$$\vec{v} \cdot \vec{\omega} = |\vec{v}||\vec{\omega}| \cos\theta$$

$\vec{v} \cdot \vec{\omega} \leq ||v|| \cdot ||\omega|| \longrightarrow$ Schwart Inequality

$||v+\omega|| \leq ||v|| + ||\omega|| \longrightarrow$ Triangle Inequality

Row picture:

$$\underset{A}{\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}} \underset{x}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{b}{\begin{bmatrix} 0 \\ 3 \end{bmatrix}}$$

$$Ax = b$$

Column picture:

$$x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Elimination:

Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 6 & 4 & 8 & 3 \\ 3 & 4 & 2 & 7 \end{array}\right]$$

Make these zeroes

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -8 & 2 & -9 \\ 0 & -2 & -1 & 1 \end{array}\right] \quad \left[\begin{array}{l} R_2 = R_2 - 6R_1 \\ R_3 = R_3 - 3R_1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ 0 & -8 & 2 & -9 \\ 0 & 0 & -\frac{3}{2} & \frac{14}{9} \end{array}\right] \quad \left[R_3 = R_3 - \tfrac{1}{4}R_2\right]$$

$$x + 2y + z = 2$$
$$-8y + 2z = -9$$
$$-\tfrac{3}{2}z = \tfrac{14}{9}$$

find z

substitute z and ~~put in~~ find y.

" z, y and find x.

* If we get 0 as pivots exchange ROWS (not columns)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 4 & -2 \\ 0 & 1 & 1 \\ -2 & -3 & 7 \end{bmatrix}$$

Do the same operation identity matrix $\longrightarrow$

$$\underbrace{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}}_{E_{21}} \times \begin{bmatrix} x & x & x \\ 0 & x & x \\ x & x & x \end{bmatrix} = \begin{bmatrix} & & \\ & \sim & \\ & & \end{bmatrix} \quad [R_2 = R_2 - R_1]$$

zero in this position

$$\left(E_{32} \times \left(E_{31} \times \left(E_{21} A\right)\right)\right) = U$$

On the left

$$(E_{32} E_{31} E_{21}) A = U$$

The elimination matrices.

$$\Rightarrow E \cdot A = U$$

$$\underset{I}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \longrightarrow \underset{P_{23}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}}$$

(Permutation Matrix)

Matrix has three operation

i) Subtract Row

ii) Multiply Row by Scalar

iii) Swap Rows

(*) We can not ~~do~~ swap columns by multiplying on the left. But we can DO column operations by multiplying on the right. But we always do row operations.

Remember $A \times B \neq B \times A$ where A, B are matrices.

Inverse Matrix

$$\cancel{\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\begin{array}{cc} A & I \\ \left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] \end{array}$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 7 & -3 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$I \qquad A^{-1}$$

(Gauss Jordan)

$$E[A\ I] = [I\ A^{-1}]$$

$$(A^{-1})^T = (A^T)^{-1}$$

Transposing and inversing can be done in any order.

But

$$(AB)(B^{-1}A^{-1}) = I$$

↑ Order matters

Matrix Multiplication

$$\begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix} \times \begin{bmatrix} \cdot \\ - \\ - \end{bmatrix} = \begin{bmatrix} - & - \\ - & - \end{bmatrix}$$

$A\,(m \times n) \quad B\,(n \times p) \quad C\,(m \times p)$

$A^{-1} \cdot A = I$

if A is invertible and non-singular.

Basic Matrix Factorization

$$\boxed{A = L \times U}$$

$$\underset{E_{21}}{\begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix}} = \underset{U}{\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}}$$

An elimination matrix is easy to inverse.
It add backs what it removes.

$$\underset{E_{21}^{-1}}{\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}}$$

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$$A = L \times U$$

$$\begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}}_{E_{21}^{-1}} \times \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

Operations to do on U to extract diagonals

$$= \underbrace{\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}}_{L} \times \underbrace{\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}}_{D} \times \underbrace{\begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix}}_{U}$$

D → Diagonal Matrix

For (3×3) [no row exchange case]

$$E_{32} E_{31} E_{21} A = U$$

$$\Rightarrow A = \underbrace{(E_{21}^{-1} E_{31}^{-1} E_{32}^{-1})}_{L} \times \underbrace{U}_{U}$$

Reverse order

Example:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ \underbrace{10} & -5 & 1 \end{bmatrix}$$

$$E_{32} \qquad E_{21} \qquad = \quad E$$

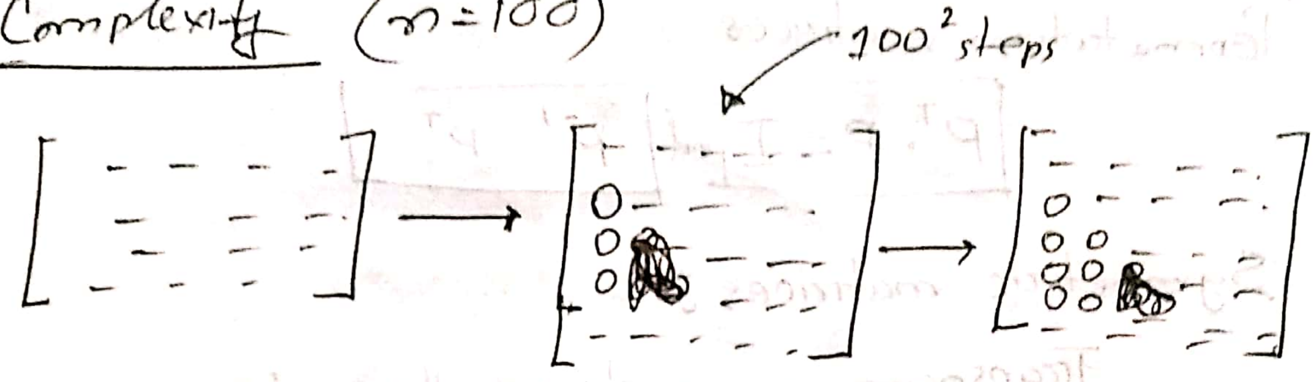
Notice (A 10 is produced)

(in reverse order) $\quad (-1) \quad (-1)$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$E_{21}^{-1} \qquad E_{32}^{-1} \qquad = \quad L$$

Complexity $(n=100)$



Let, think individually.

For an operation, $R_2 = R_2 - L \times R_1$ where L is a multiplier

It will cost 100 steps to multiply with 100 elements of R_1 and then 100 steps of subtraction with each element of R_2. Considering both as a single operation, it takes 100 steps to produce R_2 with 0 at front.

This has to be done for first 99 rows (99 is close to 100), for the first column only. For second column we need to do it, $R_3 = R_3 - L \times R_2$ which will cost 99 steps and needs to be done 98 (close to 99 times). This has to be done upto $R_{100} = R_{100} - L \times R_{99}$.

$$\text{So, total cost} = 100^2 + 99^2 + \cdots + 1^2$$

$$= \frac{n(n+1)(2n+1)}{6}$$

$$\therefore O(n^3)$$

Permutation matrices

$$\boxed{P^T \cdot P = I} \text{ and } \boxed{P^{-1} = P^T}$$

Symmetric matrices :

Transposing won't change the matrix.

Let, A be a symmetric matrix

then, $\boxed{A^T = A}$

Chapter-3

Vector Space

A bunch of vectors

R^2 = all 2 dim vectors (x-y) plane

R^3 = all vectors with 3 components

R^n = all vectors with n components.

A vector space has to be closed on multiplication and (be in same space) & addition of vectors i.e linear combinations.

Another vector space in a vector space

Example of subspaces in vector space

(i) a line in R^2 through origin.
(ii) Origin $(0,0)$
(iii) ~~Another R^2~~ all R^2.

→ Why is a line not passing through origin is a vector space?

Because multiply by 0 and we get $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ but it is not part of ~~the~~ our initial vector. Hence, the multiplication condition doesn't meet. (Closure under multiplication)

So, ALL VECTOR SUBSPACES MUST CONTAN ORIGIN.
(to fulfil multiplication condition)

Let v and w be two vectors.

So, vector space will be $cv+dw$ where c and d can be any number.

Basis Vectors : (i) Independent
(ii) Contribute to span ⊛

→ I wrote more about vector sub-spaces later on.

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NULL SPACE

All solutions of x for equation, $Ax = 0$.

Calculating NULL SPACE

$$A = \begin{bmatrix} 1 & 2 & 2 & 2 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 8 & 10 \end{bmatrix}$$

$\downarrow$ through elimination

Echelon $\rightarrow$ (staircase form)

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Two pivots

(*) Rank = No. of pivots

$$\left.\begin{aligned} x_1 + 2x_2 + 2x_3 + 2x_4 &= 0 \\ 2x_3 + 4x_4 &= 0 \end{aligned}\right]$$

Taking, $x_4 = 0$ and $x_2 = 1$

$$c\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + d\begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

$x_4 = 1$; $x_2 = 0$

$r = 2$

$(n-r) = 4-2 =$ free variables no

Row reduced echolan form,

(*) zeroes ~~befor~~ above and below pivots

(*) pivots = 1

$$\begin{array}{cccc} \begin{bmatrix} \boxed{1} & 2 & 2 & 2 \\ 0 & 0 & \boxed{2} & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{matrix} P & & P & \end{matrix} \end{array} \longrightarrow \begin{bmatrix} 1 & 2 & 0 & -2 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\downarrow$$

$$\text{Rref} \longrightarrow \begin{bmatrix} 1 & 2 & 0 & -2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} I & F \\ 0 & 0 \end{bmatrix} \longleftarrow \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

I

Free part F

$$\text{Null Space} = \begin{bmatrix} -F \\ I \end{bmatrix}$$

$$\boxed{Ax = b}$$

complete solution

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & 2 & b_1 \\ 2 & 4 & 6 & 6 & b_2 \\ 3 & 6 & 8 & 8 & b_3 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & 2 & b_1 \\ 0 & 0 & 2 & 4 & b_2 - 2b_1 \\ 0 & 0 & 2 & 4 & b_3 - 3b_1 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{cccc|c} 1 & 2 & 2 & 2 & b_1 \\ 0 & 0 & 2 & 4 & b_2 - 2b_1 \\ 0 & 0 & 0 & 0 & b_3 - b_2 - b_1 \end{array}\right]$$

$$b_3 \cancel{=b_2} - b_1 - b_2 = 0$$

$$\therefore \boxed{b_3 = b_1 + b_2}$$

Solvability

$Ax = b$ when b is in $C(A)$

Comb. of rows of A gives zero row.

Comb. of entries of b give 0.

To find complete solution for $Ax=b$

① $X_{particular}$: (i) Set all free variables to zero | $x_2 = 0$, $x_4 = 0$

(ii) Solve $Ax=b$ for pivot variables

(care not zero, but values of b)

$$x_1 + 2x_3 = 1 \qquad \therefore x_1 = -2$$

$$2x_3 = 3 \qquad \therefore x_3 = 3/2$$

$$X_p = \begin{bmatrix} -2 \\ 0 \\ 3/2 \\ 0 \end{bmatrix}$$

$$X = X_p + X_n \longrightarrow (X\text{-null space})$$

$$A X_p = b$$

$$A X_n = 0$$

$$A(x_p + X_n) = b$$

$$\therefore X = \begin{bmatrix} -2 \\ 0 \\ 3/2 \\ 0 \end{bmatrix} + c \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + d \begin{bmatrix} 2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$$

If $r=n$, then (no free variables).

$$X_{NULL} = \{\text{Zero Vector}\}$$

$$\therefore X = X_p$$

So, there is either 0 or 1 solution.

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If $r = m$,

* can solve $Ax = b$ for every b that exists
* left with $(n-r)$ free variables

$r = m = n$

(+) $r_{NULL} = \{0 \text{ vector}\}$

(+)

$r = m = n$	$r = n < m$	$r = m \leq n$
Rref $\rightarrow R = I$	$R = \begin{bmatrix} I \\ 0 \end{bmatrix}$	$R = \begin{bmatrix} I & F \end{bmatrix}$ might be combined
1 solution	0 or 1 solution	Infinity or 1 solution

$r < m,\ r < n$

$R = \begin{bmatrix} I & F \\ 0 & 0 \end{bmatrix}$ (0 or ∞ solutions)

– – Mid Syllabus (Lecture-13) – –

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 6 & 4 \\ 3 & 8 & 6 \end{bmatrix} \xrightarrow{R_2 = R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 0 \\ 3 & 8 & 6 \end{bmatrix} \xrightarrow{R_3 = R_3 - 3R_1}$$

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 0 \\ 0 & 8 & 0 \end{bmatrix} \xrightarrow{R_3 = R_3 - 4R_1} \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\downarrow R_1 = R_2 \cdot R_1$$

$$\begin{bmatrix} 1 & 0 & 2 \\ & & \\ & & \end{bmatrix}$$

somehow

$$R = \text{R-Ref}: \begin{bmatrix} \boxed{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}} & \begin{matrix} 2 \\ 0 \end{matrix} \\ \begin{matrix} 0 & 0 \end{matrix} & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} I & F \\ 0 & 0 \end{bmatrix}$$

(i) $r = n = m$ [No. of equations equal to the no. of unknowns]

$\therefore R = I$

We will have exactly one solution

(ii) $r = m < n$

Less equations than no. of unknowns.

Solutions: Infinitely many

$$R = [I \;\; F]$$

(iii) $r = n < m$ [More equations than no. of unknowns]

Solutions: At most one. (One or zero) (zero because an extra equation might be conflicting

$$R = \begin{bmatrix} I \\ 0 \end{bmatrix}$$

---

(4) $r < n,\ r < m$

$$R = \begin{bmatrix} I & F \\ 0 & 0 \end{bmatrix}$$

<u>Solution</u>: 0 or infinite solution

Upto MIT lecture-8

#

Imp.

The 4 fundamental sub-spaces

Find all solutions depending on b_1, b_2 and b_3

$$x - 2y - 2z = b_1$$
$$2x - 5y - 4z = b_2$$
$$4x - 9y - 8z = b_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -2 & b_1 \\ 2 & -5 & -4 & b_2 \\ 4 & -9 & -8 & b_3 \end{array}\right]$$

$$\downarrow$$

$$\left[\begin{array}{ccc|cc} 1 & 0 & -2 & \cancel{b_1} & 5b_1 - 2b_2 \\ 0 & 1 & 0 & \cancel{b_2} & 2b_1 - b_2 \\ 0 & 0 & 0 & \cancel{b_3} & b_3 - b_2 - 2b_1 \end{array}\right]$$

$$\downarrow$$

$$2b_1 + b_2 - b_3 = 0$$

$$X_p = \begin{bmatrix} 5b_1 - 2b_2 \\ 2b_1 - b_2 \\ 0 \end{bmatrix} \qquad N(A) = c \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$X_{complete} = X_p + N(A) = \begin{bmatrix} 5b_1 - 2b_2 \\ 2b_1 - b_2 \\ 0 \end{bmatrix} + c \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

No non-zero solution for $Ax = 0$.

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ ← This vector doesn't contribute to sub-space

→ Span: (a plane in R^3)

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ → Span: R^3

(*) The vectors which are independent and span a particular subspace are called basis vectors.

(*) The particular number of vectors required to define a space is called the dimension of that space.

Ex: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ has span R^3

3 vectors to define R^3. So, it has dimension no. 3.

(*) Rank of the matrix is the dimension of the column space of the matrix.

(*) The no. of free columns is the dimension of the null space of the matrix.

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Find the row reduced echelon form (rref) first if it is required to find the rank, dimension, null space or column space of a matrix.

$$A = \begin{bmatrix} 2 & -1 & -3 \\ -4 & 2 & 6 \end{bmatrix} \longrightarrow R = \begin{bmatrix} 1 & -1/2 & -3/2 \\ 0 & 0 & 0 \end{bmatrix}$$

Column Space: (straight line)

$\text{Dim} = r = 1$ (Dimension, rank)

$\text{Span} = \left\{ \begin{bmatrix} 2 \\ -4 \end{bmatrix} \right\} \rightarrow$ straight line

$\text{Basis} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

Null space:

$\text{Dim} = n - r = 2$

Notation for span

$\text{Span} = \left\{ \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3/2 \\ 0 \\ 1 \end{bmatrix} \right\}$ ~~R^2 in~~ [plane in R^3]

$\text{Basis} = \begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3/2 \\ 0 \\ 1 \end{bmatrix}$

$$A^T = \begin{bmatrix} 2 & -4 \\ -1 & 2 \\ -3 & 6 \end{bmatrix} \qquad R = \begin{bmatrix} 1 & -2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$C(A^T)$:

Dim = r = 1

This is also the row space of matrix A.

$$\text{Span} = \left\{ \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix} \right\} \rightarrow \text{st. line}$$

$$\text{Basis} = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$$

$N(A^T)$: [Left NULL Space of A]

Dim = n − r = 2 − 1 = 1

No. of special solutions.

$$\text{Span} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\} \rightarrow \text{st. line}$$

$$\text{Basis} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Reason

$$\begin{bmatrix} 1 & -2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 - 2x_2 = 0$$

(2,1) is a solution for (x_1, x_2)

Those spaces are called the four fundamental sub-spaces of a matrix.

$$(A^T y)^T = 0^T \rightarrow \text{Column full of zeroes}$$

$$\Rightarrow y^T (A^T)^T = 0^T$$

$$\Rightarrow y^T \cdot A = 0^T$$

→ Null-space at left side, so left null-space.

Rank is the no. of pivots i.e. no. of independent columns.

* Learn to derive the 4 fundamental sub-spaces for a given problem.

#1 Wednesday

Rank-1 Matrices

19-02-20
Wednesday

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} \qquad A = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$= uv^T$ [u, v are column vectors]

Dim of $C(A) = 1$ (column space)

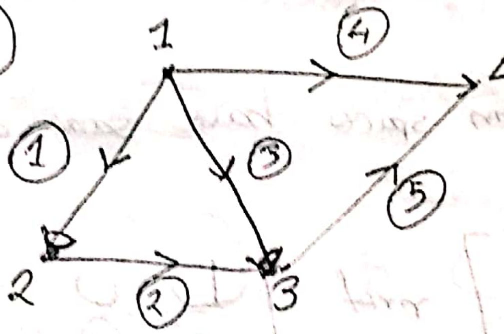
Dim of $N(A) = 2$ $(n-r)$ (null space)

Basis $= \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Graphs = {nodes, edges}

Incidence Matrix

Electric Potential of a system



(+ve) for incoming

(−ve) for outgoing

Incidence Matrix

$$I = \begin{array}{c} \text{Nodes} \\ \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \end{array} \text{Edges}$$

Each row has 2 values (-1) (starting point) and $(+1)$ (ending point).

$$Ax = 0$$

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ x_4 - x_3 \\ x_4 - x_1 \\ x_3 - x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Proof of Ohm's Law.

$I \propto V$

4 fundamental subspaces

i) Row space $R(A)$ — Dim : r $\quad [R(A) = C(A^T)]$

ii) Column space $C(A)$ — Dim : r

iii) Null space $N(A)$ (Right Null Space) — Dim : $n-r$

iv) Null space of $A^T = N(A^T)$ (Left NULL Space) — Dim : $m-r$

Row space and column space have same dimension.

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row space $\quad c\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} + d\begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

$$\boxed{A^T \cdot y = 0}$$

$$\Rightarrow (A^T \cdot y)^T = 0^T$$

$$\Rightarrow y^T A = 0$$

$y^T \rightarrow$ Left NULL space

Vector Sub-space

Sub-space → basically a collection of vectors with linear properties

(i) Subspace contains 0 vector.

(ii) Closure under multiplication

↳ $c \cdot \vec{x} = c\vec{x}$ will be in subset.

Suppose we have a vector $\vec{x}$ in our subspace. By multiplying it with any arbitary constant c, the result we get will be contained by the subspace.

$\vec{x}$ in V
$c\vec{x}$ will also be in V

(iii) Closure under addition.

$\vec{a}$ in V
$\vec{b}$ in V
then $(\vec{a}+\vec{b})$ will also be in V

If this condition is fulfilled then closure property is fulfilled.

Example: $\left\{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in R^2 \,\middle|\, x_1 \geq 0\right\}$ is NOT a subspace (Multi-closure is not fulfilled)

but $\left\{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in R^2\right\}$ is a subspace.

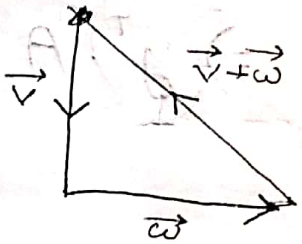
#2

Orthogonality

11-03-2020
Wednesday

$$\vec{x} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \qquad \vec{y} = \begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$$

$$\vec{x} \cdot \vec{y} = 0$$



$$v^2 + w^2 = (v+w)^2 \quad [a^2+b^2=c^2]$$

$$\|v\|^2 + \|w\|^2 = \|v+w\|^2$$

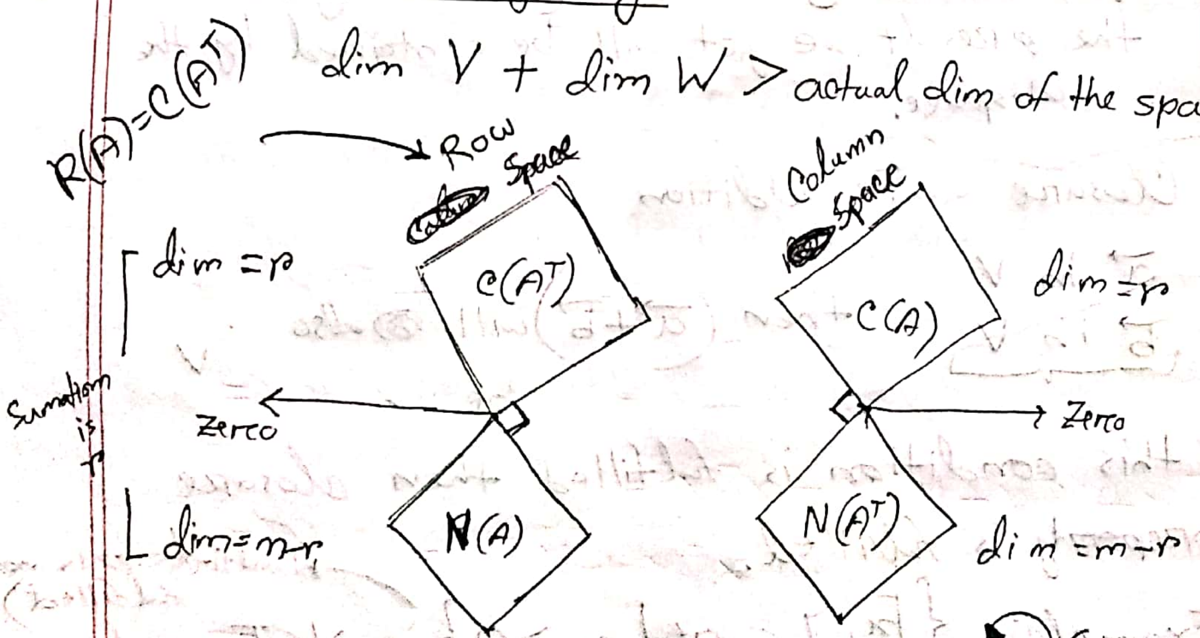
$$\Rightarrow v^T v + w^T w = (v+w)^T (v+w)$$

$$\Rightarrow v^T v + w^T w = v^T w + v^T v + w^T w + w^T v$$

$$\Rightarrow v^T w + w^T v = 0$$

Condition for orthogonality:

dim V + dim W > actual dim of the space.



A is (m X n) $\quad C(A)$ is contained in R^m (Space by all columns is R to the power no. of rows)

$R(A)$ is contained in R^n

~~My definition~~/Interpretation

Two sub-spaces are orthogonal if and only if every vector of the first sub-space is perpendicular to every vector of the next one.

Teacher's: Any two randomly vector from the given sub-space must be ~~orthogonal to m perpl~~ perpendicular for orthogonality.

Projections

$$b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Projection of on z-axis = $\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$ P_1

length is 3.

Projection on (x-y) axis is $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ P_2

length is $\sqrt{5}$.

Projection matrix for P_1 is

$$P_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If you multiply b with projection matrix then we get the projection.

For, $P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$