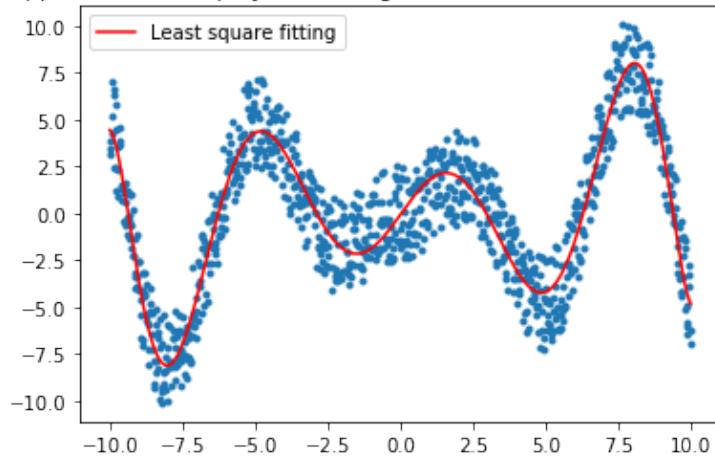
Least Square Approximation at polynomial degree= 9 Lambda index = 39 (Lambda: 2**9)



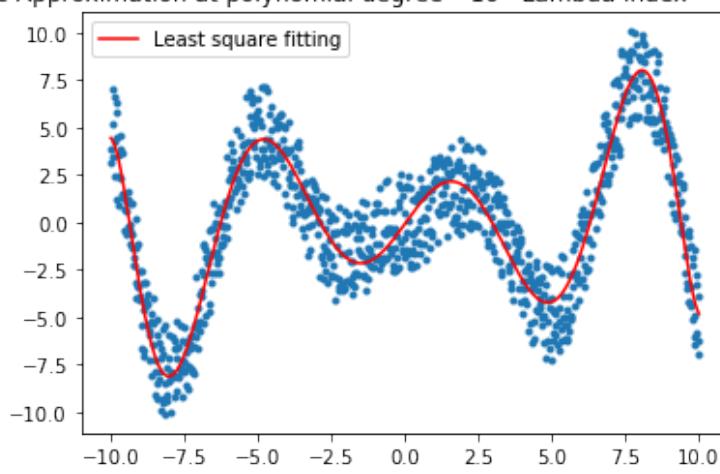
Graph: Error



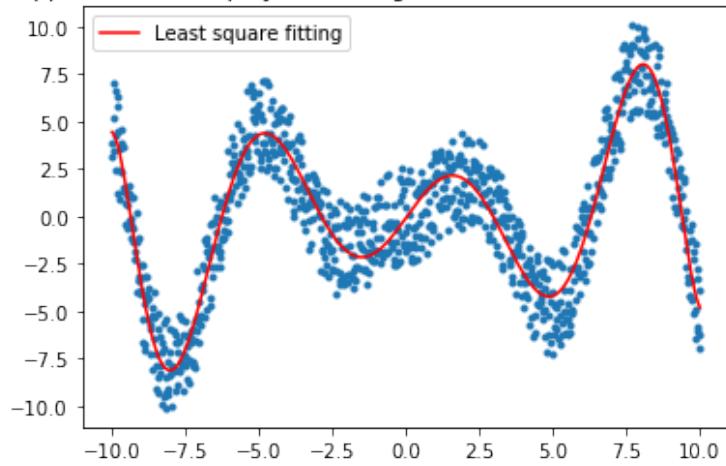
Least Square Approximation at polynomial degree= 10 Lambda index = 40 (Lambda: 2^{**-10})



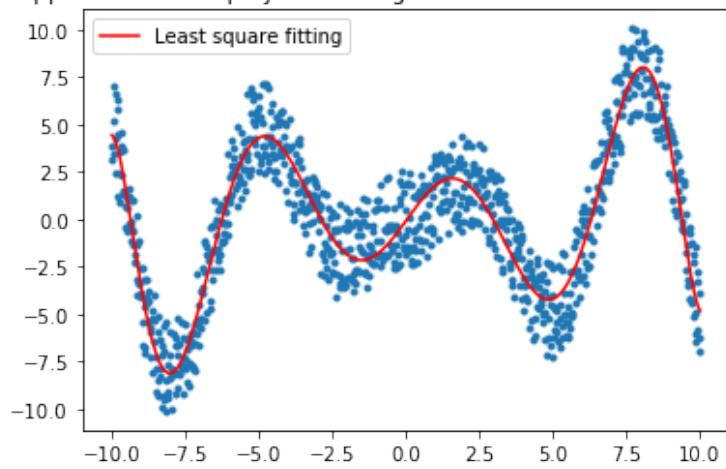
Least Square Approximation at polynomial degree= 10 Lambda index = 41 (Lambda: 2^{**-9})



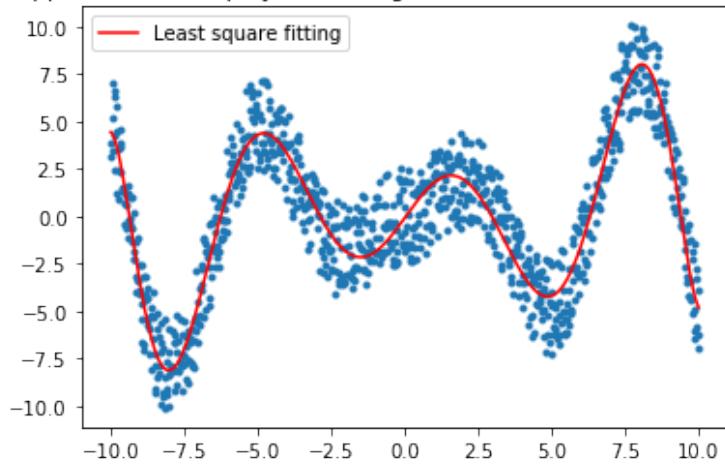
Least Square Approximation at polynomial degree= 10 Lambda index = 42 (Lambda: 2**-8)



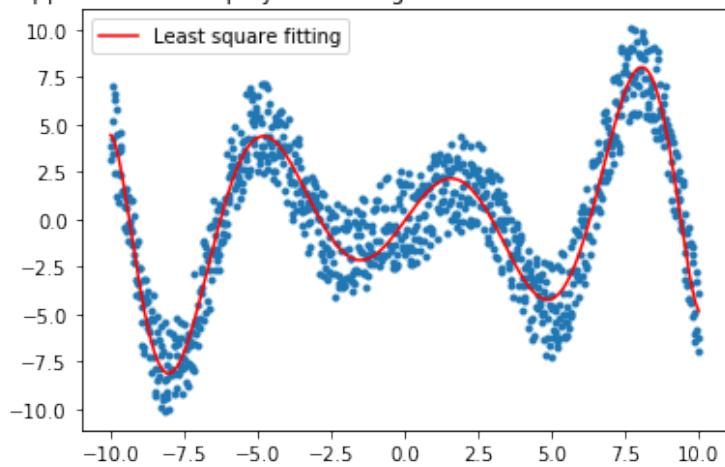
Least Square Approximation at polynomial degree= 10 Lambda index = 43 (Lambda: 2**-7)



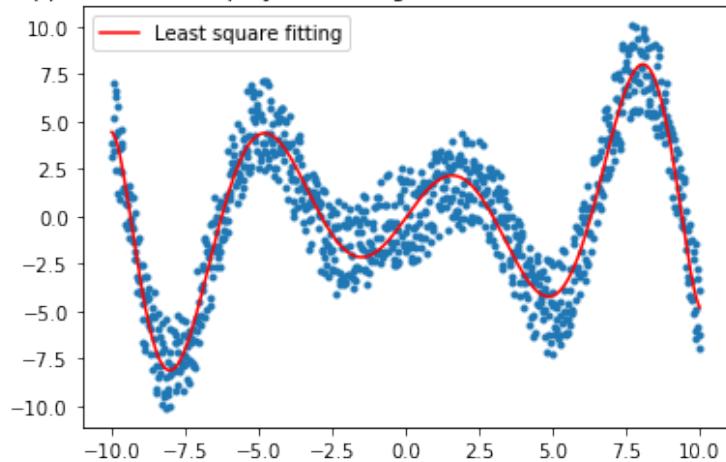
Least Square Approximation at polynomial degree= 10 Lambda index = 44 (Lambda: 2**-6)



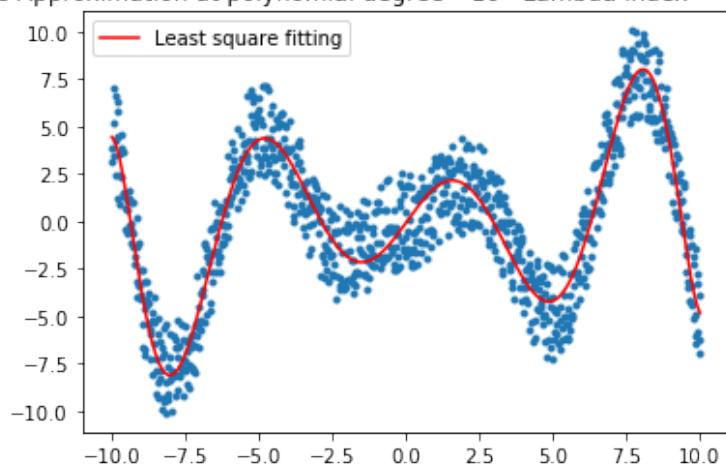
Least Square Approximation at polynomial degree= 10 Lambda index = 45 (Lambda: 2**-5)



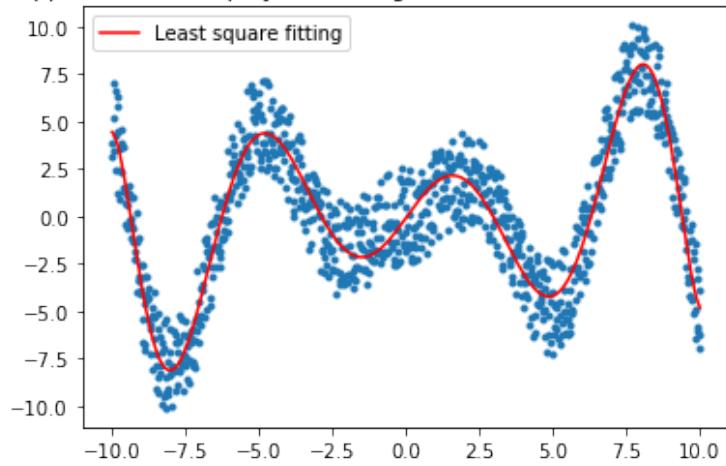
Least Square Approximation at polynomial degree= 10 Lambda index = 46 (Lambda: 2**-4)



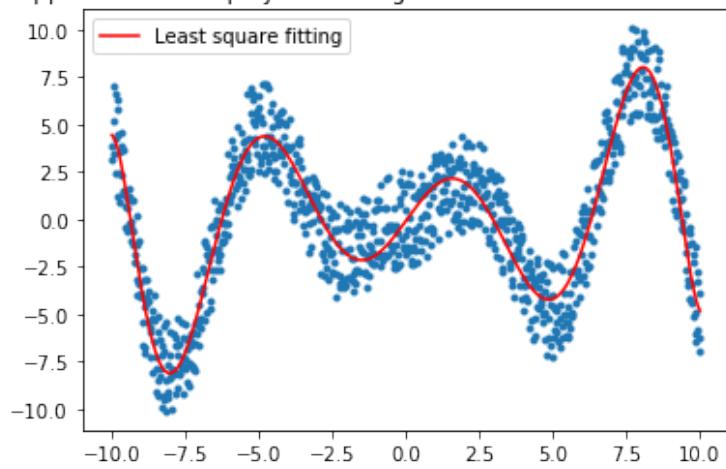
Least Square Approximation at polynomial degree= 10 Lambda index = 47 (Lambda: 2**-3)



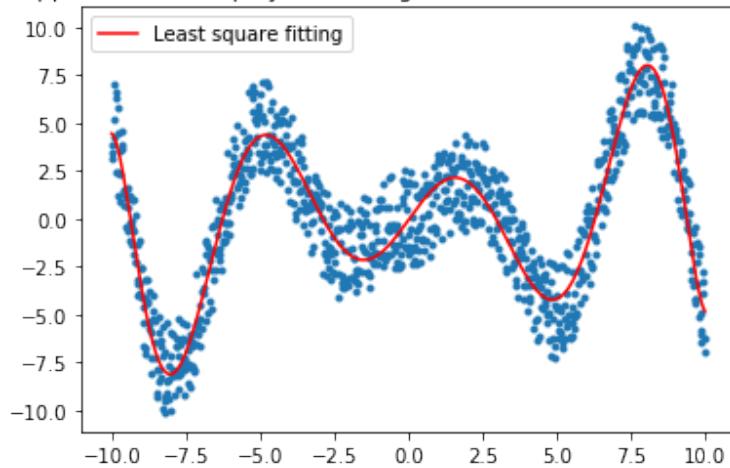
Least Square Approximation at polynomial degree= 10 Lambda index = 48 (Lambda: 2**-2)



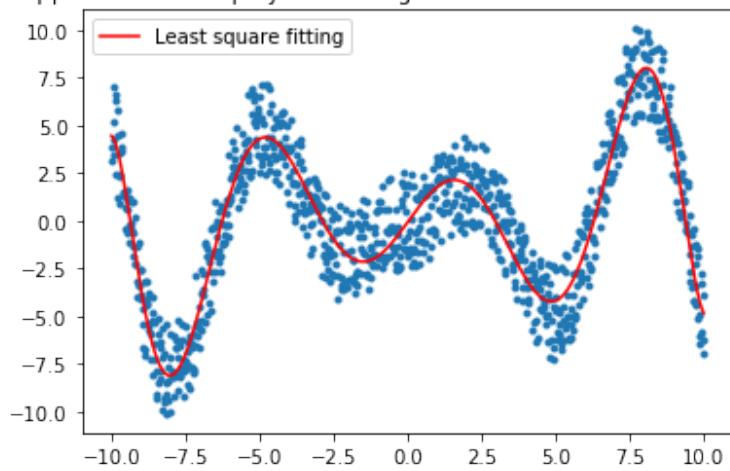
Least Square Approximation at polynomial degree= 10 Lambda index = 49 (Lambda: 2**-1)



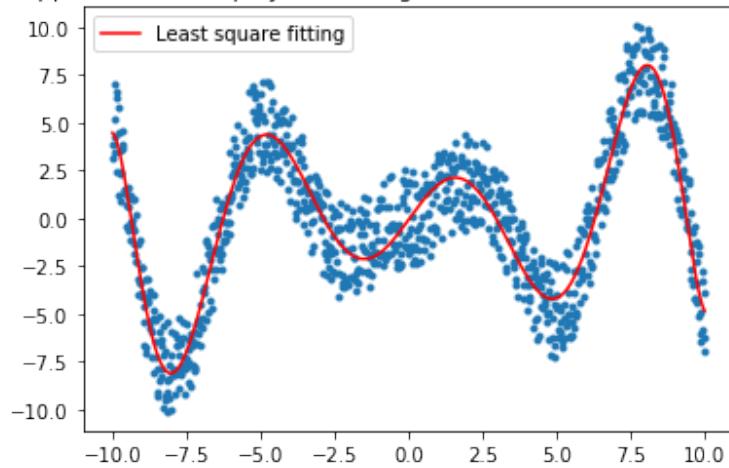
Least Square Approximation at polynomial degree= 10 Lambda index = 50 (Lambda: 2**0)



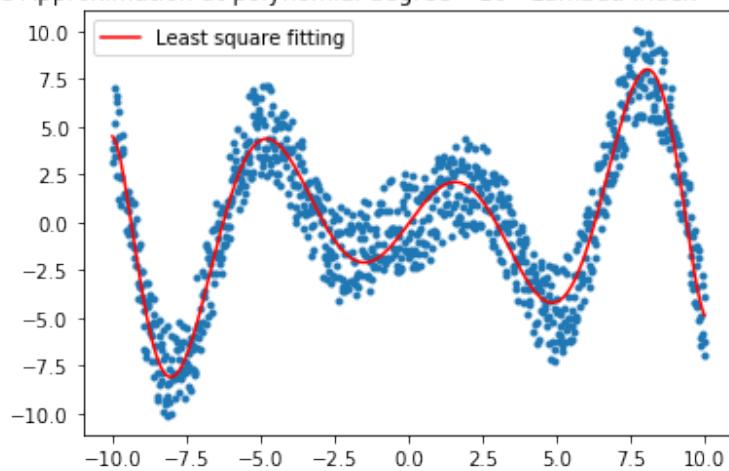
Least Square Approximation at polynomial degree= 10 Lambda index = 51 (Lambda: 2**1)



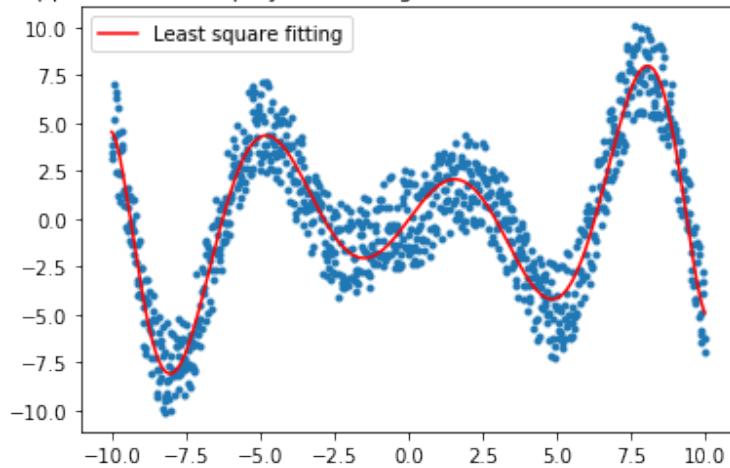
Least Square Approximation at polynomial degree= 10 Lambda index = 52 (Lambda: 2**2)



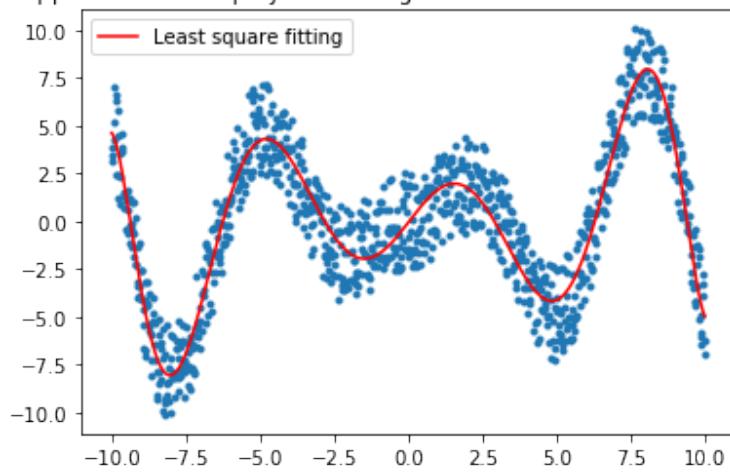
Least Square Approximation at polynomial degree= 10 Lambda index = 53 (Lambda: 2**3)



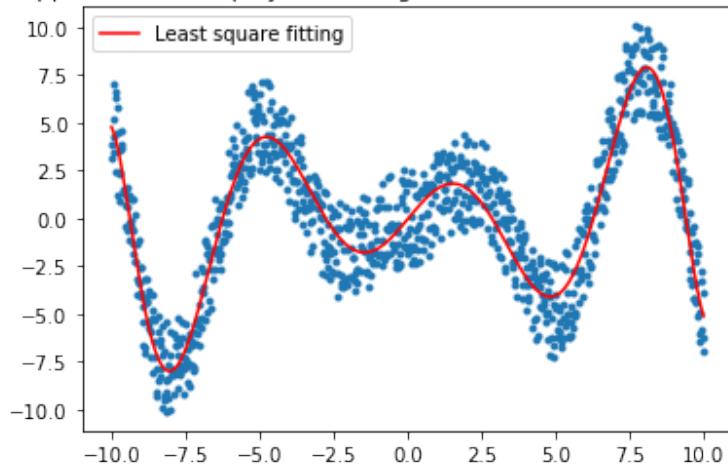
Least Square Approximation at polynomial degree= 10 Lambda index = 54 (Lambda: 2**4)



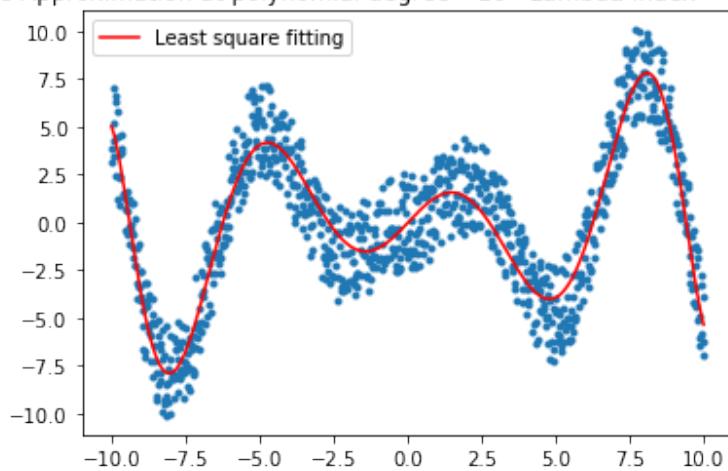
Least Square Approximation at polynomial degree= 10 Lambda index = 55 (Lambda: 2**5)



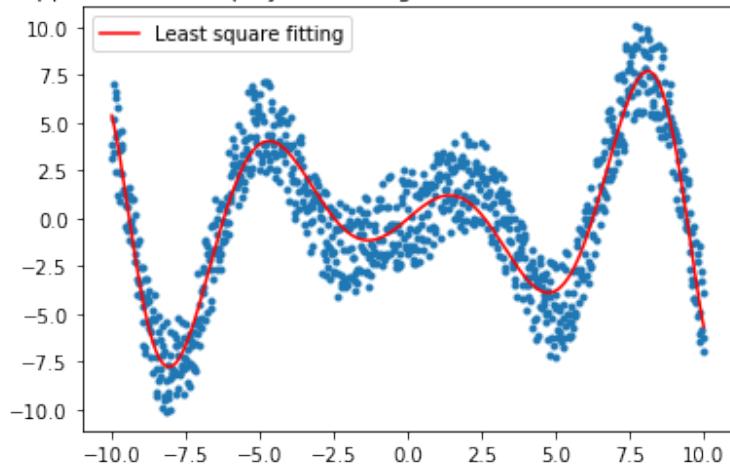
Least Square Approximation at polynomial degree= 10 Lambda index = 56 (Lambda: 2**6)



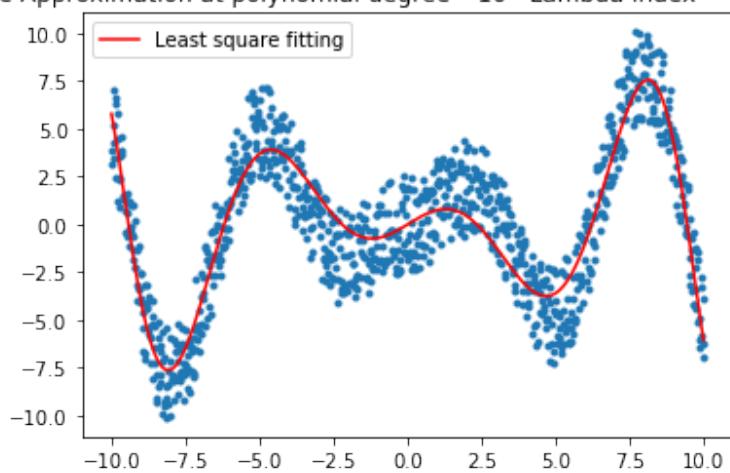
Least Square Approximation at polynomial degree= 10 Lambda index = 57 (Lambda: 2**7)

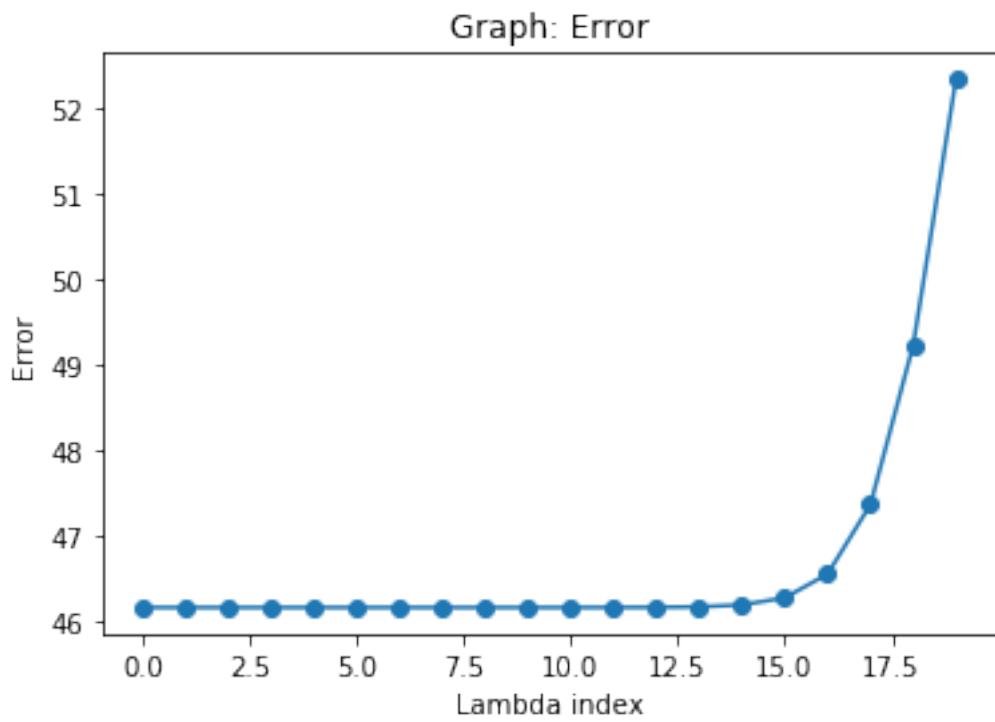


Least Square Approximation at polynomial degree= 10 Lambda index = 58 (Lambda: 2**8)

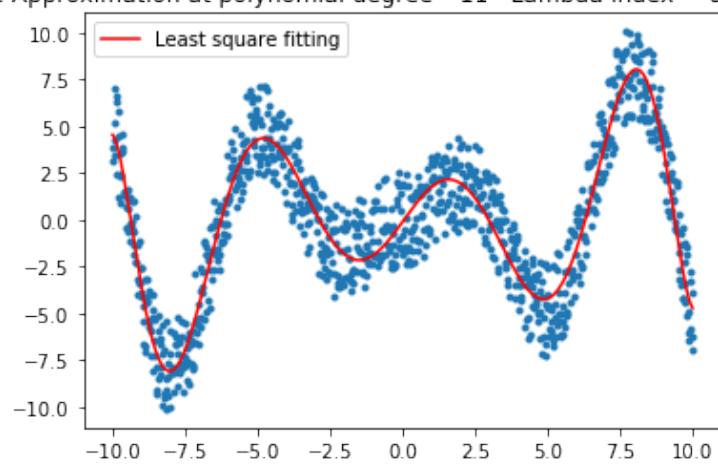


Least Square Approximation at polynomial degree= 10 Lambda index = 59 (Lambda: 2**9)

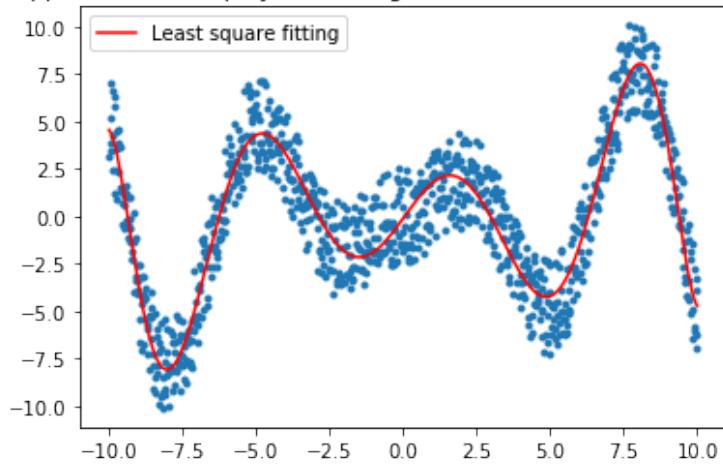




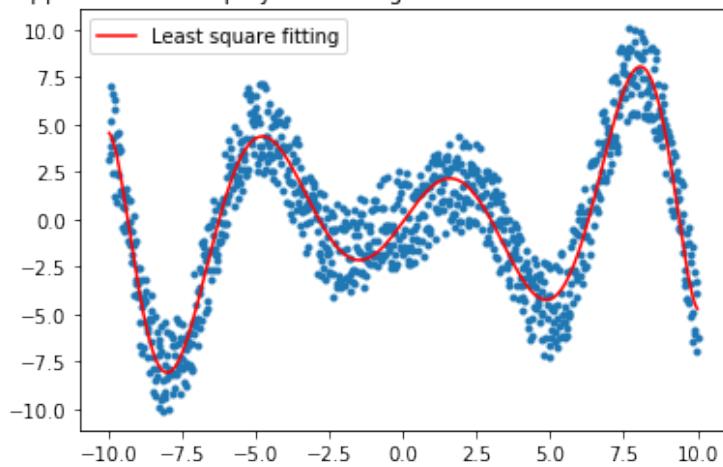
Least Square Approximation at polynomial degree= 11 Lambda index = 60 (Lambda: 2^{**-10})



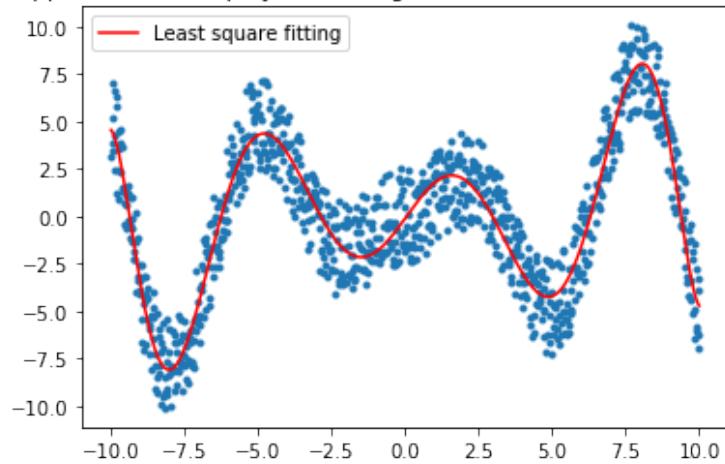
Least Square Approximation at polynomial degree= 11 Lambda index = 61 (Lambda: 2**-9)



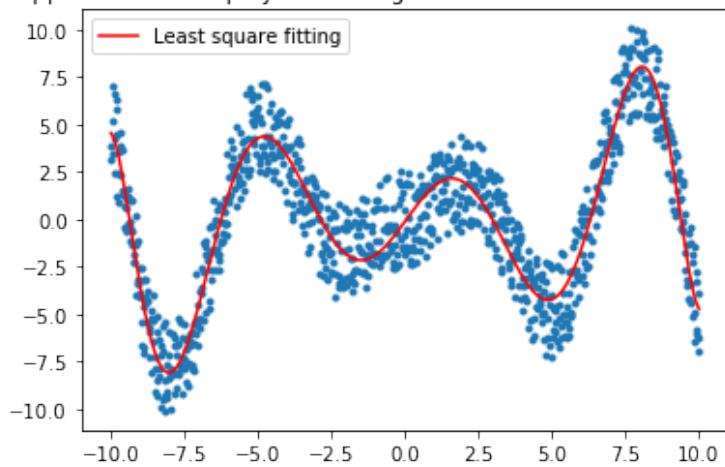
Least Square Approximation at polynomial degree= 11 Lambda index = 62 (Lambda: 2**-8)



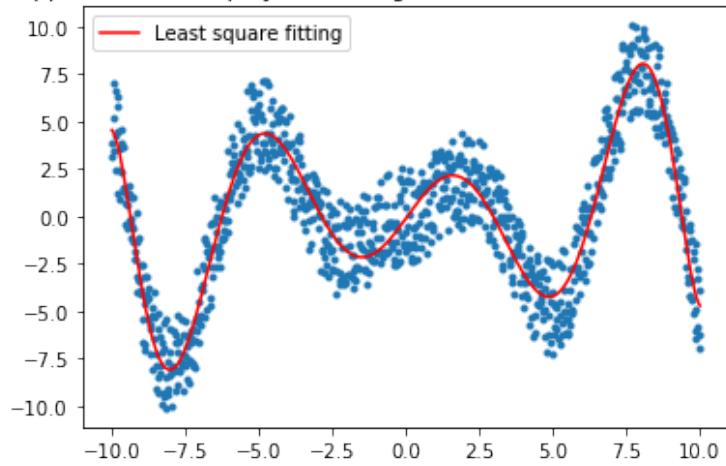
Least Square Approximation at polynomial degree= 11 Lambda index = 63 (Lambda: 2**-7)



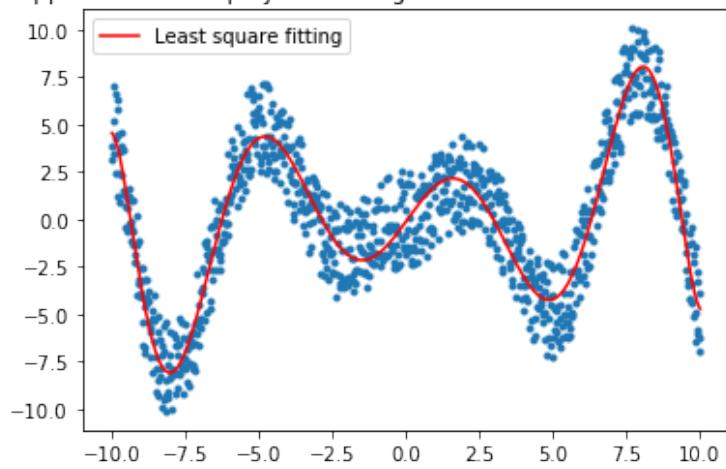
Least Square Approximation at polynomial degree= 11 Lambda index = 64 (Lambda: 2**-6)



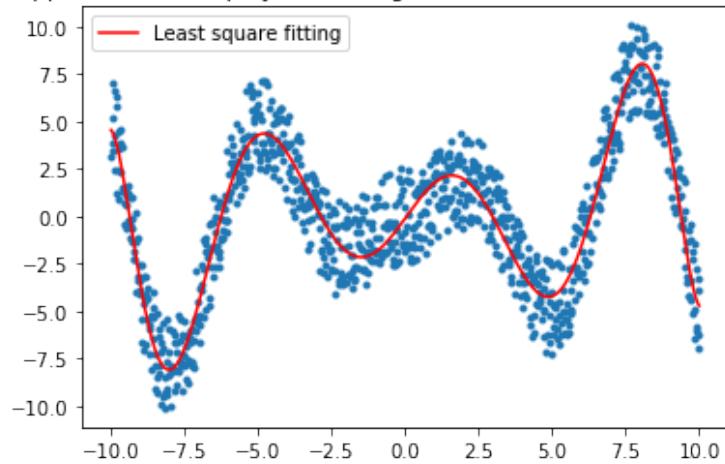
Least Square Approximation at polynomial degree= 11 Lambda index = 65 (Lambda: 2**-5)



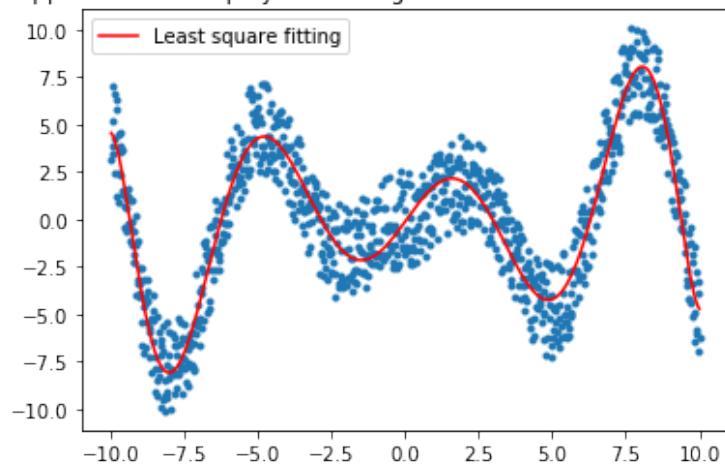
Least Square Approximation at polynomial degree= 11 Lambda index = 66 (Lambda: 2**-4)



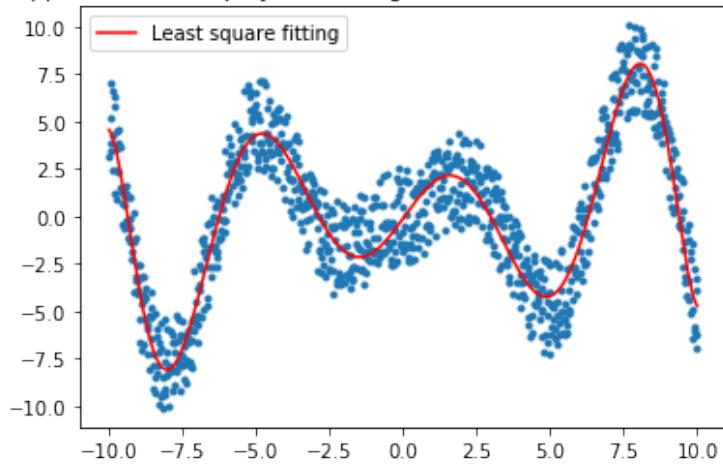
Least Square Approximation at polynomial degree= 11 Lambda index = 67 (Lambda: 2**-3)



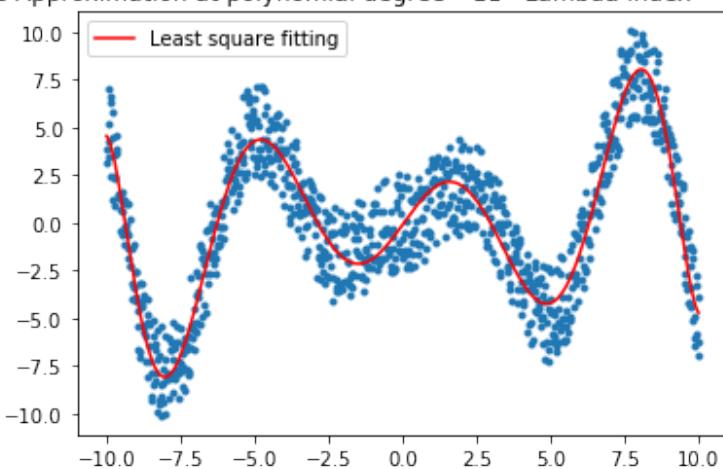
Least Square Approximation at polynomial degree= 11 Lambda index = 68 (Lambda: 2**-2)



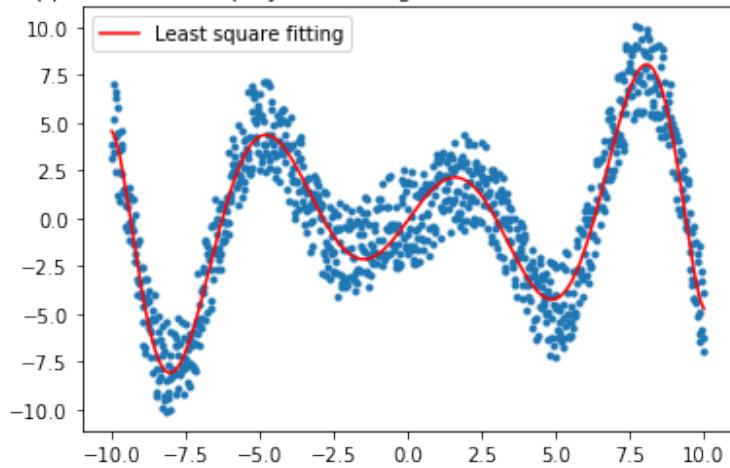
Least Square Approximation at polynomial degree= 11 Lambda index = 69 (Lambda: 2**-1)



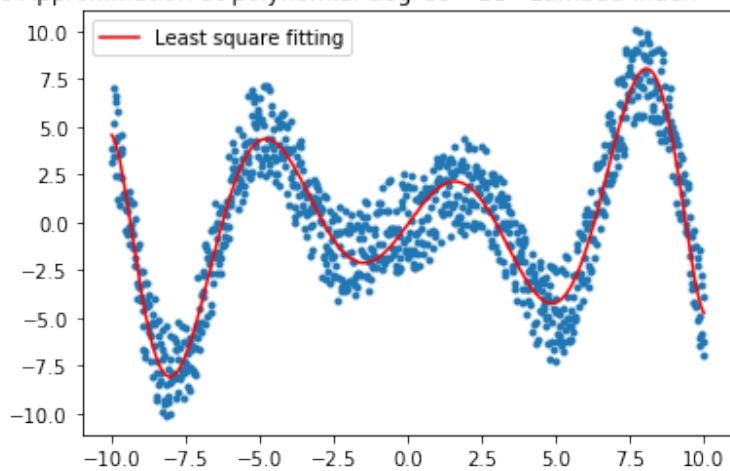
Least Square Approximation at polynomial degree= 11 Lambda index = 70 (Lambda: 2**0)



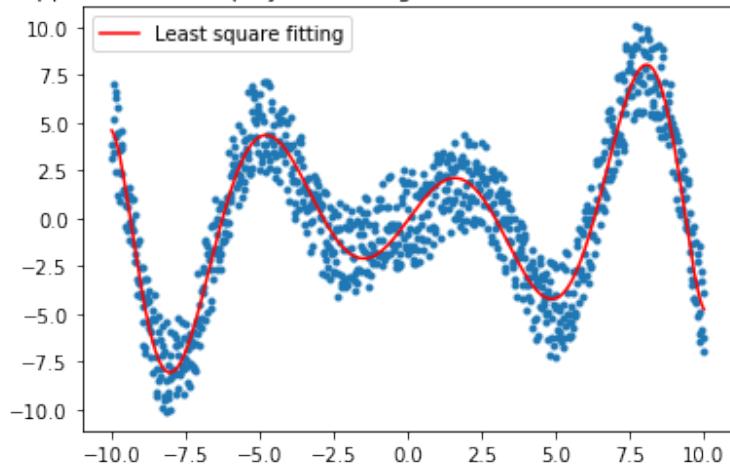
Least Square Approximation at polynomial degree= 11 Lambda index = 71 (Lambda: 2**1)



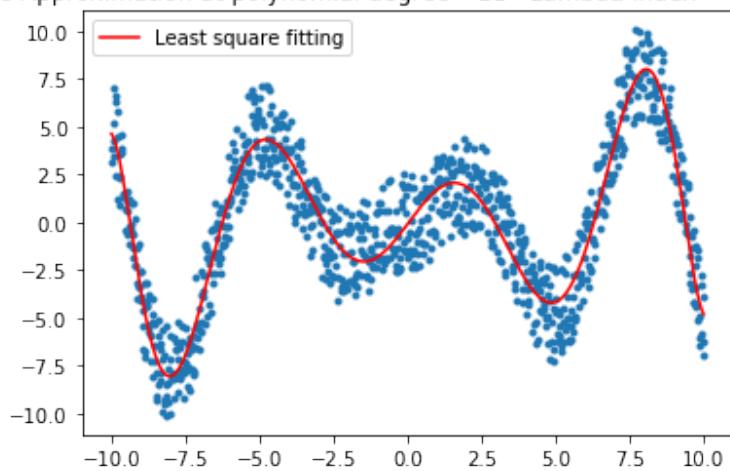
Least Square Approximation at polynomial degree= 11 Lambda index = 72 (Lambda: 2**2)



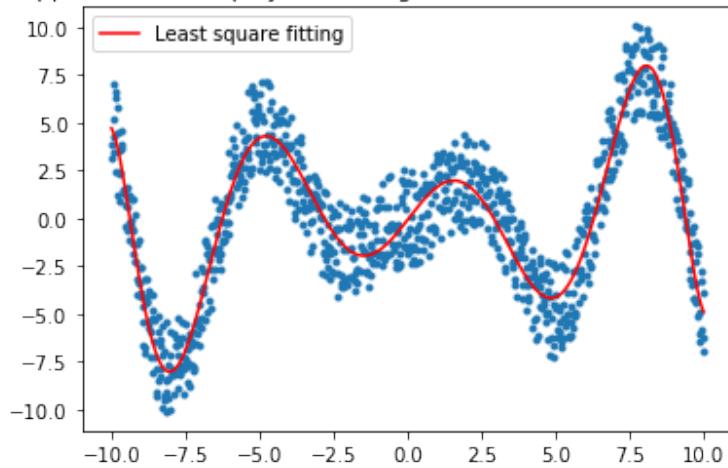
Least Square Approximation at polynomial degree= 11 Lambda index = 73 (Lambda: 2**3)



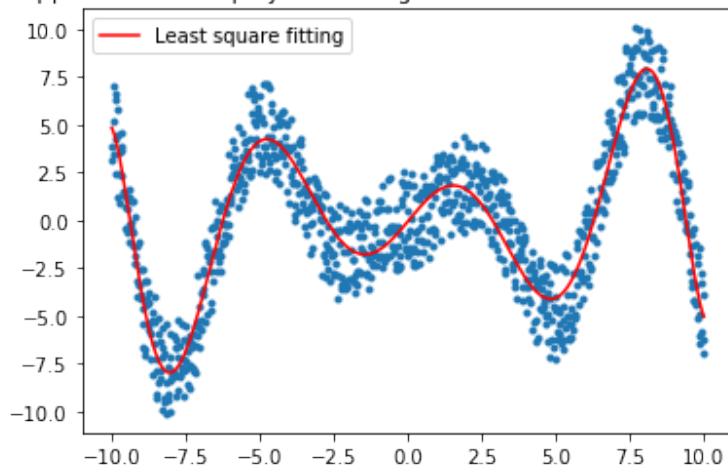
Least Square Approximation at polynomial degree= 11 Lambda index = 74 (Lambda: 2**4)



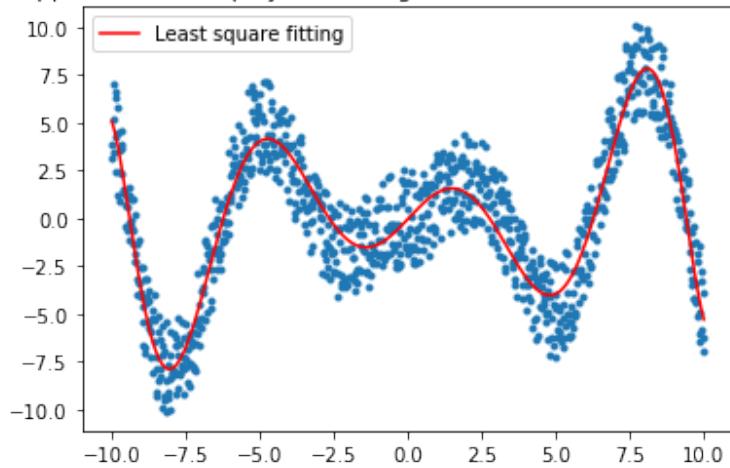
Least Square Approximation at polynomial degree= 11 Lambda index = 75 (Lambda: 2**5)



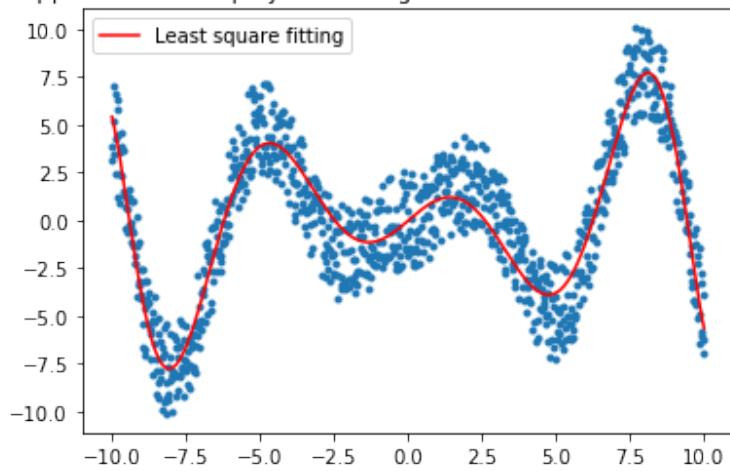
Least Square Approximation at polynomial degree= 11 Lambda index = 76 (Lambda: 2**6)



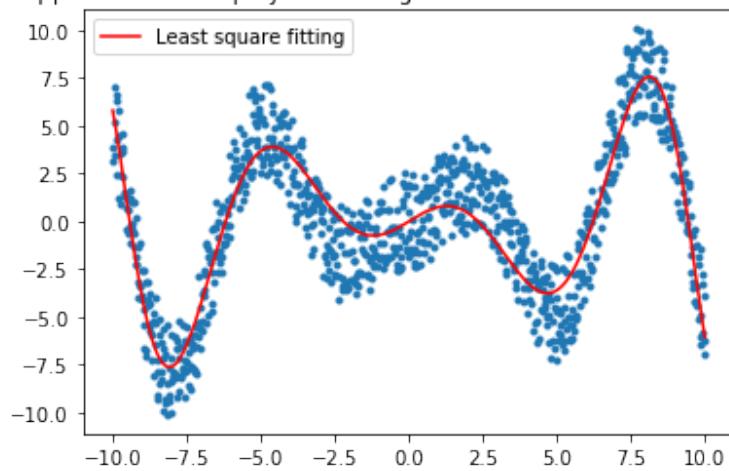
Least Square Approximation at polynomial degree= 11 Lambda index = 77 (Lambda: 2**7)



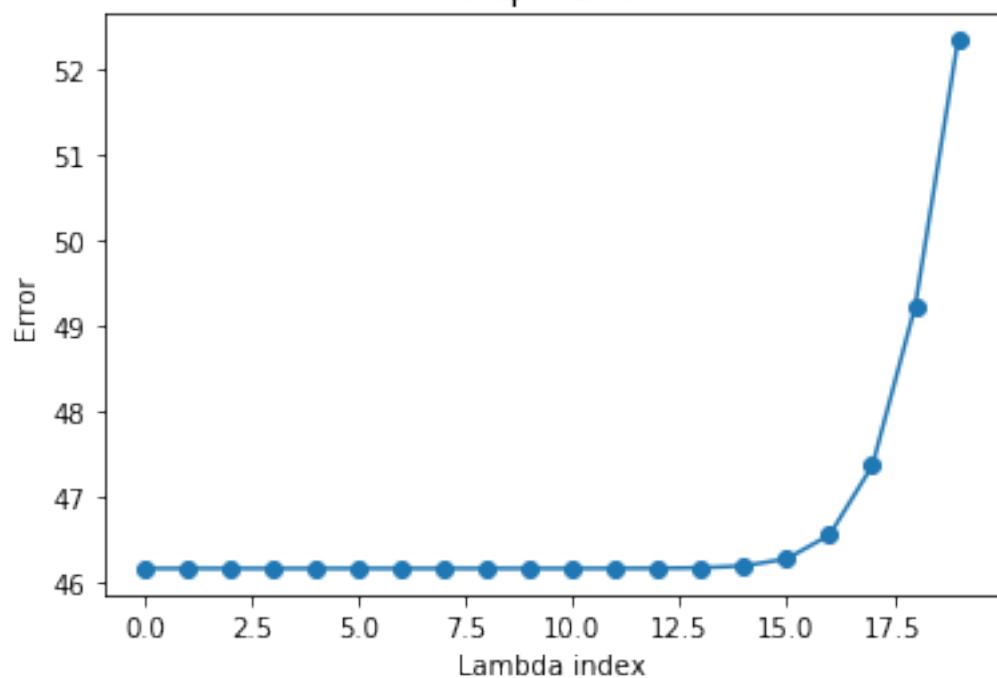
Least Square Approximation at polynomial degree= 11 Lambda index = 78 (Lambda: 2**8)



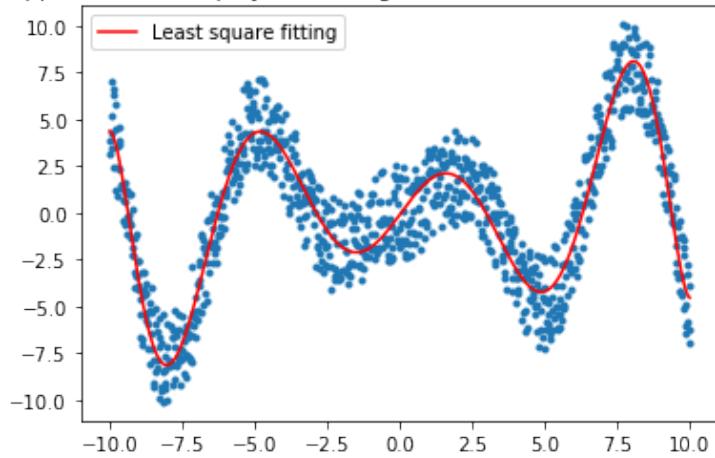
Least Square Approximation at polynomial degree= 11 Lambda index = 79 (Lambda: 2**9)



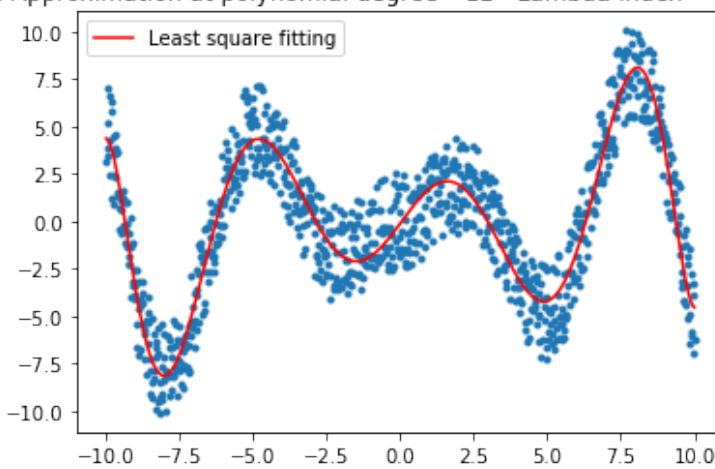
Graph: Error



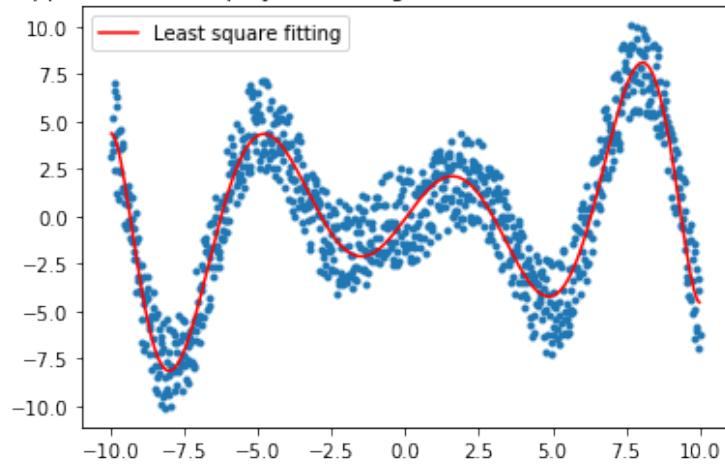
Least Square Approximation at polynomial degree= 12 Lambda index = 80 (Lambda: 2**-10)



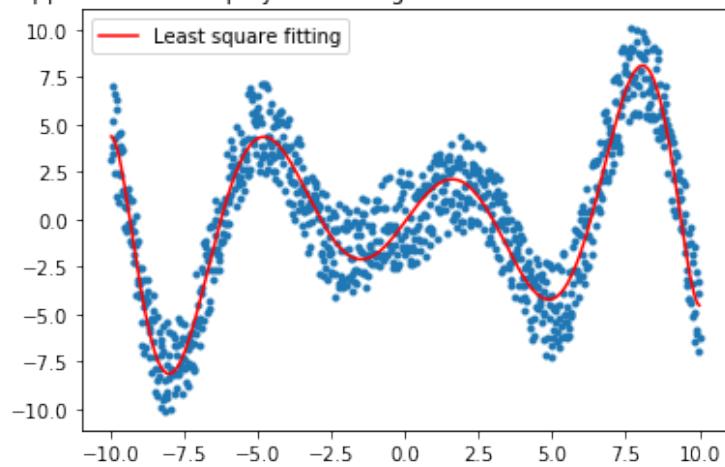
Least Square Approximation at polynomial degree= 12 Lambda index = 81 (Lambda: 2**-9)



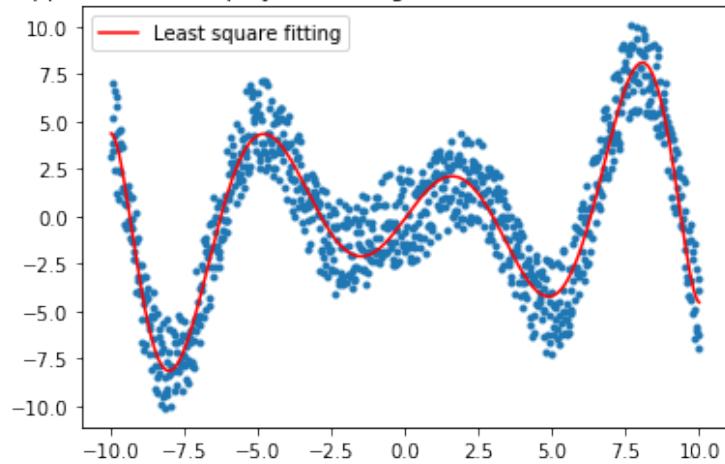
Least Square Approximation at polynomial degree= 12 Lambda index = 82 (Lambda: 2**-8)



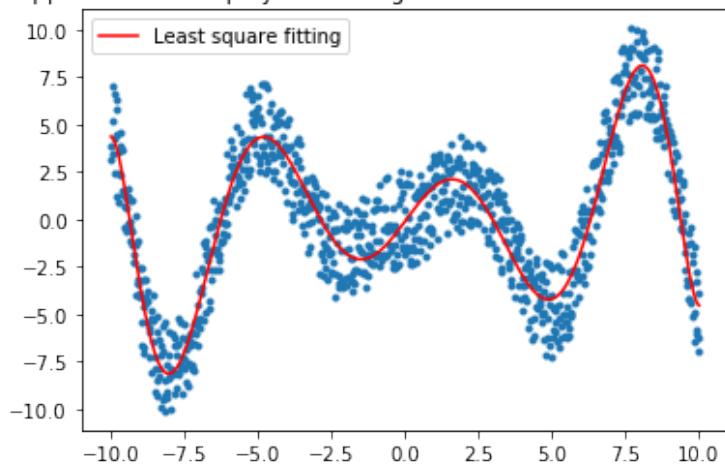
Least Square Approximation at polynomial degree= 12 Lambda index = 83 (Lambda: 2**-7)



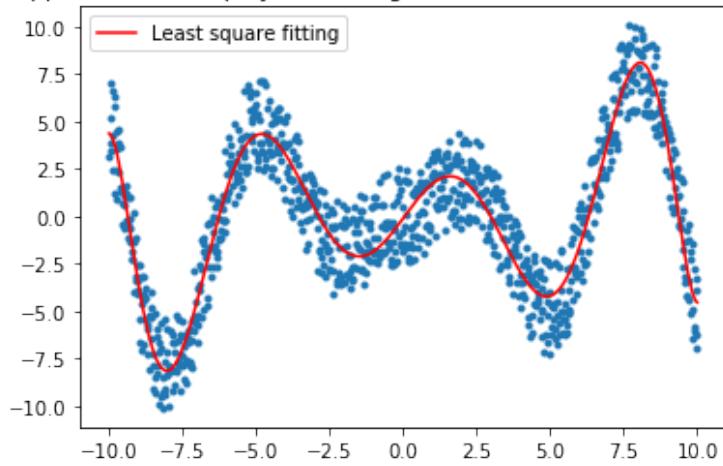
Least Square Approximation at polynomial degree= 12 Lambda index = 84 (Lambda: 2**-6)



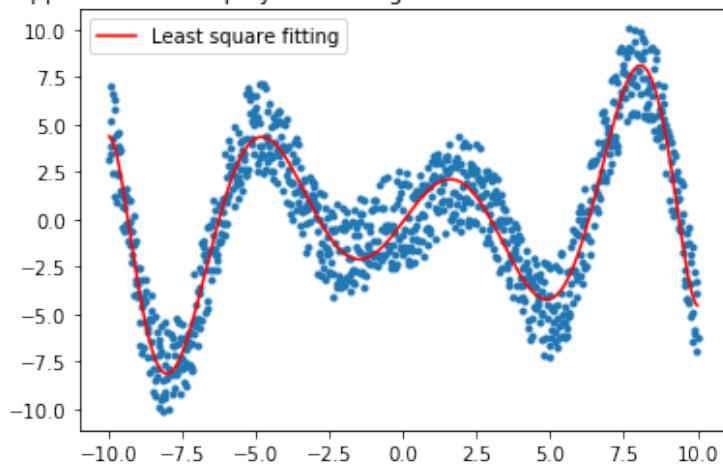
Least Square Approximation at polynomial degree= 12 Lambda index = 85 (Lambda: 2**-5)



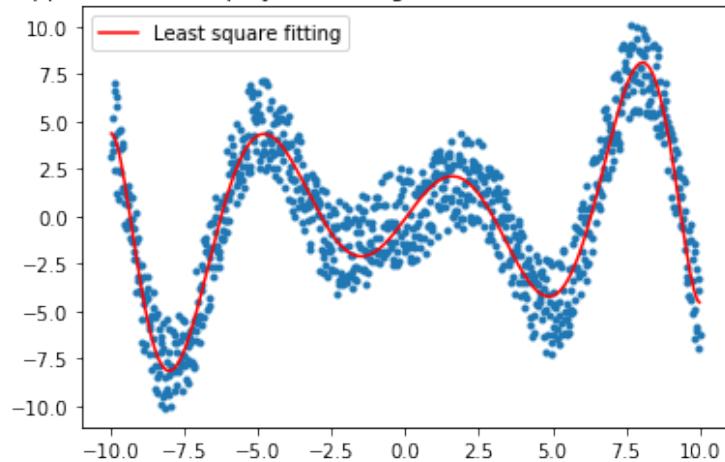
Least Square Approximation at polynomial degree= 12 Lambda index = 86 (Lambda: 2**-4)



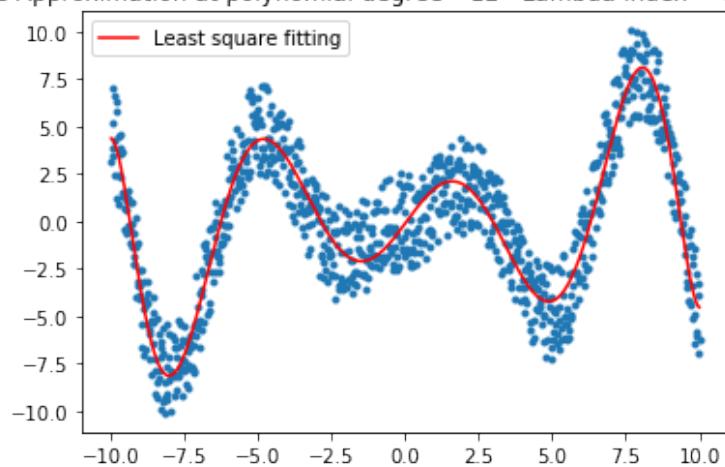
Least Square Approximation at polynomial degree= 12 Lambda index = 87 (Lambda: 2**-3)



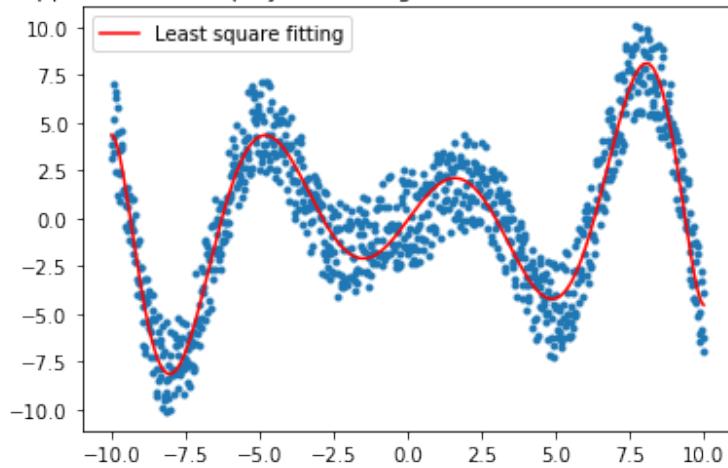
Least Square Approximation at polynomial degree= 12 Lambda index = 88 (Lambda: 2**-2)



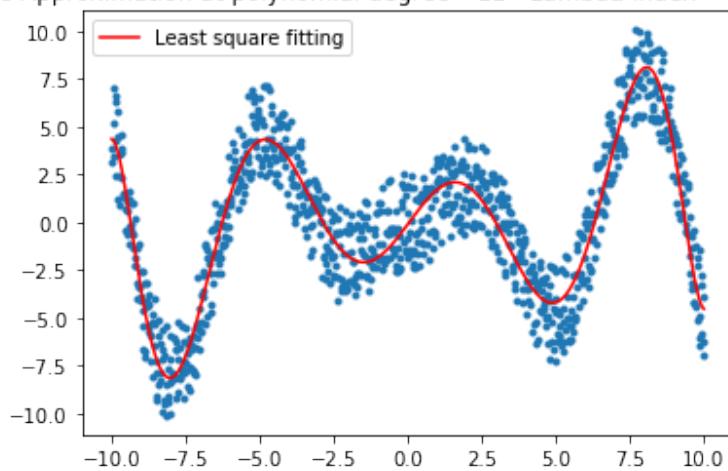
Least Square Approximation at polynomial degree= 12 Lambda index = 89 (Lambda: 2**-1)



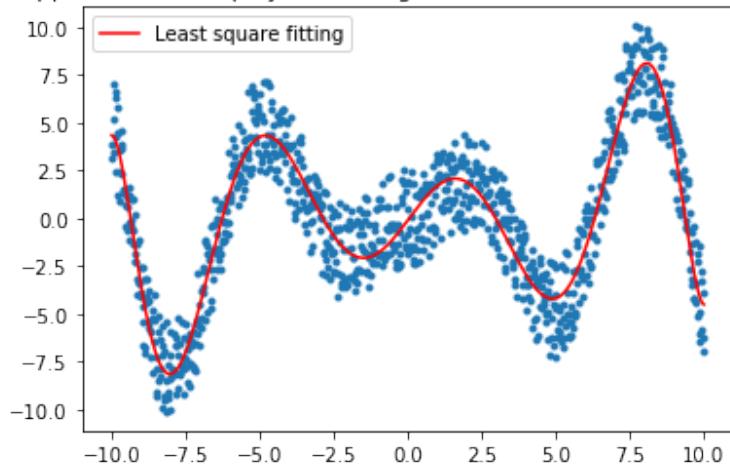
Least Square Approximation at polynomial degree= 12 Lambda index = 90 (Lambda: 2**0)



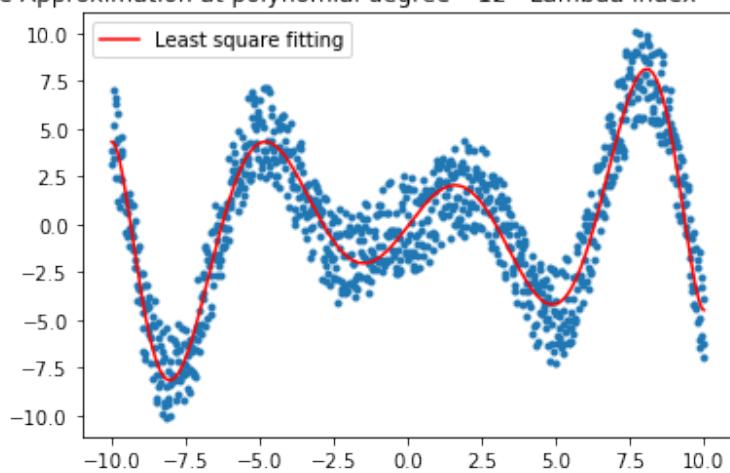
Least Square Approximation at polynomial degree= 12 Lambda index = 91 (Lambda: 2**1)



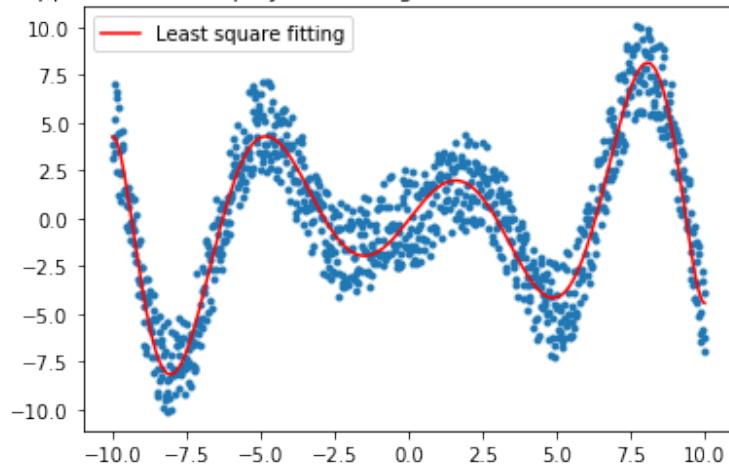
Least Square Approximation at polynomial degree= 12 Lambda index = 92 (Lambda: 2**2)



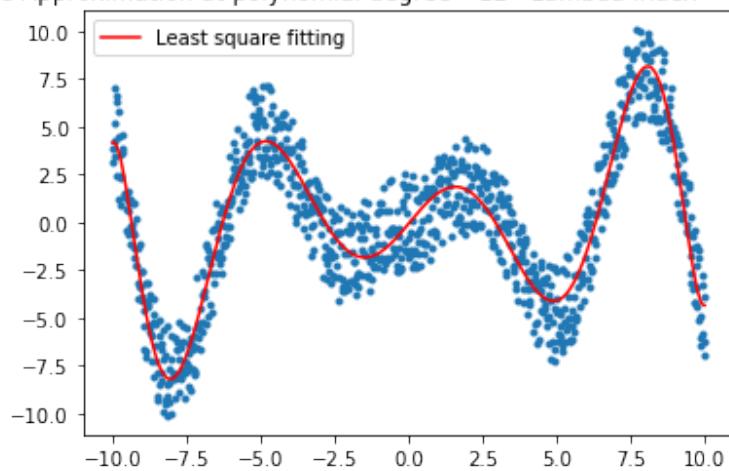
Least Square Approximation at polynomial degree= 12 Lambda index = 93 (Lambda: 2**3)



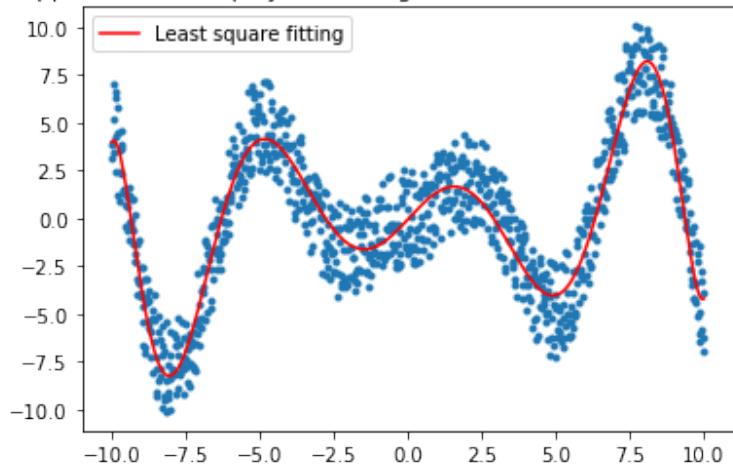
Least Square Approximation at polynomial degree= 12 Lambda index = 94 (Lambda: 2**4)



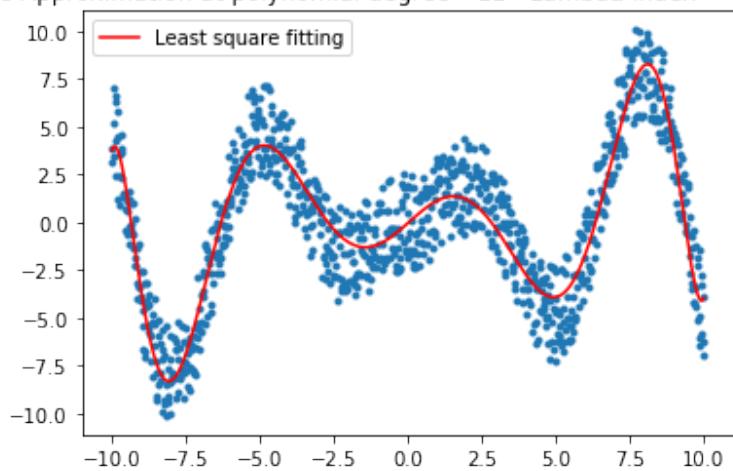
Least Square Approximation at polynomial degree= 12 Lambda index = 95 (Lambda: 2**5)



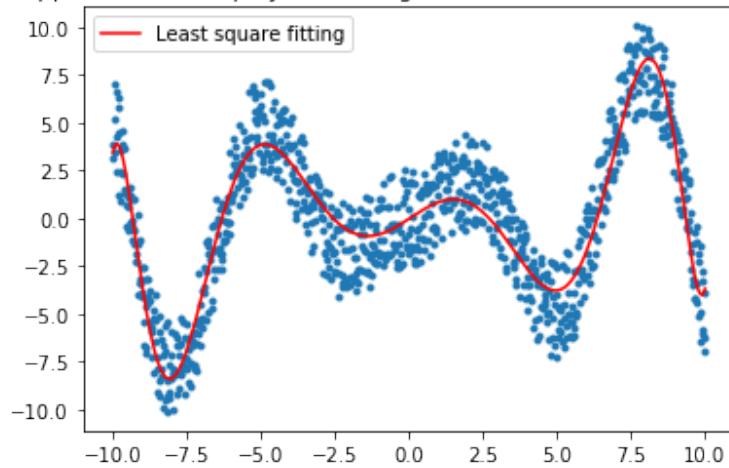
Least Square Approximation at polynomial degree= 12 Lambda index = 96 (Lambda: 2**6)



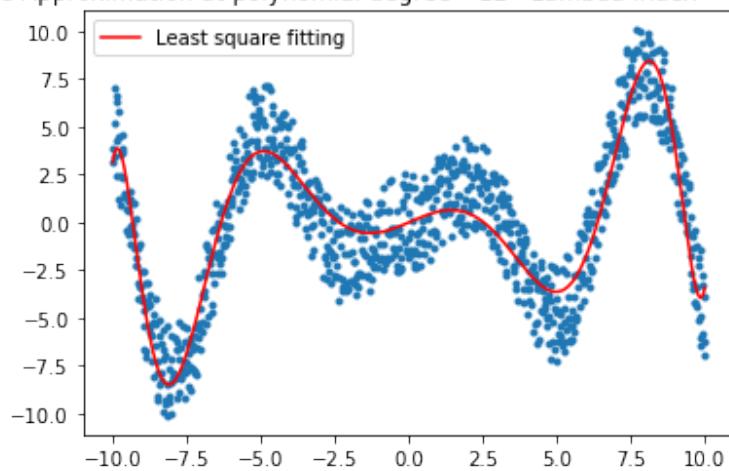
Least Square Approximation at polynomial degree= 12 Lambda index = 97 (Lambda: 2**7)

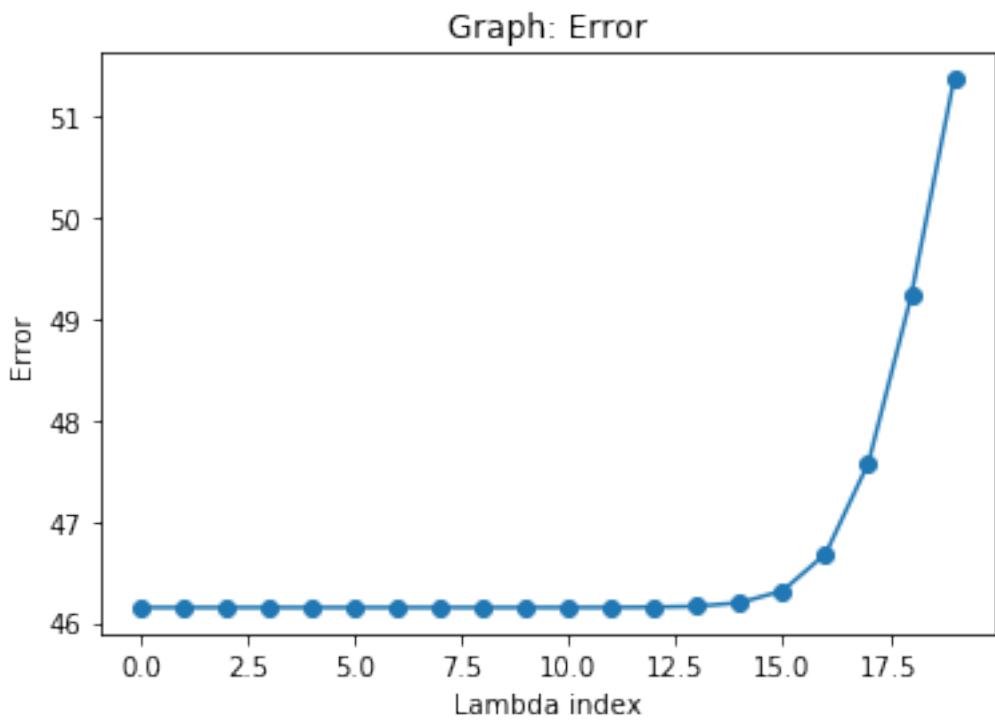


Least Square Approximation at polynomial degree= 12 Lambda index = 98 (Lambda: 2**8)

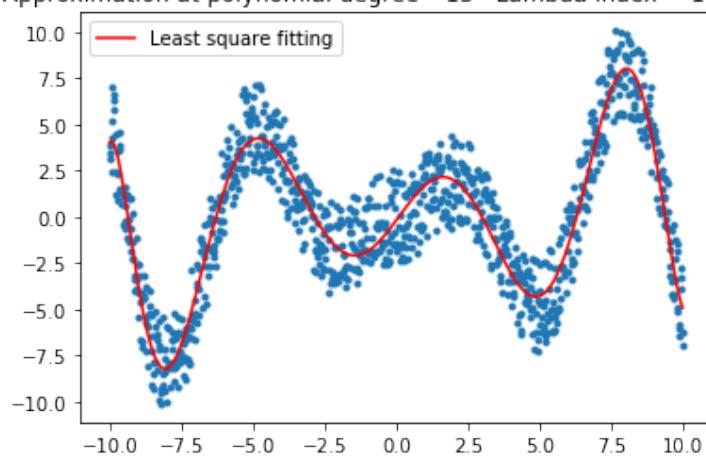


Least Square Approximation at polynomial degree= 12 Lambda index = 99 (Lambda: 2**9)

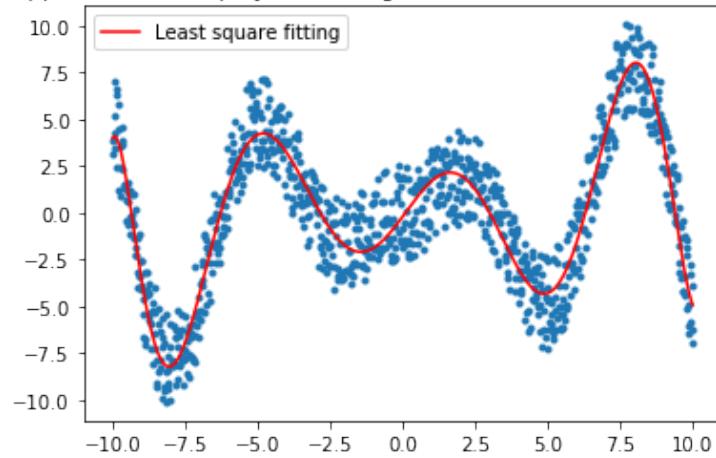




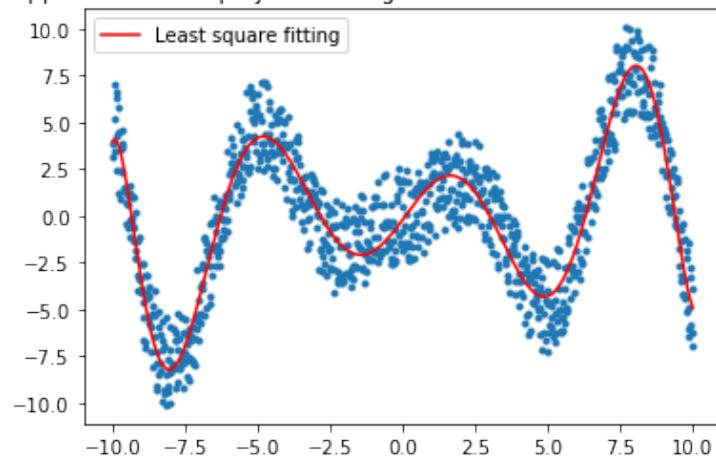
Least Square Approximation at polynomial degree= 13 Lambda index = 100 (Lambda: 2**-10)



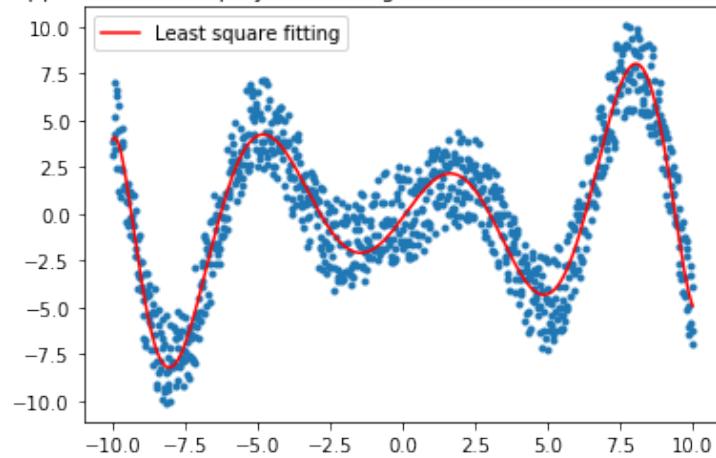
Least Square Approximation at polynomial degree= 13 Lambda index = 101 (Lambda: 2**-9)



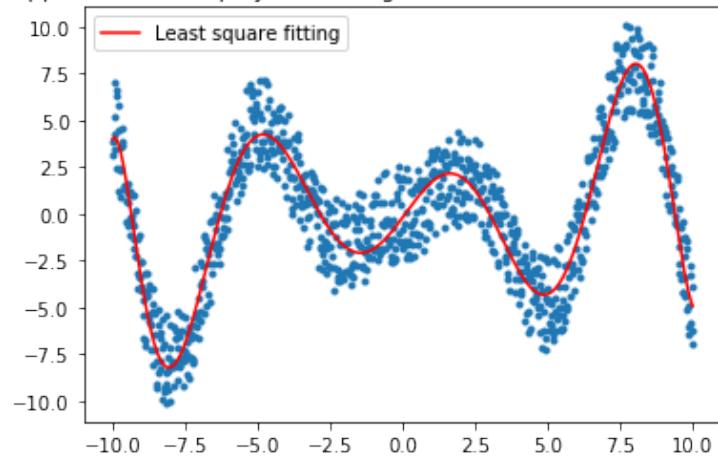
Least Square Approximation at polynomial degree= 13 Lambda index = 102 (Lambda: 2**-8)



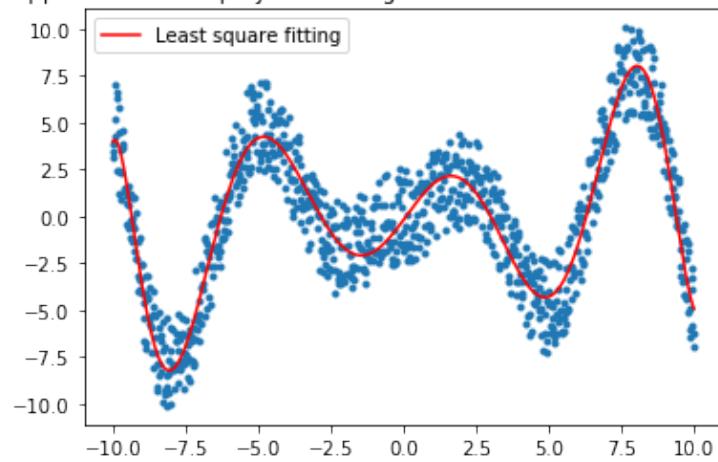
Least Square Approximation at polynomial degree= 13 Lambda index = 103 (Lambda: 2**-7)



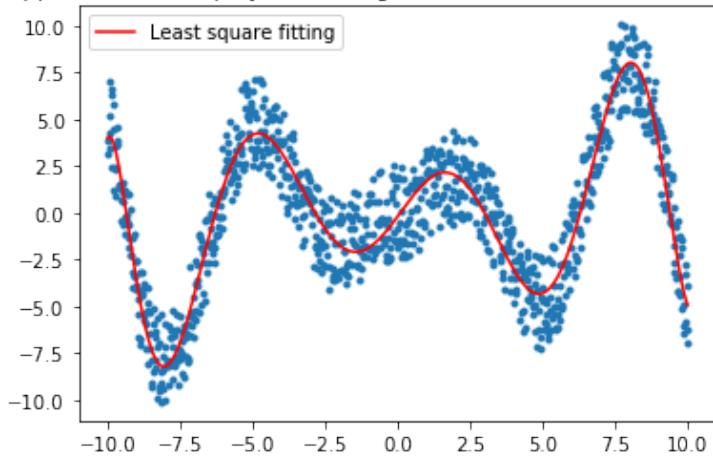
Least Square Approximation at polynomial degree= 13 Lambda index = 104 (Lambda: 2**-6)



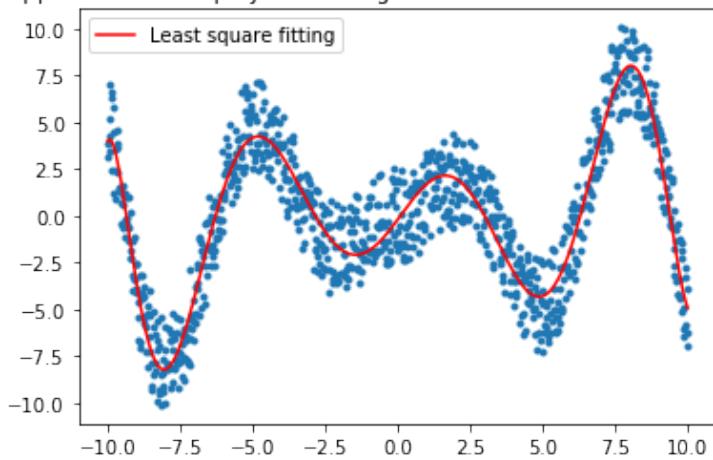
Least Square Approximation at polynomial degree= 13 Lambda index = 105 (Lambda: 2**-5)



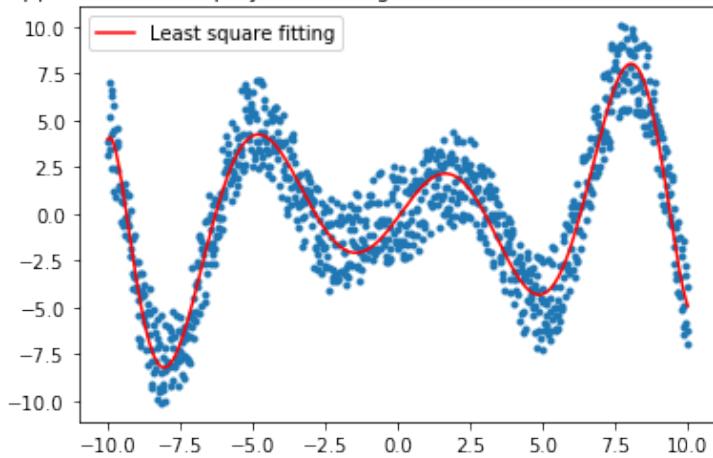
Least Square Approximation at polynomial degree= 13 Lambda index = 106 (Lambda: 2**-4)



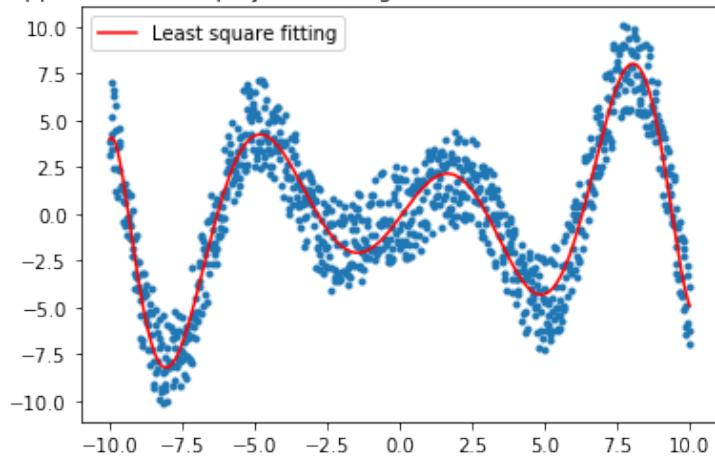
Least Square Approximation at polynomial degree= 13 Lambda index = 107 (Lambda: 2**-3)



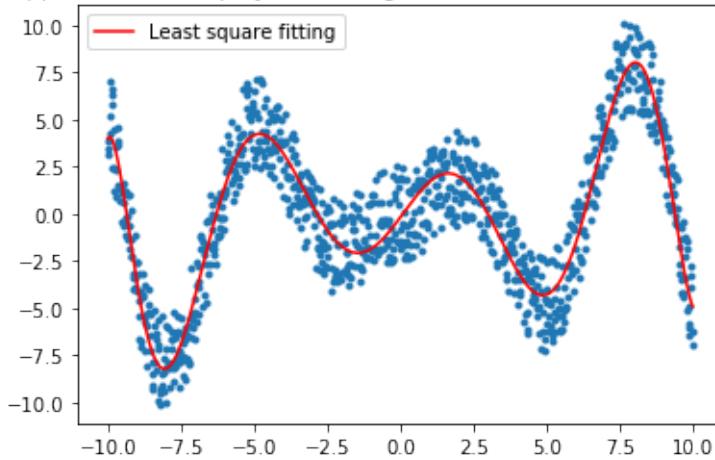
Least Square Approximation at polynomial degree= 13 Lambda index = 108 (Lambda: 2**-2)



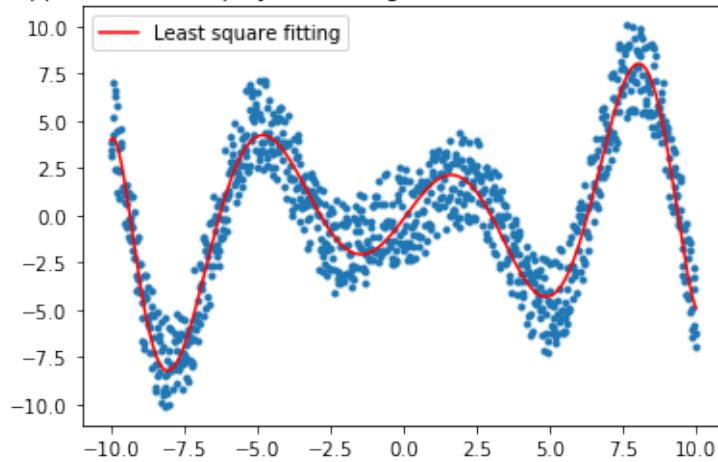
Least Square Approximation at polynomial degree= 13 Lambda index = 109 (Lambda: 2**-1)



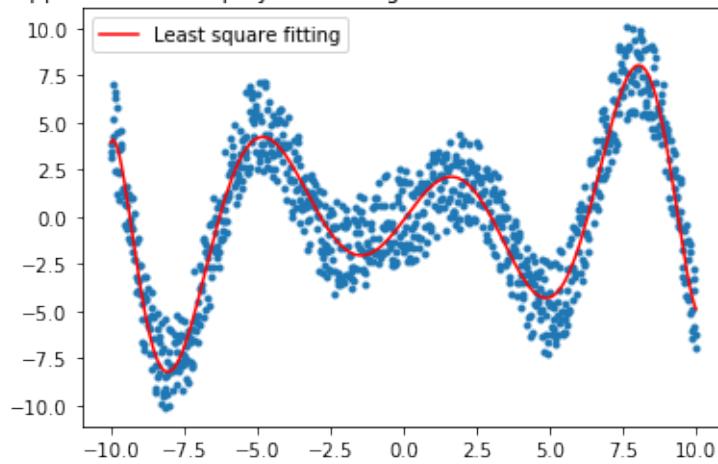
Least Square Approximation at polynomial degree= 13 Lambda index = 110 (Lambda: 2**0)



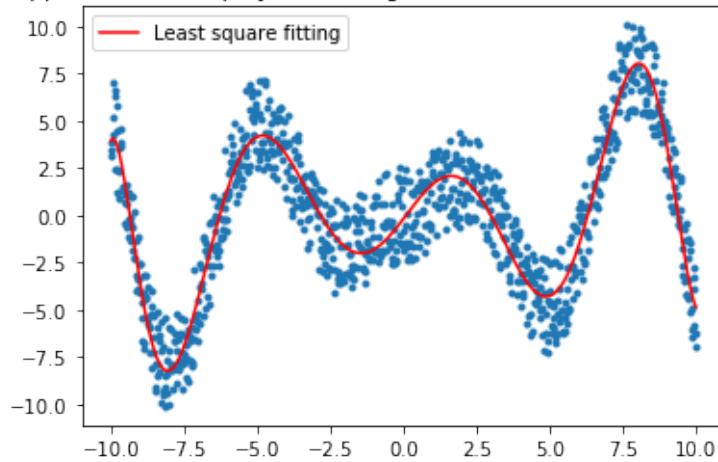
Least Square Approximation at polynomial degree= 13 Lambda index = 111 (Lambda: 2**1)



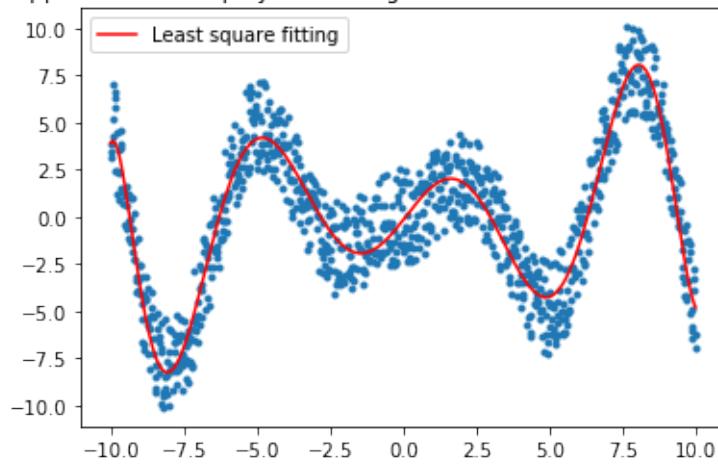
Least Square Approximation at polynomial degree= 13 Lambda index = 112 (Lambda: 2**2)



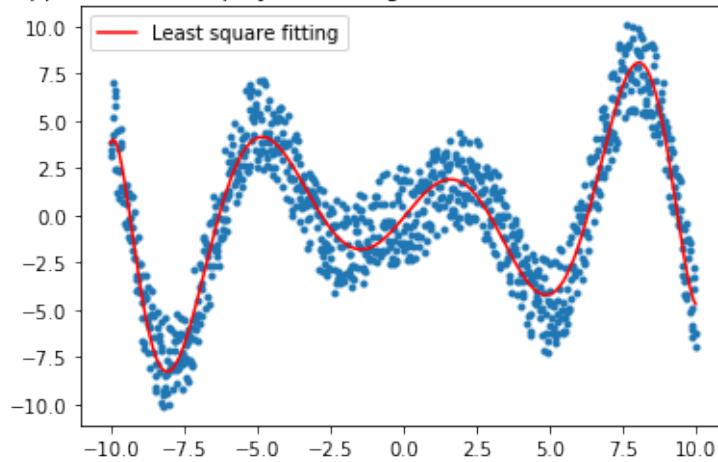
Least Square Approximation at polynomial degree= 13 Lambda index = 113 (Lambda: 2**3)



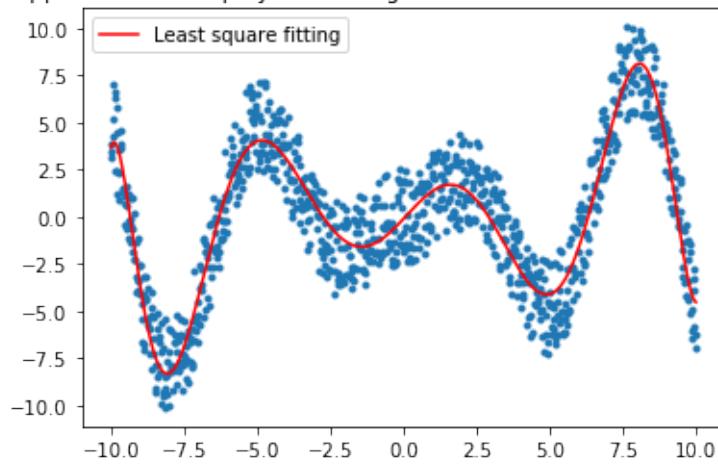
Least Square Approximation at polynomial degree= 13 Lambda index = 114 (Lambda: 2**4)



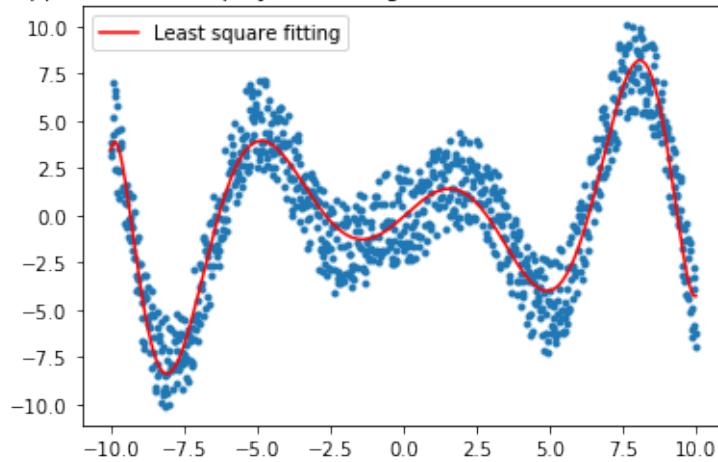
Least Square Approximation at polynomial degree= 13 Lambda index = 115 (Lambda: 2**5)



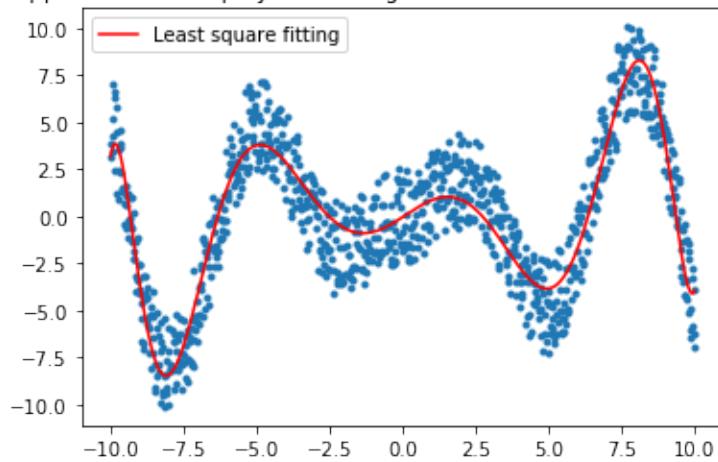
Least Square Approximation at polynomial degree= 13 Lambda index = 116 (Lambda: 2**6)



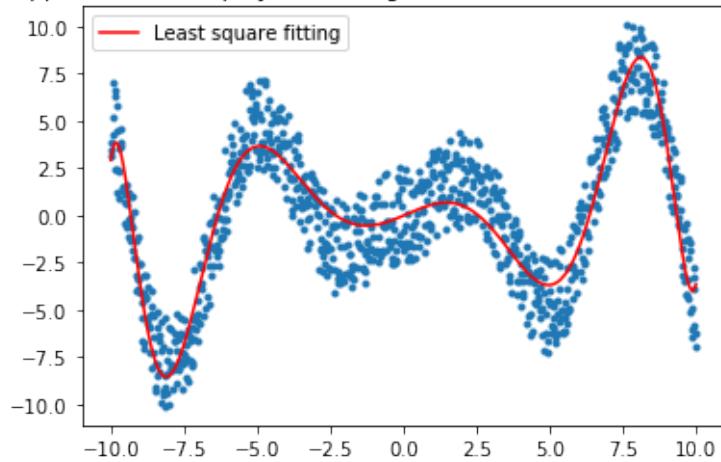
Least Square Approximation at polynomial degree= 13 Lambda index = 117 (Lambda: 2**7)



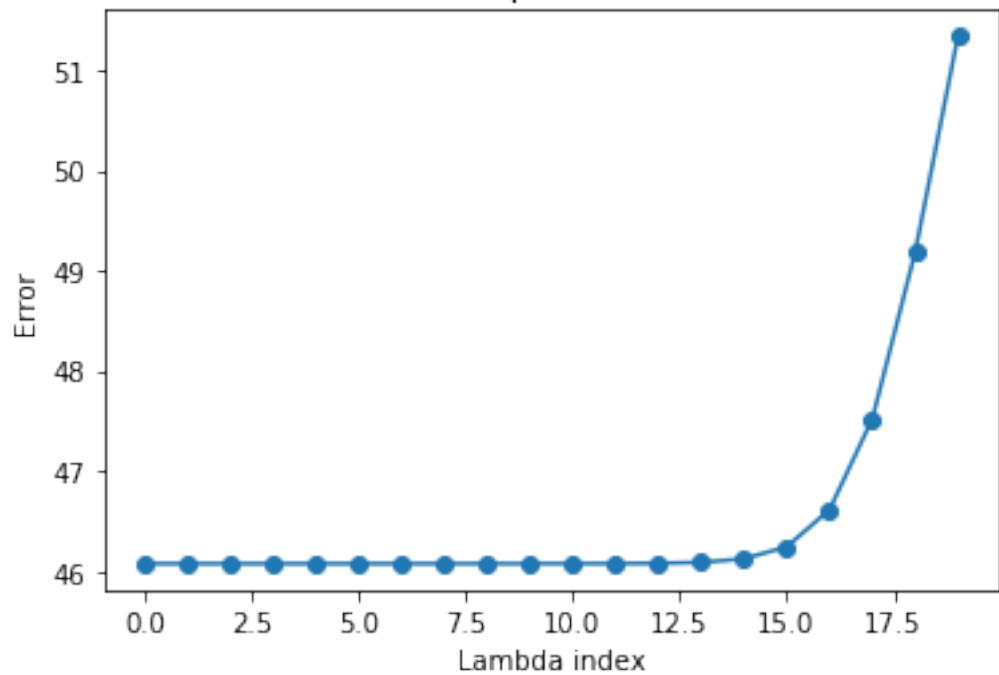
Least Square Approximation at polynomial degree= 13 Lambda index = 118 (Lambda: 2**8)



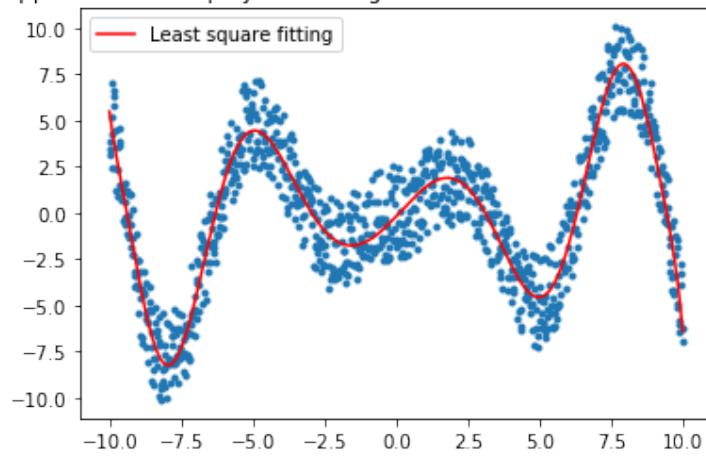
Least Square Approximation at polynomial degree= 13 Lambda index = 119 (Lambda: 2**9)



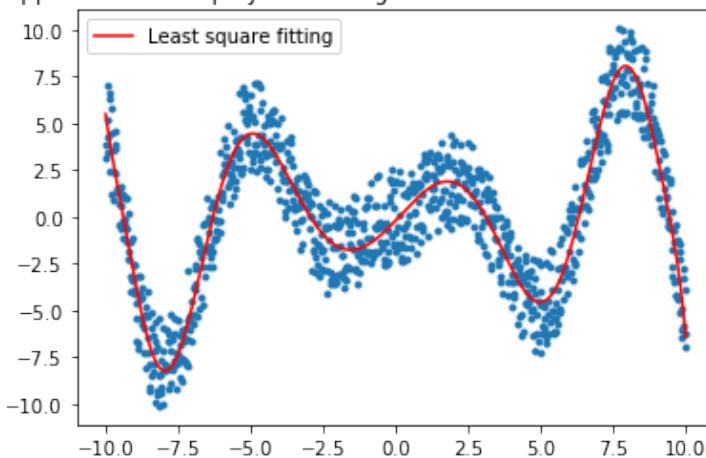
Graph: Error



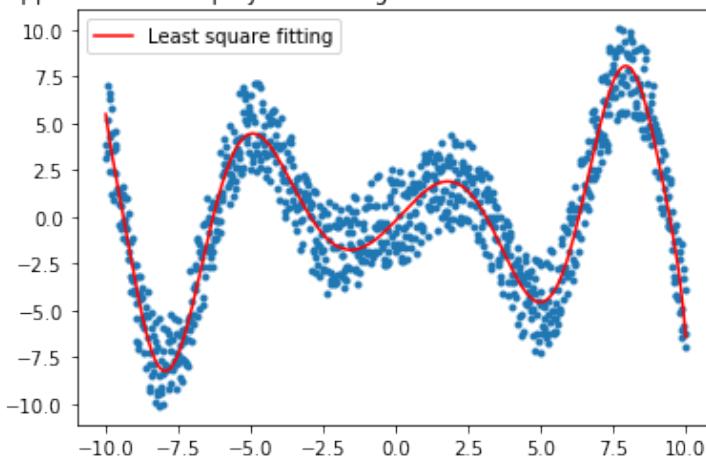
Least Square Approximation at polynomial degree= 14 Lambda index = 120 (Lambda: 2**-10)



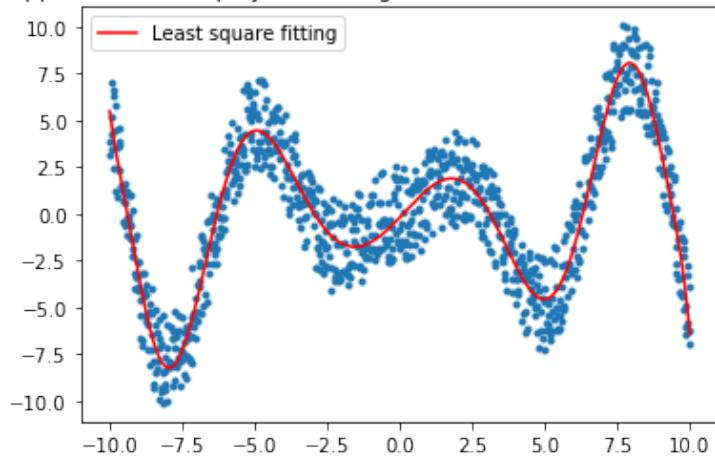
Least Square Approximation at polynomial degree= 14 Lambda index = 121 (Lambda: 2**-9)



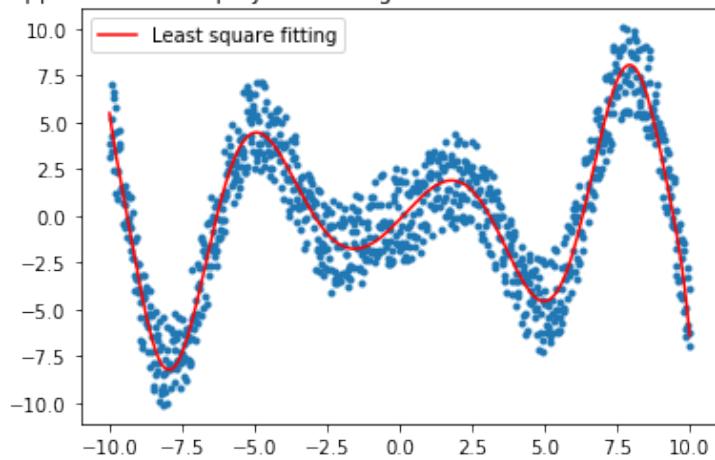
Least Square Approximation at polynomial degree= 14 Lambda index = 122 (Lambda: 2**-8)



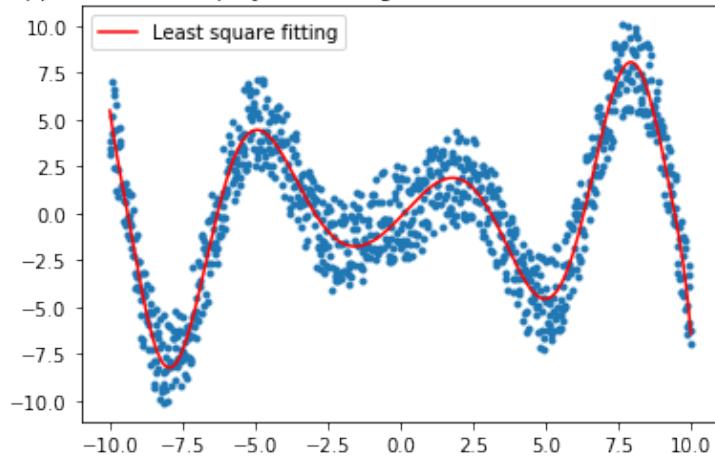
Least Square Approximation at polynomial degree= 14 Lambda index = 123 (Lambda: 2**-7)



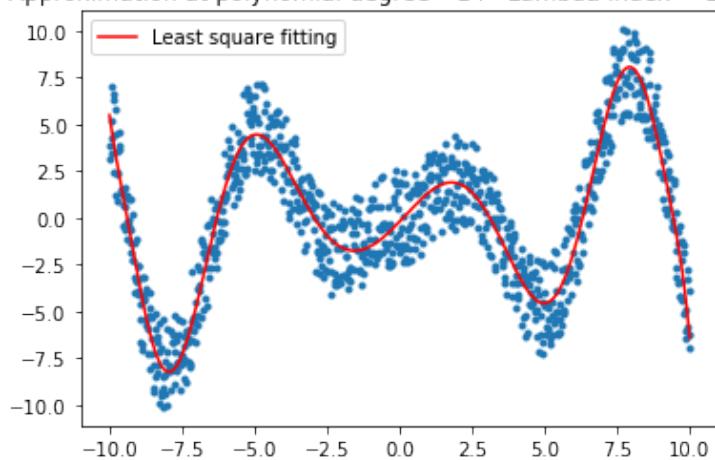
Least Square Approximation at polynomial degree= 14 Lambda index = 124 (Lambda: 2**-6)



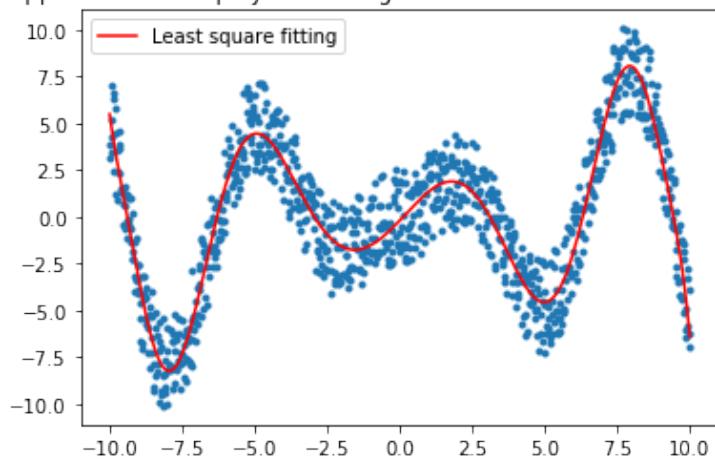
Least Square Approximation at polynomial degree= 14 Lambda index = 125 (Lambda: 2**-5)



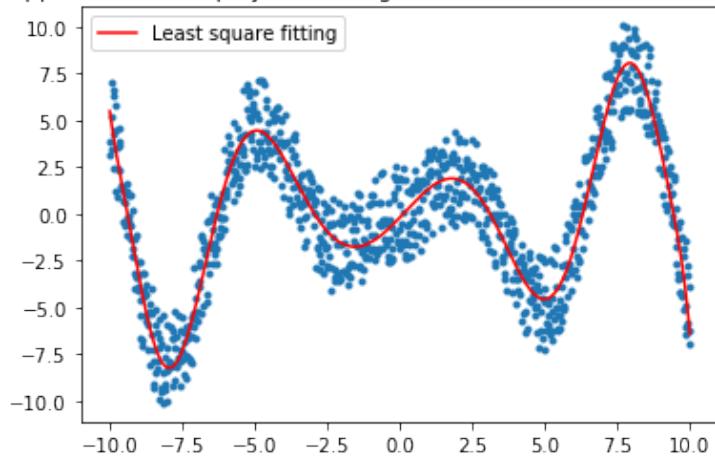
Least Square Approximation at polynomial degree= 14 Lambda index = 126 (Lambda: 2**-4)



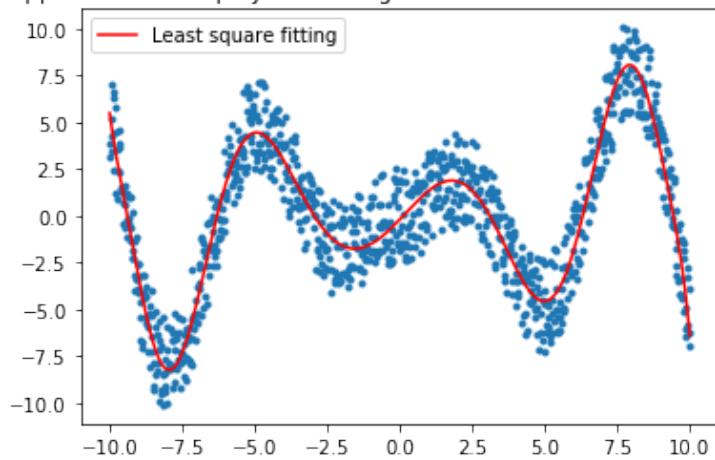
Least Square Approximation at polynomial degree= 14 Lambda index = 127 (Lambda: 2**-3)



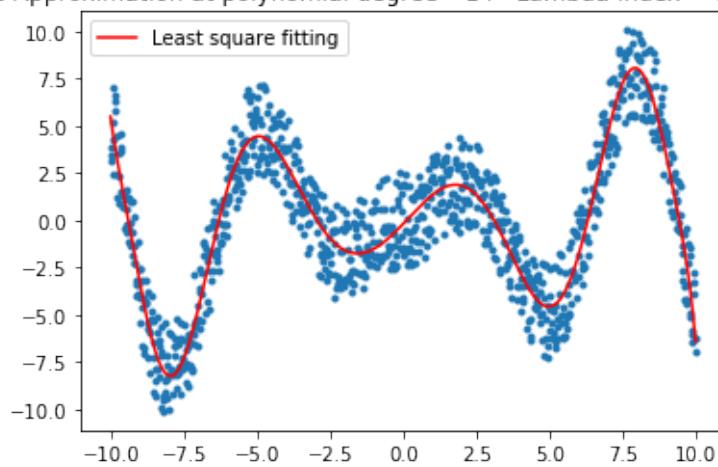
Least Square Approximation at polynomial degree= 14 Lambda index = 128 (Lambda: 2^{**-2})



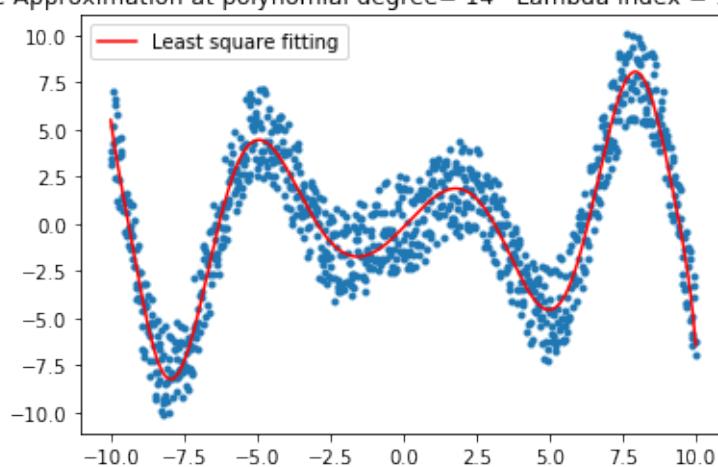
Least Square Approximation at polynomial degree= 14 Lambda index = 129 (Lambda: 2^{**-1})



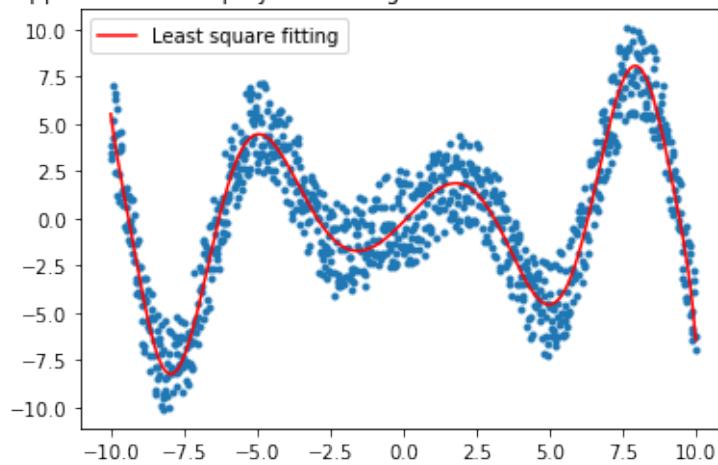
Least Square Approximation at polynomial degree= 14 Lambda index = 130 (Lambda: 2**0)



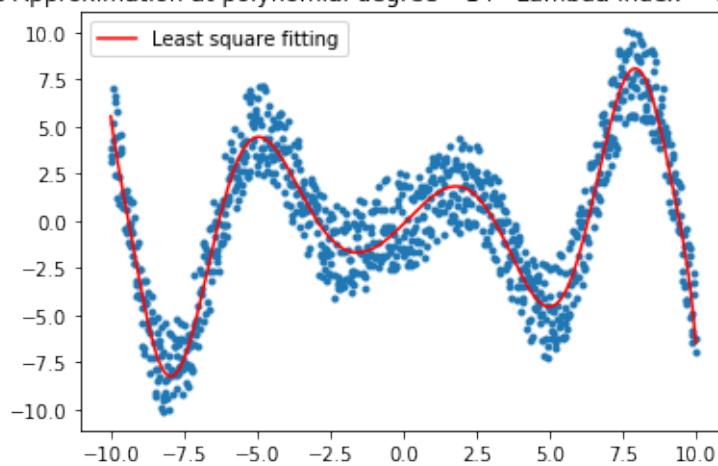
Least Square Approximation at polynomial degree= 14 Lambda index = 131 (Lambda: 2**1)



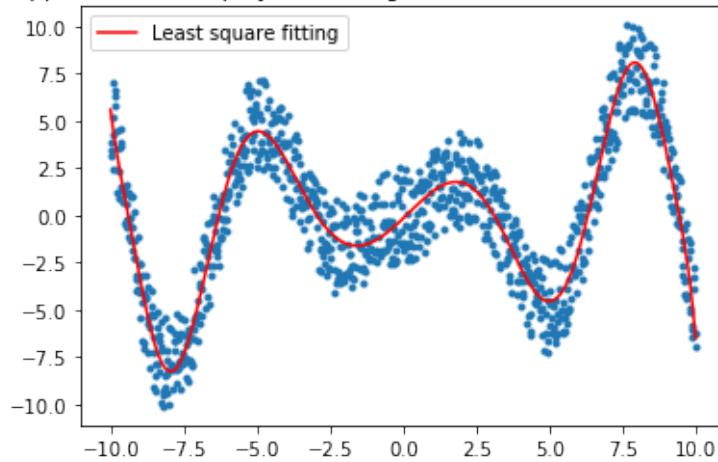
Least Square Approximation at polynomial degree= 14 Lambda index = 132 (Lambda: 2**2)



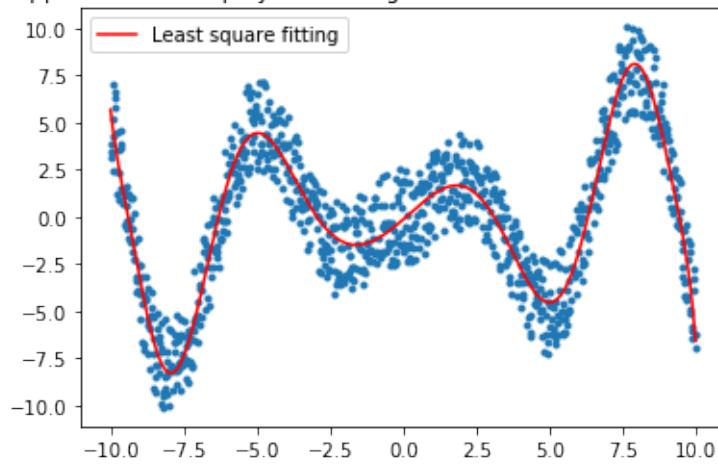
Least Square Approximation at polynomial degree= 14 Lambda index = 133 (Lambda: 2**3)



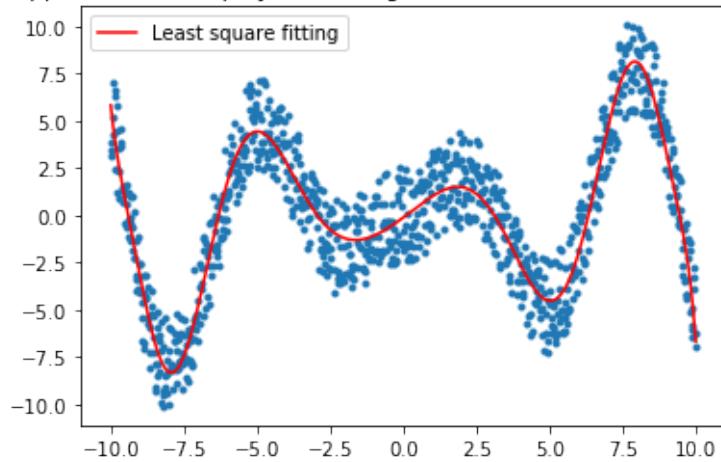
Least Square Approximation at polynomial degree= 14 Lambda index = 134 (Lambda: 2**4)



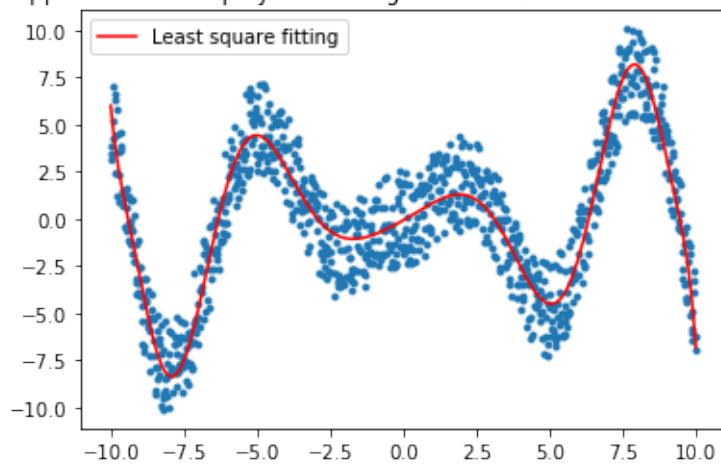
Least Square Approximation at polynomial degree= 14 Lambda index = 135 (Lambda: 2**5)



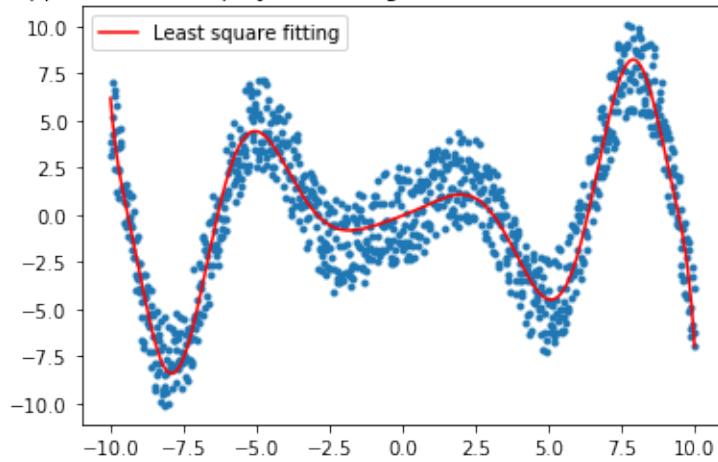
Least Square Approximation at polynomial degree= 14 Lambda index = 136 (Lambda: 2**6)



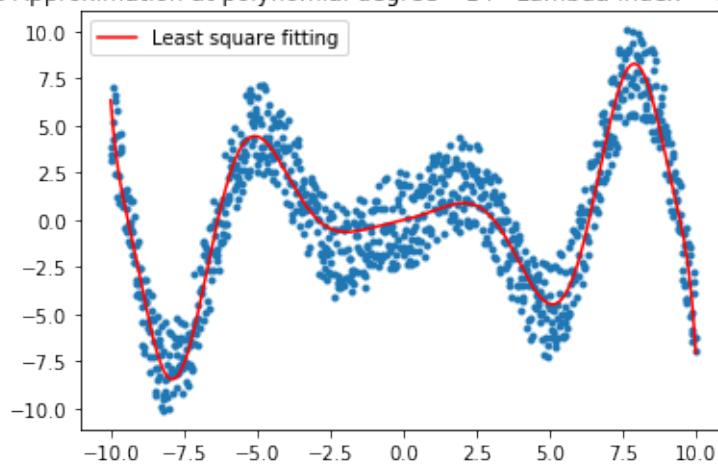
Least Square Approximation at polynomial degree= 14 Lambda index = 137 (Lambda: 2**7)

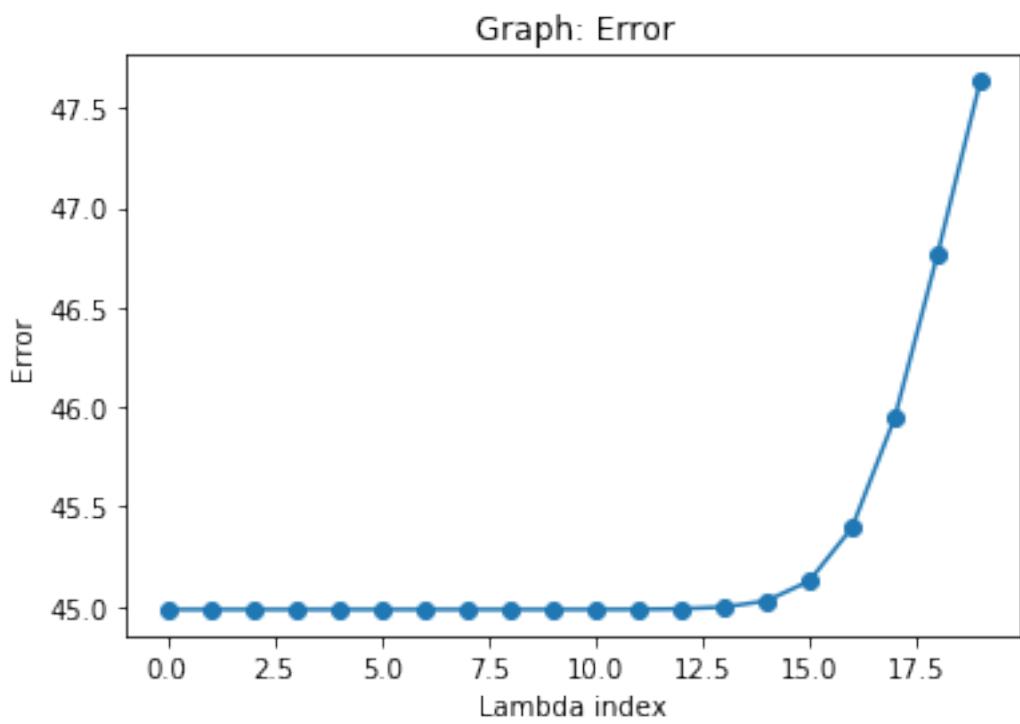


Least Square Approximation at polynomial degree= 14 Lambda index = 138 (Lambda: 2**8)

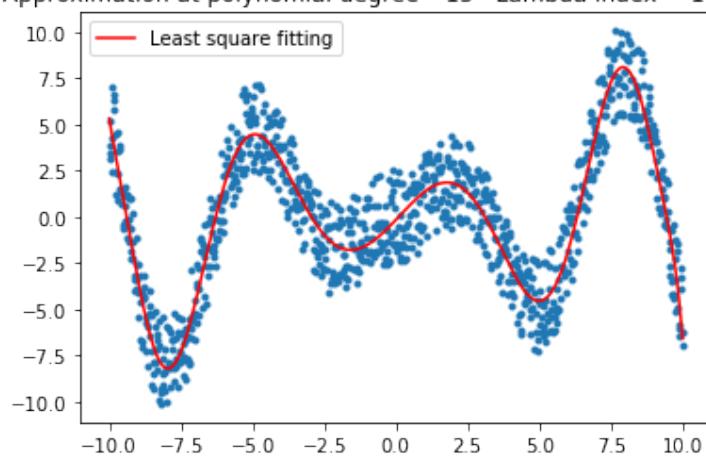


Least Square Approximation at polynomial degree= 14 Lambda index = 139 (Lambda: 2**9)

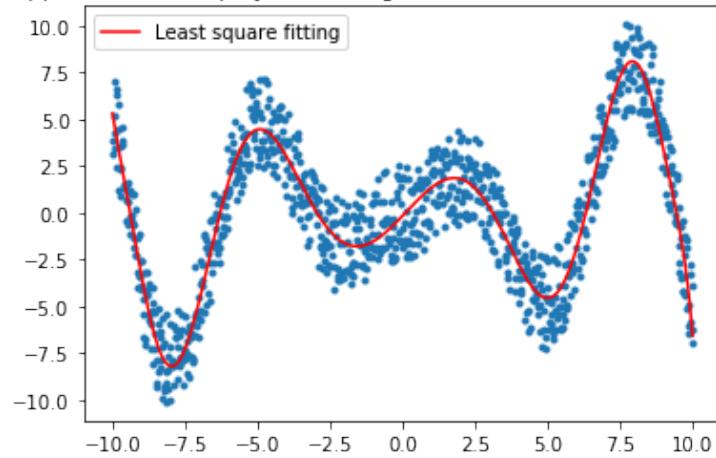




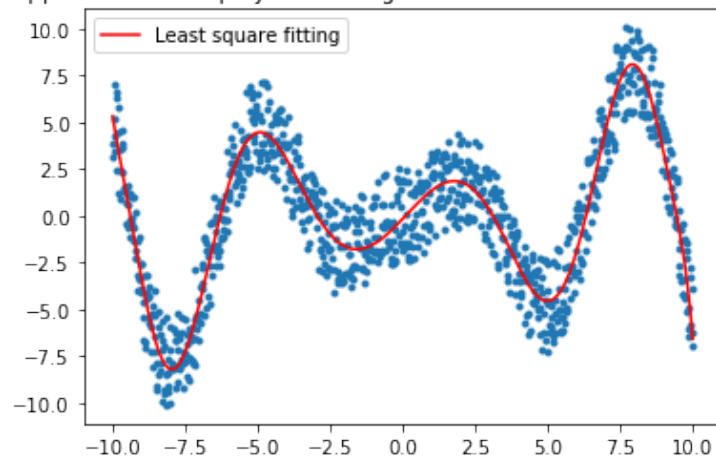
Least Square Approximation at polynomial degree= 15 Lambda index = 140 (Lambda: 2**-10)



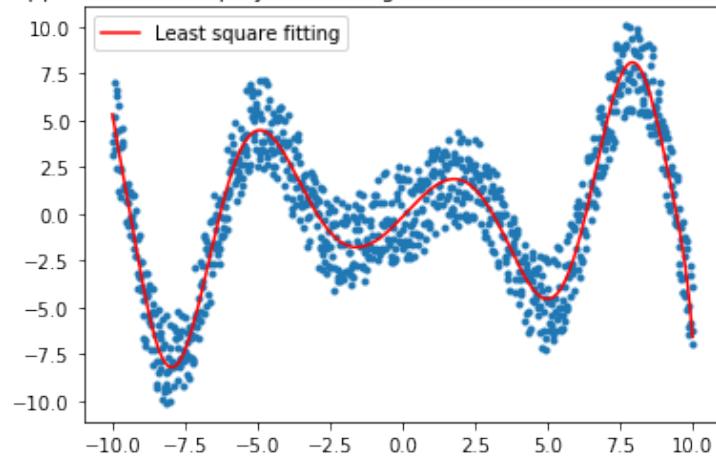
Least Square Approximation at polynomial degree= 15 Lambda index = 141 (Lambda: 2**-9)



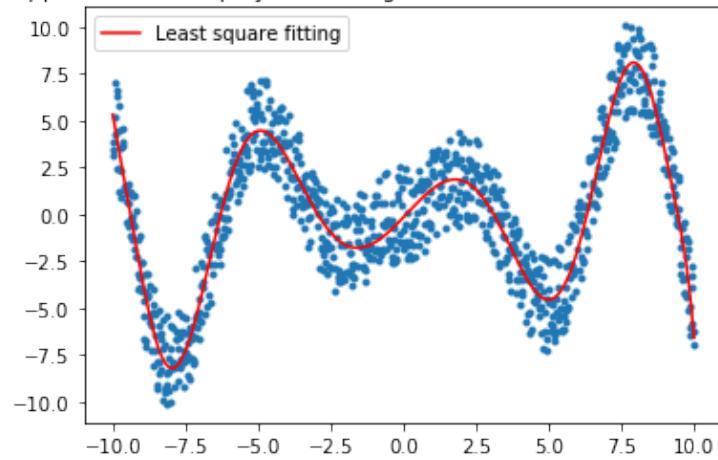
Least Square Approximation at polynomial degree= 15 Lambda index = 142 (Lambda: 2**-8)



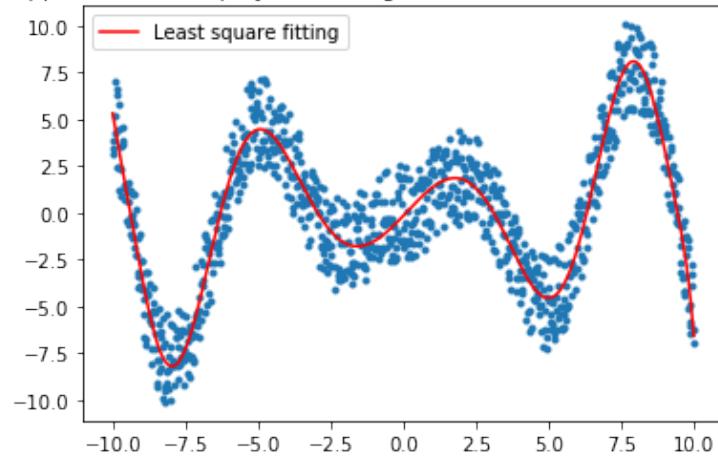
Least Square Approximation at polynomial degree= 15 Lambda index = 143 (Lambda: 2**-7)



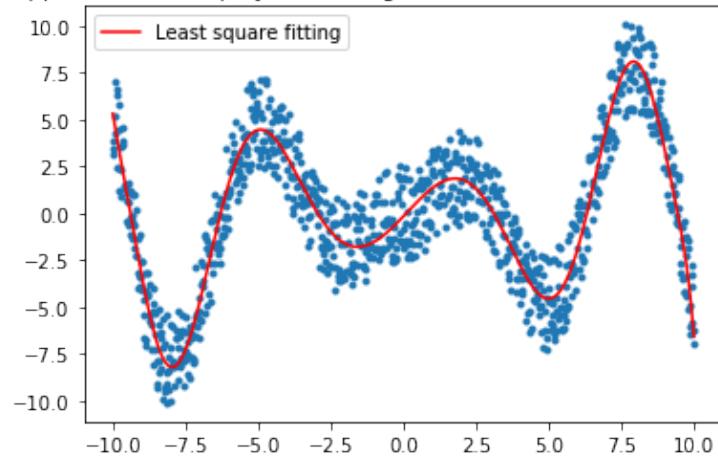
Least Square Approximation at polynomial degree= 15 Lambda index = 144 (Lambda: 2**-6)



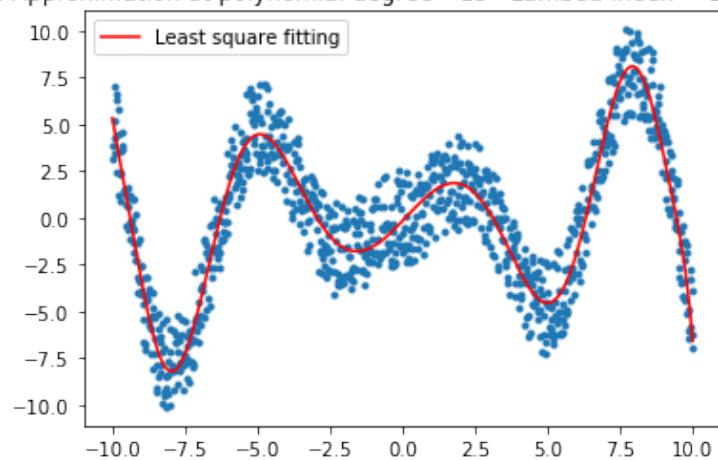
Least Square Approximation at polynomial degree= 15 Lambda index = 145 (Lambda: 2**-5)



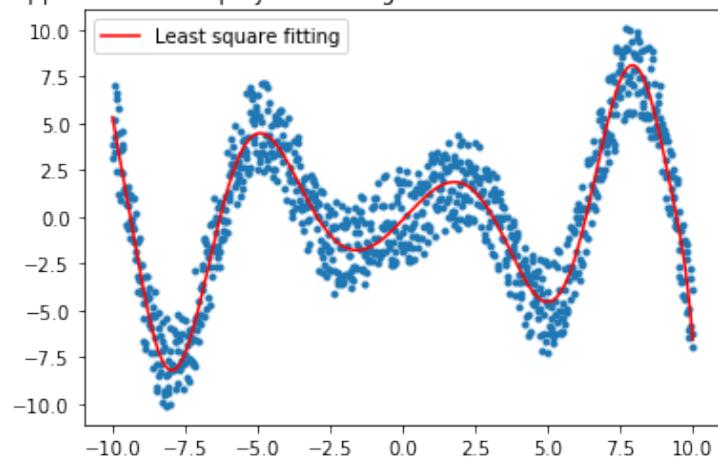
Least Square Approximation at polynomial degree= 15 Lambda index = 146 (Lambda: 2**-4)



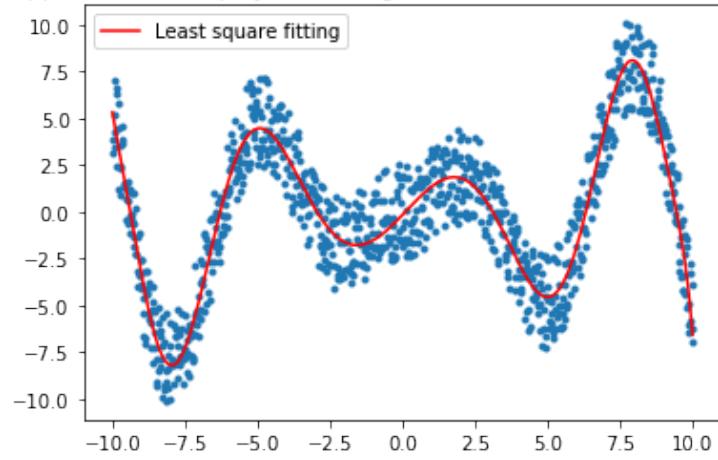
Least Square Approximation at polynomial degree= 15 Lambda index = 147 (Lambda: 2**-3)



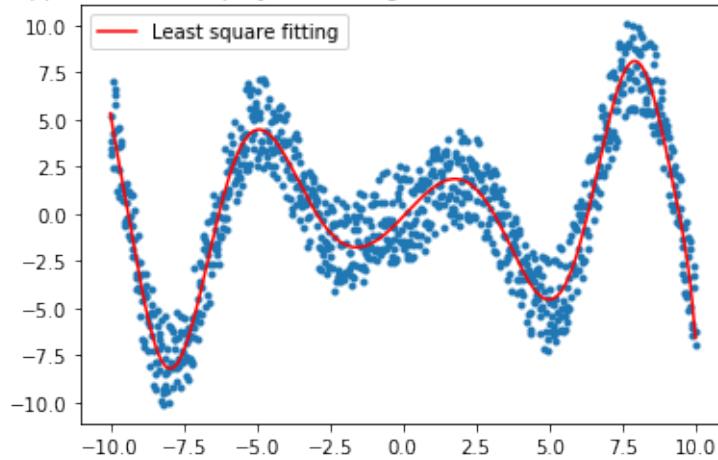
Least Square Approximation at polynomial degree= 15 Lambda index = 148 (Lambda: 2**-2)



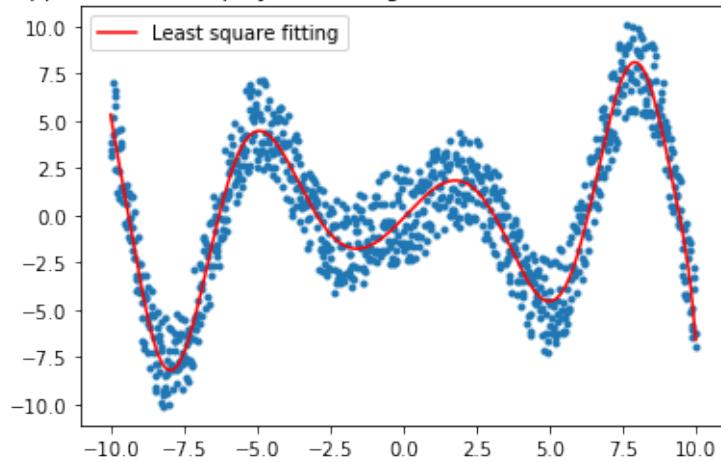
Least Square Approximation at polynomial degree= 15 Lambda index = 149 (Lambda: 2**-1)



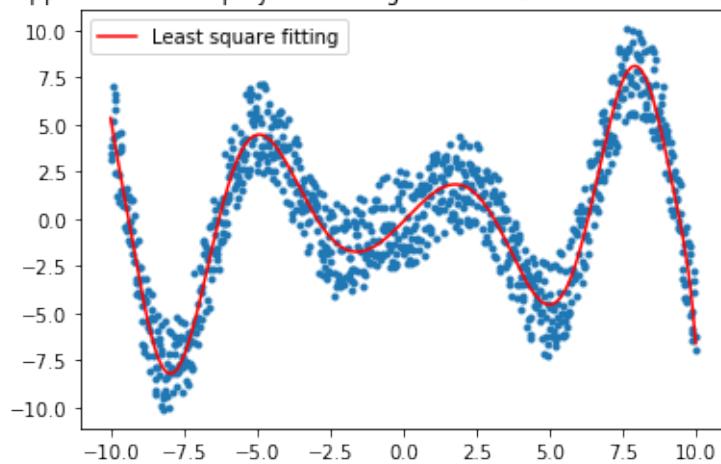
Least Square Approximation at polynomial degree= 15 Lambda index = 150 (Lambda: 2**0)



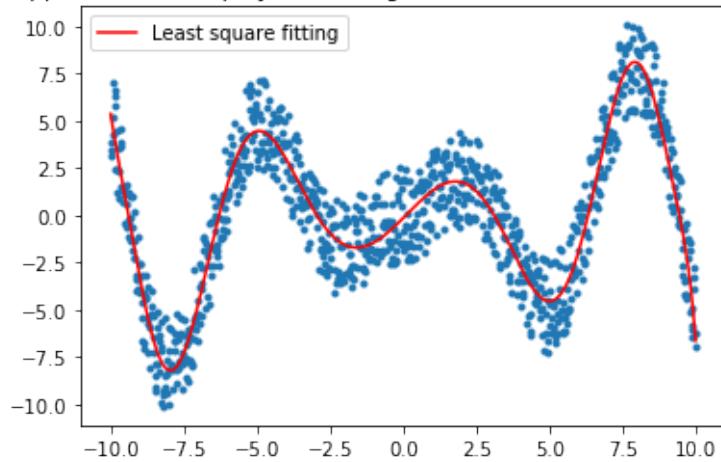
Least Square Approximation at polynomial degree= 15 Lambda index = 151 (Lambda: 2**1)



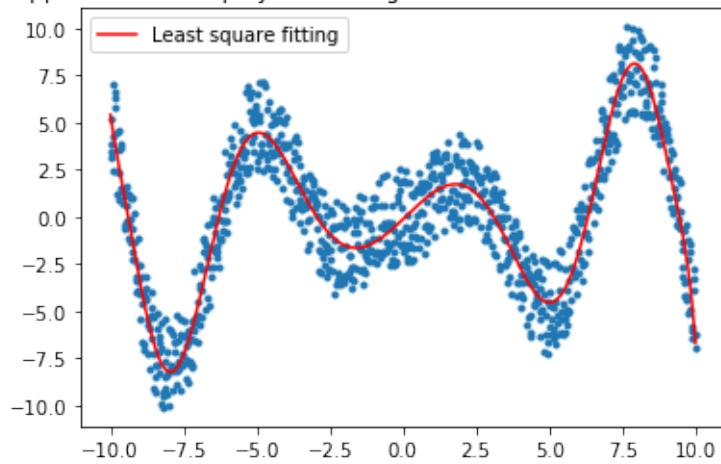
Least Square Approximation at polynomial degree= 15 Lambda index = 152 (Lambda: 2**2)



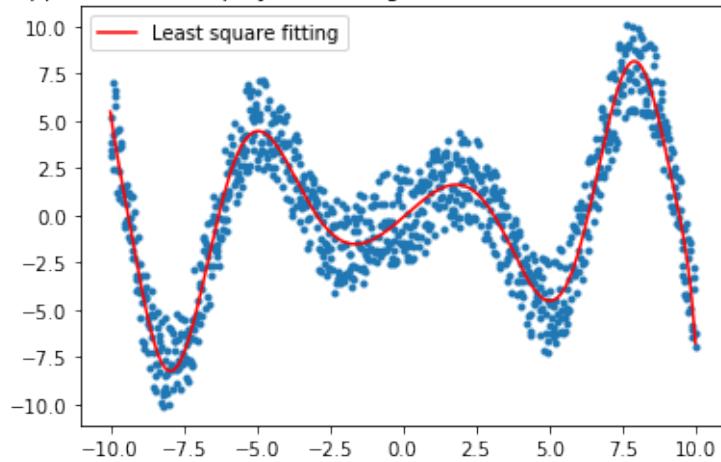
Least Square Approximation at polynomial degree= 15 Lambda index = 153 (Lambda: 2**3)



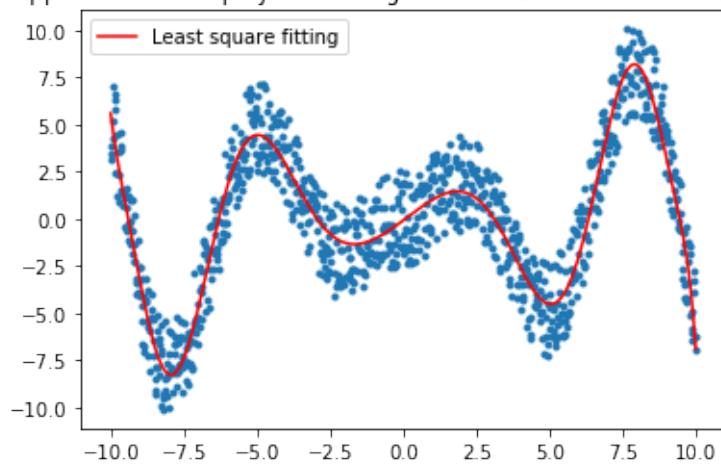
Least Square Approximation at polynomial degree= 15 Lambda index = 154 (Lambda: 2**4)



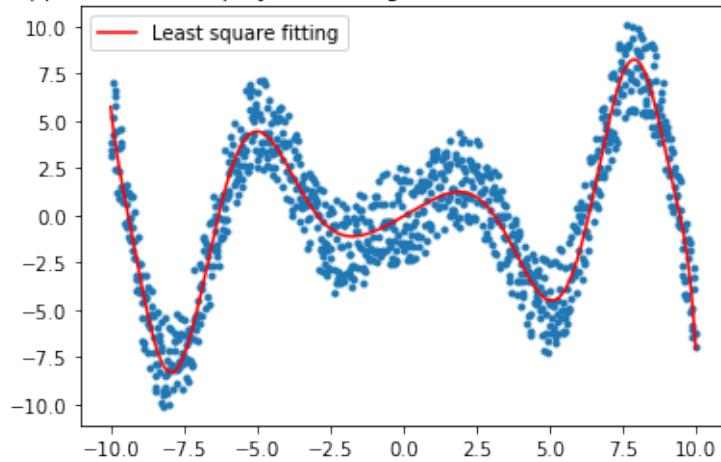
Least Square Approximation at polynomial degree= 15 Lambda index = 155 (Lambda: 2**5)



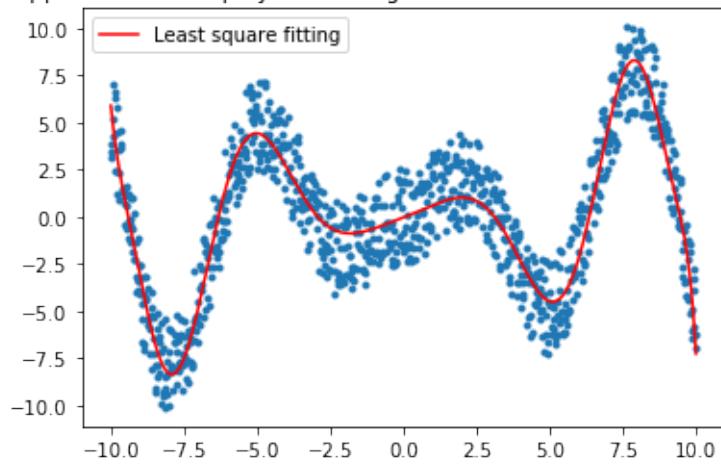
Least Square Approximation at polynomial degree= 15 Lambda index = 156 (Lambda: 2**6)



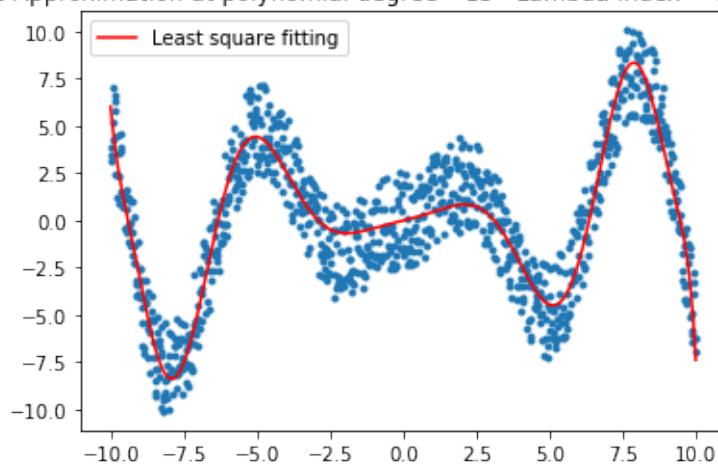
Least Square Approximation at polynomial degree= 15 Lambda index = 157 (Lambda: 2**7)



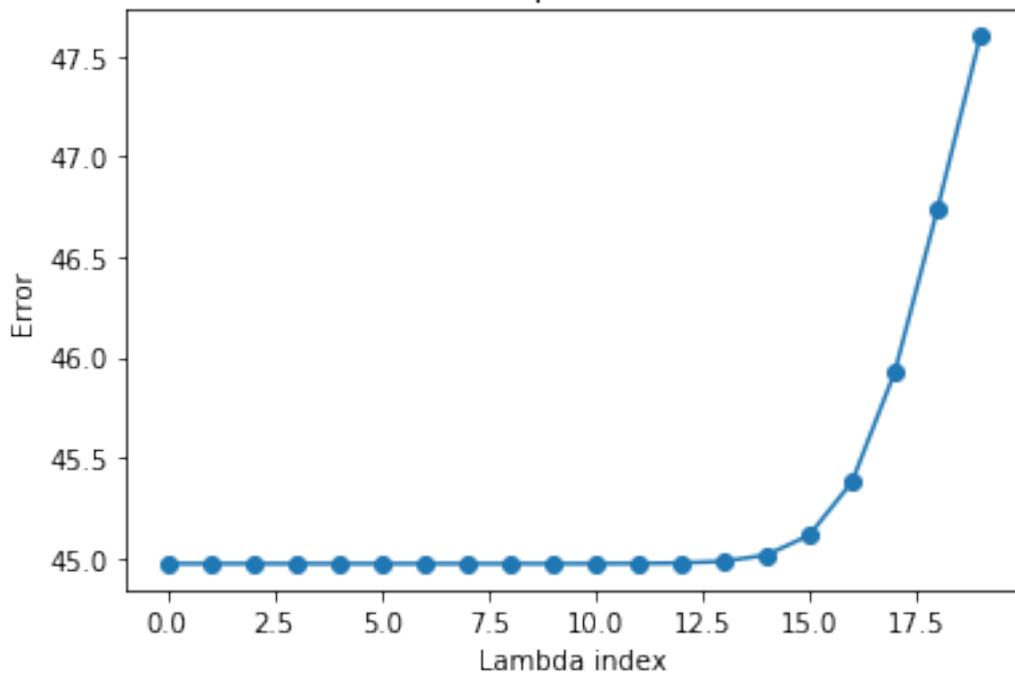
Least Square Approximation at polynomial degree= 15 Lambda index = 158 (Lambda: 2**8)



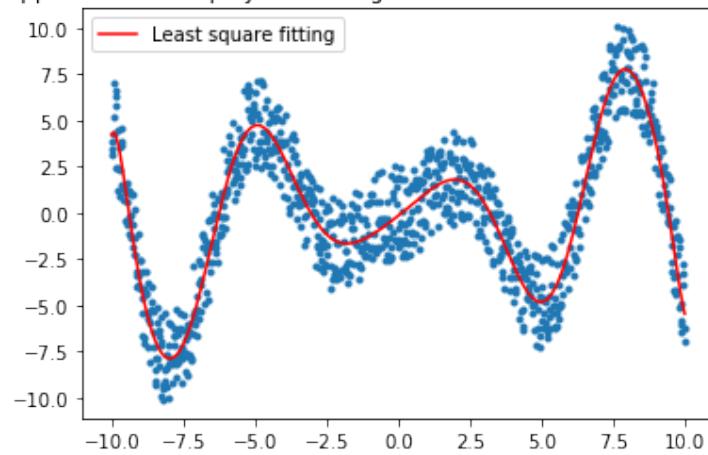
Least Square Approximation at polynomial degree= 15 Lambda index = 159 (Lambda: 2**9)



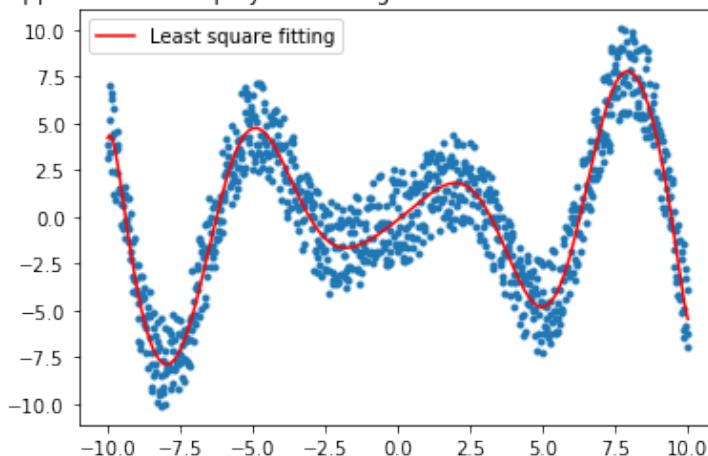
Graph: Error



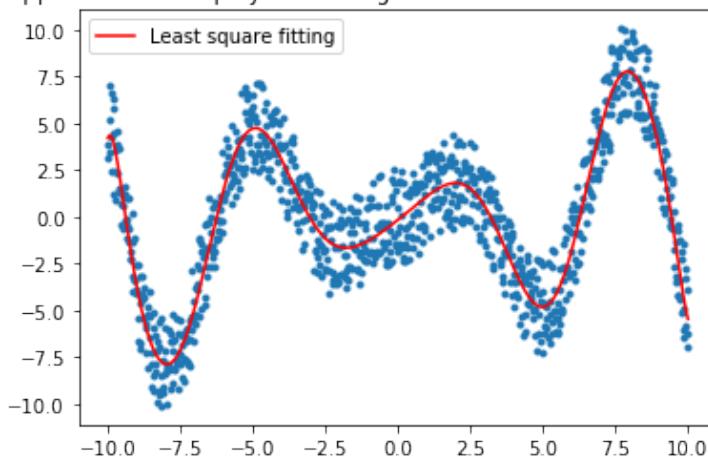
Least Square Approximation at polynomial degree= 16 Lambda index = 160 (Lambda: 2**-10)



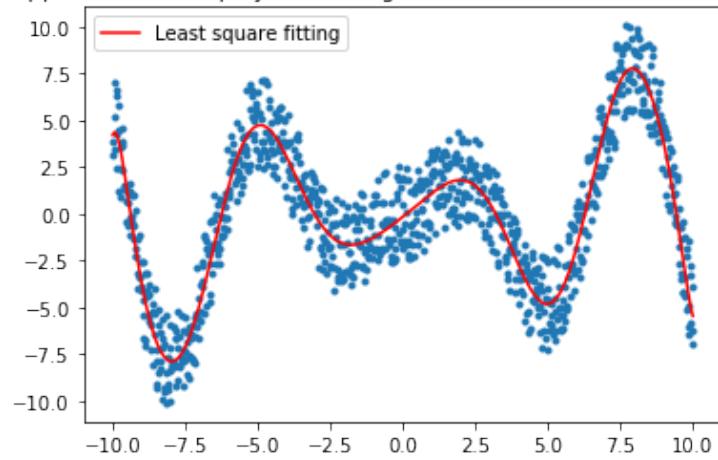
Least Square Approximation at polynomial degree= 16 Lambda index = 161 (Lambda: 2**-9)



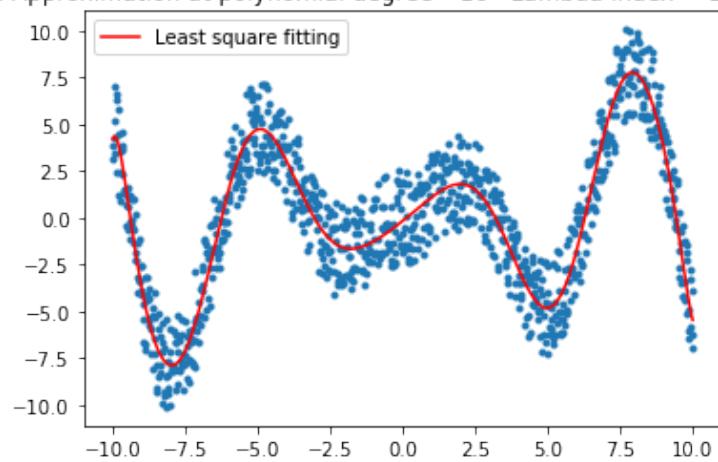
Least Square Approximation at polynomial degree= 16 Lambda index = 162 (Lambda: 2**-8)



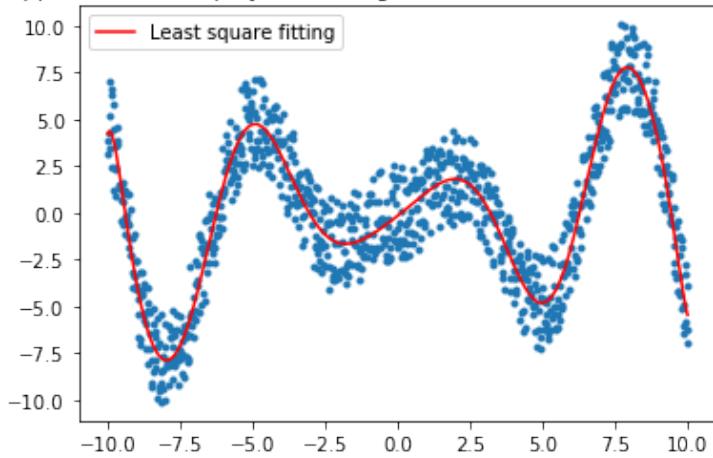
Least Square Approximation at polynomial degree= 16 Lambda index = 163 (Lambda: 2**-7)



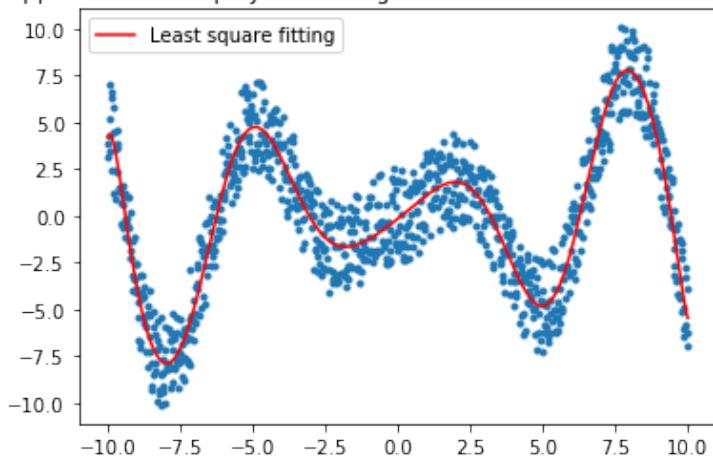
Least Square Approximation at polynomial degree= 16 Lambda index = 164 (Lambda: 2**-6)



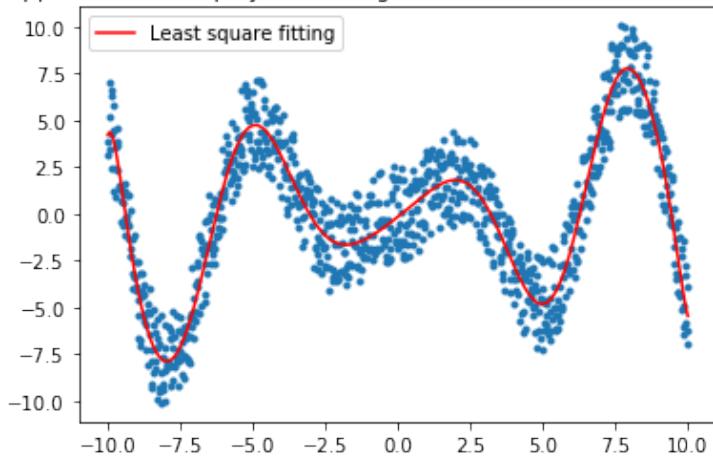
Least Square Approximation at polynomial degree= 16 Lambda index = 165 (Lambda: 2**-5)



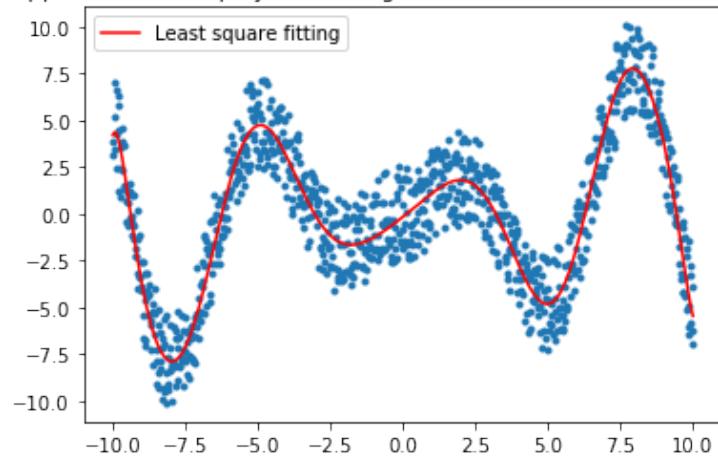
Least Square Approximation at polynomial degree= 16 Lambda index = 166 (Lambda: 2**-4)



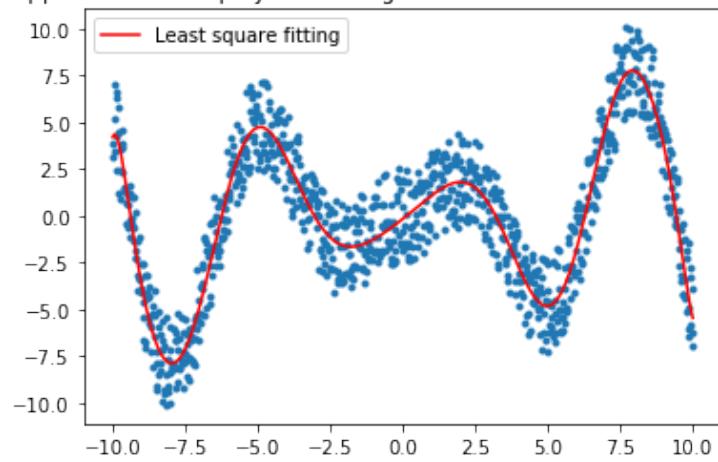
Least Square Approximation at polynomial degree= 16 Lambda index = 167 (Lambda: 2**-3)



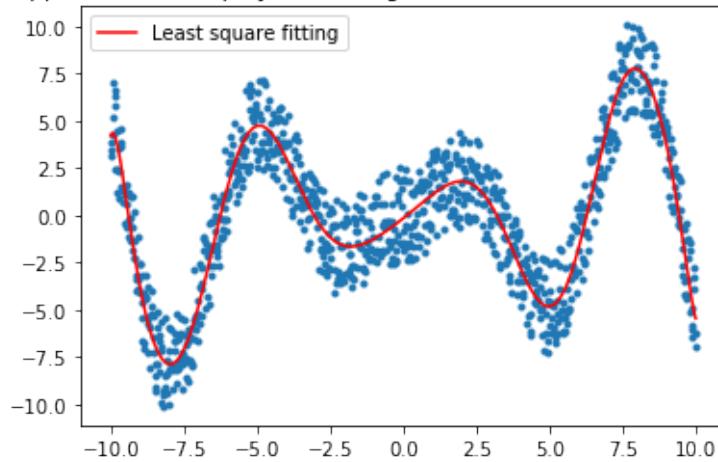
Least Square Approximation at polynomial degree= 16 Lambda index = 168 (Lambda: 2^{**-2})



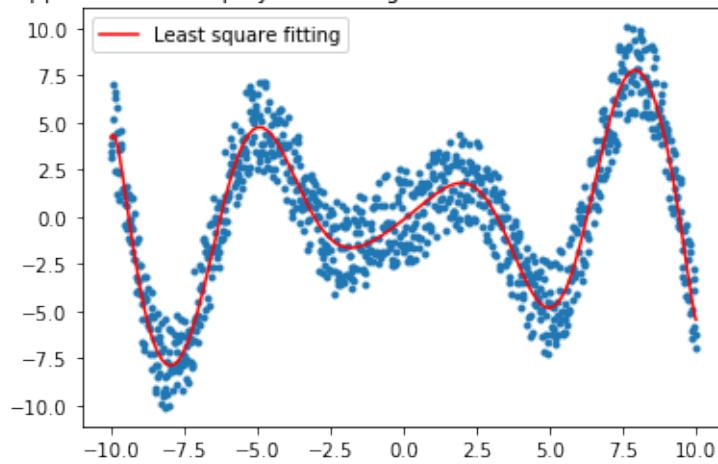
Least Square Approximation at polynomial degree= 16 Lambda index = 169 (Lambda: 2^{**-1})



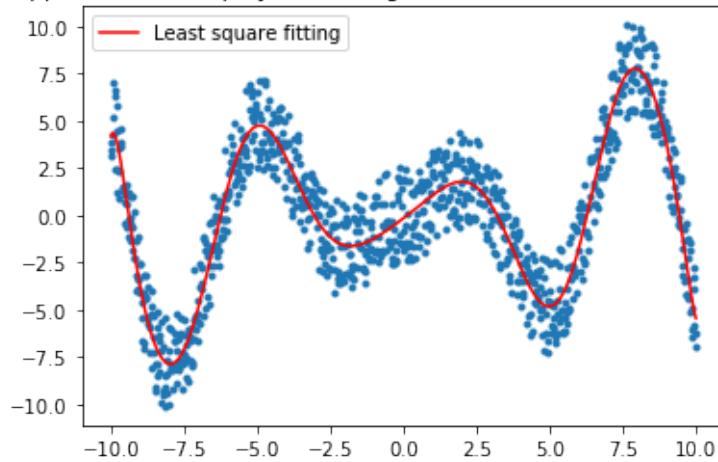
Least Square Approximation at polynomial degree= 16 Lambda index = 170 (Lambda: 2**0)



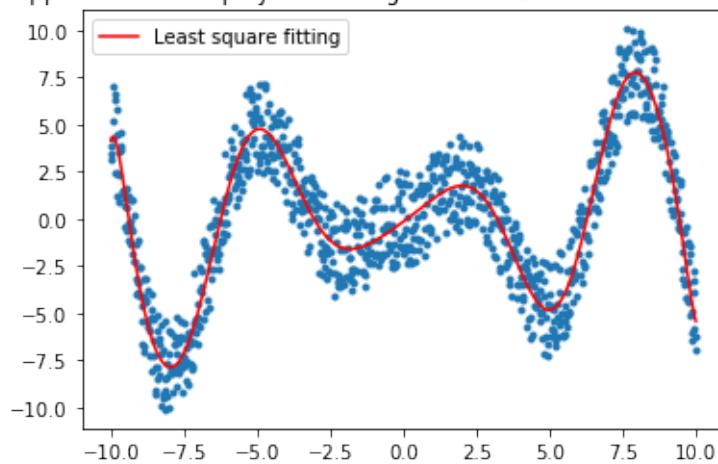
Least Square Approximation at polynomial degree= 16 Lambda index = 171 (Lambda: 2**1)



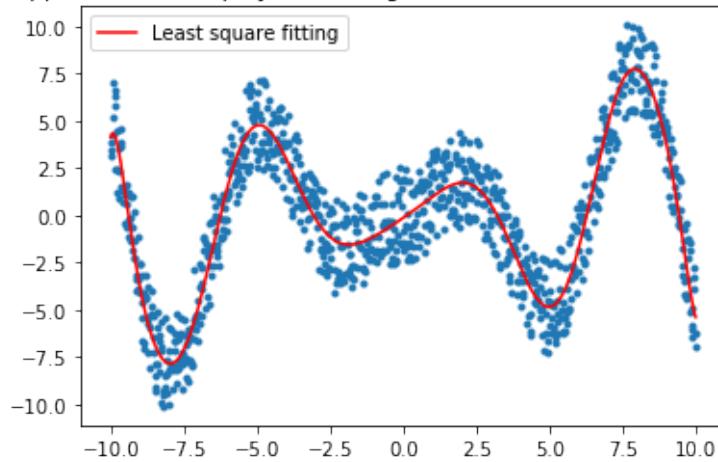
Least Square Approximation at polynomial degree= 16 Lambda index = 172 (Lambda: 2**2)



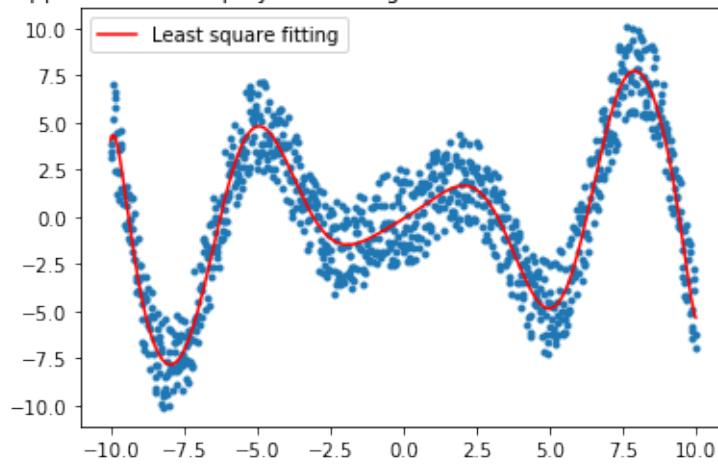
Least Square Approximation at polynomial degree= 16 Lambda index = 173 (Lambda: 2**3)



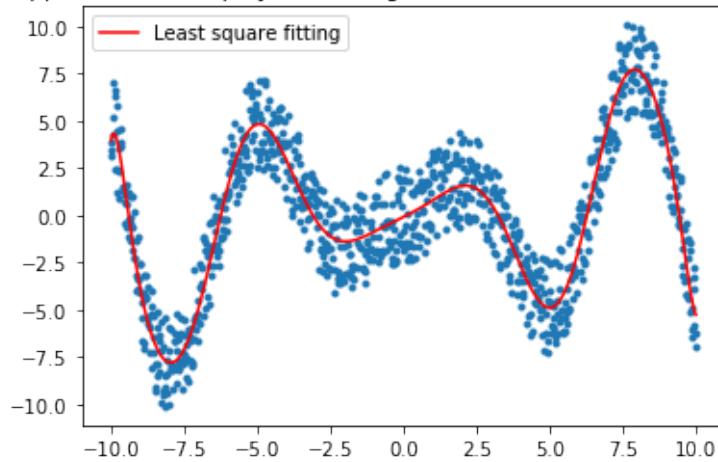
Least Square Approximation at polynomial degree= 16 Lambda index = 174 (Lambda: 2**4)



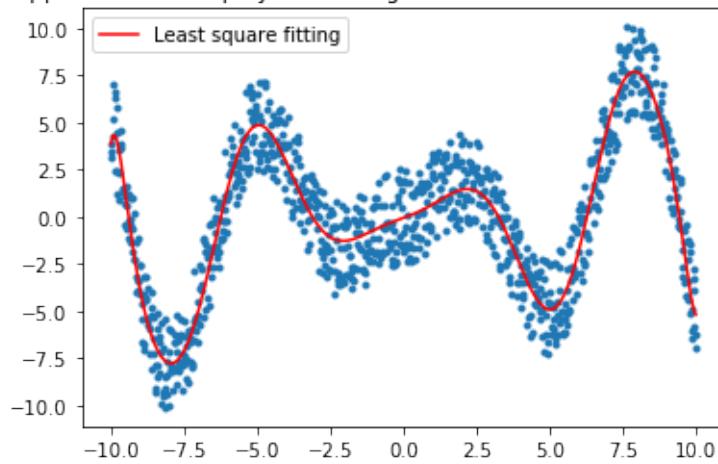
Least Square Approximation at polynomial degree= 16 Lambda index = 175 (Lambda: 2**5)



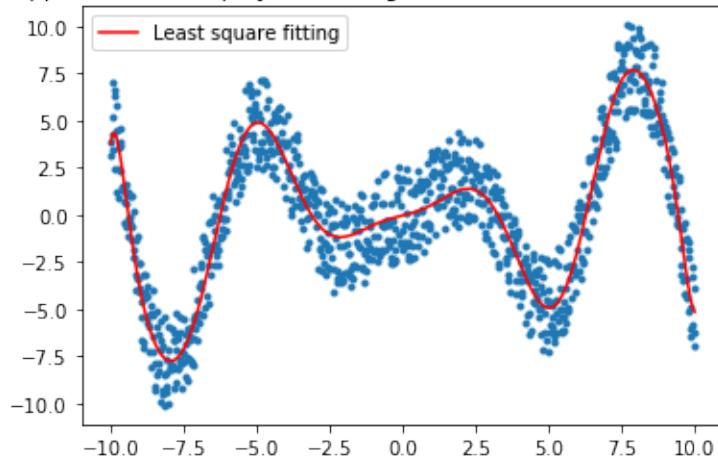
Least Square Approximation at polynomial degree= 16 Lambda index = 176 (Lambda: 2**6)



Least Square Approximation at polynomial degree= 16 Lambda index = 177 (Lambda: 2**7)



Least Square Approximation at polynomial degree= 16 Lambda index = 178 (Lambda: 2**8)



Least Square Approximation at polynomial degree= 16 Lambda index = 179 (Lambda: 2**9)

