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Section: CSE-1A

CSE 4549: Simulation and Modelling.

Ans. to Q. no. 2(a)

Here, $Z_i = (13 \times Z_{i-1} + 13) \bmod 16$

Here, $a = 13$
$c = 13$
and $m = 16$

The conditions to have full period are:

(i) c and m must be relatively prime to each other.
Here, $c = 13$ and $m = 16$. They are co-prime.
So, this condition is fulfilled.

(ii) $(a-1)$ and m must have a prime number q which can divide both of them,
Here, $(a-1) = 13 - 1 = 12$ and ~~16~~ $m = 16$ and both can be divided by prime number 2.
This condition is fulfilled.

(iii) $a = 4k + 1$ where $k = 1, 2, 3, \ldots$
Here, $a = 13 = 4 \cdot 3 + 1$.
So, this condition is fulfilled.

Since, all 3 conditions are fulfilled, we can achieve full cycle for this LCG. (Ans.)

Ans. to Q.no. 2(b)

$$Z_i = (4951 \times Z_{i-1} + 247) \bmod 256$$

Here, $a = 4951$
$c = 247$
$m = 256.$

For full cycle, the conditions are:

(i) c and m must be relatively prime.
$c = 247$ and $m = 256$ which are relatively prime.
So, condition is fulfilled.

(ii) $(a-1)$ and m must have common divisor q which has to be a prime number, and divide both.
$(a-1) = (4951-1) = 4950$ and $m = 256$ has $q = 2$ as prime divisor, which can divide both. Condition fulfilled.

(iii) $a = 4k+1$, $k = 1, 2, 3 \ldots$
$4951 = 4 \cdot 1237 + 3$ which does not meet condition.

So, condition (iii) is not fulfilled and this LCG CAN NOT have full ~~cycle~~ period. (Ans.)

Ans. to Q. no 3(a)

We can use the inverse transformation method to develop the random value generator.

Given, $f(x) = \begin{cases} 1/3 & 0 \le x \le 2 \\ 1/24 & 2 \le x \le 10 \\ 0 & o/w \end{cases}$

$\therefore$ CDF is $F(x) = \begin{cases} \int_0^x 1/3 \, dx & 0 \le x \le 2 \\ \int_2^x 1/24 \, dx & 2 \le x < 10 \\ 0 & o/w \end{cases}$

$$F(x) = \begin{cases} \left[\frac{x}{3}\right]_0^x \\ \left[\frac{x}{24}\right]_2^x \\ o/w \end{cases}$$

For the first condition, let, $u = F(x)$

$$u = \frac{x}{3} - \frac{0}{3}$$

$$\Rightarrow x = 3u$$

$$\Rightarrow F^{-1}(x) = 3u$$

$$\Rightarrow F^{-1}(u) = 3u \quad [0 \le u \le 0.2]$$

Again, $u = \left[\frac{x}{24}\right]_2^x$

$$= \frac{x}{24} - \frac{2}{24}$$

$$u = \frac{x-2}{24}$$

$$\Rightarrow x = 24u + 2$$

$$\Rightarrow F^{-1}(u) = 24u + 2 \quad [0.2 < u \leq 1]$$

Algorithm

(i) Generate random number $u \sim U(0,1)$. The random number can be generated with LCG. $Z_i = (aZ_{i-1} + c) \bmod m$ taking a 2^b value for m.

(ii) Put u in the inverse function of $F(X)$.

(iii) Return the output and repeat steps (i) to (iii) for random variates.

$$F^{-1}(u) = \begin{cases} 3u & [0 < u \leq 0.2] \\ 24u + 2 & [0.2 < u < 1] \\ 0 & o/w \end{cases}$$

If $u=0.665$, then

$$F^{-1}(u)=24\times 0.665+2$$
$$=17.96$$

If $u=0.225$, then,

$$F^{-1}(0.225)=24\times 0.225+2$$
$$=7.4$$

If $u=0.125$, then

$$F^{-1}(0.125)=3\times 0.125$$
$$=0.375$$

If $u=0.965$, then

$$F^{-1}(0.965)=24\times 0.965+2$$
$$=25.16$$

If $u=0.115$, then

$$F^{-1}(0.115)=3\times 0.115$$
$$=0.345$$

If $u=0.445$, then,

$$F^{-1}(0.445)=24\times 0.445+2$$
$$=12.68$$

If $u=0.555$ then,

$$F^{-1}(0.555)=24\times 0.555+2$$
$$=15.32$$

Ans. to Q no. 3(b)

$$f(x) = \begin{cases} \frac{1}{2}(x-2) & 2 \le x \le 3 \\ \frac{1}{2}(2-x/3) & 3 \le x \le 6 \\ 0 & o/w \end{cases}$$

$\boxed{(i)}$ First we find majoring function,

$f(2) = \frac{1}{2}(x-2) = 0$

$f(3) = \frac{1}{2}(3-2) = 1/2$

So, when, $2 \le x \le 3$, $t(x) = 1/2$

And, when, $3 \le x \le 6$,

$f(3) = \frac{1}{2}(2-1) = \frac{1}{2}$

$f(6) = \frac{1}{2}(2-2) = 0$

So, majoring function for both cases is

$$t(x) = 1/2$$

Now, $c = \int_2^6 t(x)dx = \int_2^6 1/2\, dx = \left[\frac{x}{2}\right]_2^6 = \frac{6-2}{2} = 2$

So, $r(x) = \frac{t(x)}{2} = \frac{1}{4}$

Algorithm,

(i) We generate $u_1 \sim U(0,1)$.

(ii) Then we generate $Y \sim r(x)$ using u_1.

$$u_1 = \frac{y-a}{b-a}$$

$$\Rightarrow y = a + u_1(b-a)$$
$$= 2 + 4u_1$$

In this way we can generate values of y.

(iii) Then for each value of Y,

we ~~gent~~ generate $u_2 \sim U(0,1)$ independent of Y.

If $u_2 \leq \frac{f(Y)}{t(Y)}$, then $X = Y$ i.e. we accept Y.

or else, we reject Y.

$\boxed{\text{(ii)}}$ Let, $u_1 = 0.9, 0.8, 0.11, 0.51, 0.2$

$u_2 = 0.8, 0.3, 0.1, 06, 0.5$

From u_1, $y = 5.6, 5.2, 2.44, 4.04, 2.88$

For, the first number,

$$0.8 \leq \frac{f(5.6)}{t(5.6)} = \frac{1/15}{0.5} = 0.133$$

$$\Rightarrow 0.8 \leq 0.133 \text{ (false)}$$

This is false, so, Y is rejected.

For second no,

$$0.3 \leq \frac{2/15}{0.5} = 0.267$$

False, and Y is rejected.

Third no, $0.1 \leq \dfrac{f(2.44)}{0.5} = 0.11.$

True, hence, 2.44 is accepted.

$$068 \leq \frac{f(4.04)}{t(4.04)} = \frac{49/150}{0.5} = 0.6533$$

True, hence, 4.04 is accepted.

$$0.5 \leq \frac{f(2.88)}{t(2.88)} = \frac{0.44}{0.5} = 0.22$$

False, hence Y is rejected.

~~Accepted values, 0.11,~~

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Ans. to Q. no. 1 (a)

The goals and objectives of this simulation is to ~~to~~ find information regarding a coal train station where average, maximum time a train spends, proportion of times the station is busy, idle and hogged out, and average and maximum no. of trains in the queue, is to be found. Through this simulation we can make decisions and derive conclusions, about the system.

Ans. to Q. no. 1 (b)

State Variables:

(i) System Status, $x(t)$

(ii) No. of trains in queue, $q(t)$

Output Variables:

(i) Average time in system

(ii) Maximum time in system

(iii) Proportion of time ~~unloading eq.~~ system is busy

(iv) Proportion of time system is idle

(v) Proportion of time system is hogged out

(vi) Avg. no. of trains in queue.

(vii) ~~Avg~~ Max. no. of trains in queue.

Ans to Q.no.1 (c)

Set of events :

(i) Train Arrival (A)

(ii) Train Departure (D)

(iii) Hog out (H)

(iv) Termination (T)

Ans to Q.no. 1(d)

The system states X_x is $\{0,1\}$ $\begin{pmatrix} 0 - \text{idle} \\ 1 - \text{active} \end{pmatrix}$

The queue length X_s is $\{0, 1, 2 \ldots\}$

State space is $X_{x,s} = \{(0,0), (0,1), (0,2), (0,3) \ldots$
$(1,0), (1,1), (1,2) \ldots\}$

So, any combination of 0,1 with ~~real~~ ~~no~~ a positive integer is possible.

$(0,1), (0,2) \ldots$ means hogged out states.

Ans. to Qu. 1(e)

State equation:

$$x(t^+) = \begin{cases} q(t)==0\,?\ 1 : x(t) & \text{Arrival} \\ q(t)>0\,?\ x(t) : 0 & \text{Departure} \\ x(t).\text{hog\_out}()\,?\ 0 : x(t) & \text{Hogged Out} \\ x(t) & \text{o/w} \end{cases}$$

$$q(t^+) = \begin{cases} x(t)==0\,?\ q(t) : q(t)+1 & \text{arrival} \\ \max(0, q(t)-1) & \text{departure} \\ x(t).\text{hogout}()\,?\ q(t)+1 : q(t) & \text{Hogout} \\ q(t) & \text{o/w} \end{cases}$$

$x(t).\text{hogout}()$ means if the train taking service is hogged out or not.

Output Equations:

(i) Average time in system,

Time in system, $w_i = d_i + s_i$ → service time

d_i → delay time

a_i be arrival and b_i be departure time.

$$d_i = \max(0, b_i - a_i)$$

$$\bar{w} = \frac{\sum_{i=0}^{n-1} w_i}{n} \quad \text{[Job average]}$$

(ii) Maximum time in system, mx_w

Let, mx_w be initialized at 0.

$$mx_w = \max(mx_w, w_i)$$

i.e. maximum ~~wait~~ wait time is there at end of simulation.

(iii) System busy time, for $(\tau, 0)$ time

$$\bar{B} = \frac{\int_0^{\tau} x_1(t)\,dt}{\tau} \quad \text{(Time Average)}$$

x_1 time when $x_1(t) = 1$.

$$\cancel{x_1(t) = \max(x_1(t), 0)}$$

(iv) System idle time, $(\tau, 0)$

$$\bar{I} = \frac{\int_0^{\tau} x_0(t)\,dt}{\tau}$$

~~where, $x_0(t) = \max \min(x(t), 1)$.~~

~~i.e. time~~

where, $x_0(t)$ time when $x_0(t) = 0$.

(v) Hog at time.

$$\bar{H} = \frac{\int_0^{\tau} x_H(t)\,dt}{\tau}$$

When X_H is countd when $x(t) = 0$

$q(t) > 0$.

(vi) Average no. trains,

$$\bar{N} = \frac{\int_0^{\tau} l(t)\,d(t)}{\tau}$$

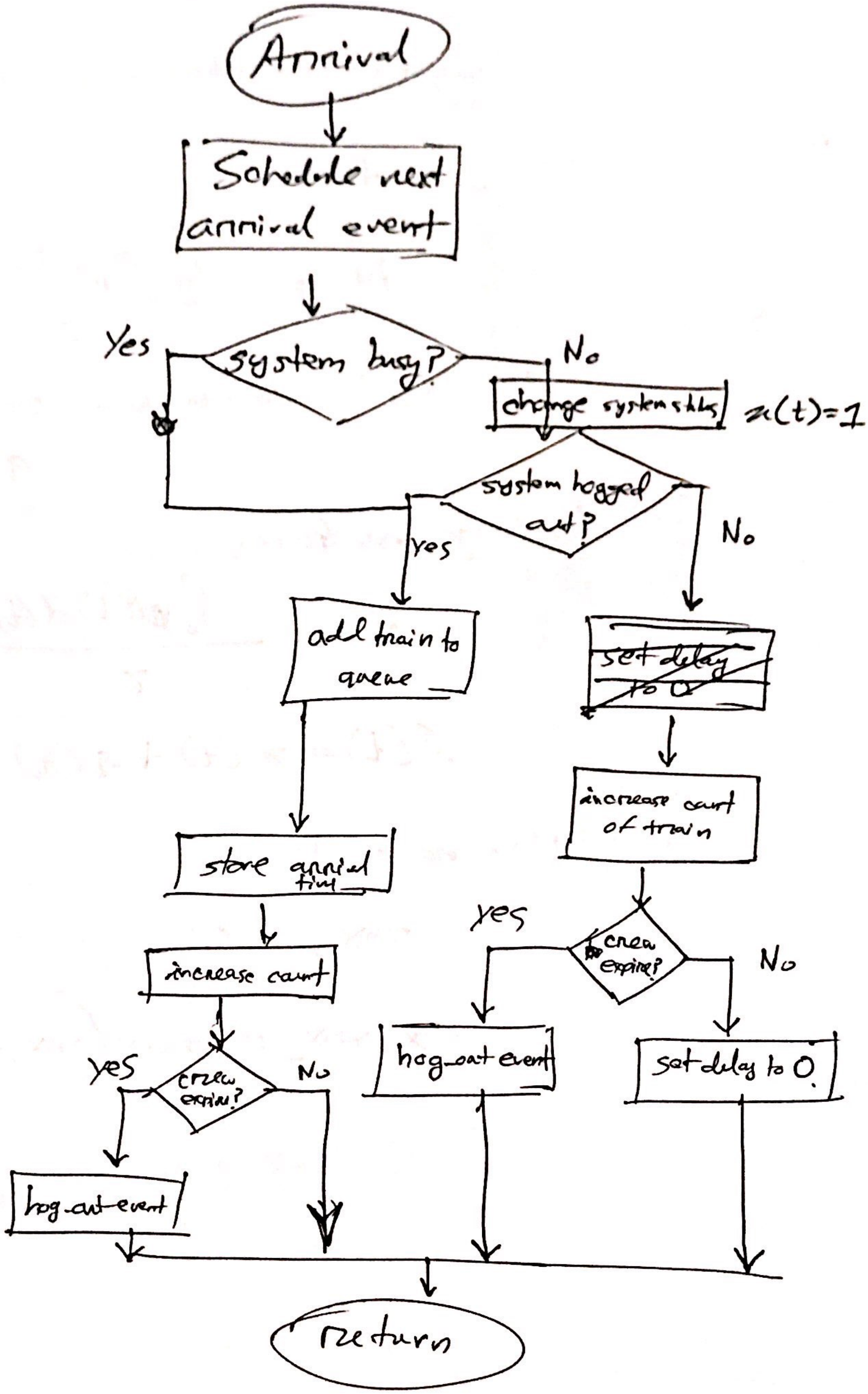
$$l(t) = x(t) + q(t)$$

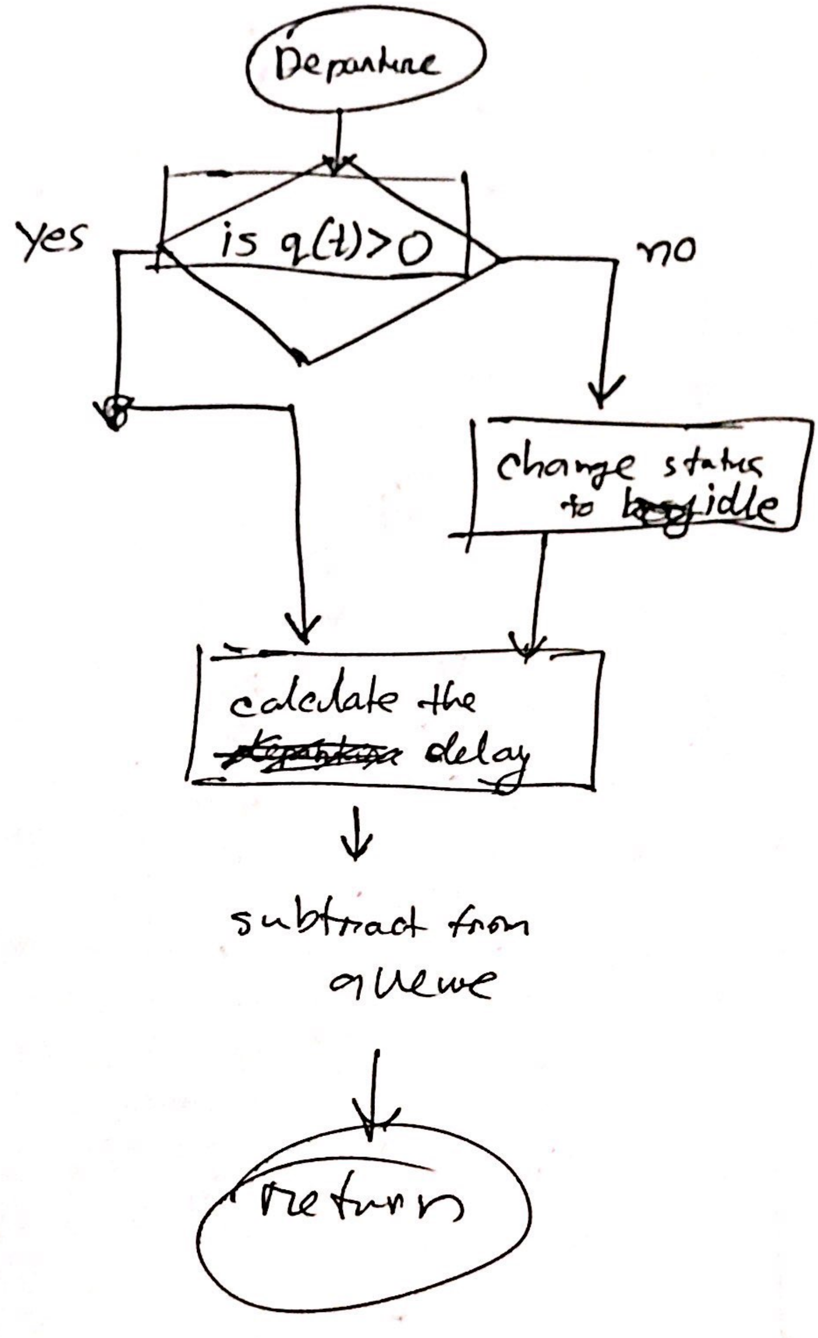
(vii) Max no of trains,

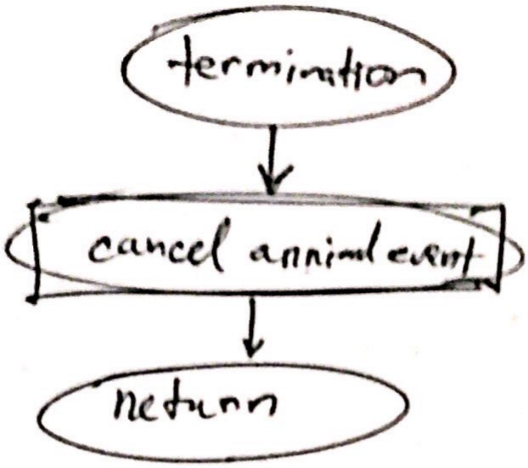
$$mx_n = 0.$$

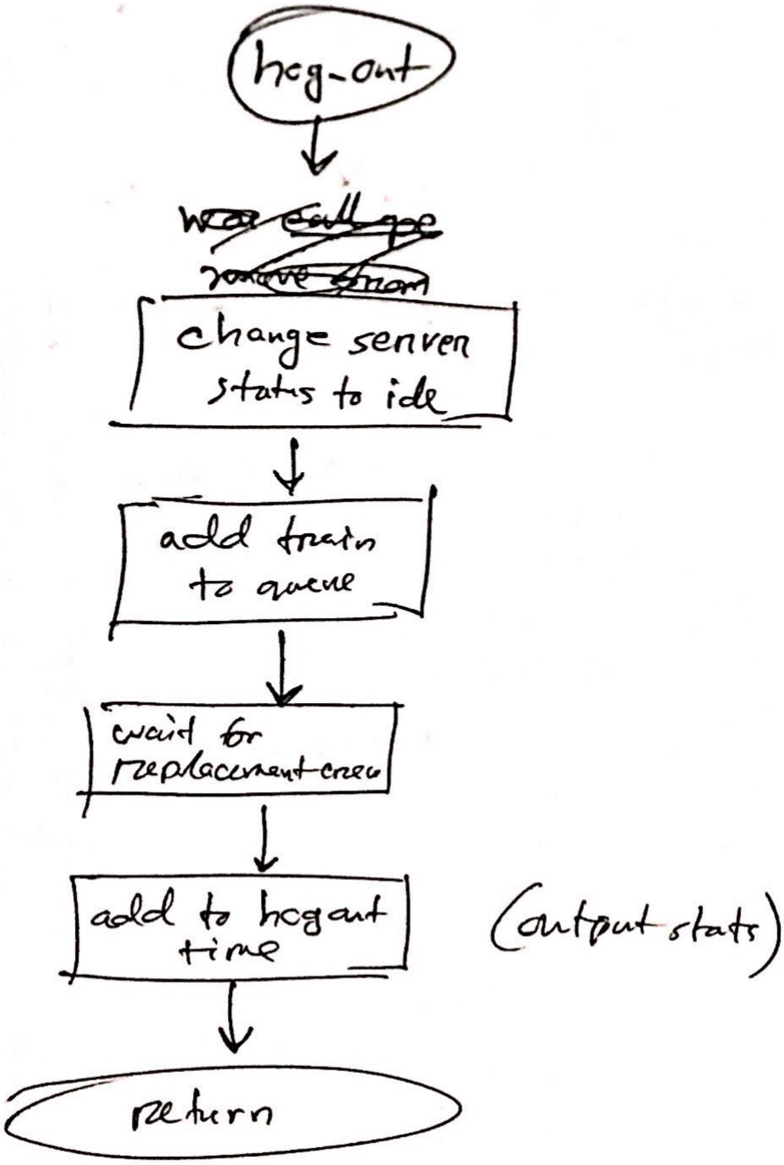
$$mx_n = \max(mx_n, l(t)).$$

Ans to. Q no 1(f)


Arrival
Schedule next arrival event
Yes
system busy?
No
change system status
x(t)=1
system hogged out?
yes
No
add train to queue
~~set delay to 0~~
increase count of train
store arrival time
increase count
yes
crew expire?
No
yes
crew expire?
No
hog-out event
hog-out event
set delay to 0.
Return


Departure
is q(t)>0
yes
no
change status to ~~busy~~ idle
calculate the ~~departure~~ delay
subtract from queue
return


termination
cancel arrival event
return


hog-out
change server status to ide
add train to queue
wait for replacement crew
add to hogout time
(output stats)
return