Question 1 **[9 marks]**

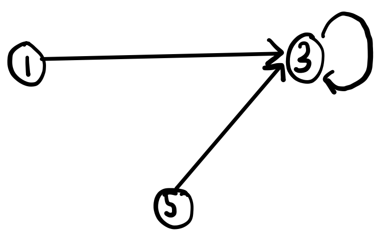
Let $D = \{1,3,5\}$. Define R on D where $x, y \in D, xRy$ if $3x + y$ is a multiple of 6.

i) Find the element of R.
ii) Determine the domain and range of R.
iii) Draw the digraph of the relation
iv) Determine whether the relation R is assymetric?

i) $R = \{(1,3),(3,3),(5,3)\}$

ii) Domain $= \{1,3,5\}$
Range $= \{3\}$

(iii)



(iv). • Relation R is not irreflexive because $(3,3) \in R$.
• Relation R is anti symmetric because $(1,3) \in R$, while $(3,1) \notin R$. Then, $(5,3) \in R$, while $(3,5) \notin R$.
∴ Relation R is not assymetric because R is anti symmetric but not irreflexive

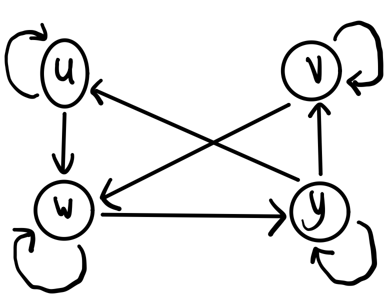
Question 3 **[15 marks]**

Let B = {u, v, w, y} and R={(u,u), (u,w), (v,v), (v,w), (w,w), (w,y), (y,u), (y,v), (y,y)}

i) Construct the matrix of relation, M_R for the relation R on B
ii) List in-degrees and out-degrees of all vertices.
iii) Determine whether the relation R on the set B is a partial order relation. Check all variance Justify for answer.

i) $M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

(ii)



For u,
in-degrees: 2
out-degrees: 2

For v,
in-degrees: 2
out-degrees: 2

For w,
in-degrees: 3
out-degrees: 2

For y,
in degrees : 2
out degrees : 3

iii) Determine whether the relation R on the set B is a partial order relation. Check all variance Justify for answer.

$R = \{(u,u),(u,w),(v,v),(v,w),(w,w),(w,y),(y,u),(y,v),(y,y)\}$

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

- R is reflexive because has 1's on the main diagonal.
  $\forall a \in B, (a,a) \in R.$
  - $(u,u) \in R$
  - $(v,v) \in R$
  - $(w,w) \in R$
  - $(y,y) \in R$

- R is antisymmetric, because
  $\forall a,b \in B, (a,b) \in R$, while $(b,a) \notin R$
  - $(u,w) \in R$, while $(w,u) \notin R$
  - $(v,w) \in R$, while $(w,v) \notin R$
  - $(w,y) \in R$, while $(y,w) \notin R$
  - $(y,u) \in R$, while $(u,y) \notin R$
  - $(y,v) \in R$, while $(v,y) \notin R$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$M_R \times M_R = N$

$M_R = [m_{ij}] \quad N = [n_{ij}]$

$\forall i \, \forall j$, if $(n_{ij} = 1)$, then $(m_{ij} = 0)$

$(n_{14} = 1) \land (m_{14} = 0)$

$(y,u) \in R$ and $(u,w) \in R$, but $(y,w) \notin R$

- Therefore, R is not transitive.

Question 2 **[8 marks]**

Suppose R is an equivalence relation on the set $A=\{x,y,z\}$. $(x,y) \in R$ and $(y,z) \in R$. List all possible member of R and justify your answer.

Equivalence relation = reflexive, symmetric, transitive

$R = \{ (x,x), (y,y), (z,z), (x,y), (y,x), (x,z), (z,x), (y,z), (z,y) \}$

= shows reflexive

$(x,y), (y,x) \in R$
$(x,z), (z,x) \in R$
$(y,z), (z,y) \in R$
$\Rightarrow$ shows symmetric

$(x,y), (y,x) \in R, (x,x) \in R$
$(y,x), (x,z) \in R, (y,z) \in R$
$(x,z), (z,x) \in R, (x,x) \in R$
$(y,z), (z,y) \in R, (y,y) \in R$
$\Rightarrow$ shows transitive

Question 4 **[6 marks]**

Let

$$f: \underset{\text{Domain}}{[1,\infty)} \to \underset{\text{Range}}{[0,\infty)}, f(x) = (x-1)^2.$$

Determine whether the function f is **one–one**, **onto**, or **bijective**.
Show full working and justify your answer.

$f(x) = (x-1)^2 \Rightarrow f = \{ (\underset{x}{x}, \underset{y}{(x-1)^2}) \}$

assume, $f(x_1) = f(x_2)$

$(x_1 - 1)^2 = (x_2 - 1)^2$

$\sqrt{(x_1 - 1)^2} = \sqrt{(x_2 - 1)^2}$

$(x_1 - 1) + 1 = (x_2 - 1) + 1$

$\boxed{x_1 = x_2}$

shows that f is one-to-one.

let $y = (x-1)^2$

$\sqrt{y} = x - 1$

$\sqrt{y} + 1 = x$ → since $x \geq 1$

$\sqrt{y} + 1 \geq 1$

$\sqrt{y} \geq 0$

$\therefore$ y is true for all positive number, thus, it is equal with the range $[0,\infty)$, making the function onto y.

$\therefore$ Function f is bijective because it is both one-to-one and onto.

(assume is continuous)

... onto and one to one
$f(x)$ is bijective function.

Question 5

(a) $g(x) = \frac{3}{2}x - 1$

If $g(x) = y$:

$y = \frac{3}{2}x - 1$

$\frac{2(y+1)}{3} = x$

$g^{-1}(x) = \frac{2(x+1)}{3}$

(b) $gf(x) = \frac{3}{2}(9x+4) - 1$

$gf(x) = \frac{27}{2}x + 5$

(c) $fg(x) = 9(\frac{3}{2}x - 1) + 4$

$fg(x) = \frac{27}{2}x - 5$

(d) $fgg(x) = \frac{27}{2}(\frac{3}{2}x - 1) - 5$

$= \frac{81}{4}x - \frac{37}{2}$

Question 6

(a) $P_t = P_{t-1} + \frac{1}{4}P_{t-2}$, $t \geq 2$. $P_0 = 4°F$, $P_1 = 5°F$

(b) $P_0 = 4°F$

$P_1 = 5°F$

$P_2 = 5 + \frac{1}{4}(4) = 6°F$

$P_3 = 6 + \frac{1}{4}(5) = 7.25°F$

$P_4 = 7.25 + \frac{1}{4}(6) = 8.75°F$

$P_5 = 8.75 + \frac{1}{4}(7.25) = 10.5625°F$

Question 7

(a)
```
input: n
output: s(n)
s(n) : {
   if (n==1)
      return 2;
   else
      return s(n-1)*s(n-1) - 1;
}
```

(b)

S(4)
n ≠ 1
return S(3)*S(3)-1

S(4)=63
8×8-1 = 63

S(3)
n≠1
return S(2)*S(2)-1

S(3) = 8
3×3-1 = 8

$\therefore S_4 = 63$

```
return S(n-1)*S(n-1)-1;
}
```

(b)

S(4)
$n \neq 1$
return S(3)*S(3)-1

S(4)=63
$8 \times 8 - 1 = 63$

S(3)
$n \neq 1$
return S(2)*S(2)-1

S(3) = 8
$3 \times 3 - 1 = 8$

S(2)
$n \neq 1$
return S(1)*S(1)-1

S(2) = 3
$2 \times 2 - 1 = 3$

S(1)
$n = 1$
return 2

S(1)=2
2

$\therefore S_4 = 63$