تاریخ: ___/___/___ M T W T F S

Assignment 1

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Course :-

Design & Analysis of Algorithm

Topic :-

Recurrence using Master Theorem

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Batch :- 22 - Evening

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Assignment 1

★ Solve the following recurrence using Master Theorem.

1- $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

Sol.

Put the values of "a" and "b" in the given formula.

∴ Formula :-

$n^{\log_b^a}$

$a = 3$, $b = 2$, $f(n) = n^2$

$= n^{\log_2^3}$

$= n^{1.5}$

∴ Now Compare $f(n)$ and $n^{\log_b^a}$

$n^2 > n^{1.5}$

Case 3 applied.

$f(n) = \Omega\left(n^{\log_b^{a+\epsilon}}\right)$

$a.\left[f\left(\frac{n}{b}\right)\right] \leq c.f(n)$, $c > 1$

$3.\frac{n^2}{4} \leq cn^2$

$T(n) = \Theta(f(n)) = \Theta(n^2)$

تاریخ: ____/____/____

M T W T F S

2- $T(n) = 16T\left(\frac{n}{4}\right) + n^2$

$a = 16, \quad b = 4, \quad f(n) = n^2$

$\therefore \quad n^{\log_b^a}$

$= \quad n^{\log_4^{16}}$

$= \quad n^2$

$\therefore$ Compare $f(n)$ and $n^{\log_b^a}$

$n^2 = n^2$

$\Rightarrow$ Case 2 applied

$T(n) = \Theta\left(n^{\log_b^a} \log n\right)$

$= \Theta\left(n^2 \log n\right)$

3- $T(n) = T\left(\frac{2n}{5}\right) + n$

~~a = 1~~

$T(n) = T\left(\frac{n}{\frac{5}{2}}\right) + n$

$a = 1, \quad b = \frac{5}{2}, \quad f(n) = n$

$\therefore \quad n^{\log_{5/2}^1}$

$= n^0$

$\Rightarrow$ Compare $f(n)$ and $n^{\log_b^a}$

$n > n^0$

تاریخ: ___/___/___ M T W T F S

Case 3 will be applied.

$$a.f\left(\frac{n}{b}\right) \le c.f(n)$$

$$1.\left(\frac{2n}{5}\right) \le cn$$

$$T(n) = \theta(f(n))$$

$$= \theta(n)$$

4- $T(n) = 3T\left(\frac{n}{2}\right) + n$

$a = 3, \; b = 2, \; f(n) = n$

$\therefore \; n^{\log_b^a}$

$= n^{\log_2^3}$

$= n^{1.5}$

Compare $f(n)$ and $n^{\log_b^a}$

$n < n^{1.5}$

⇒ Case 1 applied

$$T(n) = \theta(n^{\log_b^a})$$

$$T(n) = \theta(n^{1.5}) \approx \theta(n)$$

$$T(n) = \theta(n)$$

5- $T(n) = 3T\left(\frac{n}{3}\right) + 1$

$T(n) = 3T\left(\frac{n}{3}\right) + n^0$

$a = 3, \; b = 3, \; f(n) = n^0$

$= n^{\log_3^3}$

$= n^1$

$\therefore$ Compare $f(n)$ and $n^{\log_b^a}$

$n^0 < n$

$\Rightarrow$ Case 1 applied

$T(n) = \Theta(n^{\log_b^a})$

$T(n) = \Theta(n)$

6- $T(n) = 16T\left(\frac{n}{4}\right) + n$

$a = 16, \; b = 4, \; f(n) = n$

$\therefore$ $n^{\log_b^a}$

$= n^{\log_4^{16}}$

$= n^2$

$\Rightarrow$ Compare $f(n)$ and $n^{\log_b^a}$

$n < n^2$

$\Rightarrow$ Case 1 applied

$T(n) = \Theta(n^{\log_b^a})$

$T(n) = \Theta(n^2)$

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7- $T(n) = T\left(\frac{n}{2}\right) + n$

$a = 1, \quad b = 2, \quad f(n) = n$

$\therefore \quad n^{\log_b^a}$

$= n^{\log_2^1}$

$= n^0$

$\Rightarrow$ Compare $f(n)$ and $n^{\log_b^a}$

$n > n^0$

$\Rightarrow$ Case 3 applied

$a.f\left(\frac{n}{b}\right) \le c.f(n)$

$1.\frac{n}{2} \le c.n$

$T(n) = \theta(f(n))$

$T(n) = \theta(n)$

8- $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2, \quad b = 4, \quad f(n) = n^{0.51}$

$\therefore \quad n^{\log_b^a}$

$= n^{\log_4^2}$

$= n^{0.5}$

$\Rightarrow$ Compare $f(n)$ and $n^{\log_b^a}$

$n^{0.51} > n^{0.5}$

$\Rightarrow$ Case 3 applied

$$a \cdot f\left(\frac{n}{b}\right) \le c \cdot f(n)$$

$$2 \cdot \left(\frac{n}{4}\right)^{0.51} \le c \cdot n^{0.51}$$

$$T(n) = \Theta(f(n))$$

$$T(n) = \Theta(n^{0.51})$$

9- $T(n) = 4T\left(\frac{n}{2}\right) + n$

$a = 4, \quad b = 2, \quad f(n) = n$

$\Rightarrow n^{\log_b^a}$

$= n^{\log_2^4}$

$= n^2$

$\Rightarrow$ Compare $f(n)$ and $n^{\log_b^a}$

$$n < n^2$$

$\Rightarrow$ Case 1 applied

$$T(n) = \Theta(n^{\log_b^a})$$

$$T(n) = \Theta(n^2)$$

تاریخ: ___/___/___ M T W T F S

10- $T(n) = 3T\left(\frac{n}{3}\right) + n$

$a = 3, \quad b = 3, \quad f(n) = n$

$\therefore \; n^{\log_b^a}$

$= n^{\log_3^3}$

$= n^1$

$\Rightarrow$ Compare $f(n)$ with $n^{\log_b^a}$

$n = n^1$

$\Rightarrow$ Case 2

$T(n) = \Theta(n^{\log_b^a} \lg n)$

$T(n) = \underline{\Theta(n \log n)}$

11) $T(n) = nT\left(\frac{n}{2}\right) + n$

$a = n, \quad b = 2, \quad f(n) = n$

$\therefore \; n^{\log_b^a}$

$= n^{\log_2^n}$

$\therefore$ Compare $f(n)$ and $n^{\log_b^a}$

$n < n^{\log_2^n}$

$\Rightarrow$ Case 1 applied

$T(n) = \Theta(n^{\log_2^n})$

12- $T(n) = 2T\left(\frac{n}{2}\right) + 2^n$

$a = 2, \quad b = 2, \quad f(n) = 2^n$

$\therefore \quad T(n) = n^{\log_b^a}$

$= n^{\log_2^2}$

$= n$

$\therefore$ Compare $f(n)$ with $n^{\log_b^a}$

$2^n > n$

Case 3 applied

$a.f\left(\frac{n}{b}\right) \leq c f(n)$

$2.\left(\frac{n}{2}\right) \leq c 2^n$

$2.2^{\frac{n}{2}} \leq c2^n$

$2^{1+\frac{n}{2}} \leq c2^n$

$2^{\frac{n+2}{2}} \leq c2^n$

$T(n) = \theta(f(n))$

$T(n) = \theta(2^n)$

13- $T(n) = 4T\left(\frac{n}{2}\right) + \lg n$

$a = 4, \quad b = 2, \quad f(n) = \lg n$

$T(n) = n^{\log_b^a}$

$= n^{\log_2^4}$

$= n^2$

Compare $f(n)$ and $n^{\log_b^a}$

$\lg n < n^2$

Case 1 applied

$T(n) = \theta(n^{\log_b^a})$

$T(n) = \theta(n^2)$

14- $T(n) = 4T\left(\frac{n}{2}\right) + \frac{n}{\lg n}$

$a = 4, b = 2, f(n) = \frac{n}{\lg n}$

$T(n) = n^{\log_b^a}$

$= n^{\log_2^4}$

$= n^2$

Compare $f(n)$ and $n^{\log_b^a}$

$\frac{n}{\log n} < n^2$

Case 1 applied

$T(n) = \theta(n^{\log_b^a})$

$T(n) = \theta(n^2)$

15- $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

16- $T(n) = 4T\left(\frac{n}{2}\right) + n\log n$

$a = 4$, $b = 2$, $f(n) = n\log n$

$T(n) = n^{\log_b^a}$

$= n^{\log_2^4}$

$= n^2$

Compare $f(n)$ and $n^{\log_b^a}$

$n\log n < n^2$

Case 1 applied

$T(n) = \theta(n^{\log_b^a})$

$T(n) = \theta(n^2)$

17- $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$

$a = 6$, $b = 3$, $f(n) = n^2 \lg n$

$T(n) = n^{\log_b^a}$

$T(n) = n^{\log_3^6}$

$= n^{1.6}$

Compare $f(n)$ and $n^{\log_b^a}$

$n^2 \log n > n^{1.6}$

Case 3 applied

$T(n) = \theta(f(n))$

$T(n) = \theta(n^2 \lg n)$

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18- $T(n) = 64T\left(\frac{n}{8}\right) + n^2 \log n$

$a = 64$, $b = 8$, $f(n) = n^2 \log n$

$T(n) = n^{\log_b^a}$

$= n^{\log_8^{64}}$

$= n^2$.

Compare $f(n)$ and $n^{\log_b^a}$

$n^2 \lg n > n^2$

Case 3 applied

$T(n) = \theta(f(n))$

$T(n) = \theta(n^2 \lg n)$

19- $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$a = 4$, $b = 2$, $f(n) = n^2$

$T(n) = n^{\log_b^a}$

$= n^{\log_2^4}$

$= n^2$

Compare $f(n)$ and $n^{\log_b^a}$

$n^2 = n^2$

Case 2 applied

$T(n) = \theta(n^{\log_b^a} \lg n)$

$T(n) = \theta(n^2 \lg n)$

20- $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7, \quad b = 3, \quad f(n) = n^2$

$T(n) = n^{\log_b^a}$

$= n^{\log_3^7}$

$= n^{1.77}$

Compare $f(n)$ and $n^{\log_b^a}$

$n^2 > n^{1.77}$

Case 3 applied

$a \cdot f\left(\frac{n}{b}\right) \leq c f(n)$

$7 \cdot f\left(\frac{n}{3}\right) \leq c f(n)$

$7 \cdot \frac{n^2}{9} \leq c \cdot n^2$

So,

$T(n) = \theta(f(n))$

$T(n) = \theta(n^2)$

21- $T(n) = 3T\left(\frac{n}{4}\right) + n\lg n$

$a = 3, \quad b = 4, \quad f(n) = n\lg n$

$T(n) = n^{\log_b^a}$

$= n^{\log_4^3}$

$= n^{0.79}$

تاریخ: ___/___/___ M T W T F S

Compare $f(n)$ and $n^{\log_b^a}$

$$n \lg n > n^{0.79}$$

Case 3 applied

$$T(n) = \Theta(f(n))$$
$$T(n) = \Theta(n \lg n)$$

22- $T(n) = 2T\left(\frac{n}{2}\right) + n \log n$

$$a = 2 \quad , b = 2 \quad , f(n) = n \log n$$

$$T(n) = n^{\log_b^a}$$
$$= n^{\log_2^2}$$
$$= n^1$$
$$= n$$

Compare $f(n)$ and $n^{\log_b^a}$

$$n \log n > n$$

Case 3 applied

$$T(n) = \Theta(f(n))$$
$$T(n) = \Theta(n \log n)$$