1) Back Substitution / Iterative Method.

2) Substitution Method

$$T(n) = 2T\left(\frac{n}{2}\right) + n \rightarrow (A) \qquad T(1) = 1.$$

1. Guess the solution
2. Use mathematical induction to find constants and show that solution works.

How to make guess:
1) you've already seen it
2) Use iterative method
3) Tree method to make a guess.

Guess : $T(n) = O(n\log n)$

Now, the substitution method requires us to prove:

$$T(n) \le cn\log n. \text{ (for constant } c>0)$$

from definition of Big-Oh

$$f(n) \le c\,g(n)$$

$T(\frac{n}{2})$ is the unknown in (A), if we solve it we can solve (A).

Lets test base case first. Our claim should hold

$\boxed{n=1}$ $\quad T(1) \le c\,1\log 1$

$1 \le c \times 0 \qquad$ (FAIL)

lets check for

$\boxed{n=2}$ $\quad T(2) \le c\,2\log 2$

find $T(2)$ from (A)

$2T(1) + 2 \le c\,2\,(1) \qquad$ ($\because$ log base 2)

$2+2 \le 2c \implies 4 \le 2c$ True for all $\boxed{c \geq 2}$

Induction Step.

$n = 3, 4, \ldots, m$

Let's assume it is true for m

$n = m$

$$T(m) \le cm \log m \rightarrow B$$

Possible values of m

$$2 \le m \le n$$

m will also take value $\frac{n}{2}$ at some point.

$m = \frac{n}{2}$

from B

$$\boxed{T\left(\frac{n}{2}\right) \le c\frac{n}{2}\log\frac{n}{2}} \rightarrow C$$

put C in A.

Because of $\le$ sign in C, equality in A will change to $\le$

$$T(n) \le \cancel{2}\left(c\frac{n}{\cancel{2}}\log\frac{n}{2}\right) + n$$

$$\le cn\log n - cn\log 2 + n$$

$$= [cn\log n] - \underbrace{cn} + \underbrace{n}$$

overall ans will be less than cn logn

$$\le cn\log n$$

So, for $n_0 = 2$ and any $c \geq 2$

$T(n) \le cn\log n$

So, $T(n) = O(n\log n)$.