Exercise 1

- Write the relation R as $(x,y) \in R$

(a) The relation R on {1, 2, 3, 4} defined by $(x,y) \in R$ if $x^2 \geq y$.

(b) The relation R on {1,2,3,4,5} defined by $(x,y) \in R$ if 3 divides x-y.

$x^2 \geq y$

a) $R = \{ (1,1), (2,1), (2,2), (2,3), (2,4),$
$(3,1), (3,2), (3,3), (3,4), (4,1), (4,2),$
$(4,3), (4,4) \}$

b) $3 \mid (x-y)$

$R = \{ (1,1), (1,4), (2,2), (2,5), (3,3), (4,1), (4,4),$
$(5,2), (5,5) \}$

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Exercise 12

The relation *R* on the set {1,2,3,4,5} defined by the rule *(x,y)∈R* if *x+y≤6*

(i) List the elements of R
(ii) Find the domain of R
(iii) Find the range of R
(iv) Is the relation of R reflexive, symmetric, assymmetric, antisymmetric, transitive, and/or equivalence relation or partial order?

INSPIRING CREATIVE AND INNOVATIVE MINDS 13

$x + y \leq 6$

i) R = {(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)}

ii) Domain = {1, 2, 3, 4, 5}

iii) Range = {1, 2, 3, 4, 5}

iv)

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad M_R^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad M_R \otimes M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

1. = not all 0's or 1's ⇒ not reflexive
2. $M_R \neq M_R^T$ → symmetric
3. (1,2) ∈ R, (2,1) ∈ R ⇒ not antisymmetric
4. not antisymmetric → not assymmetric
5. (3,2), (2,4) ∈ R, (3,4) ∉ R ⇒ not transitive
6. not transitive ⇒ not equivalence & not partial order

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Exercise 13

The relation *R* on the set {1,2,3,4,5} defined by the rule *(x,y)∈R* if 3 divides *x-y*

(i) List the elements of R
(ii) Find the domain of R
(iii) Find the range of R
(iv) Is the relation of R refelxive, symmetric, assymmetric, antisymmetric, transitive, and/or equivalence relation or partial order?

INSPIRING CREATIVE AND INNOVATIVE MINDS 14

i) R = {(1,1), (1,4), (2,2), (2,5), (3,3), (4,1), (4,4), (5,2), (5,5)}

ii) Domain = {1, 2, 3, 4, 5}

iii) Range = {1, 2, 3, 4, 5}

iv)

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \quad M_R^T = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \quad M_R \otimes M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

1. = reflexive
2. $M_R = M_R^T$ → symmetric
3. (1,4) ∈ R, (4,1) ∈ R ⇒ not antisymmetric
4. not antisymmetric ⇒ not assymetri
5. $M_R \otimes M_R = M_R$ ⇒ transitive
6. not antisymmetric → not partial order
7. reflexive, symmetric, transitive ⇒ equivalence