Lecture 08

27/05/2021

11:30 pm

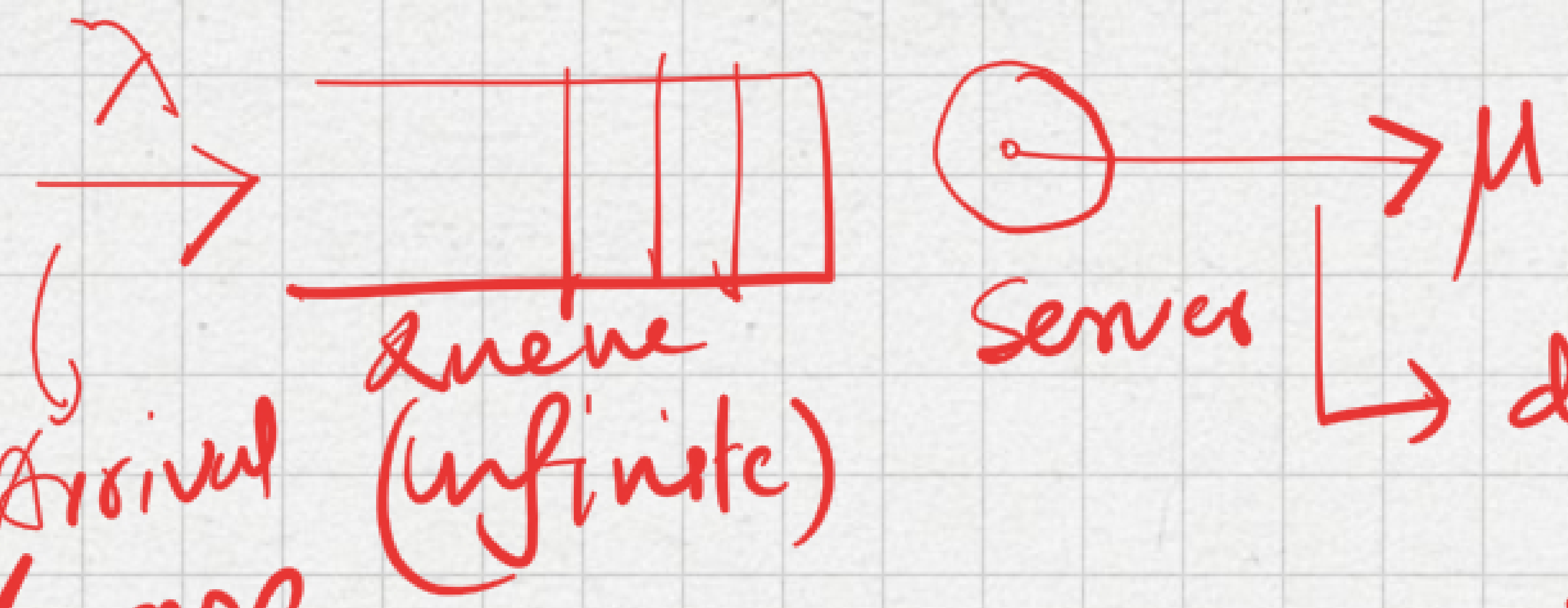
Simulation & Modeling

- ✓ Simulation Model
- ✓ Analytical Solution
- ✓ Results Presentation

$\boxed{\text{* Continuous-time Markov Chain}}$

~~Skip/Avoid~~

SS&S : Analytical solution



Arrival process

↳ Interarrival time

→ exponential

Memoryless

↳ departure

Poisson

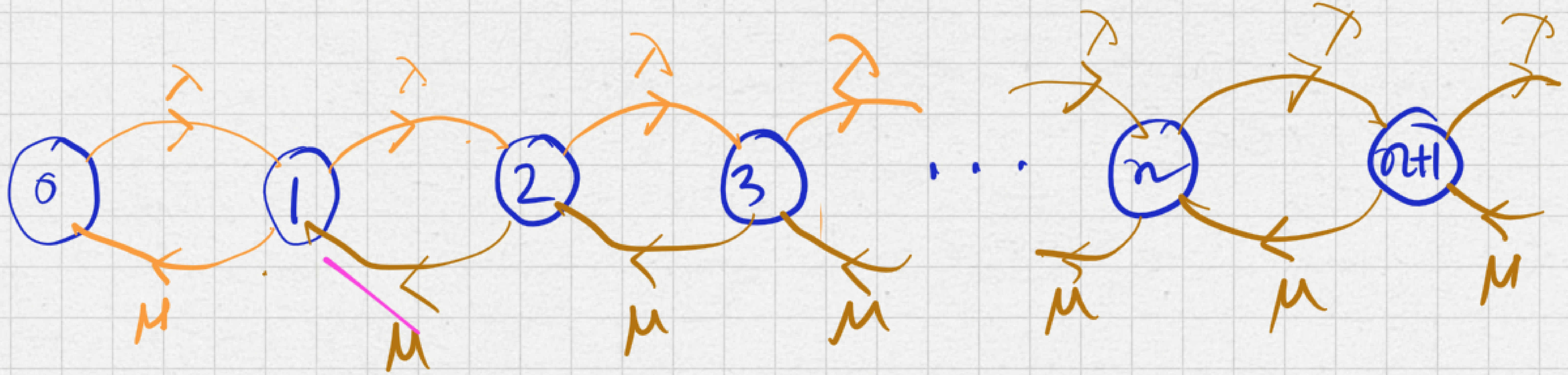
→ Inter departure time / service time

→ exponential

Memoryless

M/M/1 → # of server

Memoryless



state transition diagram

$\lambda \to$ arrival rate

$\mu \to$ service rate

1 hour in state 1 on an avg μ jobs departs

birth death process

birth rate depends on population

$\lambda_i \to$ arrival rate at state i

$\mu_i \to$ service rate " " "

state space

0, 1, 2, ...

state var

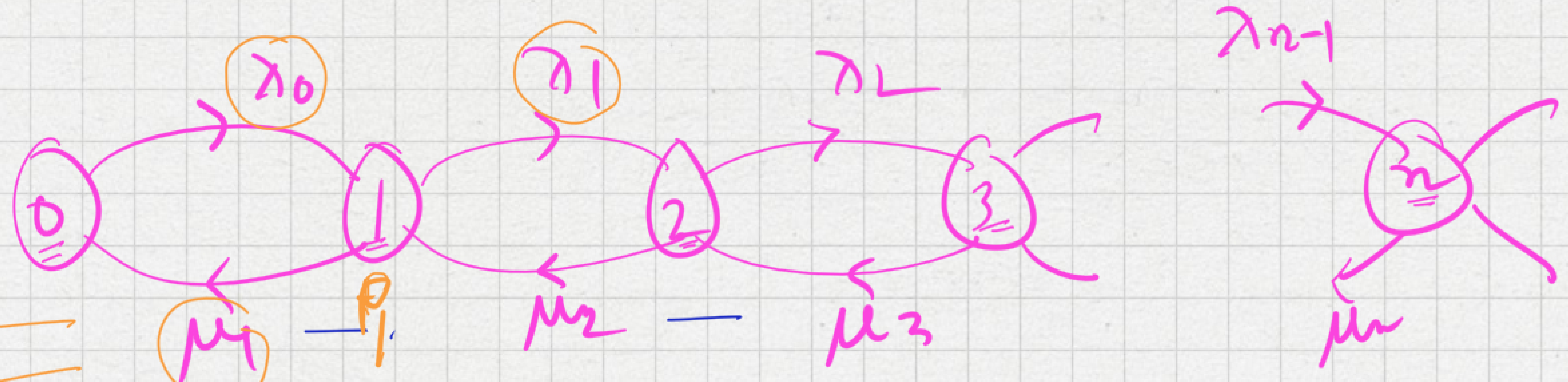
$e(t)$

$e(t) = 0$

$\to$ both queue & server empty

$e(t) = 1$

server busy queue empty



⇒ $\boxed{\text{steady-state}}$ scenario

↳ average values

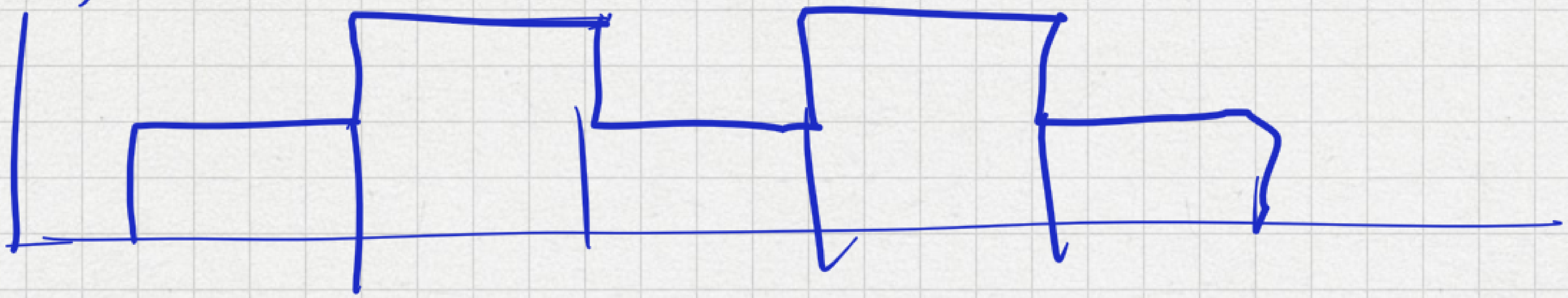
P_0

$P_i \triangleq$ steady state prob. that the system is in state i

→ function of time we visit state i

$$P_N(i) = \begin{cases} P_i & i = 0,1,2.. \\ 0 & o/w \end{cases}$$

$$E[N] = \sum_{i=0}^{\infty} i \cdot P_i$$



* average # of jobs N
* Average system time W / T

1 hour

$P_0 = 0.1$

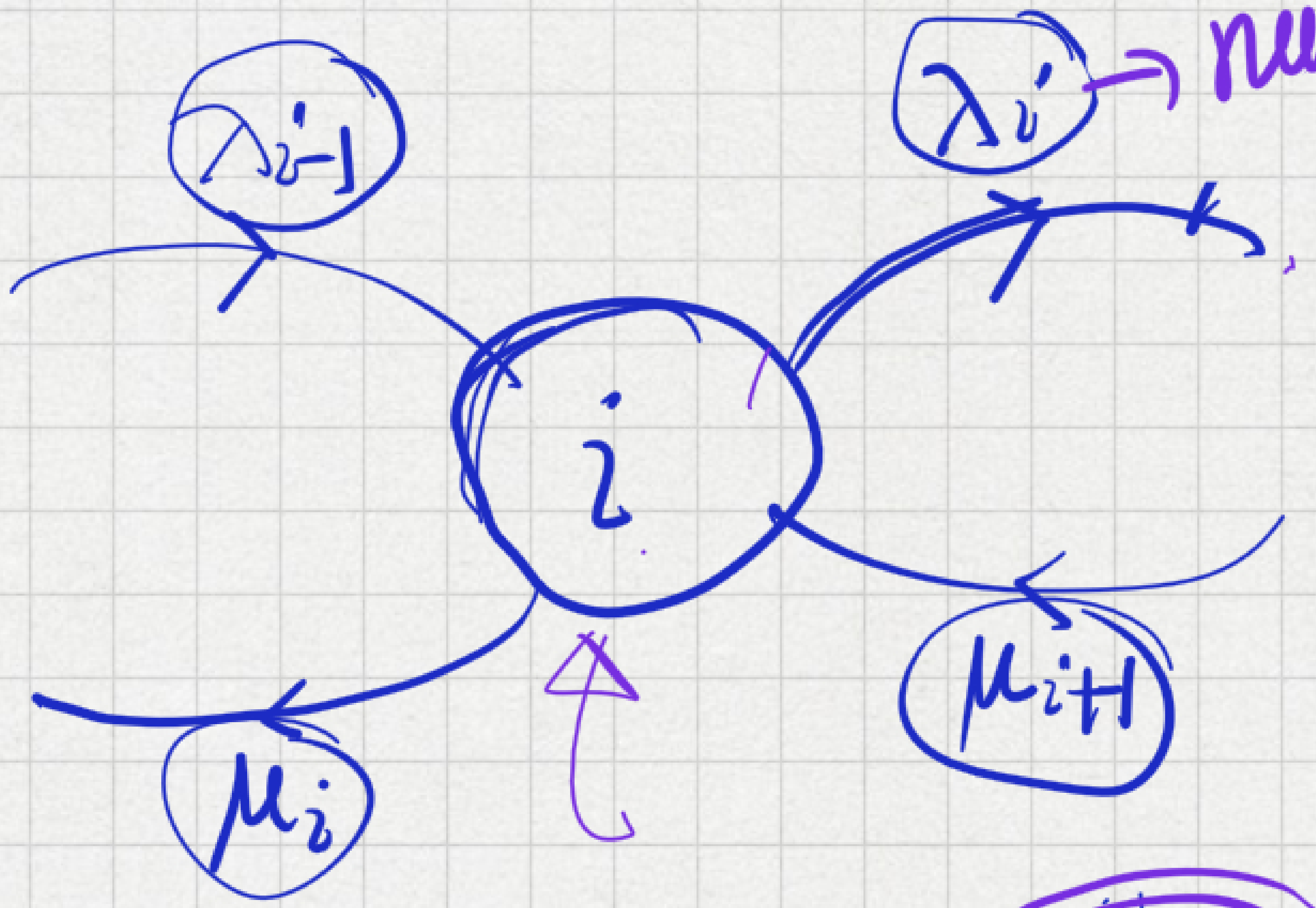
6 min in 0 state avg

steady-state

for each state

(I) Rate at which enter the state

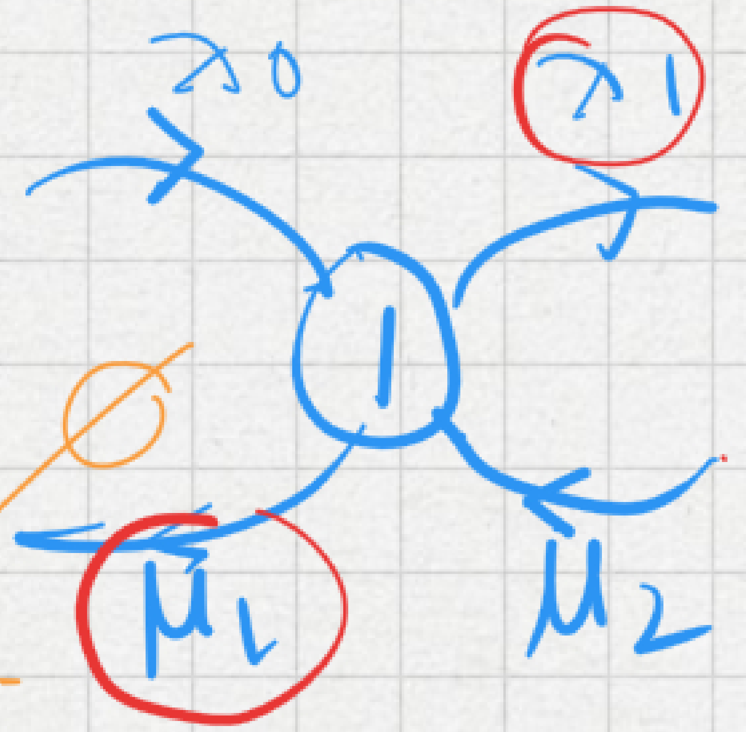
(II) Rate . . . leave " "



$i \neq 0$

P_i	λ_i
P_i	μ_i

State	Rate of leaving = Rate of ent
0	$\lambda_0 P_0 = \mu_1 P_1$ — (I)
1	$(\lambda_1 + \mu_1) P_1 = \mu_2 P_2 + \lambda_0 P_0$ — (II)
2	$(\lambda_2 + \mu_2) P_2 = \mu_3 P_3 + \lambda_1 P_1$ — (III)
	⋮
n, $n \geq 1$	$(\lambda_n + \mu_n) P_n = \mu_{n+1} P_{n+1} + \lambda_{n-1} P_{n-1}$



Systems of linear equations

Add each eq to its previous one

0 $\lambda_0 P_0 = \mu_1 P_1$

1 $\lambda_1 P_1 = \mu_2 P_2$

2 $\lambda_2 P_2 = \mu_3 P_3$

$\vdots$

n $\lambda_n P_n = \mu_{n+1} P_{n+1}$

$$\Rightarrow P_1 = \frac{\lambda_0}{\mu_1} P_0$$

$$\Rightarrow P_2 = \frac{\lambda_1}{\mu_2} \cdot P_1 = \frac{\lambda_1 \lambda_0}{\mu_2 \mu_1} \cdot P_0$$

$$\Rightarrow P_3 = \frac{\lambda_2}{\mu_3} P_2 = \frac{\lambda_2 \lambda_1 \lambda_0}{\mu_3 \mu_2 \mu_1} P_0$$

$$P_n = \frac{\lambda_{n-1}}{\mu_n} P_{n-1} = \frac{\lambda_{n-1} \lambda_{n-2} \cdots \lambda_1 \lambda_0}{\mu_n \mu_{n-1} \cdots \mu_2 \mu_1} P_0$$

$$\sum_{n=0}^{\infty} P_n = 1$$

$$\boxed{P_0 + P_0 \sum_{n=1}^{\infty} \frac{\lambda_{n-1}\,\lambda_{n-2} \quad \lambda_1 \lambda_0}{\mu_n\,\mu_{n-2} \quad \mu_2 \mu_1} = 1}$$

$n=1, \quad \frac{\lambda_0}{\mu_1}$

$n=2 \quad \frac{\lambda_1 \lambda_0}{\mu_2 \mu_1}$

$$P_0 = \frac{1}{1 + \sum_{n=1}^{\infty} \frac{\lambda_{n-1}\,\lambda_{n-2} \cdots \lambda_1 \lambda_0}{\mu_n\,\mu_{n-1} \cdots \mu_2 \mu_1}}$$

$$P_n = \frac{\lambda_{n-1}\,\lambda_{n-2} \cdots \lambda_1 \lambda_0}{\mu_n\,\mu_{n-1} \cdots \mu_2 \mu_1} \cdot \frac{1}{1 + \sum_{n=1} \frac{\lambda_{n-1} \cdots \lambda_0}{\mu_n \cdots \mu_1}}$$

$< \infty$

$$\boxed{\begin{aligned} \lambda_i &= \lambda \\ \mu_i &= \mu \end{aligned}}$$

$$\boxed{P_n = \left(\frac{\lambda}{\mu}\right)^n \cdot P_0}$$

$$\boxed{P_0 = \frac{1}{1 + \sum_{n=1}^{\infty} \left(\frac{\lambda}{\mu}\right)^n}}$$

$$= \frac{1}{\left(\frac{\lambda}{\mu}\right)^0 + \left(\frac{\lambda}{\mu}\right)^1 + \left(\frac{\lambda}{\mu}\right)^2 + \cdots}$$

$\frac{\lambda}{\mu} < 1$

$\frac{\lambda}{\mu} < 1$

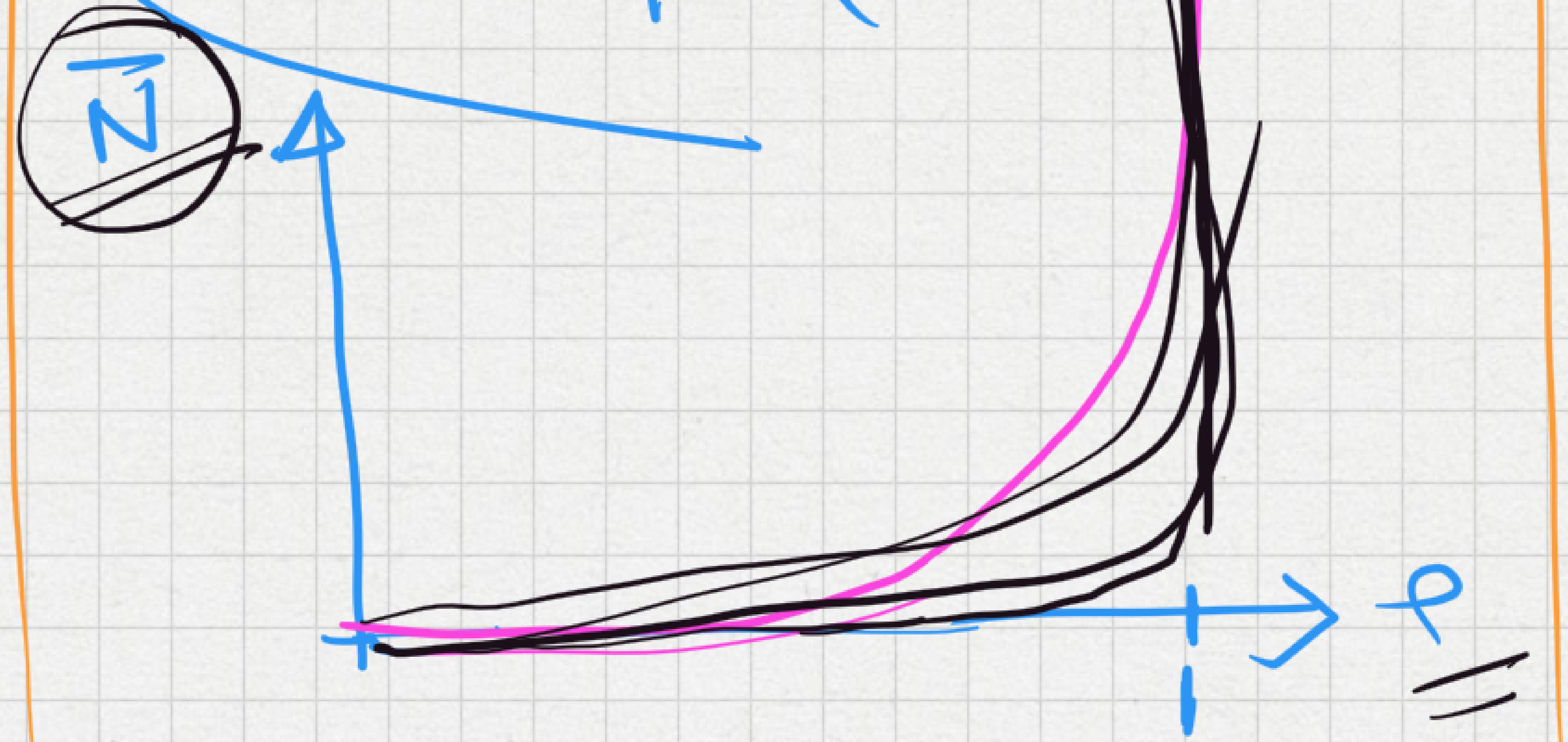
$\boxed{\Rightarrow \lambda < \mu}$

$\rho \triangleq \frac{\lambda}{\mu} \rightarrow$ traffic intensity

$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \rho$

$$\boxed{P_n = \rho^n (1-\rho)}$$

$$\bar{N} = \sum_{n=0}^{\infty} n \cdot P_n = \sum_{n=0}^{\infty} n \rho^n (1-\rho) = \frac{\rho}{1-\rho}$$



0.1

1

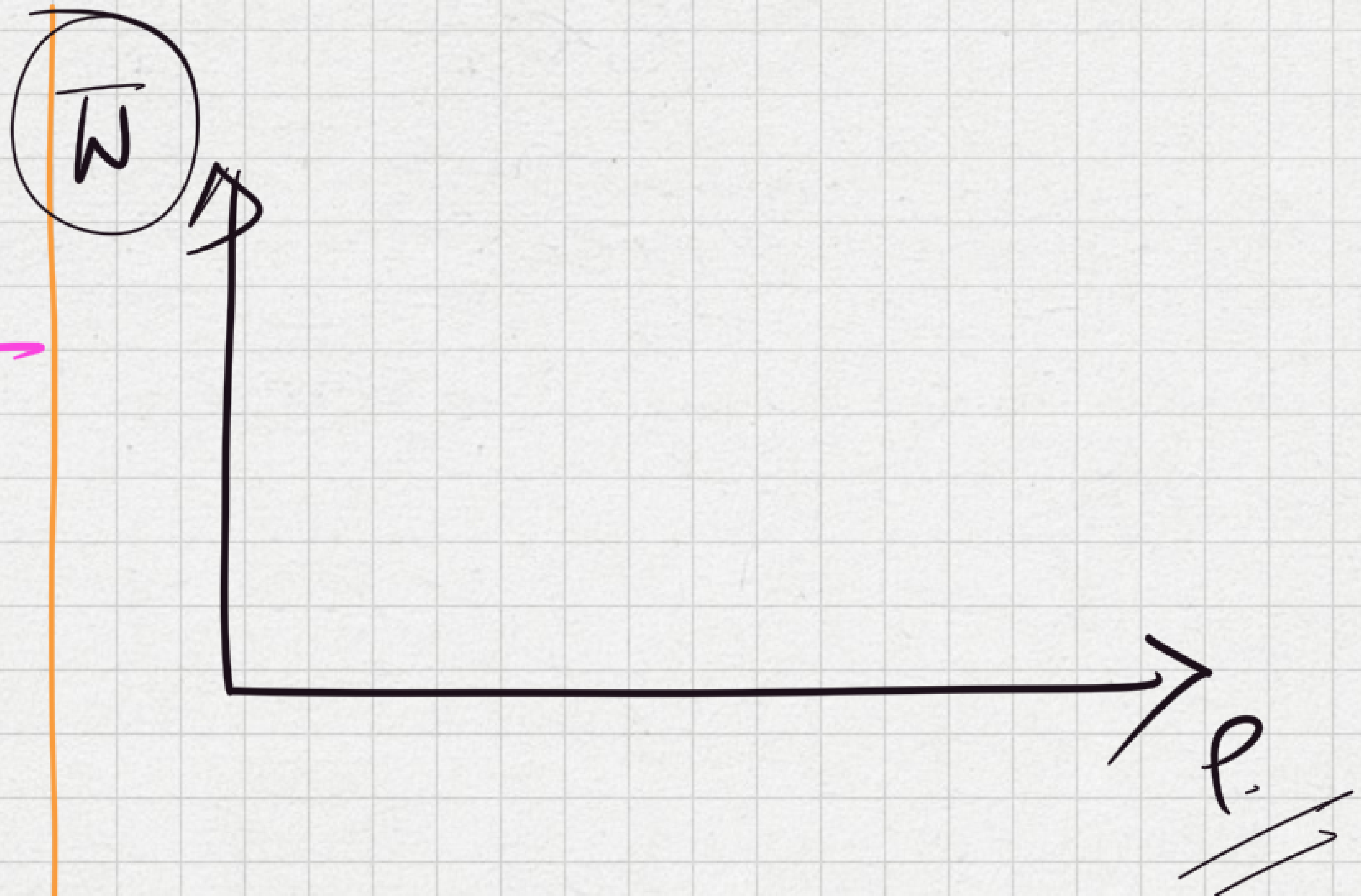
Little formula

W/T

$$\bar{W} = \frac{1}{\lambda}\bar{N}$$

$$= \frac{1}{\lambda}\frac{\rho}{1-\rho} \qquad \rho \to \frac{\lambda}{\mu}$$

$$= \frac{1}{\mu(1-\rho)}$$



Run the C codes given in the book

λ μ $\Rightarrow$ match with ρ

task

numerically solve the eq for different values of ρ

$\to$ draw graph

Single value of P

run the prog.

10 times

$\bar{N}_1 \ \bar{N}_2 \ldots \bar{N}_{10}$

$$\boxed{E[\bar{N}] = \frac{\bar{N}_1 + \cdots + \bar{N}_2}{10}}$$

Sample mean

expected value of the Sample mean

P-10