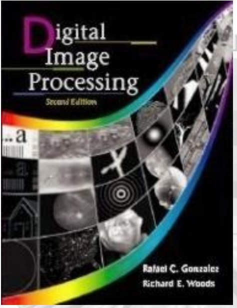
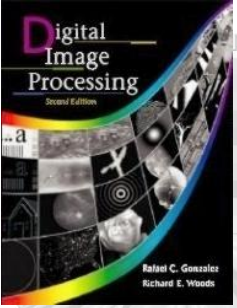
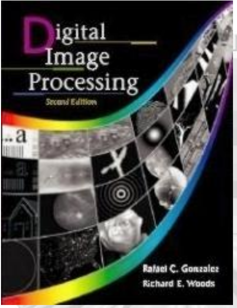


Chapter 10
Image Segmentation

Md. Hasanul Kabir, Ph.D
Professor, Dept. of CSE
Islamic University of Technology



- Review
  - Segmentation is to subdivide an image into its constituent regions or objects.
  - Segmentation should stop when the objects of interest in an application have been isolated.



Chapter 10
Image Segmentation

- Segmentation algorithms generally are based on the following properties of intensity values:
  – Discontinuity: partition an image based on abrupt changes in intensity such as edges
  – Similarity : partition an image into regions that are similar according to a set of predefined criteria

Chapter 10 :Image Segmentation

Detection of Discontinuities

-Detect the three types of gray-level discontinuities: points, line, edges

- The most common way of detecting the discontinuities is to run a mask through the image

-The respons of the mask at any point is given by

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9$$

FIGURE 10.1 A general 3 × 3 mask.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

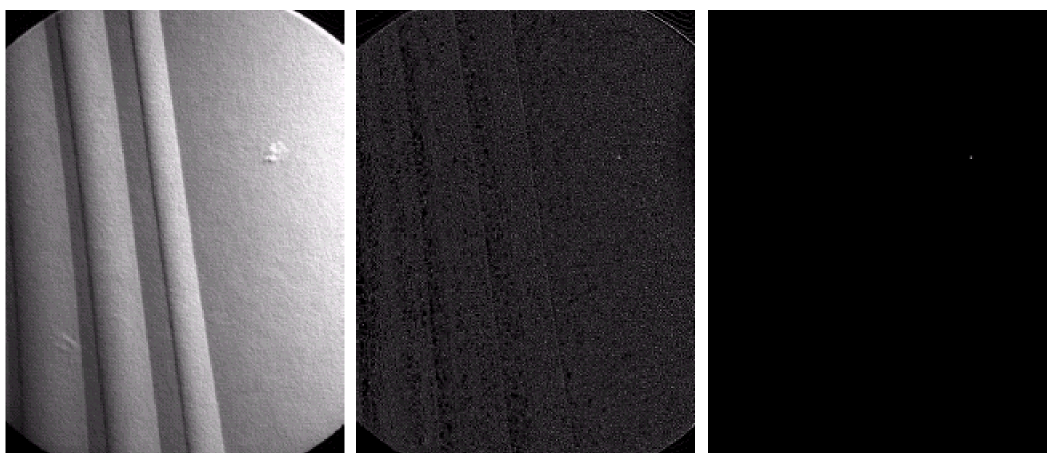
Chapter 10 :Image Segmentation

Point Detection

-A point has been detected at the location on which the mask is centered if $|R| \geq T$, where T is a nonnegative threshold

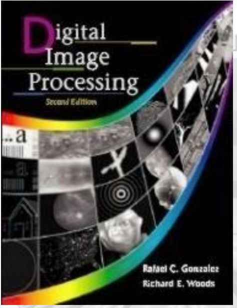
-Example: use Laplacian operator

−1	−1	−1
−1	8	−1
−1	−1	−1



a
b c d

FIGURE 10.2 (a) Point detection mask. (b) X-ray image of a turbine blade with a porosity. (c) Result of point detection. (d) Result of using Eq. (10.1-2). (Original image courtesy of X-TEK Systems Ltd.)



Chapter 10 :Image Segmentation

Line Detection

- Run the all the masks each of which represents different direction and select the direction of the masks that gives the best response.

FIGURE 10.3 Line masks.

Horizontal

−1	−1	−1
2	2	2
−1	−1	−1

+45°

−1	−1	2
−1	2	−1
2	−1	−1

Vertical

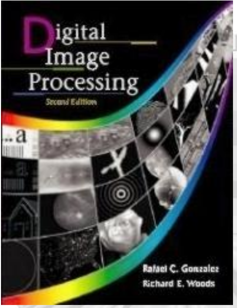
−1	2	−1
−1	2	−1
−1	2	−1

−45°

2	−1	−1
−1	2	−1
−1	−1	2

-Horizontal mask would respond more strongly to line(one pixel thick) oriented horizontally.

-The preferred direction of each mask is weighted with a larger coefficient than other possible directions.



Chapter 10 :Image Segmentation

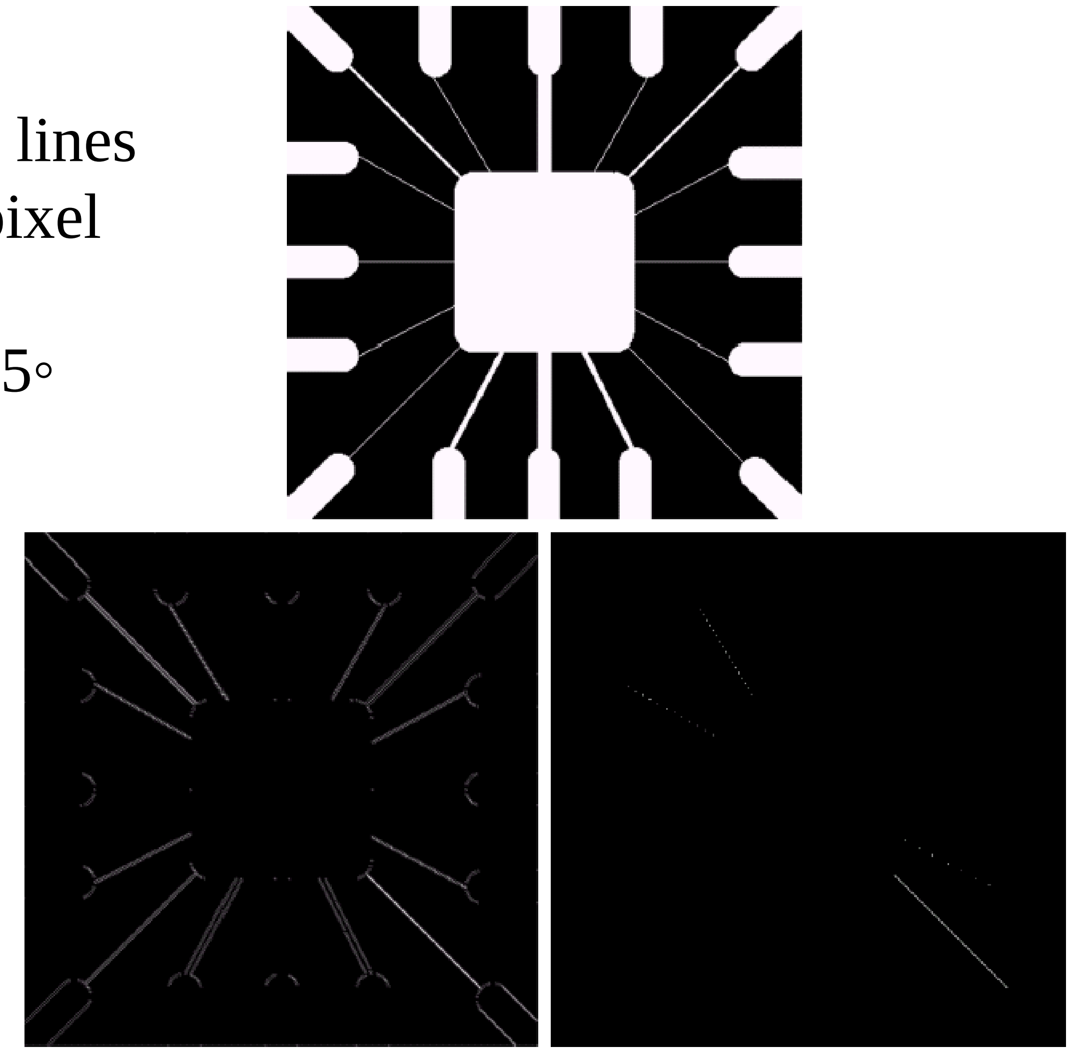
Line Detection

-If we want to detect all the lines in a certain direction, we run the mask representing the line direction through the image and threshold the absolute value of the response.

-The points that are left are the strongest responses, which, for lines one pixels thick, correspond closest to the direction defined the maks.

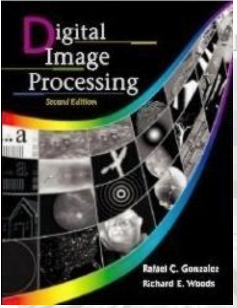
Line Detection Example

Detect all the lines that are one pixel thick and are oriented at -45∘



a
b c

FIGURE 10.4 Illustration of line detection. (a) Binary wire-bond mask. (b) Absolute value of result after processing with −45° line detector. (c) Result of thresholding image (b).



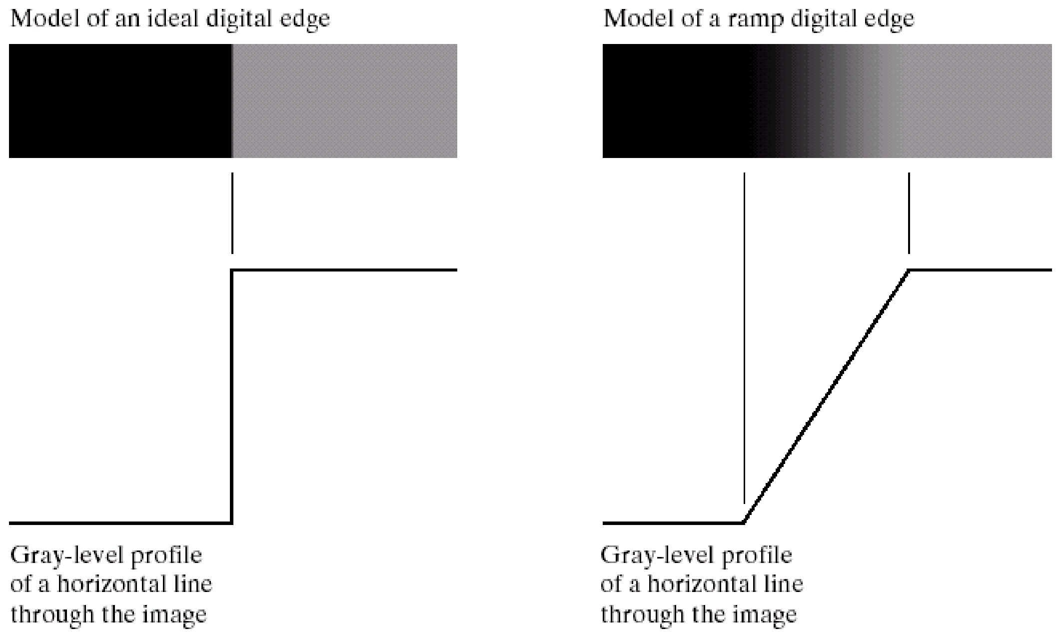
Chapter 10 :Image Segmentation

Edge Detection

- The most common approach for detecting meaningful discontinuities in gray level.
- Discuss edge detection approaches based on:
  - First order derivative(Gradient operator)
  - Second order derivative(Laplacian operator)
- Basic Formulation
  - An edge is a set of connected pixels that lie on the boundary between two regions
  - An edge is a “local” concept whereas a regions boundary, owing to the way it is defined, is a more global idea.

Chapter 10 :Image Segmentation

Ideal edge and ramp edge



a b

FIGURE 10.5 (a) Model of an ideal digital edge. (b) Model of a ramp edge. The slope of the ramp is proportional to the degree of blurring in the edge.

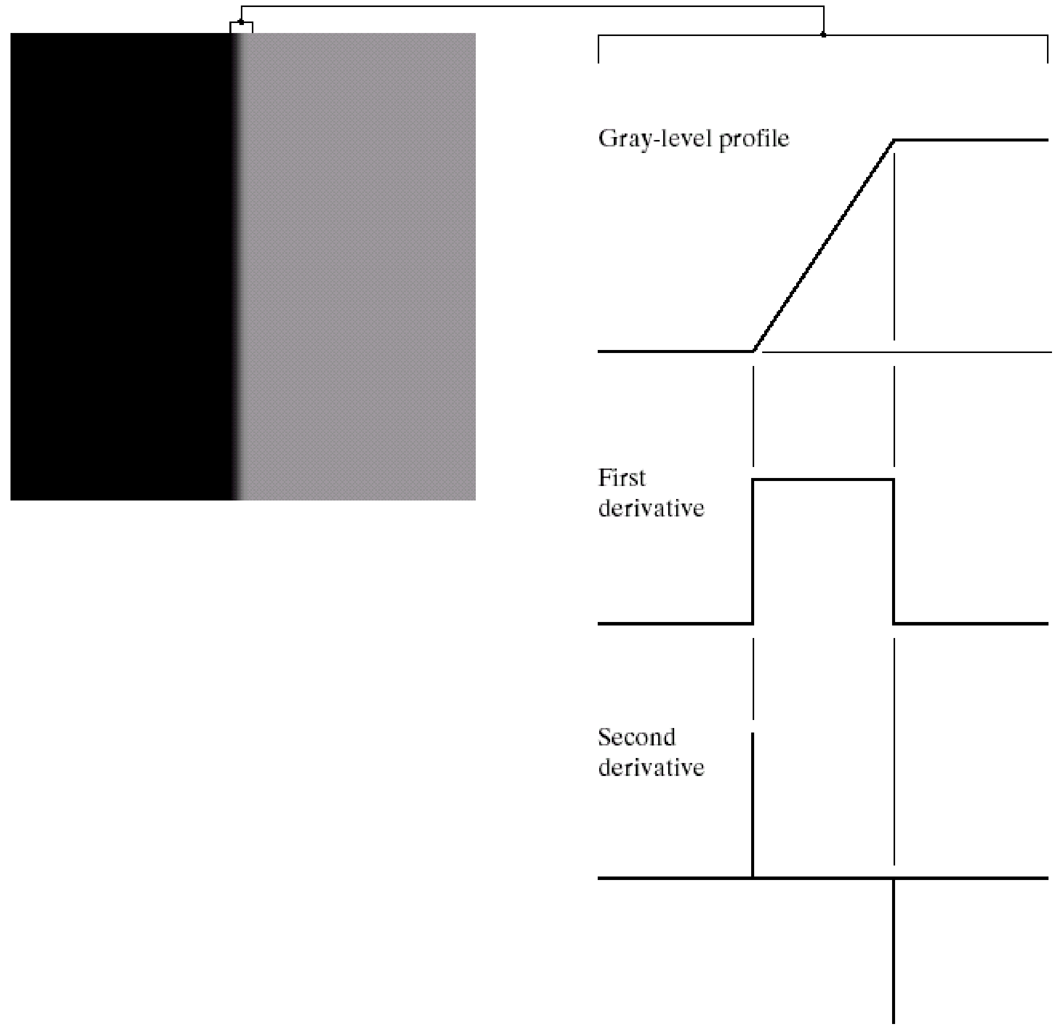
The degree of blurring is determined by factors such as the quality of the image acquisition system, the sampling rate, and illumination conditions.

Ramp (or thick) Edge

- The slope of the ramp is inversely proportional to the degree of blurring in the edges
- No longer have a thin(on pixel thick) path.
- Instead, an edge point now is any point contained in the ramp, and an edge would then be a set of such points that are connected.
- The thickness is determined by the length of the ramp.
- Then length is determined by the slop, which is in turn determined by the degree of blurring.
- Blurred edges tend to be thick and sharp edges tend to be thin.

Chapter 10 :Image Segmentation

First and second derivatives

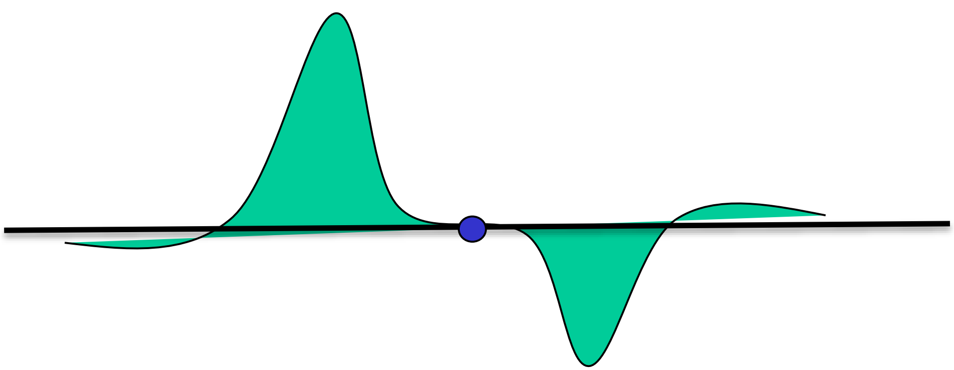


a b

FIGURE 10.6
(a) Two regions separated by a vertical edge.
(b) Detail near the edge, showing a gray-level profile, and the first and second derivatives of the profile.

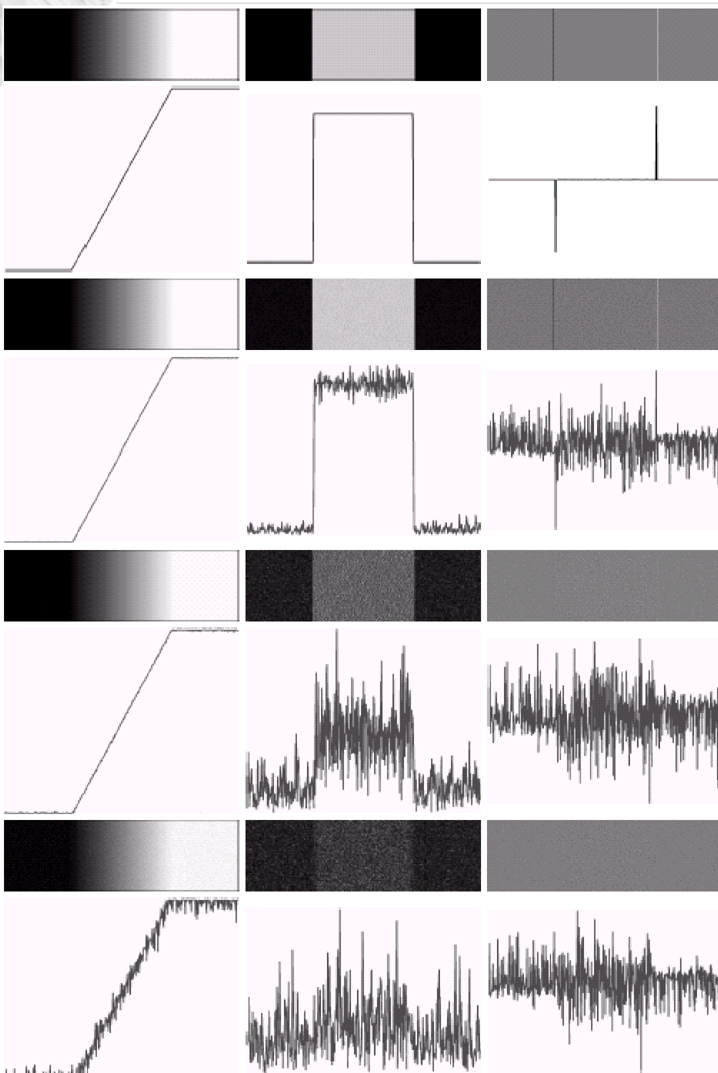
Zero Crossing

- **Zero Crossing Property:** An imaginary and negative values of the second derivative would cross zero near the midpoint of the edge.
- Quite useful for locating the centers of thick edges.



Chapter 10 :Image Segmentation

d



•First column: images and gray-level profiles of a ramp edge corrupted by random Gaussian noise of mean 0 and σ = 0.0, 0.1, 1.0 and 10.0, respectively.

• Second column: first derivative images and gray-level profiles.

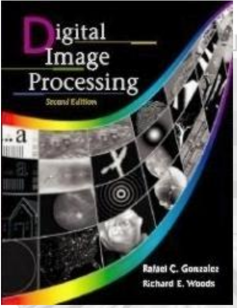
• Third column : second derivative images and gray-level profiles.

FIGURE 10.7 First column: images and gray-level profiles of a ramp edge corrupted by random Gaussian noise of mean 0 and $\sigma = 0.0, 0.1, 1.0$, and 10.0, respectively. Second column: first-derivative images and gray-level profiles. Third column: second-derivative images and gray-level profiles.

a
b
c
d

Considerations for the edge detection

- Zero crossing is very sensitive to noise in the image.
- Image smoothing should be serious consideration prior to the use of derivatives in applications where noise is likely to be present.
- To determine a point as an edge point
  - ✓ the transition in grey level associated with the point has to be significantly stronger than the background at that point.
  - ✓ use threshold to determine whether a value is “significant” or not.
  - ✓ the point’s two-dimensional first-order derivative must be greater than a specified threshold.
- Need more operations to assemble edge segments into longer edges.



Chapter 10 :Image Segmentation

Gradient Operator

- First derivatives.

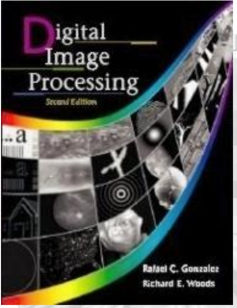
$$\nabla \mathrm{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

The magnitude of the gradient.

$$\nabla f = mag(\nabla \mathrm{f}) = [G_x^2 + G_y^2]^{1/2} = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

- Approximation of the gradient magnitude.

$$\nabla f \approx |G_x| + |G_y|$$

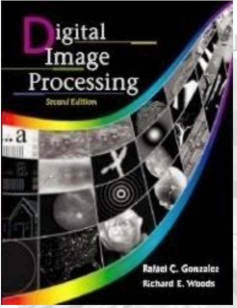


Gradient Operator

- Gradient direction
  - ✓ Let α (x,y) represent the direction angle of the vector ∇f at (x,y)

$$\alpha(x,y) = \tan^{-1}(G_y/G_x)$$

  - ✓ The direction of an edge at (x,y) is perpendicular to the direction of the gradient vector at that point



Chapter 10 :Image Segmentation

Gradient Masks

a
b c
d e
f g

FIGURE 10.8 A 3 × 3 region of an image (the z's are gray-level values) and various masks used to compute the gradient at point labeled z_5.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

−1	0
0	1

0	−1
1	0

Roberts

−1	−1	−1
0	0	0
1	1	1

−1	0	1
−1	0	1
−1	0	1

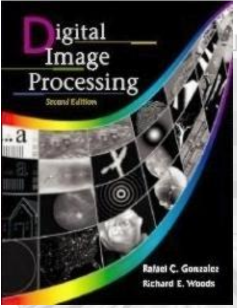
Prewitt

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel

What are the differences among these masks?



Masks for detecting diagonal edges

Prewitt

0	1	1
−1	0	1
−1	−1	0

−1	−1	0
−1	0	1
0	1	1

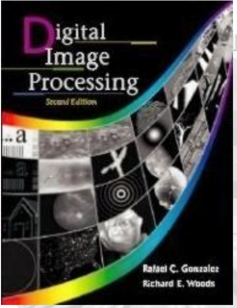
Sobel

0	1	2
−1	0	1
−2	−1	0

−2	−1	0
−1	0	1
0	1	2

a b
c d

FIGURE 10.9 Prewitt and Sobel masks for detecting diagonal edges.

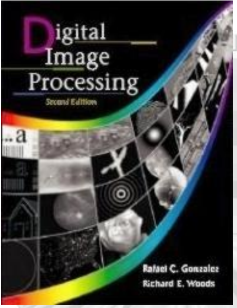


Chapter 10 :Image Segmentation

Example 1/3

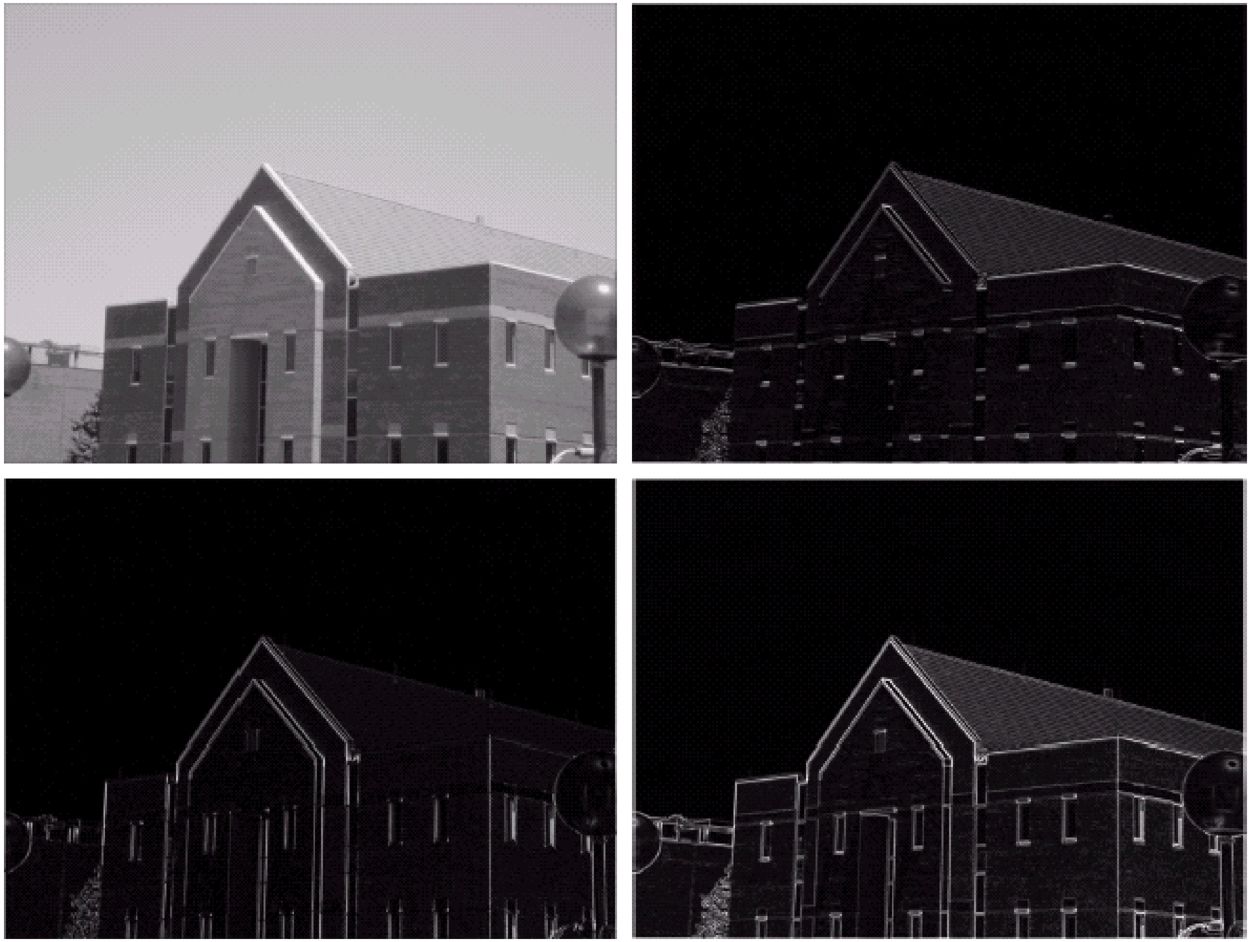
a b
c d

FIGURE 10.10
(a) Original image. (b) $|G_x|$, component of the gradient in the x-direction.
(c) $|G_y|$, component in the y-direction.
(d) Gradient image, $|G_x| + |G_y|$.



Chapter 10 :Image Segmentation

Example 2/3 : with smoothing filter



a b
c d

FIGURE 10.11 Same sequence as in Fig. 10.10, but with the original image smoothed with a 5×5 averaging filter.

Example 3/3: diagonal mask

a b

FIGURE 10.12 Diagonal edge detection. (a) Result of using the mask in Fig. 10.9(c). (b) Result of using the mask in Fig. 10.9(d). The input in both cases was Fig. 10.11(a).

Laplacian Operator

Laplacian operator

(linear operator)

$$\nabla^2 f = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)]$$

FIGURE 10.13 Laplacian masks used to implement Eqs. (10.1-14) and (10.1-15), respectively.

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

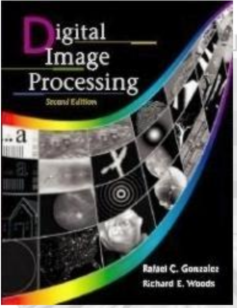
Laplacian of Gaussian (LoG)

- Laplacian operator is very sensitive to noise(need smoothing before applying Laplacian)
- Combine Laplacian operator with Gaussian smoothing operator.

$$h(r) = -e^{-\frac{r^2}{2\sigma^2}}$$

$$\nabla^2 h(r) = -\left[\frac{r^2 - \sigma^2}{\sigma^4}\right] e^{-\frac{r^2}{2\sigma^2}}$$

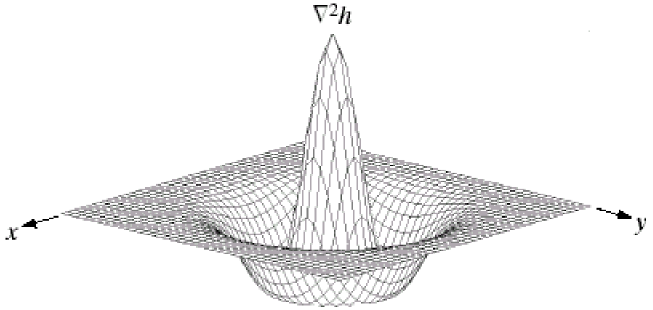
, where r2 = x2+y2, and σ is the standard deviation

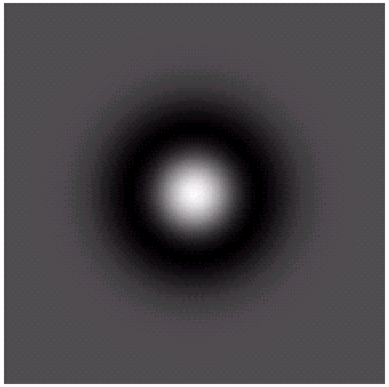


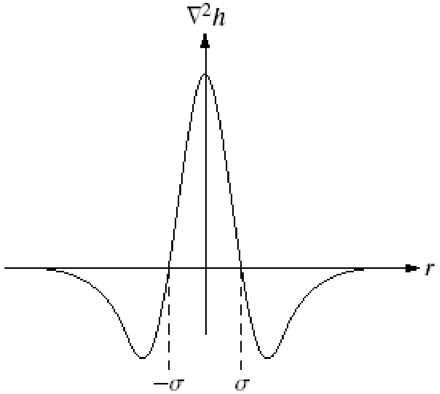
Chapter 10 :Image Segmentation

Laplacian of Gaussian (LoG)

* Mexican Hat







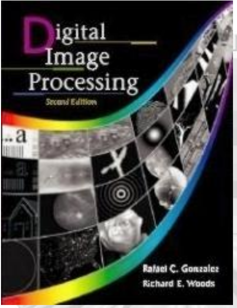
0	0	–1	0	0
0	–1	–2	–1	0
–1	–2	16	–2	–1
0	–1	–2	–1	0
0	0	–1	0	0

a	b
c	d

FIGURE 10.14 Laplacian of a Gaussian (LoG). (a) 3-D plot. (b) Image (black is negative, gray is the zero plane, and white is positive). (c) Cross section showing zero crossings. (d) 5 × 5 mask approximation to the shape of (a).

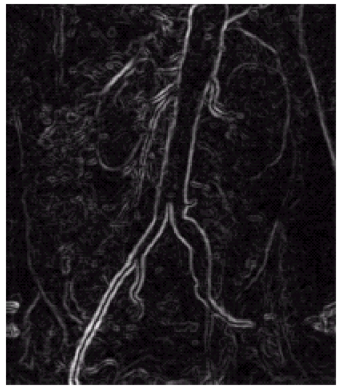
Laplacian of Gaussian (LoG)

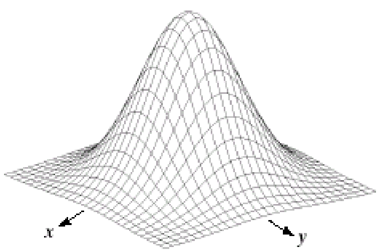
- The coefficient must be summed to zero
- The approximation is generated to capture the essential shape of Mexican hat function. So it is not unique.
- Second derivation is linear operation. Thus, $\nabla^2 f$ is the same as convolving the image with Gaussian smoothing function first and then computing the Laplacian of the result



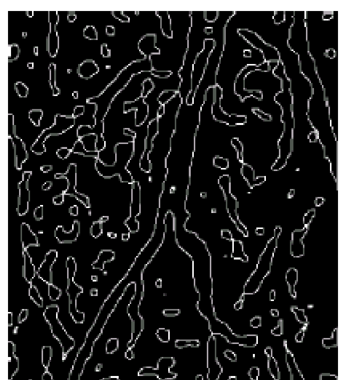
Chapter 10 :Image Segmentation

Examples





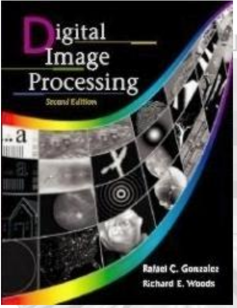
–1	–1	–1
–1	8	–1
–1	–1	–1



a). Original image
b). Sobel Gradient
c). Spatial Gaussian smoothing function
d). Laplacian mask
e). LoG
f). Threshold LoG
g). Zero crossing

Zero crossing using LoG

- Approximate the zero crossing from LoG image
- To threshold the LoG image by setting all its positive values to white and all negative values to black.
- The zero crossing occur between positive and negative values of the thresholded LoG.
- Advantages over Gradient
  - ✓ Zero crossing produces thinner edges
- Disadvantages over Gradient
  - ✓ Zero crossing creates closed loops. (spaghetti effect)
  - ✓ Need sophisticated zero crossing techniques to obtain acceptable results.
- Edge finding methods based on various implementation of the Gradient are used more frequently .



Edge Linking and boundary detection

- Edge detection algorithm are followed by linking procedures to assemble edge pixels into meaningful edges.
- Basic approaches
  - ✓ Local Processing
  - ✓ Global Processing via the Hough Transform
  - ✓ Global Processing via Graph-Theoretic Techniques

Local Processing

- Analyze the characteristics of pixels in a small neighborhood (say, 3x3, 5x5) about every edge pixels (x,y) in an image.
- All points that are similar according to a set of predefined criteria are linked, forming an edge of pixels that share those criteria.

<u>Criteria 1:</u> the strength of the response of the gradient operator used to produce the edge pixel

✓an edge pixel with coordinates (x_0,y_0) in a predefined neighborhood of (x,y) is similar in magnitude to the pixel at (x,y) if

$$|\nabla f(x,y) - \nabla f(x_0,y_0)| \leq E$$

Local Processing

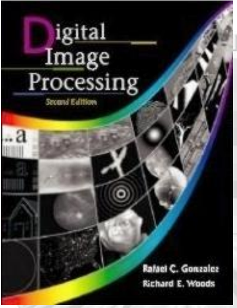
Criteria 2: the direction of the gradient vector

- ✓an edge pixel with coordinates (x_0,y_0) in a predefined neighborhood of (x,y) is similar in angle to the pixel at (x,y) if

$$|\alpha(x,y) - \alpha(x0,y0)| < A$$

Linking:

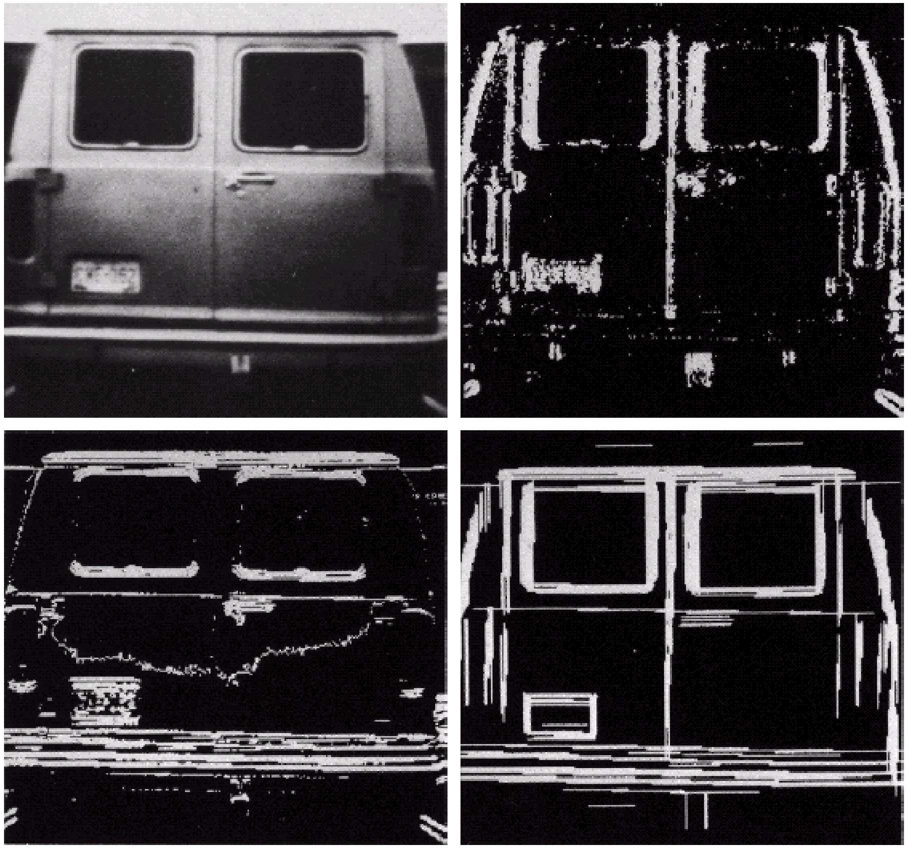
- ✓A point in the predefined neighborhood of (x,y) is linked to the pixel at (x,y) if **both magnitude and direction criteria are satisfied.**
- ✓the process is repeated at every location in the image
- ✓a record must be kept
- ✓simply by assigning a different gray level to each set of linked edge pixels.



Example for local processing 1/2

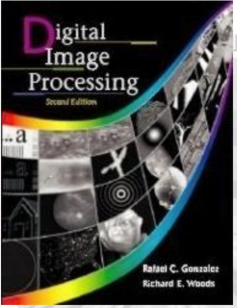
- Find rectangles whose sizes make them suitable candidates for license plates
- Procedures:
  - ✓ Apply horizontal and vertical Sobel operators
  - ✓ Eliminate isolated short segments
  - ✓ Link the segments using the following conditions:
    - Gradient value threshold E=25
    - Gradient direction threshold A=15

Example for local processing 2/2



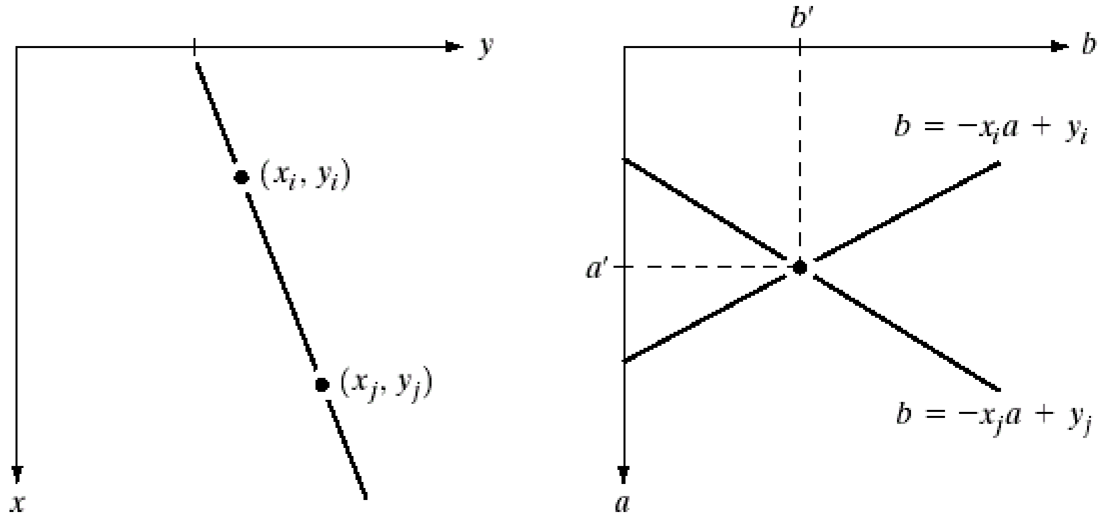
a b
c d

FIGURE 10.16 (a) Input image. (b) G_y component of the gradient. (c) G_x component of the gradient. (d) Result of edge linking. (Courtesy of Perceptics Corporation.)



Chapter 10 :Image Segmentation

Hough Transformation(Line)



a b

FIGURE 10.17
(a) *xy*-plane.
(b) Parameter space.

<image space> < parameter space>0

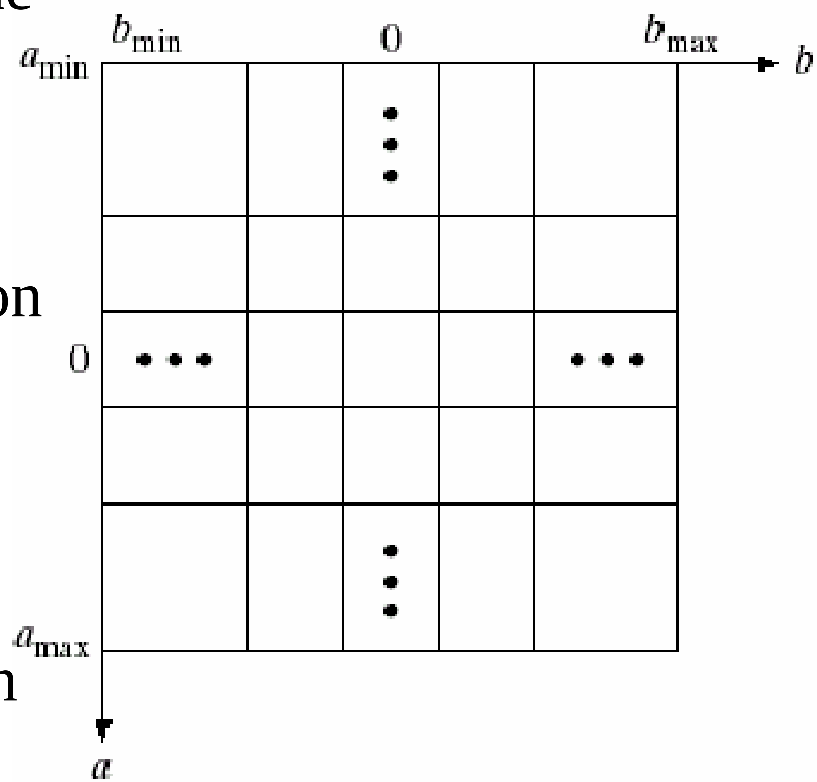
$y_i = ax_i + b$ $b = -ax_i + y_i$

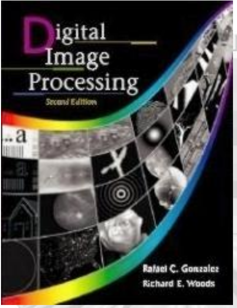
All points (xi ,yi) on the line, $y_i = ax_i + b$, must have lines in parameter space that intersect at (a’,b’)

Accumulator

- (a_{max}, a_{min}) and (b_{max}, b_{min}) are the expected ranges of slope and intercept values.
- all are initialized to zero
- if a choice of a_p results in solution b_q then we let

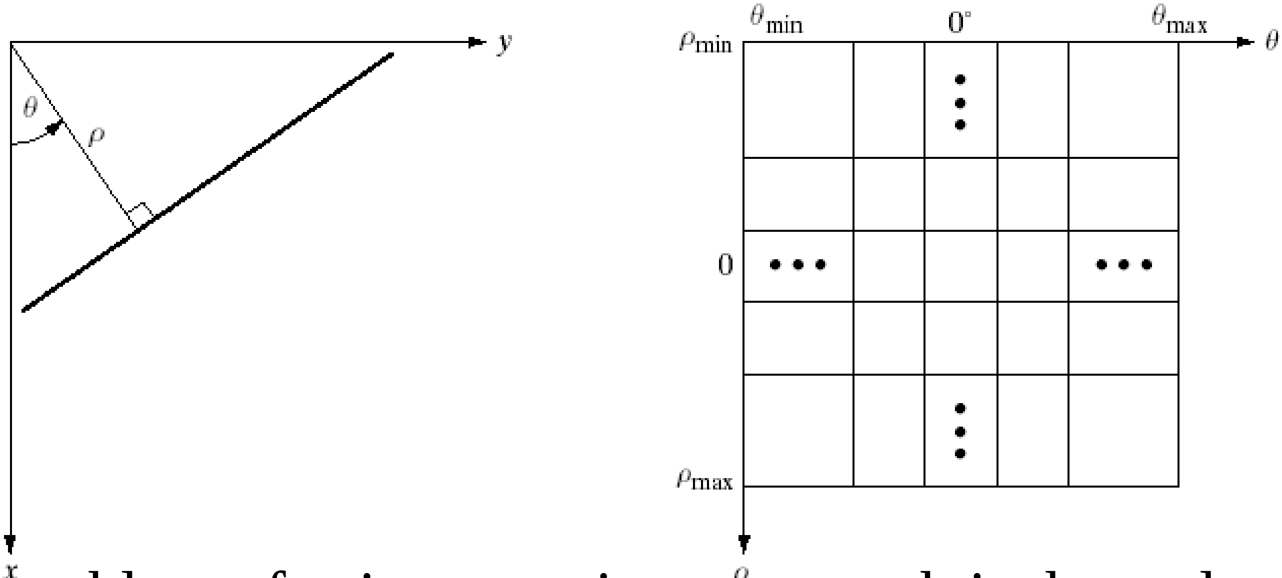
  $A(p,q) = A(p,q)+1$
- at the end of the procedure, value Q in $A(i,j)$ corresponds to Q points in the xy-plane lying on the line $y = a_i x+b_j$





Chapter 10 :Image Segmentation

ρθ-plane



a b

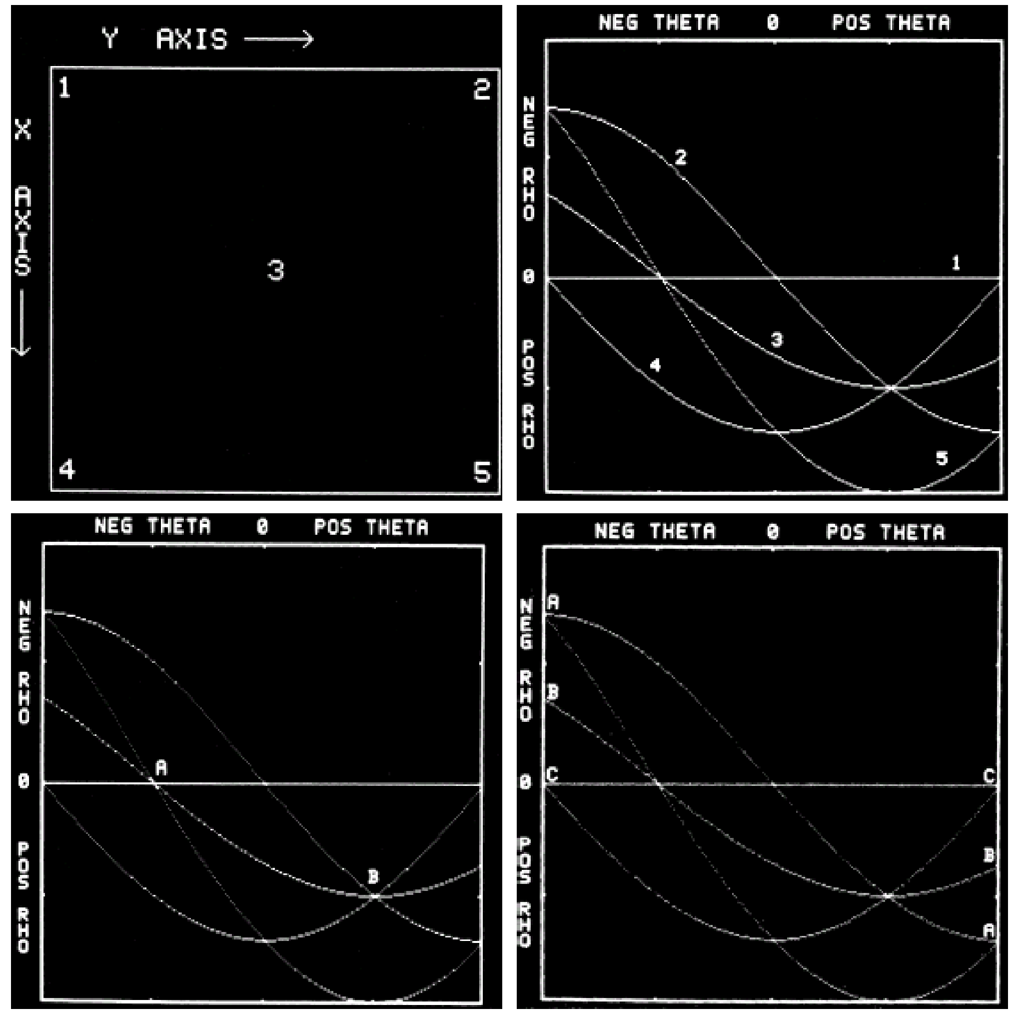
FIGURE 10.19 (a) Normal representation of a line. (b) Subdivision of the $\rho\theta$-plane into cells.

- problem of using equation y = ax + b is that value of a is infinite for a vertical line.
- To avoid the problem, use equation x cos θ+ y sin θ = ρ to represent a line instead.
- vertical line has θ = 90° with ρ equals to the positive y-intercept or θ = -90° with ρ equals to the negative y-intercept

Illustration of the Hough transform

a b
c d

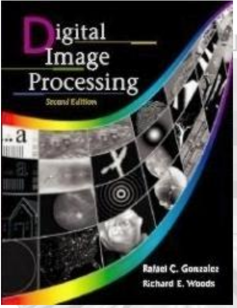
FIGURE 10.20 Illustration of the Hough transform. (Courtesy of Mr. D. R. Cate, Texas Instruments, Inc.)



range of $\rho = \pm D$
, where D is the distance between corners in the image

Edge Linking example based on HT

1. Compute the gradient of an image and threshold it to obtain a binary image.
2. Specify subdivisions in the ρθ-plane.
3. Examine the counts of the accumulator cells for high pixel concentrations.
4. Examine the relationship (principally for continuity) between pixels in a chosen cell.
   - ✓Trace the line represented by the cell
   - ✓Identify the disconnected pixels and compute the distance between them.
   - ✓Connect the disconnected segments if the distance(or gap) is less than a certain threshold.



Chapter 10 :Image Segmentation

Hough Transformation

- We can extend HT to detect any shape which can be represented by a function of the form

  $g(v,c) = 0$, where v is a vector of coordinates and c is a vector of coefficients.

- Circle detection example

$$g(v,c)= (x-c_1)^2 + (y-c_2)^2 - c_3= 0$$

  ✓Use a 3D accumulator of the form A(i,j,k)

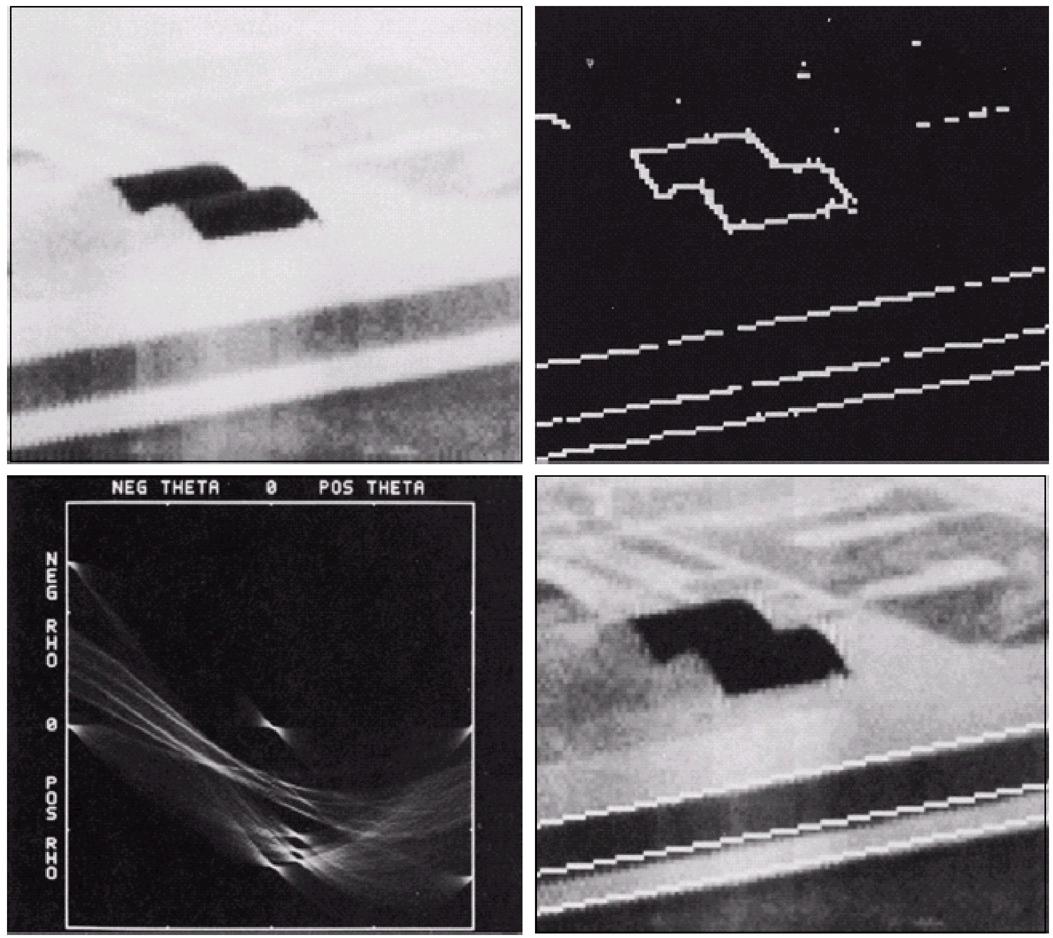
  ✓For a given edge point (x,y), find a set of parameter vectors satisfying the equation: $(c_1 - x)^2 + (c_2 -y)^2 - c_3= 0$

  ✓Increment all the cells associated with the parameter vectors.

Chapter 10 :Image Segmentation

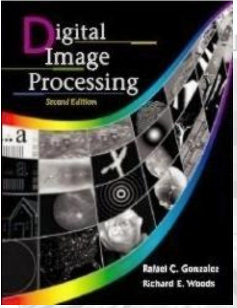
Edge Linking example based on HT

Link if
1.the pixel belonged to one of the three highest peak cells
2.No gaps were longer than 5 pixels



a b
c d

FIGURE 10.21 (a) Infrared image. (b) Thresholded gradient image. (c) Hough transform. (d) Linked pixels. (Courtesy of Mr. D. R. Cate, Texas Instruments, Inc.)

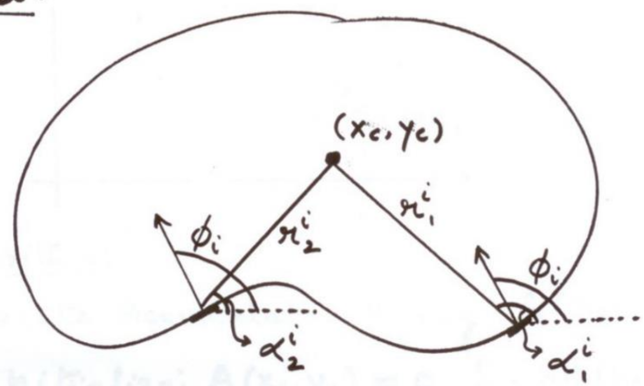


Generalized Hough Transform.

- We also can extend HT to detect arbitrary shape by representing the shape with the reference table.
  - ✓Draw the shape on an image space
  - ✓Select a point on the shape as a reference point.
  - ✓Represent each boundary point as a vector starting from the reference point
  - ✓Store them in a table.

Generalized Hough Transform.

Model :



Edge Direction	$\bar{r} = (r, \alpha)$
ϕ_1	$\bar{r}_1^1, \bar{r}_2^1, \bar{r}_3^1$
ϕ_2	$\bar{r}_1^2, \bar{r}_2^2$
ϕ_i	$\bar{r}_1^i \vdots \bar{r}_2^i$
ϕ_n	$\bar{r}_1^n, \bar{r}_2^n$

Fig. R-Table

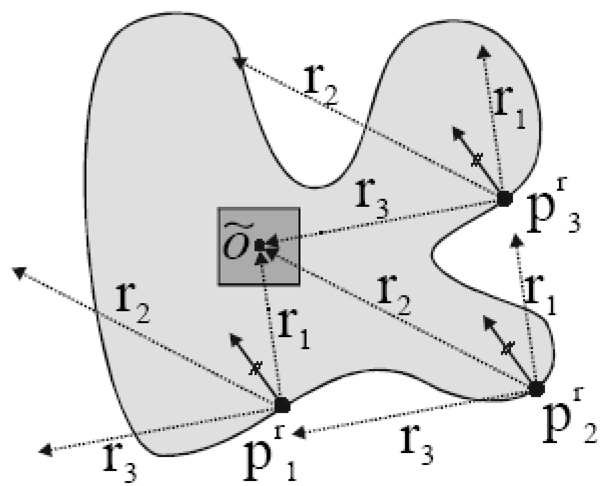
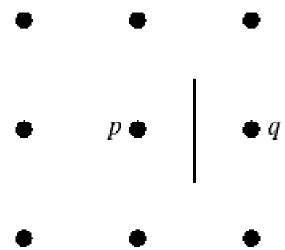


Fig. Vote in Hough Space

Chapter 10 :Image Segmentation

Global Processing via Graph Theoretic Techniques

FIGURE 10.22 Edge element between pixels *p* and *q*.



Cost of a path:

$$c = \sum_{i=2}^{k} c(n_{i-1}, n_i)$$

Edge element (boundary between two pixels p and q:

$$c(p,q) = H - [f(p) - f(q)]$$

Global Processing via Graph Theoretic Techniques

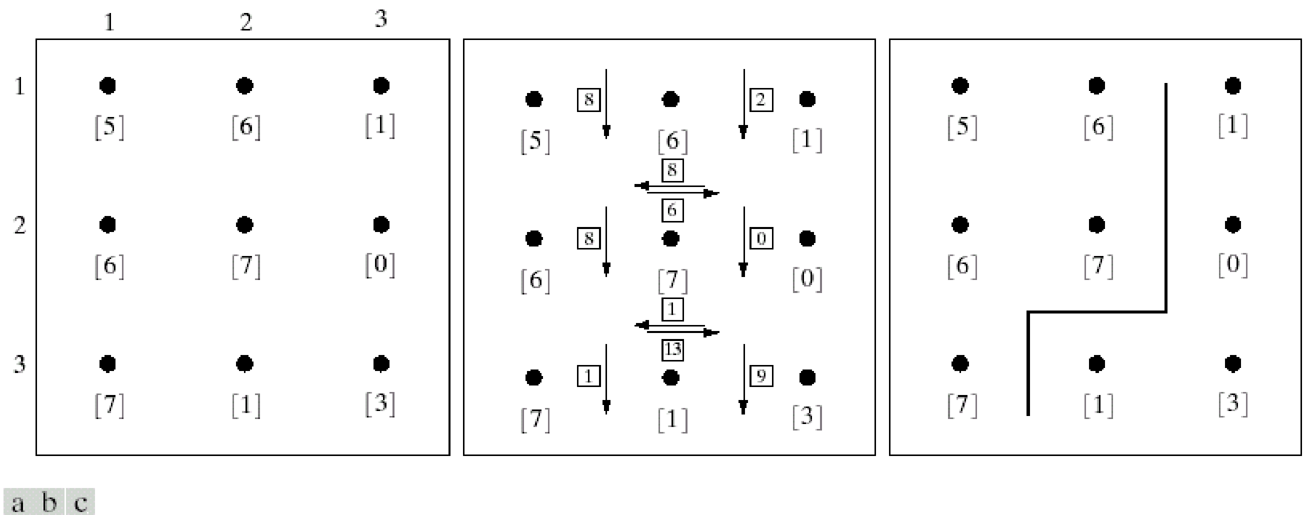


FIGURE 10.23 (a) A 3 × 3 image region. (b) Edge segments and their costs. (c) Edge corresponding to the lowest-cost path in the graph shown in Fig. 10.24.

Chapter 10 :Image Segmentation

Global Processing via Graph Theoretic Techniques

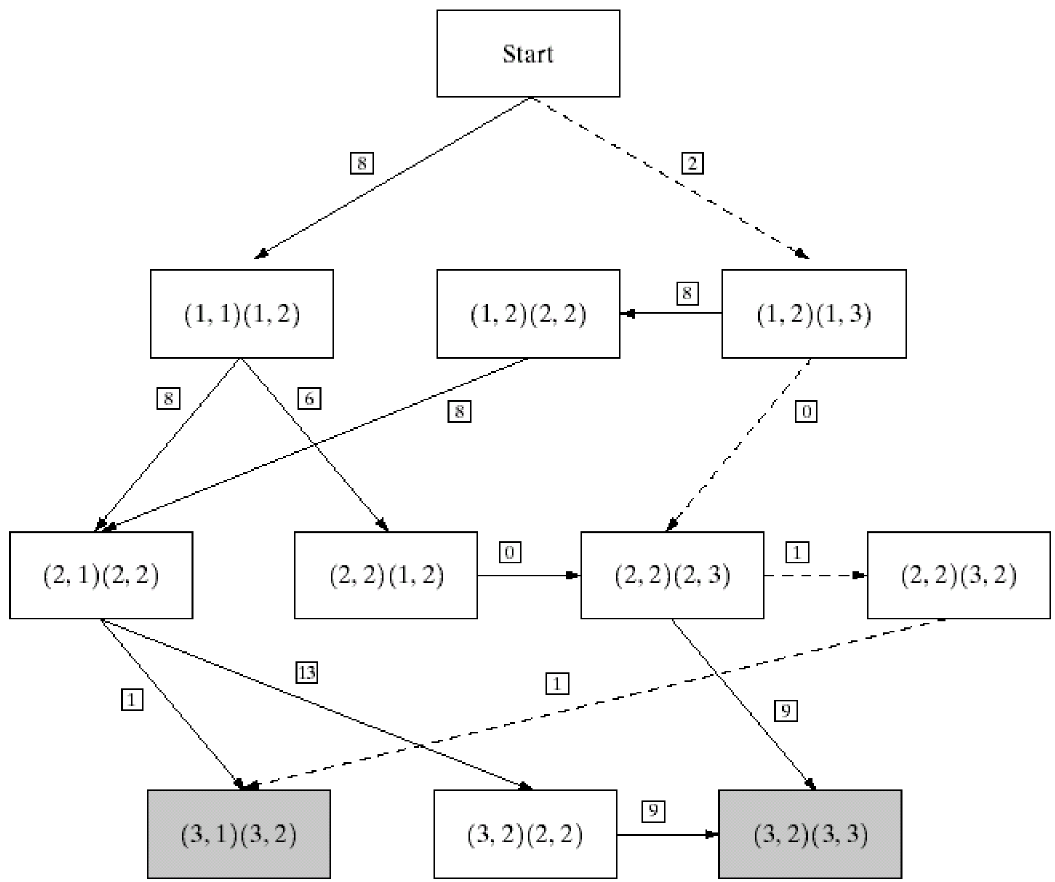
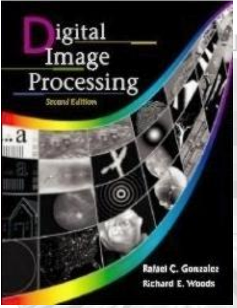


FIGURE 10.24 Graph for the image in Fig. 10.23(a). The lowest-cost path is shown dashed.



Chapter 10 :Image Segmentation

Global Processing via Graph Theoretic Techniques

FIGURE 10.25 Image of noisy chromosome silhouette and edge boundary (in white) determined by graph search.

Chapter 10 :Image Segmentation