

Counting Inversions

Inversion

$$A[i] > A[j]$$

where $i < j$

Example

7, 6, 3, 8

for 7

2 inversions

for 6

1 inversion

for 3

0 inversions

for 8

0 inversions as there are no elements to its right.

Total inversions = 3

Divide Step : separate list into 2 pieces

Conquer : recursively count inversions in each half.

Combine : Count inversions where a_i and a_j are in different halves and return sum of three quantities

- * Assume each half is sorted
- * count inversions where a_i & a_j are in different halves
- * Merge 2 sorted halves into sorted whole
(to maintain sorted invariant)

MS-In (A, p, r)

{ if $n = 1$
return 0

else

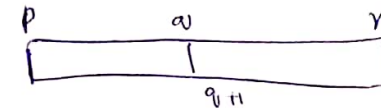
$X = \text{MS-In}(A, p, q)$

$Y = \text{MS-In}(A, q+1, r)$

$Z = \text{M-In}(A, p, q, r)$

Inversions = $X + Y + Z$

return inversions }



Merge-Inversions (A, p, q, r)

{ $n_1 = q - p + 1$

$n_2 = r - q$

$L[1 \dots n_1]$

$R[1 \dots n_2]$

for $i = 1 \rightarrow n_1$
 $L[i] = A[p+i-1]$

for $j = 1 \rightarrow n_2$
 $R[j] = A[q+j]$

$i = 1, j = 1$

for $k = p$ to r

{ if $L[i] \leq R[j]$
 $A[k] = L[i], i++$

else

$A[k] = R[j], j++$

inversions += $\text{len}(L) - i + 1$ }

return inversions

