DISCRETE STRUCTURE ASSIGNMENT 2

GROUP 8

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SECI1013: DISCRETE STRUCTURES

SESSION 2025/2026 – SEMESTER 1

ASSIGNMENT 2 (CHAPTER 2 – RELATION, FUNCTION & RECURRENCE

INSTRUCTIONS:

a. This assignment must be conducted in a group. Please clearly write the group members' names & matric numbers on the front page of the submission.
b. Solutions for each question must be readable and neatly written on plain A4 paper. Every step or calculation should be properly shown. Failure to do so will result in the rejection of the submission of the assignment.
c. This assignment consist of 7 questions (60 marks), contributing 5% of overall course marks.

Question 1 **[9 marks]**

Let D = {1,3,5}. Define R on D where $x, y \in D, xRy$ if $3x + y$ is a multiple of 6.

i) Find the element of R.
ii) Determine the domain and range of R.
iii) Draw the digraph of the relation
iv) Determine whether the relation R is assymetric?

Question 2 **[8 marks]**

Suppose R is an equivalence relation on the set $A=\{x,y,z\}$. $(x,y) \in$ R and $(y,z) \in$ R. List all possible member of R and justify your answer.

Question 3 **[15 marks]**

Let B = {u, v, w, y} and R={(u,u), (u,w), (v,v), (v,w), (w,w), (w,y), (y,u), (y,v), (y,y)}

i) Construct the matrix of relation, M_R for the relation R on B
ii) List in-degrees and out-degrees of all vertices.
iii) Determine whether the relation R on the set B is a partial order relation. Check all variance Justify for answer.

Question 1 **[9 marks]**

Let *D* = {1,3,5}. Define R on *D* where $x, y \in D, xRy$ if 3*x* + *y* is a multiple of 6.

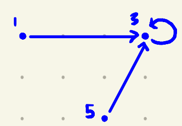
i) Find the element of R.
ii) Determine the domain and range of R.
iii) Draw the digraph of the relation
iv) Determine whether the relation R is assymetric?

i) $D = \{(1,3), (3,3), (5,3)\}$

ii) Domain = $\{1,3,5\}$

Range = $\{3\}$

iii)



iv) No, as $(3,3) \in R$, assymmetric said that relation M_{ii} must be 0 and if $M_{ij} = 1$, then $M_{ji} = 0$.

Question 2 **[8 marks]**

Suppose R is an equivalence relation on the set *A*={*x*,*y*,*z*}. (*x*,*y*) ∈ R and (*y*,*z*) ∈ R. List all possible member of R and justify your answer.

$R = \{(x,y), (y,x), (x,z), (z,x), (y,z), (z,y)$
$(x,x), (y,y), (z,z)\}$

$(x,x), (y,y), (z,z) \in R$ reflexive

$$M_A = \begin{array}{c|c} & x \quad y \quad z \\ \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array} = M_A^T \begin{array}{c|c} & x \quad y \quad z \\ \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

symmetric

It is equivalence set

$$\begin{array}{c|c} & x \quad y \quad z \\ \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array} \odot \begin{array}{c|c} & x \quad y \quad z \\ \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{c|c} & x \quad y \quad z \\ \begin{matrix} x \\ y \\ z \end{matrix} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

transitive

R is an equivalence relation because it is reflexive, symmetric and trans

Reflexive : (x,x) ∈ R, (y,y) ∈ R, (z,z) ∈ R

Transitive : (x,y) ∈ R, (y,z) ∈ R, thus (x,z) ∈ R

Symmetric : (x,y) ∈ R thus (y,x) ∈ R

Question 3 **[15 marks]**

Let B = {*u*, *v*, *w*, *y*} and R={(*u*,*u*), (*u*,*w*), (*v*,*v*), (*v*,*w*), (*w*,*w*), (*w*,*y*), (*y*,*u*), (*y*,*v*), (*y*,*y*)}

i) Construct the matrix of relation, M_R for the relation R on B

ii) List in-degrees and out-degrees of all vertices.

iii) Determine whether the relation R on the set B is a partial order relation. Check all variance Justify for answer.

i)
$$M_R = \begin{array}{c} \\ u \\ v \\ w \\ y \end{array} \begin{array}{c} \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

ii)

in	out
$u_{in} = 2$	$u_{out} = 2$
$v_{in} = 2$	$v_{out} = 2$
$w_{in} = 3$	$w_{out} = 2$
$y_{in} = 2$	$y_{out} = 3$

iii) $(u,u), (v,v), (w,w), (y,y) \in R$ reflexive

$$M_R = \begin{array}{c} \\ u \\ v \\ w \\ y \end{array} \begin{array}{c} \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$(u,w) \in R$, $(w,u) \notin R$

$(v,w) \in R$, $(w,v) \notin R$

$(w,y) \in R$; $(y,w) \notin R$

$(y,u) \in R$, $(u,y) \notin R$

$(y,v) \in R$; $(v,y) \notin R$

Antisymmetric

$$\begin{array}{c} \\ u \\ v \\ w \\ y \end{array} \begin{array}{c} \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array} \otimes \begin{array}{c} \\ u \\ v \\ w \\ y \end{array} \begin{array}{c} \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array} = \begin{array}{c} \\ u \\ v \\ w \\ y \end{array} \begin{array}{c} \begin{array}{cccc} u & v & w & y \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$M_R \times M_R \neq M_n$

$(m_{uy} = 0) \wedge (n_{uy} = 1)$

Not transitive

Not partial order relation.

Question 4 **[6 marks]**

Let

$$f: [1, \infty) \to [0, \infty), f(x) = (x-1)^2.$$

Determine whether the function f is **one–one**, **onto**, or **bijective**.
Show full working and justify your answer.

Let $f(x_1) = f(x_2)$,

$(x_1 - 1)^2 = (x_2 - 1)^2$

$(x_1 - 1) = (x_2 - 1)$

$x_1 = x_2$

It is one to one

Let $(x-1)^2 = \text{any } y$

$x - 1 = \sqrt{y}$

$x = 1 + \sqrt{y}$

since x is $[1, \infty)$
so it is onto
as $x = 1 + \sqrt{y}$

It is bijective as both onto and one to one achieved.

Question 5 **[9 marks]**

Let f and g be functions from the positive integers to the positive integers defined by

$$f(x) = 9x + 4, g(x) = \frac{3}{2}x - 1.$$

a) Find the inverse of $g(x)$.

b) Find the composition $(g \circ f)(x)$.

c) Find the composition $(f \circ g)(x)$.

d) Find the composition $(f \circ g \circ g)(x)$.

a) $y = \frac{3}{2}x - 1$

$y + 1 = \frac{3}{2}x$

$x = \frac{2y+2}{3}$

$g^{-1}(x) = \frac{2x+2}{3}$

b) $(g \circ f)(x) = g(9x+4)$

$= \frac{3}{2}(9x+4) - 1$

$= \frac{27}{2}x + 6 - 1$

$= \frac{27}{2}x + 5$

c) $(f \circ g)(x) = f(\frac{3}{2}x - 1)$

$= 9(\frac{3}{2}x - 1) + 4$

$= \frac{27}{2}x - 9 + 4$

$= \frac{27}{2}x - 5$

d) $(f \circ g \circ g)(x) = f \circ g(\frac{3}{2}x - 1)$

$= f(\frac{3}{2}(\frac{3}{2}x - 1) - 1)$

$= f(\frac{9}{4}x - \frac{3}{2} - 1)$

$= f(\frac{9}{4}x - \frac{5}{2})$

$= 9(\frac{9}{4}x - \frac{5}{2}) + 4$

$= \frac{81}{4}x - \frac{45}{2} + 4$

$= \frac{81}{4}x - \frac{37}{2}$

Question 6 **[6 marks]**

In a reactor, two intermediates mix to form product P. The initial temperatures are $P_0 = 4.0^\circ F$ and $P_1 = 5.0^\circ F$. Engineers observe that, for $t \geq 2$minutes, the update rule is:

"The new temperature is the previous temperature plus one-quarter of the temperature two minutes ago."

a) Write the recurrence relation that models this.

b) Using your recurrence, list $P_0, P_1, \dots, P_5$ (exact values preferred).

a) $P_n = P_{n-1} + 1/4\, P_{n-2}$, $n \geq 2$ with $P_0 = 4.0$ and $P_1 = 5.0$

b) $P_0 = 4.0$

$P_1 = 5.0$

$P_2 = P_1 + 1/4\, P_0$

$= 5.0 + 1/4(4.0)$

$= 6.0^\circ F$

$P_3 = P_2 + 1/4\, P_1$

$= 6.0 + 1/4(5.0)$

$= 7.25^\circ F$

$P_4 = P_3 + 1/4\, P_2$

$= 7.25 + 1/4(6.0)$

$= 8.75^\circ F$

$P_5 = P_4 + 1/4\, P_3$

$= 8.75 + 1/4(7.25)$

$= 10.5625^\circ F$

Question 7 **[7 marks]**

Given the recurrence relation below,

$$s_1 = 2, s_n = s_{n-1}^2 - 1 \text{ for } n \geq 2.$$

a) Write a recursive algorithm to calculate the n^{th} term of the sequence

b) Trace the recursive steps to compute s_4. Show your working in a diagram.

a)
```
s(n)
{ if(n=1)
      return 2;
  else
  return s²(n-1) - 1;
}
```

b) s(4)

return $s^2(3) - 1$ return $8^2 - 1 = 63$

s(3)

return $s^2(2) - 1$ return $3^2 - 1 = 8$

s(2)

return $s^2(1) - 1$ return $2^2 - 1 = 3$

s(1)

return 2 return 2

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