Fact 1/ CIF math solve করতে দিলে, just ~~demo~~

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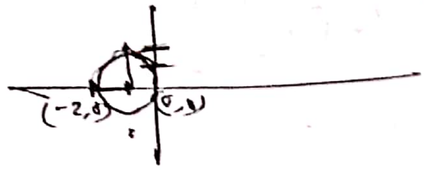
denominator কোন value তে 0 হয় তার দিকে খেয়াল রাখব।

$(-1, 0)$



$\sqrt{1+4} = \sqrt{5}$

Fact 2/ Cauchy theorem (CT) বলছে, যদি কোনো function এর pole closed curve C এর ভেতর না থাকে তাহলে তার integration relative to C [$\oint$ or $\int_C$] zero হবে।

P-10 Ex 1 (a) $y = x^2 + 1$

$\Rightarrow dy = 2x\,dx + 0 = 2x\,dx$

$$\int_0^2 (3x + x^2 + 1)\,dx + (2x^2 + 2 - x)\,2x\,dx$$

$$= \int_0^2 (x^2 + 3x + 1)\,dx + (4x^3 + 4x - 2x^2)\,dx$$

$$= \left[\frac{x^3}{3} + \frac{3x^2}{2} + x\right]_0^2 + \left[4\frac{x^4}{4} + \frac{4x^2}{2} - 2\frac{x^3}{3}\right]_0^2$$

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$$= \left[\frac{8}{3} + \frac{12}{2} + 2\right] + \left[4\cdot\frac{16}{\cancel{4}} + 4^2\frac{4}{\cancel{2}} - 2\cdot\frac{8}{3}\right]$$

$$= \left[\frac{8}{3} + 6 + 2\right] + \left[16 + 8 - \frac{16}{3}\right] \quad \text{(Ans.)}$$

(b) $$\frac{x-0}{0-2} = \frac{y-1}{1-5} \Rightarrow \frac{x}{-\cancel{2}} = \frac{y-1}{-\cancel{4}\,2}$$

$$\Rightarrow 2x = y-1 \Rightarrow \boxed{y = 2x+1}$$

$$\boxed{dy = 2\,dx}$$

$\boxed{\cancel{dy = 2}}\ dx$

same procedure

(c) $x = 0 \Rightarrow dx = 0$ for $(0,1)$ to $(0,5)$

$$\int_1^5 y\,dx + 2y\,dy = \int_1^5 2y\,dy = 2\left[\frac{y^2}{2}\right]_1^5$$

$$= 2\left[\frac{25}{2} - \frac{1}{2}\right] = \boxed{24}$$

$y = 5 \Rightarrow dy = 0$ for $(0,5)$ to $(2,5)$

$$\int_0^2 (3x+5)\,dx = 3\left[\frac{x^2}{2}\right]_0^2 + 5\,[x]_0^2$$

$$= 3\left[\frac{4}{\cancel{2}}\right] + 5\cdot 2$$

$$= 6 + 10 = \boxed{16}$$

Adding the required value

$24 + 16$

$= \boxed{40}$ Ans.

Theme: (d) same procedure

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(2) (a) $x = 4\cos\theta \qquad y = 3\sin\theta$

$dx = -4\sin\theta \, d\theta \qquad dy = 3\cos\theta \, d\theta$

$$\int_0^{2\pi} (4\cos\theta + 6\sin\theta)(-4\sin\theta \, d\theta) + (3\sin\theta - 8\cos\theta)(3\cos\theta \, d\theta)$$

$$= \int_0^{2\pi} -16\cos\theta\sin\theta - 24\sin^2\theta + 9\sin\theta\cos\theta - 24\cos^2\theta$$

যেখানে যেখানে দরকার সেখানে $d\theta$ আছে

$$= \int_0^{2\pi} -7\sin\theta\cos\theta - 24(1)$$

$$= -7\left[\sin\theta\int\cos\theta - \int\cos\theta\int\cos\theta\right] - 24\left[\theta\right]_0^{2\pi}$$

$$= -7\left[\sin^2\theta - \int\sin\theta\cos\theta\right] - 48\pi$$

আগের HSC

$$= -7\left[\frac{\sin^2\theta}{2}\right]_0^{2\pi} - 48\pi$$

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$$I = \int \sin\theta \cos\theta \, d\theta$$

$$= \sin\theta \int \cos\theta - \int \left\{ \cos\theta \int \cos\theta \right\}$$

$$= \sin^2\theta - \int \cos\theta \sin\theta$$

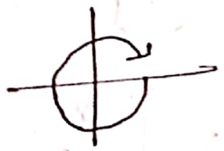
$$= \sin^2\theta - I$$

$$\Rightarrow 2I = \sin^2\theta$$

$$\Rightarrow I = \boxed{\frac{\sin^2\theta}{2}}$$

→ বাদেও HSC

$$= -7[0] - 48\pi = \boxed{-48\pi}$$

(b) Now if we took clockwise  , then the limit will be 2π to 0,

$$= -7\left[\frac{\sin^2\theta}{2}\right]_{2\pi}^{0} - \cancel{48}\,24\,[\theta]_{2\pi}^{0}$$

$$= 0 - \cancel{48}\,24\,[0 - 2\pi] = \boxed{48\pi}$$

কোনো logic নাই মনে হলো তাই করলাম

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(3) ~~same limit~~, $dy = 4x\,dx$; $dz = dx + i\,dy$

$$\int_1^2 \left[x^2 - i\,(2x^2)^2\right] dz$$

$$= \int_1^2 \left[x^2 - i\,4x^4\right](dx + i\,4x\,dx)$$

$$= \int_1^2 \left[x^2 - i\,4x^4\right](1 + i4x)\,dx$$

$$= \int_1^2 (x^2 + 4ix^3 - 4ix^4 - i^2\,16x^5)\,dx$$

$$= \int_1^2 (x^2 + 4ix^3 - 4ix^4 + 16x^5)\,dx$$

$$= \left[\frac{x^3}{3} + 4i\,\frac{x^4}{4} - 4i\,\frac{x^5}{5} + 16\,\frac{x^6}{6}\right]_1^2$$

$$\left[\frac{8}{3} + 16i - \frac{4}{5}i\,32 + \frac{16\times 64}{6}\right] - \left[\ldots\right]$$

(Ans)

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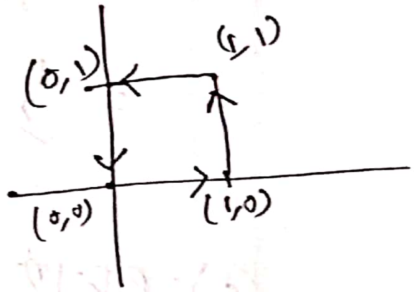
④ $|z| = \sqrt{x^2+y^2} \Rightarrow |z| = x^2+y^2$

$dz = dx + i\,dy$



$$\oint_C (x^2+y^2)(dx+i\,dy)$$

<u>case 1</u>: $(0,0)$ to $(1,0)$ $y=0,\ dy=0$

$$\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \boxed{1/3}$$

<u>case 2</u>: $(1,0)$ to $(1,1)$ $x=1,\ dx=0$

$$\int_0^1 (1+y^2)\,i\,dy = i\int_0^1 dy + i\int_0^1 y^2\,dy$$

$$= i[y]_0^1 + i\left[\frac{y^3}{3}\right]_0^1$$

$$= i + i/3 = \frac{3i+i}{3} = \boxed{\frac{4i}{3}}$$

<u>Case 3</u>: $(1,1)$ to $(0,1)$ $y=1,\ dy=0$

$$\int_0^1 (x^2+1)\,dx = \left[\frac{x^3}{3}\right]_0^1 + [x]_0^1 = 1/3 + 1 = \boxed{4/3}$$

<u>case 4</u>: $(0,1)$ to $(0,0)$ $x=0,\ dx=0$

$$\int_1^0 y^2\,i\,dy = i\left[\frac{y^3}{3}\right]_1^0 = -i\,1/3 = \boxed{-i/3}$$

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Ans: $\frac{4i}{3} + 4/3 + 1/3 \oplus - i/3$

$= \frac{1}{3}(4i + 5 - i) = \frac{1}{3}(5 + 3i) = \boxed{\frac{5}{3} + i}$

(5) দুটাই circle এর ~~radius~~ radius দিয়া হয় দেখাই (5, 2) এখানেই circle. এখানে শুধু যেখানে যুক্ত হবে range আছে -2 থেকে 2 এর বাইরে না যায়, কারণ তার বাইরে আমাদের $\boxed{\text{circle / curve}}$ exist করে না।

$z = x + iy$

$\int [(x+iy)^2 + 3(x+iy)](dx + idy)$

$\int (x^2 + 2xyi + i^2y^2 + 3x + 3iy)\, dx + idy$

$= \int (x^2 + 2xyi - y^2 + 3x + 3iy)(dx + idy)$

$= \cancel{\int x^2 dx + 2xyi\,dx}$

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$$\Rightarrow \int (x^2+2xyi-y^2+3x+3iy)\,dx + (ix^2-2xy-y^2i+3xi-3y)\,dy$$

$$\frac{x-2}{2-0} = \frac{y-0}{0-2} \Rightarrow \boxed{-x+2=y}$$

-1

$$\Rightarrow \boxed{dy=-dx}$$

$$= \int_2^0 [x^2+2x(-x+2)i-(-x+2)^2+3x+3i(-x+2)]\,dx - (ix^2-2x(-x+2)-(-x+2)^2i+3xi-3(-x+2)]\,dx$$

$$\Rightarrow \int_2^0 x^2-2x^2i+4xi-(x^2-4x+4)+3x-3xi+6i - (ix^2+2x^2-4x-(x^2-4x+4)i+3xi+3x-6)$$

$$\Rightarrow \int_2^0 [\cancel{x^2}-2x^2i+4xi-\cancel{x^2}+4x-4+3x-3xi+6i -ix^2-2x^2+4x+ix^2-i4x+i4-3xi-\cancel{3x}+6]$$

$$\int_2^0 [-2x^2i-ix^2+ix^2+4xi-3xi-i4x-3xi -2x^2]\,dx$$

$$+x+3x+4x-4+6i+i4+6$$

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$$\Rightarrow \int_2^0 \left[-2x^2 i - 6xi + 8x + \underline{2 + i10} - 2x^2\right] dx$$

$$= -2i\left[\frac{x^3}{3}\right]_2^0 - 6i\left[\frac{x^2}{2}\right]_2^0 + 8\left[\frac{x^2}{2}\right]_2^0$$

$$+ -2\left[\frac{x^3}{3}\right]_2^0 + (2+10i)\left[x\right]_2^0$$

আর দরকার না।

(b)

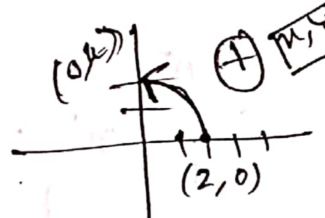
~~(2,0) to (2,2)~~

case:

(5) (a) $|z| = 2 \Rightarrow \sqrt{x^2+y^2} = 2$

$\Rightarrow x^2 + y^2 = 4$

$\Rightarrow y^2 = 4 - x^2 \Rightarrow y = \pm\sqrt{4-x^2}$



$\Rightarrow 2y\,dy = -2x\,dx$

$\Rightarrow dy = -\frac{x}{y}dx$

$= \pm\frac{x}{\sqrt{4-x^2}}dx$

$= \frac{x}{\sqrt{4-x^2}}dx$

counter clockwise বলে মনে হয় + নিতে হবে

We can always assume right ♡

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$\cancel{(x+iy)^2 + 3(x}$

$$\int_2^0 \left[(x+iy)^2 + 3(x+iy)\right](dx + i\,dy)$$

$$= \int_2^0 \left[(x+i\sqrt{4-x^2})^2 + 3(x+i\sqrt{4-x^2})\right]\left(dx + i\frac{x}{\sqrt{4-x^2}}dx\right)$$

$$= \int_2^0 \left[x^2 + 2xi\sqrt{4-x^2} + i^2(4-x^2) + 3x + 3i\sqrt{4-x^2}\right]\left(1 + i\frac{x}{\sqrt{4-x^2}}\right)dx$$

$$= \int_2^0 \left(x^2 + 2xi\sqrt{4-x^2} - 4 + x^2 + 3x + 3i\sqrt{4-x^2}\right)\left(1 + i\frac{x}{\sqrt{4-x^2}}\right)dx$$

$$\int_2^0 \cancel{x^2} + 2xi\sqrt{4-x^2} - 4 + \cancel{x^2} + \cancel{3x} + 3i\sqrt{4-x^2}$$

$$x^2 + 2xi\sqrt{4-x^2} - 4 + x^2 + 3x + 3i\sqrt{4-x^2}$$

$$+ i\frac{x^3}{\sqrt{4-x^2}} + 2\cancel{x^2}i^2 - 4i\frac{x}{\sqrt{4-x^2}} + 3i\frac{x^2}{\sqrt{4-x^2}}$$

$$+ 3i^2x + i\frac{x^3}{\sqrt{4-x^2}}$$

- আর পারছি না -

(b) $-x+2 = y \quad dy = -dx$

$$\int_2^0 (z^2 + 3z)\,dz$$

⇒ অনেকক্ষণ ধরে calculation করার পর

$$\Rightarrow -2i\left[\frac{x^3}{3}\right]_2^0 - 6i\left[\frac{x^2}{2}\right]_2^0 + 8\left[\frac{x^2}{2}\right]_2^0 + (-2)\left[\frac{x^3}{3}\right]_2^0 + (2+10i)\left[x\right]_2^0$$

N.B. ⇒ পুরোটা আরো 4 পাতা আগে আছে।
5 (a) তে দেয়া আছে।

(c) shoja,

case 1: $x = 2 \quad dx = 0$ পুরো এক point নাই
হয়ে যায়

case 2: $y = 2 \quad dy = 0$

পরে এই দুই ভাবে যোগ করতে হবে

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⑥ ~~কে~~ বাহির করিয়া আমাকে জানাইও।

⑦ (a) $z = i$ হলে, $x = 0, y = 1$ $(0,1)$

$z = 2-i$ হলে, $x = 2, y = -1$ $(2,-1)$

$$\frac{x-0}{0-\cancel{2}_{-1}} = \frac{y-1}{\cancel{1+1}} \Rightarrow x = \cancel{0} - y + 1$$

$$\Rightarrow y = -x + 1 \Rightarrow dy = -dx$$

$$\int_0^2 \left[3x(-x+1) + i(-x+1)^2\right](dx \cancel{+} - i dx)$$

$$\Rightarrow \int_0^2 \left[3x(-x+1) + i(-x+1)^2\right](1-i)\,dx \quad \text{(Ans.)}$$

(b) $dx = 2dt \quad dy = dt - 2t\,dt$

$$= (1-2t)\,dt$$

$z = i$ হলে, $x = 0$, $0 = 2t - 2 \Rightarrow t = 1$

$z = 2-i$ হলে, $x = 2$, $2 = 2t - 2 \Rightarrow t = 2$

$$\int_1^2 \left[3(2t-2)(1+t-t^2) + i(1+t-t^2)^2\right]$$

$$(2dt + i(1-2t)\,dt)$$

(Ans.)

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(8) $x^2+y^2=1 \Rightarrow y^2 = 1-x^2 \Rightarrow \boxed{y = \pm\sqrt{1-x^2}}$

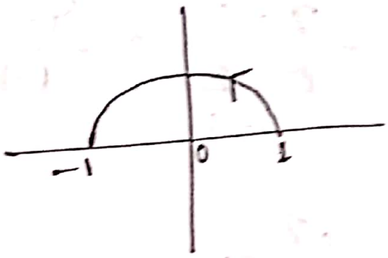
(+) ধরব কারণ (−) হলো লাগে না,

$\cancel{\frac{dy}{dx} = (1-x^2)}$

$\cancel{\Rightarrow 2x + 2y\,dy} \Rightarrow 2x\,dx + 2y\,dy = 0$

$\Rightarrow \cancel{2}y\,dy = -\cancel{2}x\,dx$

$\Rightarrow \pm\sqrt{1-x^2}\,dy = -x\,dx$



$\Rightarrow \boxed{dy = \frac{x}{\sqrt{1-x^2}}\,dx}$

x এর value -1 থেকে 1 পর্যন্ত হতে পারে।

$$\int_{1}^{-1} \overline{(x+iy)}^2\,(dx+i\,dy)$$

$$= \int_{1}^{-1} (x-iy)^2 \left(dx + i\frac{x}{\sqrt{1-x^2}}\,dx\right)$$

$$= \int_{1}^{-1} \left(x - i\sqrt{1-x^2}\right) \cancel{dx} \left(1 + i\frac{x}{\sqrt{1-x^2}}\right) dx$$

(Ans.)

(b) $(x-1)^2 + y^2 = 1 \Rightarrow y = \pm\sqrt{1-(x-1)^2}$

$= \sqrt{1-(1-t-1)^2}$

$= \sqrt{1-t^2}$

$2(x-1)dx + 2y\,dy = 0$

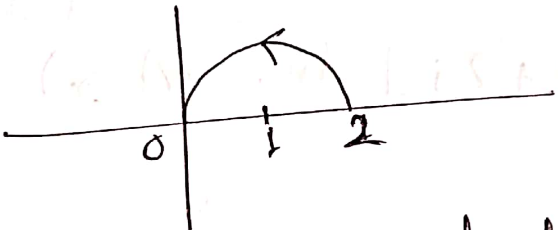
$\Rightarrow \cancel{2}y\,dy = -\cancel{2}(x-1)dx$

$\Rightarrow dy = \dfrac{(1-x)}{\sqrt{1-(x-1)^2}}dx \Rightarrow dy = \dfrac{(1-x)}{\sqrt{1-(1-x)^2}}dx$

$t = 1-x \qquad \Rightarrow -dt = dx$

$\Rightarrow x = (1-t)$

$\boxed{dy = \dfrac{-t}{\sqrt{1-t^2}}dt}$



$t = 1-0 = 1$

$t = 1-2 = -1$

এখানে, counter clockwise যাচ্ছে,

$\displaystyle\int_{-1}^{1}(x-iy)^2(dx+i\,dy)$

$= \cancel{x^2 - 2xiy + y^2}$

$= \displaystyle\int_{-1}^{1}\left((1-t) - i\sqrt{1-t^2}\right)^2\left(-dt + i\frac{-t}{\sqrt{1-t^2}}dt\right)$

$= \displaystyle\int_{-1}^{1}\left[(1-t) - i\sqrt{1-t^2}\right]^2\left[-1 - \frac{it}{\sqrt{1-t^2}}\right]dt$

(Ans.)

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(9) (a) $x^2+y^2=1$ same ভাবে করতে হবে।

(b) case 1 2 3 4 প্রমান সাথে উপর করে করতে হবে।

(c) $y=x^2$ $dy = 2dx$

$$\int_0^1 \left[5(x+iy)^4 - (x+iy)^3 + 2\right]\left[dx + i\,dy\right]$$

$$= \int_0^1 \left[5(x+ix^2)^4 - (x+ix^2)^3 + 2\right]\left[dx + i\,2dx\right]$$

$$= \int_0^1 \left[5 \ldots\ldots\right]\left[1+2i\right] dx. \quad \text{(Ans.)}$$

$dx = 2dy$

$$\int_1^0 \left[5(y^2+iy)^4 - (y^2+iy)^3 + 2\right]\left[2dy + i\,dy\right]$$

$$= \int_1^0 \left[\ldots\right]\left[2+i\right] dy. \quad \text{(Ans.)}$$

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(10) $dx = a(d\theta - \cos\theta\, d\theta)$ $\quad dy = a(0 + \sin\theta\, d\theta)$

$dx = a(1-\cos\theta)d\theta \quad dy = a\sin\theta\, d\theta$

$$\int_0^{2\pi} \left[(x+iy)^2 + 1\right]^2 (dx + i\,dy) \quad \ldots\ldots \text{(i)}$$

putting all the required value in (i) we get the answer. (Ans.)

(11) $\cancel{(x+iy)^2+2}$

sin ক্ষ বুঝলে তোমার আমার Gap থাকত না।

$(x+iy)^2 + 2(x+iy)(x-iy)^0 + (x-iy)^2 = \boxed{\text{R.H.S করে}}$

$\Rightarrow x^2 + \cancel{2xyi} - y^2 + 2(x^2 - i^2y^2) + x^2 - \cancel{2xyi} + i^2y^2 = \text{"}$

$\Rightarrow 2x^2 - \cancel{2y^2} + 2x^2 + \cancel{2y^2}$

$= (2-2i)(x+iy) + (2+2i)(x-iy)$

$\Rightarrow 4x^2 = (2-2i)(x+iy) + (2+2i)(x-iy)$

$\Rightarrow 4x^2 = 2x + 2iy - 2ix - 2i^2y + 2x - 2iy + 2ix - 2i^2y$

$\Rightarrow 4x^2 = 2x + \cancel{2iy} - \cancel{2ix} + 2y + 2x - \cancel{2iy} + \cancel{2ix} + 2y$

$\Rightarrow 4x^2 = 4x + 4y$

$\Rightarrow 4y = 4x^2 - 4x \Rightarrow y = x^2 - x$

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$dy = (2x-1)\,dx \qquad y = x^2 - x$

$z = 1$ মানে, $x = 1, y = 0$, $(1,0)$

$z = 2+2i$ মানে, $x = 2, y = 2$, $(2,2)$

$$\int_1^2 (x-iy)^2(dx+i\,dy) + (x+iy)^2(dx-i\,dy)$$

putting all the requried . . . . . (Ans.)

(12) প্রতি মানে হবে $\oint_C dz/z$ বের করে তার থেকে 2টি বাদ দিতে হবে

ⓐ $(x-2)^2 + y^2 = 16$ blah.blah.blah..

ⓑ $(x-1)^2 + y^2 = 25$ 〃

ⓒ $3+3i, \; 3-3i, \; -3+3i, \; -3-3i$

$(3,3)$ case1 $\quad (3,-3)$ case 2 $\quad (-3,3)$ case 3 $\quad (-3,-3)$ $\quad (3,3)$ case 4

(Ans)

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$$\int \frac{dz}{z}$$

$$= \int \frac{(dx+idy)}{x+iy}$$

$$= \int \frac{(dx+idy)(x-iy)}{x^2-i^2y^2} = \int \frac{(dx+idy)(x-iy)}{x^2+y^2}$$

$$= \int \left(\frac{x}{x^2+y^2} - \frac{iy}{x^2+y^2}\right)(dx+idy)$$

এটার integration কইরা আমারে দিও। ---

(13) আমরা যতটুকু জানি curve কে differentiate করলে straight line হয়। তাই circle এর equation কে differentiate করলে s আসার কথা। $x^2+y^2=2 \Rightarrow 2x\frac{dx}{dy}+2y=0$

$$\Rightarrow 2x\frac{dx}{ds} + 2y\frac{dy}{ds} = 0$$

$$\Rightarrow 2x\frac{dx}{\cancel{ds}}\frac{\cancel{ds}}{dy} + 2y\frac{\cancel{dy}}{\cancel{ds}}\frac{\cancel{ds}}{\cancel{dy}} = 0$$

$$\Rightarrow 2x\frac{dx}{dy} + 2y = 0$$

$$\Rightarrow 2\cancel{x}\left(-\frac{y}{\cancel{x}}\right) + 2y = 0$$

$$\Rightarrow -2y + 2y = 0$$

$$\Rightarrow \frac{dx}{dy} = -\frac{2y}{2x} = -y/x$$

Shit

কেউ পারলে দিও।