Assignment 2

Deadline: 17 October 2022

Problem 1

Suppose you want to organize activities for some campers. This includes a triathalon exercise: each contestant must swim 20 laps of a pool, then bike 10 miles, then run 3 miles. The contestants must use the pool one at a time. In other words, first one contestant swims the 20 laps, gets out, and starts biking. As soon as this first person is out of the pool, a second contestant begins swimming the 20 laps; as soon as he or she is out and starts biking, a third contestant begins swimming . . . and so on. Each contestant has a projected swimming time (the expected time it will take him or her to complete the 20 laps), a projected biking time (the expected time it will take him or her to complete the 10 miles of bicycling), and a projected running time (the time it will take him or her to complete the 3 miles of running). You want to decide on a schedule for the triathalon: an order in which to sequence the starts of the contestants. The completion time of a schedule is the earliest time at which all contestants will be finished with all three legs of the triathalon, assuming they each spend exactly their projected swimming, biking, and running times on the three parts.(Again, note that participants can bike and run simultaneously, but at most one person can be in the pool at any time.) What's the best order for sending people out, if one wants the whole competition to be over as early as possible?

a. Design an efficient algorithm for finding such a schedule.

b. Analyze the running time of your algorithm as a function of n, the number of contestants.

c. Prove that your algorithm works (Give informal argument).

Problem 2

Suppose there is an art gallery in form of a straight long hall. You are given an ordered set P= $\{p_1, p_2, ..., p_n\}$of real numbers that represent the positions of precious paintings or sculptures in this hallway. A guard can protect all the objects that comes within d distance on his left and right.

a) Design an algorithm for finding a placements of guards that uses the minimum number of guards to guard all the objects with positions in P.
b) Analyze the running time of your algorithm as a function of n, the number of objects that need guarding.
c) Prove that your algorithm works (Give informal argument).

Problem 3

The spread of second wave of COVID-19 in Pakistan might result in closure of academic Institutes. The management of FAST Lahore decided to save your academic year by conducting online sessions (video lectures). There are total n videos that need to be streamed one after the other. Each video v_i consists of b_i bits that needs to be sent at a constant rate over a period of t_i seconds. There is only one connection allowed so two videos can't be sent at a time. This means scheduling of videos is required (an order in which to send these videos). Whichever order is chosen, there cannot be any delays between the end of one video and the start of the next. The connection does not want its user taking up too much bandwidth, so it imposes the following constraint, using a fixed parameter r: For each natural number t > 0, the total number of bits you send over the time interval from 0 to t cannot exceed r*t. A schedule is considered valid if it satisfies the constraint imposed by the connection. You are a computer science expert and management of FAST need your services. Given a set of n video streams specified by its number of bits bi and its time duration t_i, they need to determine whether there exists a valid schedule that satisfies connection parameter r. For example you have 3 videos with (b_1,t_1)=(2000,1), (b_2,t_2)=(6000,2) and (b_3,t_3)=(2000,1) also r=5000. The schedule that runs videos in order 1, 2, 3, is valid because at time t=1 the first stream is sent and 2000 <5000*1 at time t=2 2000+3000 (half of second video)<5000*2 similar calculation can be done to check the constraint for t=3 and t=4.

a. Design an efficient algorithm that takes a set of n streams each specified by b_i and t_i along with r and determines whether a valid schedule exists or not.

b. Analyze the running time of your algorithm as a function of n.

c. Prove that your algorithm works (Give informal argument).

Problem 4

You are planning to start a new business of printing flexes. In the first step you have bought a printing machine. Every day you receive printing orders from n different customers. Customer C_i's job requires time t_i. The wait time of a customer C_i is w_i which is the amount of time the customer waited till its job completion. You have only one printing machine so you have to devise a schedule of printing. You want your customers to be happy. The happiness of the customer is dependent on the wait time. The smaller the wait time the happier your customer is. Devise a schedule that minimizes the total wait time of all the n customers. Suppose there are three customer with job time t_1=4, t_2=5, t_3=2. If you schedule (1, 2, 3) then w_1=4, w_2=9, w_3=11 total wait time is 4+9+11=24

a) Design an algorithm for finding such a schedule i.e given t_i for every customer, find a schedule that minimizes $\sum_{i=1}^{n} w_i$
b) Analyze the running time of your algorithm as a function of n.

c) Prove that your algorithm works (Give informal argument).

Problem 5

Suppose that in Problem 4, you have prioritize your customers i.e. each customer C_i also has a priority p_i that represents his/her importance. So now you want to find a schedule that minimizes the weighted sum of waiting time based on customer priority. Specifically you want to minimize $\sum_{i=1}^{n} w_i p_i$

a) Design an algorithm for finding such a schedule i.e. given t_i and p_i for every customer, find a schedule that minimize $\sum_{i=1}^{n} w_i p_i$
b) Analyze the running time of your algorithm as a function of n.
c) Prove that your algorithm works (Give informal argument).