1) $a_n = 6a_{n-1} - 9a_{n-2}$, where $a_0 = 2, a_1 = 3$

2) $a_n = 2a_{n-1} - a_{n-2}$, where $a_0 = 5, a_1 = 3$

1) $a_n = 6a_{n-1} - 9a_{n-2} \quad n \geq 2$

$a_2 = 6a_1 - 9a_0 = 6(3) - 9(2) = 0$

$a_3 = 6a_2 - 9a_1 = 6(0) - 9(3) = -27$

$a_4 = 6a_3 - 9a_2 = 6(-27) - 9(0) = -162$

$2, 3, 0, -27, -162$

2) $a_n = 2a_{n-1} - a_{n-2}, \quad n \geq 2$

$a_2 = 2a_1 - a_0 = 2(3) - 5 = 1$

$a_3 = 2a_2 - a_1 = 2(1) - 3 = -1$

$a_4 = 2a_3 - a_2 = 2(-1) - 1 = -3$

$5, 3, 1, -1, -3$

Exercise 2

A basketball is dropped onto the ground from a height of 15 feet. On each bounce, the ball reaches a maximum height 55% of its previous maximum height.

a)Write a recursive formula, a_n, that completely defines the height reached on the n_{th} bounce, where the first term in the sequence is the height reached on the ball's first bounce.

b)How high does the basketball reach after the 4_{th} bounce? Give your answer to two decimal places.

a)

$a_0 = 15$

$a_n = a_{n-1} \times \frac{55}{100}, \quad a_0 = 15, \quad n \geq 1$

$a_n = 0.55 a_{n-1}$

$a_1 = 0.55 a_0 = 0.55(15) = 8.25$

$a_2 = 0.55 a_1 = 0.55(0.55 a_0) = (0.55)^2 a_0$

$\Rightarrow a_n = (0.55)^n a_0$

b) $a_4 = (0.55)^4 a_0 = (0.55)^4 (15)$

$= 1.37 \text{ ft}$ #

Exercise 3

A grain elevator company receives 300 tons of corn per week from farmers once harvest starts. The elevator operators plan to ship out 40% of the corn on hand each week the harvest season begins.

i) If the company has 800 tons of corn on hand at the beginning of harvest, what reccurrence relation describes the amount of corn on hand at the end of each week throughout the harvest season?

ii) Using the reccurance relation obtain in (i), find the amount of corn on hand at the end of week 3.

i) 40% ⇒ shipped   60% ⇒ on hand
0.6

$$a_n = 0.6\,a_{n-1} + 300, \quad a_0 = 800, \quad n \geq 1$$

ii) $a_1 = 0.6a_0 + 300 = 0.6(800) + 300 = 780$

$a_2 = 0.6a_1 + 300 = 0.6(780) + 300 = 768$

$a_3 = 0.6a_2 + 300 = 0.6(768) + 300 = \boxed{760.8 \text{ tons}}$ #

Exercise 4

Determine s_5 if $s_0, s_1, s_2, \ldots$ is a sequence satisfying the given recurrence relation and initial conditions.

$$s_n = 2s_{n-1} + s_{n-2} - s_{n-3} \quad \text{for } n \geq 3,\ s_0 = 2,\ s_1 = -1,\ s_2 = 4$$

$$S_5 = 2S_4 + S_3 - S_2 = 2(16) + 5 - 4 = \boxed{31}$$

$$S_4 = 2S_3 + S_2 - S_1 = 2(5) + 4 - (-1) = 15$$

$$S_3 = 2S_2 + S_1 - S_0 = 2(4) + (-1) - 2 = 5$$

Exercise 5

A consumer purchased items costing RM280 with a department store credit card that charges 1.5% interest per month compounded monthly. Write a recurrence relation and initial condition for b_n, the balance of the consumer's account after n months if no further charges occur and the minimum monthly payment of RM25 is made.

$1.5\% = 0.015$

$b_0 = 280$

$$b_n = b_{n-1} + \underbrace{(0.015 \times b_{n-1})}_{\text{interest}} - 25$$

$$b_n = 1.015\, b_{n-1} - 25, \quad b_0 = 280, \quad n \geq 1$$

#

Exercise 6

- Given,

$$s_n = 3+6+9+.......+3n$$

$$s_1 = 3 \qquad s_n = s_{n-1} + 3n \quad \text{for } n \geq 2,$$

- Write a recursive algorithm to compute S_n, $n \geq 1$.

Algorithm

input : n, integer ≥ 1

output : f(n)

```
f(n) {
    if  n == 1
            return 3

    else
            return  f(n-1)+3*n
```

example :

n = 2

f(2) = 3 + 3(2)

= 9 #

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Exercise 7: Test 1 (2018/2019)

×3

1. Suppose that the number of bacteria in a colony triples every hour.
   a) Set up a recurrence relation for the number of bacteria after n hours have elapsed.
   b) If 10 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours?

2. Find a recurrence relation for the balance B(k) owed at the end of k months on a loan of RM9000 at rate of 8% if a payment of RM150 is made each month.
   [Hint: Express B(k) in terms of B(k-1); the monthly interest is (0.08/12)(B(k-1))]

3. Write a recursive algorithm for computing n^2 where n is a nonnegative integer using the fact that $(n+1)^2 = n^2 + 2n + 1$.

1.

a) let b = number of bacteria colony

$b_n = 3 b_{n-1}$

$b_1 = 3 b_0$

$b_2 = 3 b_1 = 3(3 b_0)$
$\quad = (3)^2 b_0$

$b_3 = 3 b_2 = 3((3)^2 b_0)$
$\quad = (3)^3 b_0$

$\Rightarrow b_n = 3^n b_0$ #

b) $b_0 = 10$

$b_{10} = 3^{10}(10)$
$\quad = 590490$ #

2. $b_0 = 9000$, $8\% = 0.08$

$b_k = b_{k-1} + \left(\frac{0.08}{12} \times b_{k-1}\right) - 150$

$b_k = b_{k-1}\left(1 + \frac{0.08}{12}\right) - 150$ #

3.

input : n, integer ≥ 0
output : n^2

```
f(n) {
    if n == 0
        return 0
    else
        return f(n-1) + 2n - 1
```

how got this?

$(n+1)^2 = n^2 + 2n + 1$

let $n^2 = f(n)$

$f(n+1) = f(n) + 2n + 1$

substitute n with n-1

$\Rightarrow f(n) = f(n-1) + 2(n-1) + 1$
$\quad = f(n-1) + 2n - 1$

Exercise 8

Write a recursive algorithm to find the n term of sequence defined by $a_1 = 2,\ a_2 = 3,$ and $a_n = a_{n-1}a_{n-2}$ for $n \geq 3$. Then, use the algorithm to trace a_7.

```
input : n
output : f(n)
  f(n){
    if n == 1
      return 2
    else if n == 2
      return 3
    else
      return a_{n-1} * a_{n-2} }
```

$a_7 = a_6 * a_5 = 209,952$

$a_6 = a_5 \times a_4 = 1944$

$a_5 = a_4 \times a_3 = 108$

$a_4 = a_3 \times a_2 = 6 \times 3 = 18$

$a_3 = a_2 \times a_1 = 3 \times 2 = 6$