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Ans. to Q. no. 1 (a)

(i) The signal to a monitor are colored, digital video feed.

Hence it is a multi-dimensional, multi-channel, digital signal.

There are 3 dimensions (x, y, t), $x-y$ co-ordinate, t time

There are 3 channels, r, b, g (red, blue and green)

The signal is represented as

$$I(x, y, t) = \begin{bmatrix} I_r(x, y, t) \\ I_b(x, y, t) \\ I_g(x, y, t) \end{bmatrix}$$

(ii) From mouse to CPU, the signal is digital, multi-channel, multi-dimensional

The signal has 2 channels — i) position of mouse
ii) state of buttons.

Each channel has 2 dimensions → (x, y) co-ordinate
→ Left/right status of button

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The signal can be written as

$$S_m = \begin{bmatrix} S_p(x,y) \\ S_c(l,r) \end{bmatrix}$$

(iii)

The signal is analog, single-dimensional and multi-channel.

The number of channels is 3 (x, y, z).

$$a(t) = \begin{bmatrix} a_x(t) \\ a_y(t) \\ a_z(t) \end{bmatrix}$$

Ans. to Q. no. 1(b) (i)

Digital signal processing are preferred over analog signal processing because

i) DSP is cheaper than ASP. For ASP lots of circuit, OP-Amps are required. DSP just needs an ADC convertor, ~~and~~ software and DAC converter.

ii) DSP is more convenient. Using software ~~is~~ or programming is more convinient than constructing complex circuits.

iii) DSP usually has higher precision. In ASP, there's lot of signal loss due to material resistence. In DSP, there is less loss and can perform more processing at higher precision.

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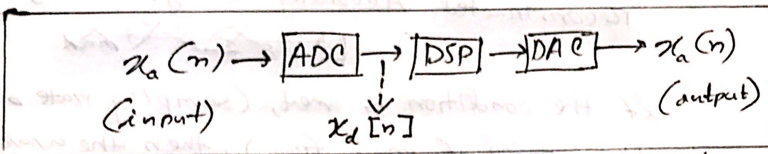
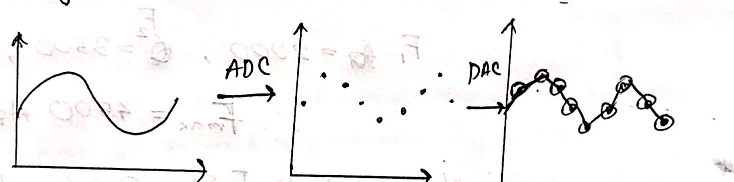
(ii)

No, we can NOT always recreate any analog signal after it has been digitalized.



During conversion, not 100% of the analog signal is retained. There is loss of input signal due to sampling rate. If the sampling rate ~~is~~ was infinite with infinite quantization levels, then we can recreate the exact original signal.



Input Analog Signal

Digital Signal

(Signal lost its values due to sampling)

As it's not possible to take infinite samples, the original analog signal can always not be recreated after digitalized.

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Ans. to Q.no. 1(c)

Nyquist Rate is the minimum sampling rate for the signal to be digitalized in order to be successfully reconstructed. According to Nyquist, $F_s = 2 \times F_{max}$.

~~$f_s \geq 2 \times f_{max}$~~

If the condition is met, (sampling rate is more than twice of max. freq.), then the analog signal can be successfully reconstructed without the aliasing problem. Or else, there will be signal loss.

(i)

$$x_a(t) = 5\cos 4000\pi t + 3\sin 7000\pi t + 8\cos 9000\pi t$$

$$F_1 = 2000\ ; \quad F_2 = 3500\ ; \quad F_3 = 4500$$

$$\therefore F_{max} = 4500\ Hz$$

$$\text{Nyquist Rate, } F_s = 2 \times F_{max}$$
$$= 2 \times 4500$$
$$= 9000\ Hz \quad \text{(Ans.)}$$

(ii)

The discrete time signal is $\left[f = \frac{F}{F_s} \right]$

$$x_d[n] = 5\cos\left(\frac{4000}{3000}\pi n\right) + 3\sin\left(\frac{7000}{3000}\pi n\right) + 8\cos\left(\frac{9000}{3000}\pi n\right)$$
$$= 5\cos\left(\frac{4}{3}\pi n\right) + 3\sin\left(\frac{7}{3}\pi n\right) + 8\cos(3\pi n)$$

(Ans.)

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Ans. to Q no 2(a)

Linear systems are systems with static linerity and memorylessness. The preference of linear systems over non-linear ones are –

i) Linears systems can be stacked together to form a more complex linear system which can be denoted as single linear system. This makes linear systems convenient to use.

ii) Linear systems have sinusoidal fidelity i.e. if ~~sine~~ sinusoid waves ~~are~~ at a particular frequency is given as input, the output will be sinusoid with the same frequency. Sinusoidal fidelity makes them easier to work with sinusoid waves which are common in Fourier decomposition.

iii) Linear systems are present everywhere, from simaltaneous equations to ~~re~~ matrix problems. Any problem can easily be represented by a linear system.

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(ii)

Correlation is an operation like convalution that takes two ~~numbers~~ inputs and combines them. It is mathematically ~~Rep~~ represented as

$$a[n] \circledast b[n] \text{ where } \circledast \text{ means correlation}$$

It is primarilary used to find the similarity between two vectors or signals. It is also used in Fourier analysis for signal decomposition.

$\cancel{a \circledast b = 0}$

Relationship with convalution is

$$a[n] \circledast b[n] = a[n] * b[-n]$$

If a signal is flipped in convalution, then it would result in ~~correla~~ correlation.

Ans. to Q.no. 2(b)

(i)

$$\text{Length of Output} = \left(\text{Input Length} + \text{Conv. Signal Length} - 1 \right)$$

$$= 720 + 64 - 1$$

$$= 783 \text{ (Ans.)}$$

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(ii)

First $(m-1)$ samples are not useful/noisy.

i.e. $(64-1) = 63$.

First 63 samples are noisy.

Ans to Q.no. 2(c) (i)

P, Q = 1, 0 [∵ 180041120]

$$x[n] = \{\underset{\uparrow}{4}, 3, 1, 7, -1, 2, 1, 0\}$$

Sum of shifted impulses is

$$x[n] = 4\delta[n] + 3\delta[n-1] + \delta[n-2] + 7\delta[n-3] - \delta[n-4] + 2\delta[n-5] + \delta[n-6] + 0\cdot\delta[n-7]$$

(ii)

Now, $x[n] * h_{s1} = \{\underset{\uparrow}{4}, 3, 1, 7, -1, 2, 1, 0\} * \{1, \underset{\uparrow}{2}, -1\}$

Using output-side algorithm,

$$\begin{array}{l} \underset{\uparrow}{4}, 3, 1, 7, -1, 2, 1, 0 \\ -1, \underset{\uparrow}{2}, 1 \\ \hline 4, \underset{\uparrow}{11}, 3, 6, 12, -7, 6, 0, -1, 0 \end{array}$$

Calculation:

$(4 \times 1) = 4$

$(4 \times 2) + (3 \times 1) = 11$

$4 \times (-1) + (3 \times 2) + (1 \times 1) = 3$

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$(-1\times 3)+(2\times 1)+(7\times 1) = 6$

$-1\times 1 + 2\times 7 + 1\times(-1) = 12$

$-1\times 7 + 2\times(-1) + 1\times 2 = -7$

$-1\times(-1) + 2\times 2 + 1\times 1 = 6$

$-1\times 2 + 2\times 1 + 1\times 0 = 0$

$-1\times 1 + 2\times 0 = -1$

$-1\times 0 = 0$

$x[n]*h_{s_1} = \{4, 11, 3, 6, 12, -7, 6, 0, -1, 0\}$

$y[n] = x[n]*h_{s_1}*h_{s_2}$

$$
\begin{array}{r}
4, \underset{\uparrow}{11}, 3, 6, 12, -7, 6, 0, -1, 0 \\
\underset{\uparrow}{3}, 2 \\
\hline
8, \underset{\uparrow}{34}, 39, 21, 42, 22, -9, 18, -2, -3, 0
\end{array}
$$

(Ans.)

Calc:

$4\times 2 = 8$

$(4\times 3)+(11\times 2) = 34$

$(11\times 3)+(3\times 2) = 39$

$(3\times 3)+(6\times 2) = 21$

$(6\times 3)+(12\times 2) = 42$

$(12\times 3)-(7\times 2) = 22$

$-7\times 3 + (6\times 2) = -9$

$6\times 3 + 0\times 2 = 18$

$0\times 3 + 2\times(-1) = -2$

$(-1)\times 3 + 0\times 2 = -3$

$0\times 3 = 0$

Ans: $y[n] = \{8, 34, 39, 21, 42, 22, -9, 18, -2, -3, 0\}$

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Ans to Q.no.3(a)

(i)

(i) The basis functions are picked orthogonal to each other. Orthogonal means perpendicular and if the vectors are plotted in a graph, they would be perpendicular.

That is why when element-wise multiplication is performed the summation will be zero. Because multiplication of two orthogonal vectors is zero.

$$\vec{a} \cdot \vec{b} = 0 \quad (\text{if orthogonal})$$

(ii)

We scale using,
$$Re\bar{X}[k] = \frac{ReX[k]}{N/2}$$

$$Im\bar{X}[k] = \frac{-ImX[k]}{N/2}$$

In Fourier decomposition, there will be two signals with $(N/2+1)$ terms. The ReX and ImX arrays contain the amplitudes of these cosine and sine functions. When we add these $(N/2+1)$ terms after multiplying with ReX amplitude array, the amplitude gets scaled to N/2 times. So, the output signal will have $(N/2)$ ~~more a~~ times the amplitude of original signal due to combining ~~(N/2)~~ $(N/2+1)$ signals during synthesis.

Hence, to get original signal value, we scale by ~~dividing~~ dividing by $(N/2)$.

The endpoints are scaled as

$$Re\,\bar{X}[0] = \frac{Re\,X[0]}{N}$$

$$Re\,\bar{X}[N/2] = \frac{Re\,X[N/2]}{N}$$

Ans. to Q.no.3(b)

(i)

The extra information is $Re\,X[0]$ and $Re\,X[N/2]$.

These two samples are useless as they are amplitudes of first and last ~~of~~ cosine waves. They are ~~also~~ always 0.

Hence, the 2 extra samples are from here.

(ii)

First difference, $y[n] = x[n] - x[n-1]$

~~If the samples of x start from 0,~~

Discrete derivative $= y[n] = \dfrac{x[n+1] - x[n-1]}{2}$

If their impulse responses have all negative indices as zero, then it will be causal.

Ans. to Q. no. 3(c)

(i)

In second method, we use $f = k/N$.

Here, $N = 32$.

So, $f = 6/32$ [As index starts from 0, 7^{th} sample has $k=6$]

$= 3/16$ (Ans.)

(ii)

If there's only one impulse in Re X, then,

$$x[i] = \sum_{i=0}^{N/2} Re\,X[0] \cos\left(\frac{2\pi \times 0 \times i}{N}\right) = Re\,X[0]$$

So, the signal will just return the amplitude.

Because, the value of cos will always be 1.

Hence, the constructed signal will have a constant value, and this will be output of inverse D.F.T.