# Back Substitution or Iteration Method:-

**Example 1:**

$$T(n) = 1 + T(n-1) \rightarrow ①$$

$$T(n-1) = 1 + T(n-2) \rightarrow ②$$

$$T(n-2) = 1 + T(n-3) \rightarrow ③$$

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fun(n)
{ if (n>1)
     return fun(n-1)
}.
```

$T(n) = 1 + T(n-1)$ ; $n>1$

Base case $n = 1$

$$T(n) = \begin{cases} 1 + T(n-1) & ; n>1 \\ 1 & ; n=1 \end{cases}$$

putting ② in ①.

$$T(n) = 1 + 1 + T(n-2)$$

$$= 2 + T(n-2) \rightarrow ④$$

putting ③ in ④.

$$T(n) = 2 + 1 + T(n-3)$$

$$T(n) = 3 + T(n-3)$$

⋮

$$= k + T(n-k)$$

I want to go upto $T(1)$ cuz its my base case.

$$n - k = 1 \Rightarrow k = n-1.$$

$$= n-1 + T(n-(n-1))$$

$$= n-1 + T(1)$$

$$= n-1+1$$

$$\boxed{T(n) = n}$$

1 EXAMPLE

fun(n) $\longrightarrow$ $T(n)$

{ if $n > 0$ $\longrightarrow$ const

for (i=1; i<n; i=i×2)

print ~~something~~ (i) $\longrightarrow$ $O(\log n)$

fun(n-1) $\longrightarrow$ $T(n-1)$

}.

$T(n) = T(n-1) + \log n \quad n > 0$

$T(n) = 1 \quad n = 0$

$\log n$
$T(n-1)$

$\log n$
$\log(n-1)$
$T(n-2)$

$\log n$
$\log(n-1)$
$\log(n-2)$
$T(n-3)$

$\log n$
$\log(n-1)$
$\log(n-2)$
$\vdots$
$T(0)$

cost at each level

$\log n$

$\log(n-1)$

$\log(n-2)$

Total work =

$\log n + \log(n-1) + \log(n-2) + \cdots + \log 2 + \log 1$

$\log(ab) = \log(a) + \log(b)$

So, $\log[n \times (n-1) \times (n-2) \times (n-3) \cdots 2 \times 1]$

$\log[n!]$

$\log[n!] \leq \log[n^n]$

$\leq n \log n$

What is upper bound of n!?

$n \times (n-1) \times (n-2) \times (n-3) \cdots 2 \times 1 \leq n \times n \times n \cdots \times n$

$n! \leq n^n$

By substitution Method

$T(n) = T(n-1) + \log n \rightarrow ①$

$T(n-1) = T(n-2) + \log(n-1) \rightarrow ②$.

$T(n-2) = T(n-3) + \log(n-2) \rightarrow ③$

put ② in ①.

$T(n) = T(n-2) + \log(n-1) + \log n \rightarrow ④$

put ③ in ④

$T(n) = T(n-3) + \log(n-2) + \log(n-1) + \log(n)$

⋮

$T(n) = T(n-k) + \overset{\log(n-(k-1))}{\log(n-k+1)} + \overset{\log(n-(k-2))}{\log(n-k+2)} + \dots + \log(n-2) + \log(n-1) + \log n$

we need $T(0)$ for base condition.

$n-k=0$.

$n=k$.

$T(n) = T(0) + \log(n-n+1) + \log(n-n+2) + \dots + \log(n-1) + \log(n)$

$= 1 + \log(1) + \log(2) + \log(3) \dots + \log(n)$

$= 1 + \log[1 \times 2 \times 3 \times \dots \times (n-1) \times (n)]$

$= 1 + \log[n!]$

$\leq 1 + \log[n^n]$

$= 1 + n\log n$

$= O(n \log n)$