Exercise #1

Two fair dice are rolled, one red and one blue, and sum of the number showing face up is computed. How may ways that

a) The sum is 4 or 11

b) At least one dice shows the number 3

c) The red dice shows the number 3

d) None of the outcome of the dice shows the number 3

a) $4 = (1,3),(2,2),(3,1) \Rightarrow 3$ ways

$11 = (6,5),(5,6) \Rightarrow 2$ ways

total 5 ways

b) red $3 = (3,1) \rightarrow (3,6) \Rightarrow 6$ ways

blue $3 = (1,3) \rightarrow 6,3) \Rightarrow 6$ ways

since $(3,3)$ is on both, we can only can count once

$\Rightarrow 12 - 1 = 11$ ways #

c) 6 ways

d) total possible outcome $= 6 \times 6 = 36$

outcome with $3 = 11$

$36 - 11 = 25$ ways #

Exercise # 2

a) How many 8-bit string that has bit 1 only one
b) How many 8-bit string that has bit 1 at least one
c) how many 8-bit string that begins and ends with bit-1
d) How many eight-bit strings have either the second or the fourth bit 1(or both)?

a) 1 _ _ _ _ _ _ _ = 1 × 8 = 8 ways

b) _ _ _ _ _ _ _ _ = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2
= 256 ⇒ total possible outcome for 8-bit

strings with zero 1's ⇒ only 1 (0 0 0 0 0 0 0 0)

⇒ 256 − 1 = 255 #

c) 1 _ _ _ _ _ _ 1 ⇒ $2^6 = 64$

d) A : _ 1 _ _ _ _ _ _ ⇒ $2^7 = 128$

B : _ _ _ 1 _ _ _ _ ⇒ $2^7 = 128$

C : _ 1 _ 1 _ _ _ _ ⇒ $2^6 = 64$

A ∨ B = 128 + 128 − 64 = 192 #

OR

A : _ 1 _ 0 _ _ _ _ ⇒ $2^6 = 64$

B : _ 0 _ 1 _ _ _ _ ⇒ $2^6 = 64$

C : _ 1 _ 1 _ _ _ _ ⇒ $2^6 = 64$

64 + 64 + 64 = 192

Exercise # 3

- How many license plates of 2 letters from A to Z, followed by 3 digits from 0 to 9 can be made, if repetition of letters is not allowed?

A − Z = 26 ⇒ $\overline{{}^{26}P_1} \times \overline{{}^{25}P_1}$ ⇒ 650

0 − 9 = 10 ⇒ $\overline{{}^{10}P_1}\ \overline{{}^{10}P_1}\ \overline{{}^{10}P_1}$ ⇒ 1000

650 × 1000 ⇒ 650 000

Exercise #4

- In how many ways can we select two books from different subjects among five distinct computer science books, three distinct mathematics books, and two distinct art books?

total books = 5 + 3 + 2 = 10
(cs, m, a)

cs & m ⇒ ${}^5C_1 \times {}^3C_1 = 15$
+
cs & a ⇒ ${}^5C_1 \times {}^2C_1 = 10$
+
a & m ⇒ ${}^3C_1 \times {}^2C_1 = 6$

total = 31

Exercise #5

Given three sets of integers; **A**={1, 3, 5}, **B** = {4, 6} and **C** = {0, 2,7,9}. How many ways are there to choose one integer from set **A**, **B**, or **C**?

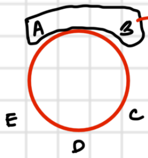
total = 3 + 2 + 4 = 9

$^{9}C_{1} = 9$ #

Exercise #6

- In how many ways can five people A, B, C, D, and E be seated around a circular table if
  - a) A and B must sit next to each other
  - b) A and B must not sit next to each other
  - c) A and B must be together and CD must be together

a)



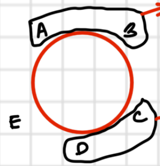
$^2P_2 \times {}^4P_4 = 48$

$= \frac{48}{4} = 12$ #

b) total possible ways $= \frac{^5P_5}{5} = 24$

$24 - 12 = 12$ #

c)



$^2P_2$

$^2P_2$

$\Rightarrow {}^2P_2 \times {}^2P_2 \times {}^3P_3 = 24$

$\frac{24}{3} = 8$ #

Exercise #7

1. How many words, with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if
    i) 4 letters are used at a time ,
    ii) all letters are used at a time
    iii) All letters are used but first letter is vowel

2. One hundred twelve people bought raffle tickets to enter a random drawing for three prizes. How many ways can three names be drawn for first prize, second and third prize?

3. In how many ways can the letter of the word 'JUDGE' be arranged such that the vowels always come together?

1. MONDAY → 6 letters

i) $^{6}P_{4} = 360$

ii) $^{6}P_{6} = 720$

iii) $\underset{^{2}P_{1}}{\_}\ \underset{^{5}P_{1}}{\_}\ \underset{^{4}P_{1}}{\_}\ \underset{^{3}P_{1}}{\_}\ \underset{^{2}P_{1}}{\_}\ \underset{^{1}P_{1}}{\_}$ ⇒ $^{2}P_{1} \times {}^{5}P_{1} \times {}^{4}P_{1} \times {}^{3}P_{1} \times {}^{2}P_{1} \times {}^{1}P_{1}$

= 240 ways#

2. 112 people for 3 prize

$^{112}P_{3} = 1367520$ ways#

3) JUDGE → $\underbrace{\overbrace{U\ E}^{^{2}P_{2}}\ \_\ \_\ \_}_{^{4}P_{4}}$

⇒ $^{2}P_{2} \times {}^{4}P_{4}$

= 48 #