Md. Farhan Ishmam
180041120

Student ID : 180041120

Name : Md Farhan Ishmam

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Md. Farhan Ishmam
180041120

Ans. to Q.no. 1(a)

Difference between horizontal trace and level curve is for the level curve's various value of $c = 0, 1, 2 \ldots$, it will produce ~~one~~ a function in xy plane. On the other hand, a horizontal trace will not have output of a ~~x~~ 2d function.

For example, $f(x,y) = \sqrt{64 - x^2 - y^2}$, we take value of ~~c~~ $c = 0, 1, 2$ and get xy graphs. For level curves. But for horizontal trace, we get different 3d graph.

Ans. to Q no. 1(b)

Two level curves of the same function cannot ~~can~~ intersect. But if the level curve is of 2 distinct functions then they can intersect.

$$\text{Ex} - \sqrt{4 - x^2 - y^2} = c_0$$

$$\sqrt{64 - x^2 - y^2} = c_1$$

~~for $c_0 = 4$ and $c_1 = 64$ both level curves will intesect at~~

These two level curves will intersect.

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180041120

Ans. to Q. no. 1(a)

(i)

$f(x,y) = \sqrt{9-x^2-y}$

The domain is when $9-x^2-y > 0$

$\Rightarrow 9 > x^2+y$

$\Rightarrow x^2+y < 9$

The domain is the area inside region $x^2+y=9$.

~~x^2+y~~

Domain is $\{x,y \in R,$ where $x^2+y<9\}$

$Dom_f =$

The range is $[0, 3)$.

$Range_f = [0,3)$

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180041120

(ii)

$$g(x,y,z) = x\sqrt{y} + \ln(z-1)$$

Here, $y>0$ and $z>1$.

Domain is ~~x, z~~ y

~~Dom g = {~~

$$Dom_g = \{x, y, z \in R \text{ where } y>0 \text{ and } z>1\}$$

And range is $(-\infty, +\infty)$ (Ans.)

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180041120

Ans. to Q.no. 2(a)

Given, $f(2,y) = y^3$

then, $f(2,3) = \lim\limits_{y \to 3} y^3$ [$\because f(x,y)$ is continuous at $(2,3)$]

$= 3^3$

$= 27$

Ans: 27

Ans. to Q.no. 2(b)

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$$

We approach the limit from $x \to 0$ and then $y \to 0$

$$\lim_{y\to 0}\left(\lim_{x\to 0} \frac{x^2}{x^2+y^2}\right) = \lim_{y \to 0} \frac{0^2}{y^2} = 0$$

Now, we approach limit from $y \to 0$ and then $x \to 0$

$$\lim_{x\to 0}\left(\lim_{y\to 0} \frac{x^2}{x^2+y^2}\right) = \lim_{x\to 0} \frac{x^2}{x^2}$$

$$= \lim_{x \to 0} 1$$

$$= 1.$$

If we take the path $x \to 0$ then $y \to 0$, we get 0.
But, if we take path $y \to 0$ then $x \to 0$, we get 1.
Limiting value is different for different paths.

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180041120

Limiting value is path dependent.
Hence, limit doesn't exist. (proved).

Ans. to Q.no 2(c)

(i)
It is not essential to use the quotient rule in $\frac{\partial}{\partial x}\left(\frac{x+y}{y+1}\right)$ because the denominator is a function of y.

$$\frac{\partial}{\partial x}\left(\frac{x+y}{y+1}\right) = \frac{1}{y+1}\frac{\partial}{\partial x}(x+y)$$

In partial derivative of x, we treat y as constant. So, it is not required to use quotient rule as denomitor is function of y, we treat is as constant.

Ans: The quotient rule shouldn't be applied.

Md. Farhan Ishmam
1800041120

Ans. to Qn. ~~2(ii)~~ 2(c)

(ii)

$f(x,y) = \cancel{x}\, xy^3 + x^2$ at $(2,-2, f(2,-2))$.

$\Rightarrow z = xy^3 + x^2$

$\Rightarrow F(x,y,z) = xy^3 + x^2 - z$

$F_x(x,y,z) = y^3 + 2x$

$F_y(x,y,z) = 3y^2 x$

$F_z(x,y,z) = -1$

$f(2,-2) = 2(-2)^3 + 2^2$

$= -16 + 4$

$= -12$

$\therefore F_x(2,-2,-12) = (-2)^3 + 2 \times 2 = -4$

$\therefore F_y(2,-2,-12) = 3 \times (-2)^2 \times 2 = 24$

$\therefore F_z(2,-2,-12) = -1.$

Equation of tangent plane is $(x-x_0)F_x + (y-y_0)F_y - (z-z_0) = 0$

$(-4)(x-2) + 24(y+2) + (-1)(z+12) = 0$

$-4x + 8 + 24y + 48 - z - 12 = 0$

$\Rightarrow 4x - 24y + z - 44 = 0$

Ans: Equation of tangent plane is

$4x - 24y + z - 44 = 0.$

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Md. Farhan Ishmam
180041120

Ans to Qno. 3(a)

Two main properties of gradients are given below:

(i) Gradient is a vector valued function of multiple variables are is denoted by ∇.

(ii) If $\nabla f(x,y)=0$, then [f is differentiable at (x,y)]

$D_u f(x,y) = 0$ for u

(ii) The maximum increase of f is $\nabla f(x,y)$.

Maximum value $D_u f(x,y) = ||\nabla f(x,y)||$

Similarly, minimum value $D_u f(x,y) = -||\nabla f(x,y)||$

Ans to Qno. 3(b) (i)

Given $\theta = \pi/4$.

$$f(x,y) = 2500 + 100(x+y^2)e^{-0.3y^2}$$

at $P(-1,1)$

$$f_x(x,y) = e^{-0.3y^2} \times 100$$

$$f_x(-1,1) = e^{-0.3} \times 100$$

$$= 74.082$$

$$f_y(x,y) = 100(x+y^2) \cdot e^{-0.3y^2} \cdot (-0.6y)$$
$$+ e^{-0.3y^2} \times 100 \times 2y$$

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180041120

$$f(-1,1) = 100(-1+1)e^{-0.3\times1^2}(-0.6\times1) + 200\times1\times e^{-0.3\times1^2}$$

$$= 200e^{-0.3}$$

$$= 148.164$$

The directional derivative ~~is~~ in direction u at $\theta = \pi/4$ is

$$D_u f(x,y) = f_x(-1,1)\cos \pi/4 + f_y(-1,1)\sin \pi/4$$

$$= \frac{74.082}{\sqrt{2}} + \frac{148.164}{\sqrt{2}}$$

$$= 157.151 \text{ (approx.) (Ans)}$$

As the unit is in 100 meter it is ~~15700 k~~

15715.1 m

= 15.71 km

(ii)

The directional derivative ~~is a unit vector~~ denotes the intensity in a particular direction. In the example, the directional derivative means the altitude of the mountain at an angle $\theta = \pi/4$ from the point P with direction u is 15.71 km. So, from point P at $\theta = \pi/4$, we ~~need~~ will have an altitude of 15.71 km.

Md Farhan Ishmam
180041120

Ans. to Q no. 3 ~~(b)~~ (c)

~~(ii)~~

~~The interpretation of given directional derivative.~~

The given function is $\frac{x}{2} + \frac{y}{4} + \frac{z}{4} = 1$

$$\Rightarrow \frac{z}{4} = 1 - \frac{x}{2} - \frac{y}{4}$$

$$\Rightarrow z = 4 - 2x - y$$

Now, the distance from origin is

$$d(x,y,z) = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$$

$$= \sqrt{x^2 + y^2 + (4-2x-y)^2}$$

~~$d_x(x,y,z) =$~~

$$= \sqrt{x^2 + y^2 + 4x^2 + y^2 + 16 - 16x - 8y + 4xy}$$

$$= \sqrt{5x^2 + 2y^2 - 16x - 8y + 4xy}$$

$$d_x(x,y,z) = \frac{10x - 16 + 4y}{2\sqrt{5x^2 + 2y^2 - 16x - 8y + 4xy}}$$

$$\Rightarrow 0 = 10x - 16 + 4y$$

$$\Rightarrow 5x + 2y - 8 = 0 \quad \text{---} \quad (i)$$

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$$d_y(x,y,z) = \frac{4y - 8 + 4x}{2\sqrt{5x^2+2y^2-16x-8y+4xy}}$$

$$\Rightarrow 0 = \frac{4y-8+4x}{2\sqrt{5x^2+2y^2-16x-8y+4xy}}$$

$$\Rightarrow 4y-8+4x=0$$

$$\Rightarrow x+y-2=0 \quad \text{——} \quad \text{(ii)}$$

Solving (i) and (ii), we get,

$x = 4/3$ and $y = 2/3$ (Ans.)