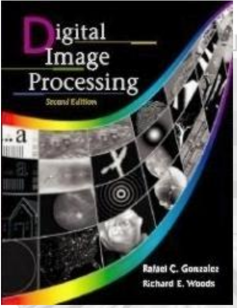

Chapter 9
Image Morphology

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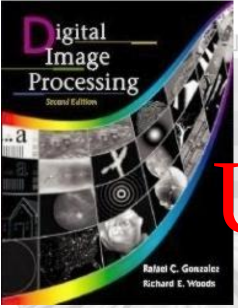
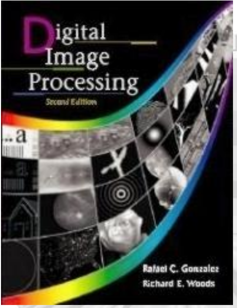


Uses of Mathematical Morphology

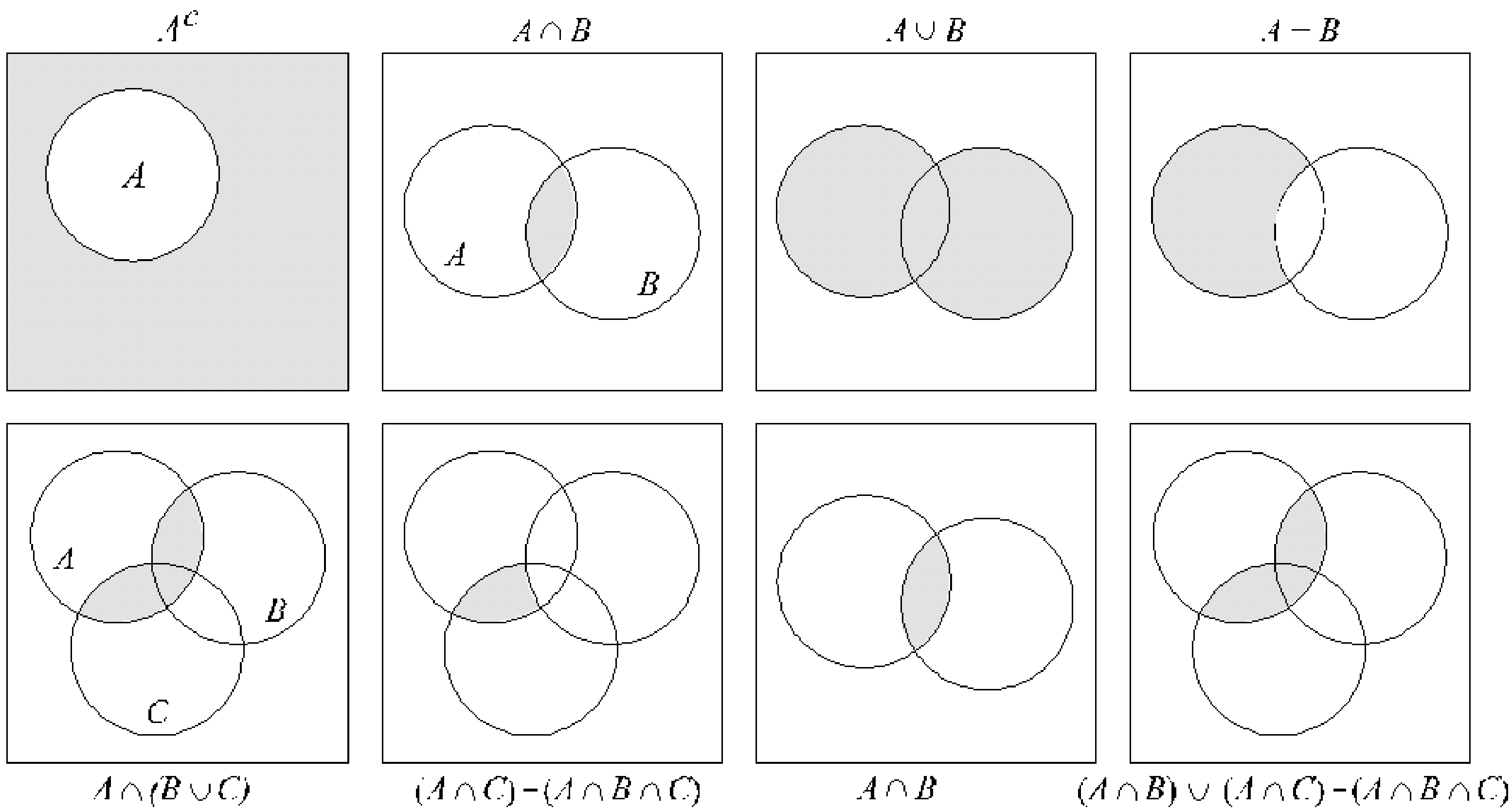
- image enhancement
- image segmentation
- image restoration
- edge detection
- texture analysis
- feature generation
- skeletonization
- shape analysis
- image compression
- component analysis
- curve filling
- general thinning
- feature detection
- noise reduction

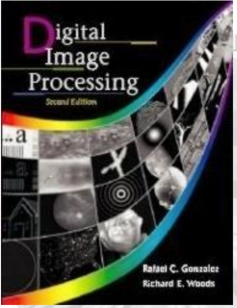


Basic Concepts of Set Theory

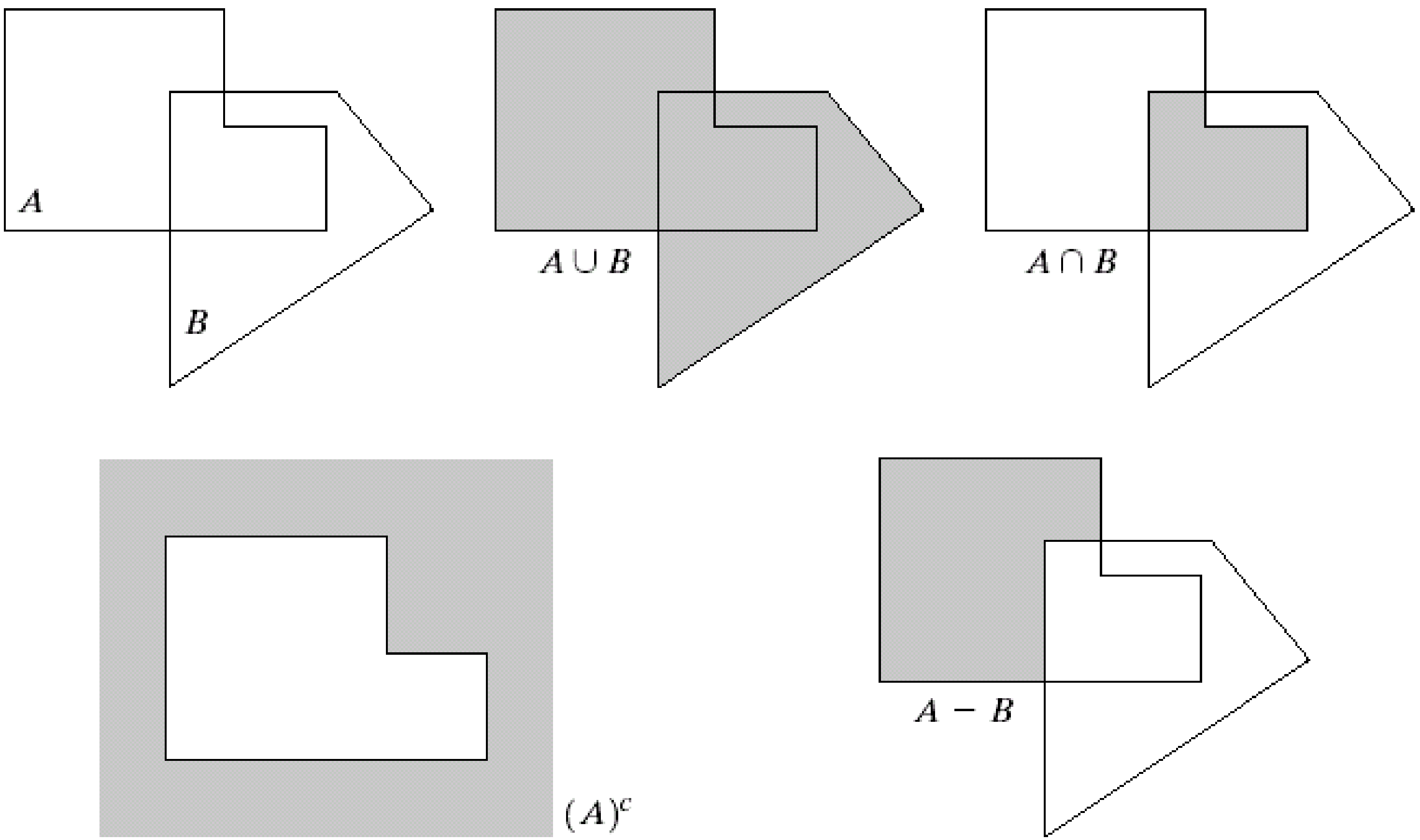
- A is a set in Z^2, a=(a_1,a_2) an element of A, a$\in$A
- If not, then a$\notin$A
- $\varnothing$: null (empty) set
- Typical set specification: C={w|w=-d, for d $\in$ D}
- A subset of B: A$\subseteq$B
- Union of A and B: C=A$\cup$B
- Intersection of A and B: D=A$\cap$B
- Disjoint sets: A$\cap$B= $\varnothing$
- Complement of A: $A^c = \{w \mid w \notin A\}$
- Difference of A and B: A-B={w|w $\in$ A, w $\notin$ B}= $A \cap B^c$
- Reflection of B: $\hat{B} = \{w \mid w = -b, b \in B\}$
- Translation of A by z=(z_1,z_2): $(A)_z = \{c \mid c = a + z, a \in A\}$

Venn Diagram





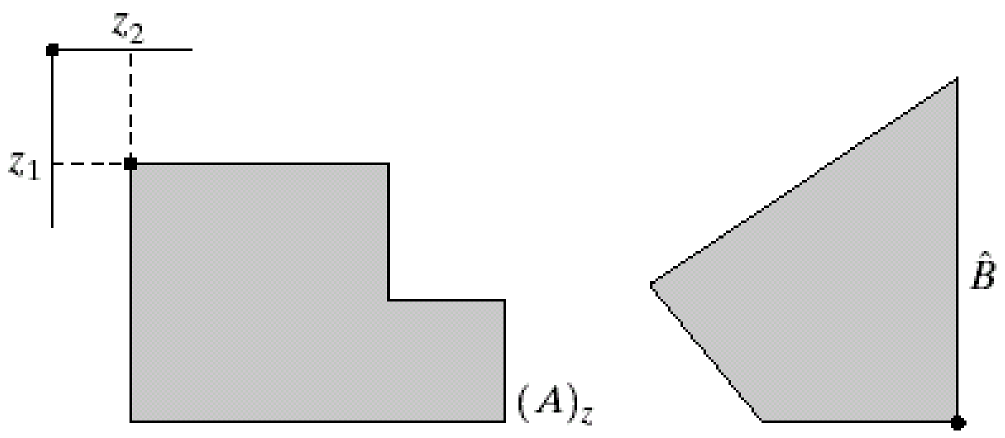
Morphological Image Processing



a b c
d e

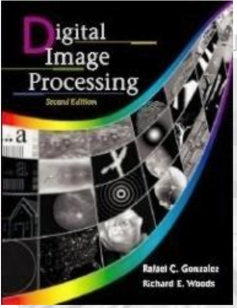
FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

Morphological Image Processing



a b

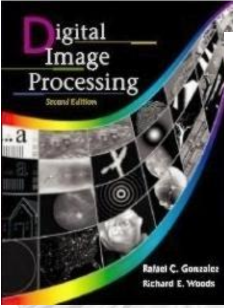
FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.



Morphological Image Processing

TABLE 9.1
The three basic logical operations.

p	q	p **AND** q **(also** $p \cdot q$**)**	p **OR** q **(also** $p + q$**)**	**NOT (**p**) (also** $\bar{p}$**)**
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0



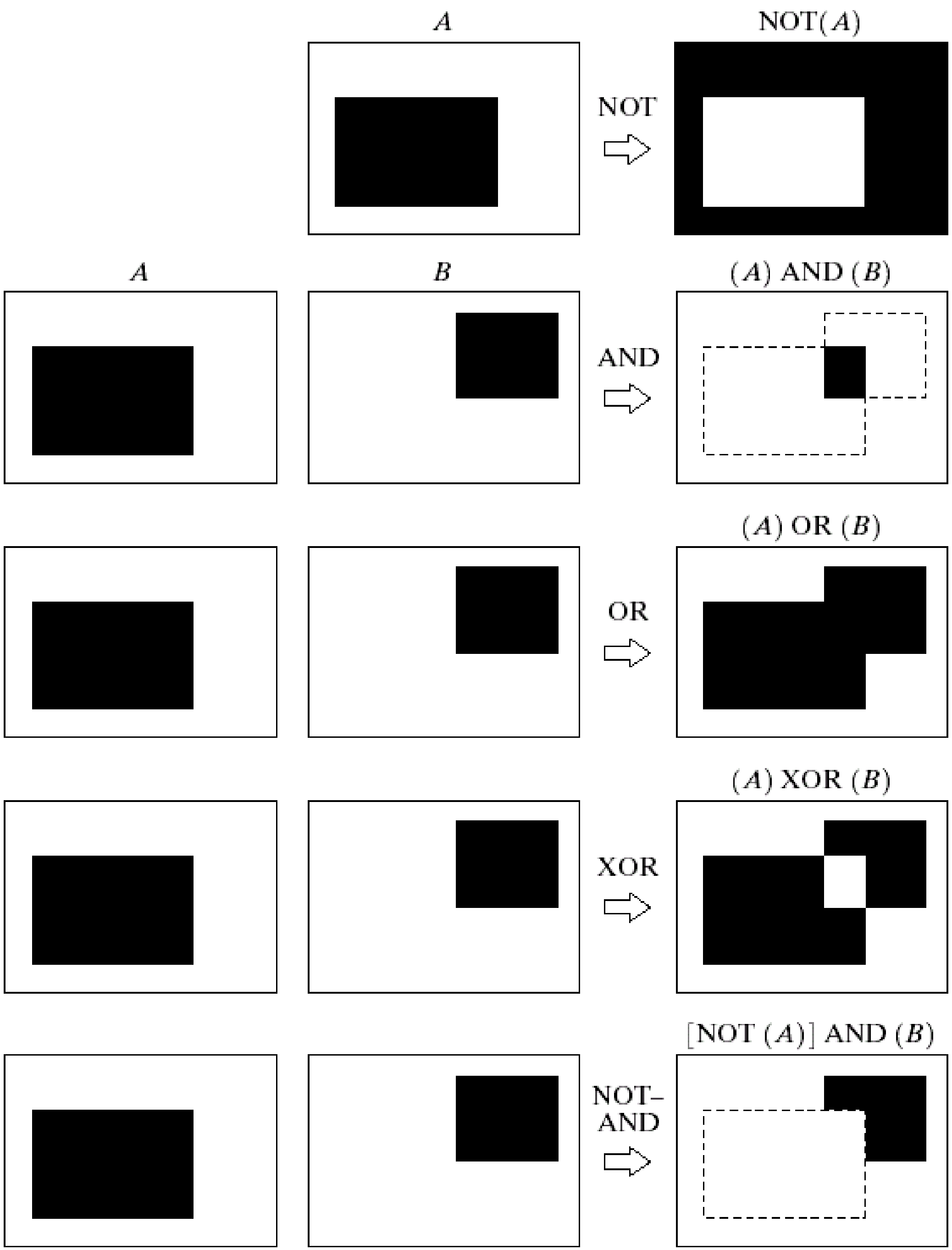
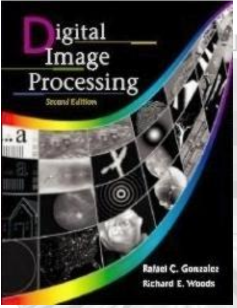
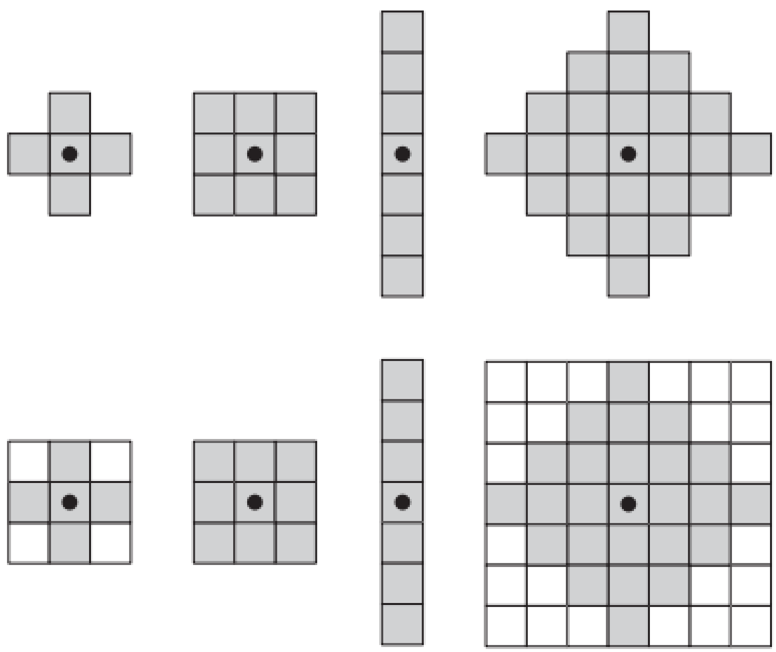


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.



Structuring Element

- Structuring element (SE): is also called the *kernel*, but we reserve this term for the similar objects used in convolutions
- Origin: the SE is typically translated to each pixel position in the image based on the origin.



1	1	
1	0	
1		0

Dilation & Erosion

- Basic definitions:
  - A,B: sets in Z^2 with components $a=(a_1,a_2)$ and $b=(b_1,b_2)$
  - Translation of A by $x=(x_1,x_2)$, denoted by $(A)_x$ is defined as:

$$(A)_x = \{c \mid c=a+x, \text{ for } a \in A\}$$

Dilation & Erosion

- More definitions:

Reflection of B: $\hat{B} = \{x \mid x = -b, \text{ for } b \in B\}$

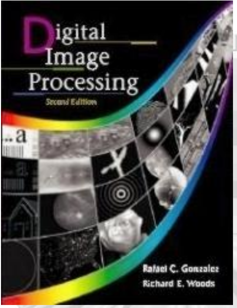
Complement of A: $A^c = \{x \mid x \notin A\}$

Difference of A & B: $A - B = \{x \mid x \in A, x \notin B\} = A \cap B^c$

Dilation

- Dilation:
  - $\varnothing$: empty set; A,B: sets in Z^2
  - Dilation of A by B:

$$A \oplus B = \{x \mid (\hat{B})_x \cap A \neq \varnothing\}$$



Dilation

- Dilation:

$$A \oplus B = \{x \mid [(\hat{B})_x \cap A] \subseteq A\}$$

B is the structuring element in dilation.

Morphological Image Processing

a b c
d e

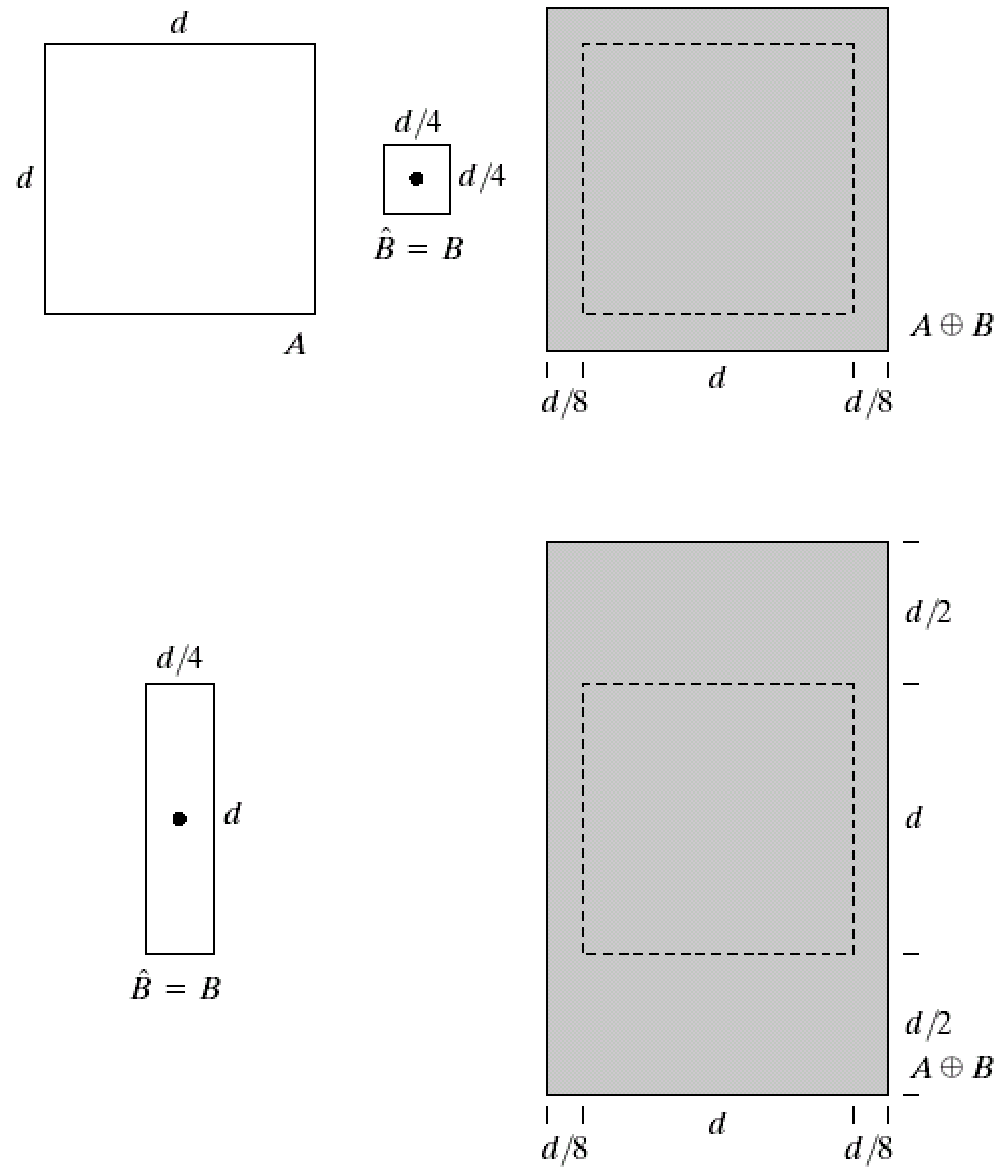
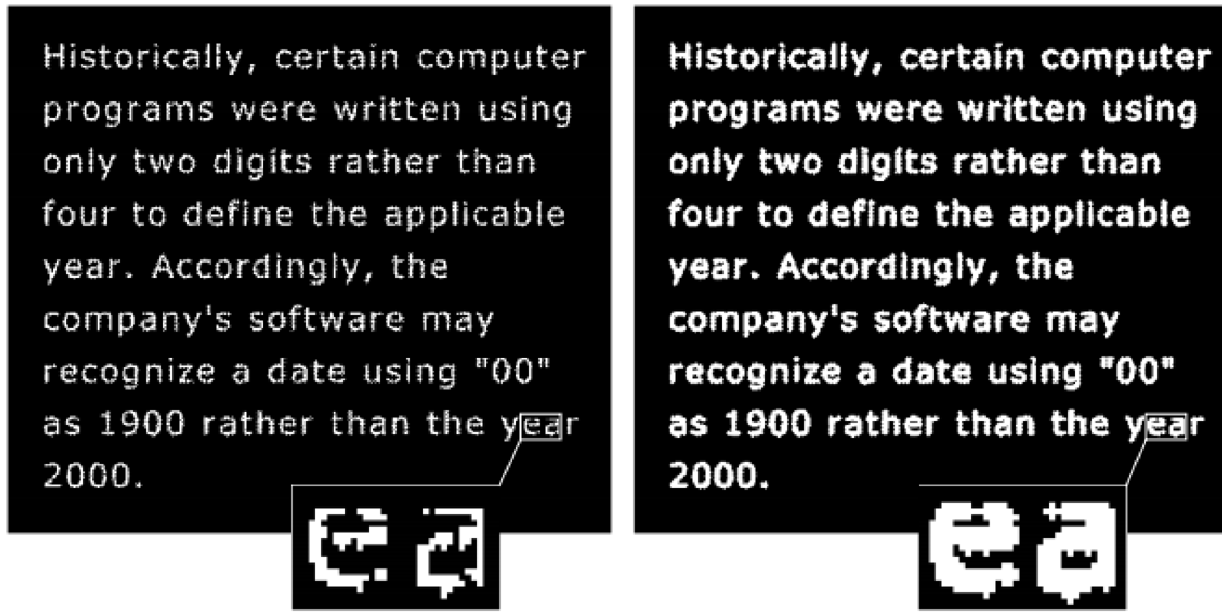


FIGURE 9.4 (a) Set A. (b) Square structuring element (dot is the center). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element.

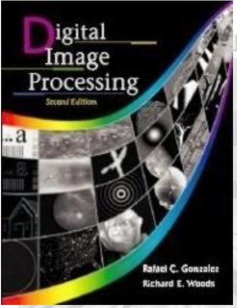
Morphological Image Processing

a c
b

FIGURE 9.7
(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.



0	1	0
1	1	1
0	1	0

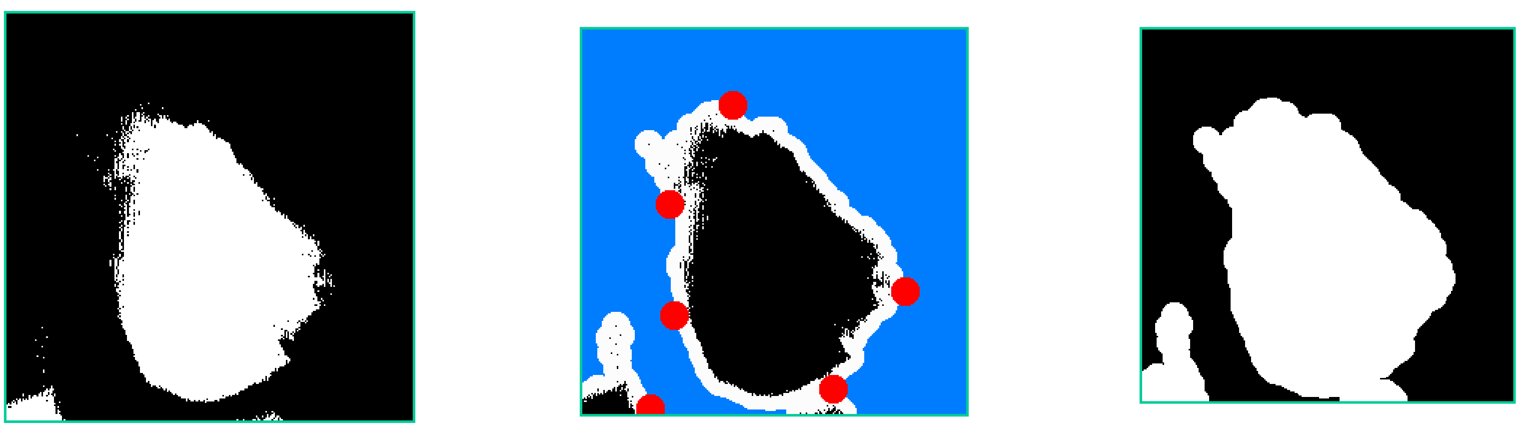


Dilation

- Binary Dilation: also called Minkowski addition. An image *F* dilated by a SE *K* is defined as:

$$D(F,K) = F \oplus K = \bigcup_{b \in K} (\{a+b \mid a \in F\})$$

- It can be regarded as an expansion operation.



Erosion

- Erosion:

$$A \ominus B = \{x \mid (B)_x \subseteq A\}$$

i.e. the erosion of A by B is the set of all points x such that B, translated by x, is contained in A.

In general:

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

Morphological Image Processing

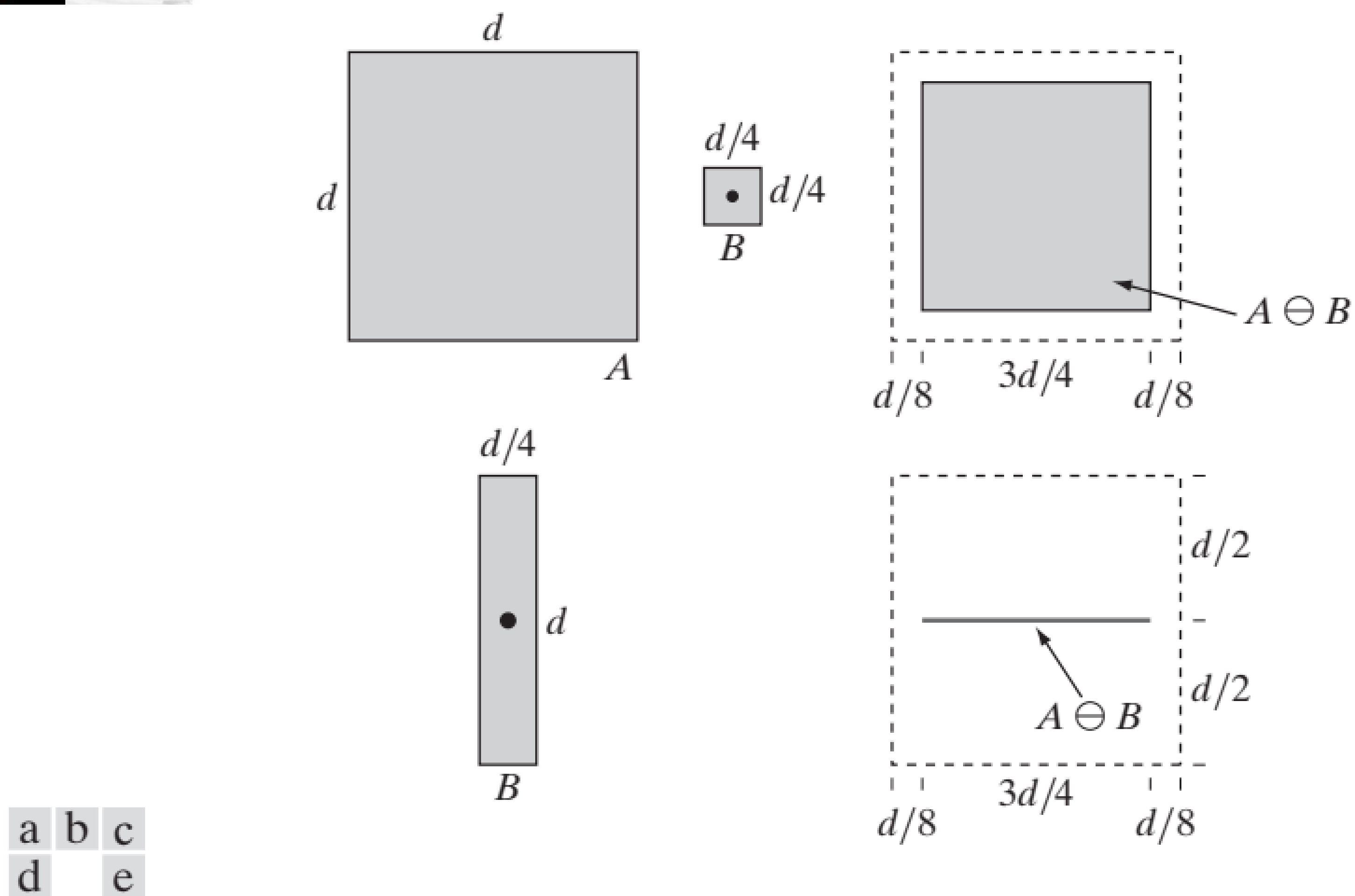
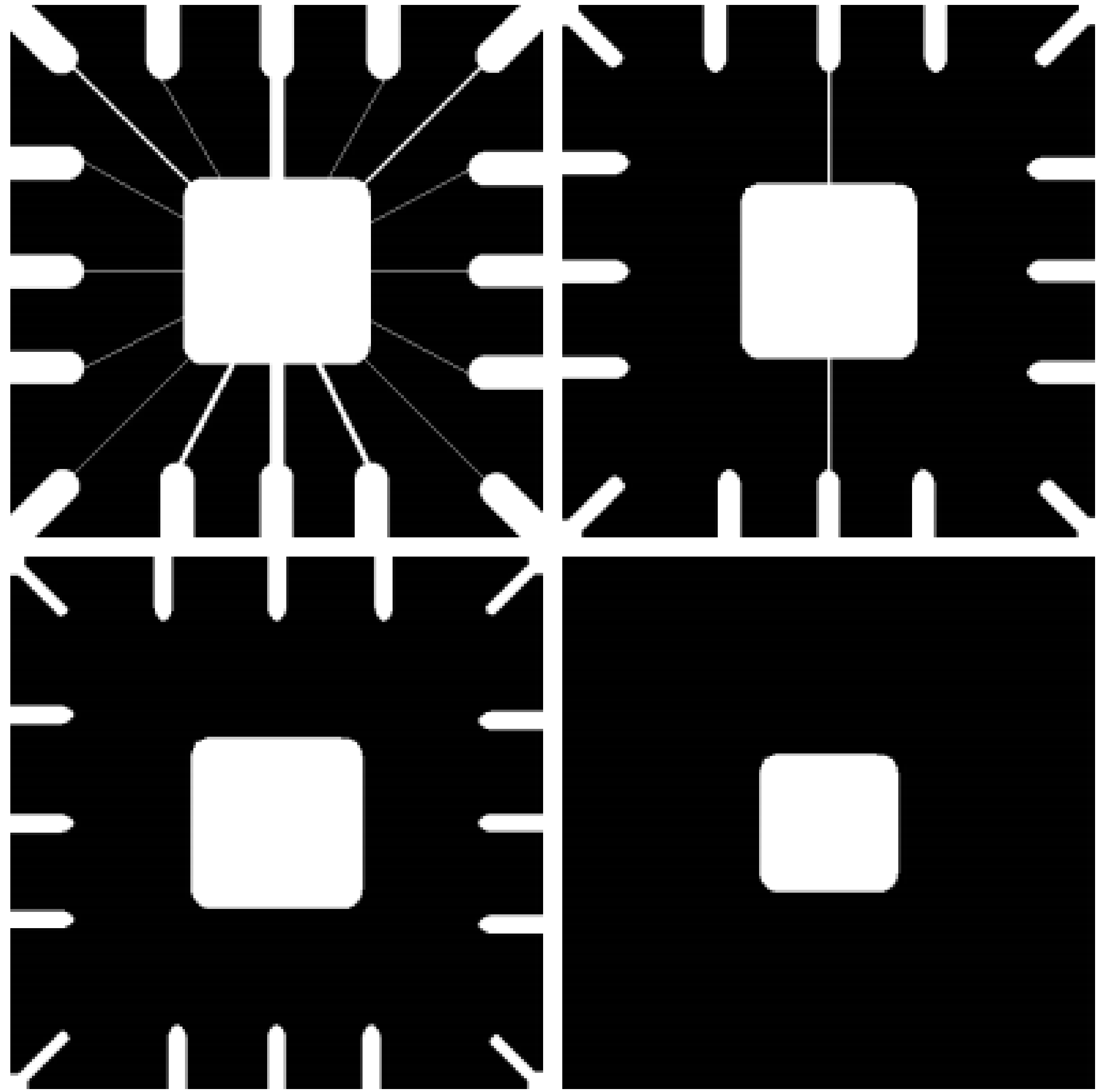


FIGURE 9.4 (a) Set A. (b) Square structuring element, B. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A by B using this element. The dotted border in (c) and (e) is the boundary of set A, shown only for reference.

Erosion

a b
c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

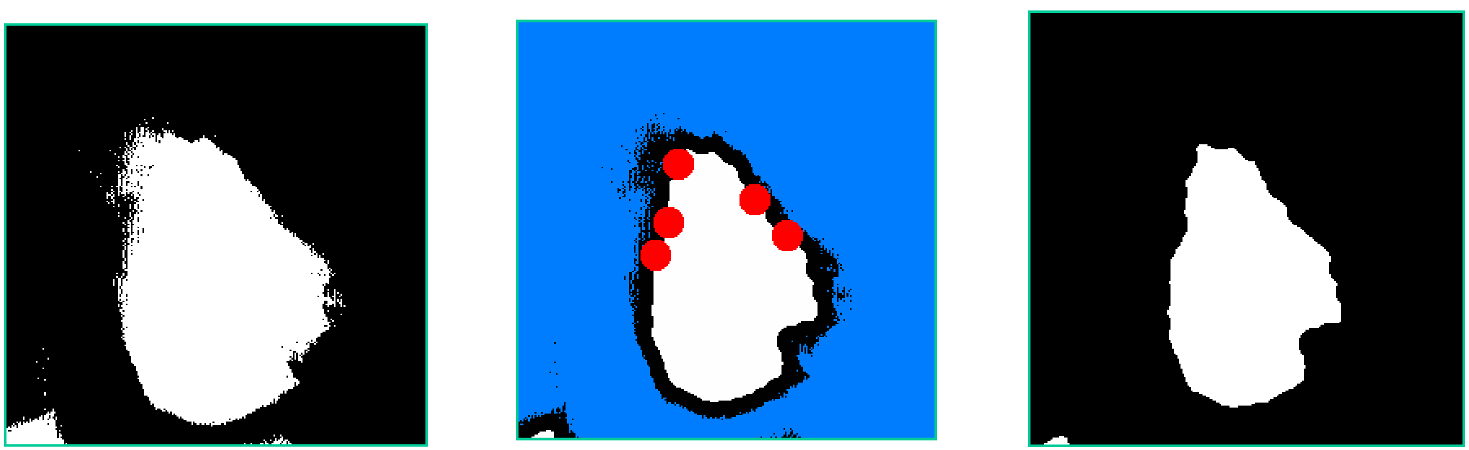


Erosion

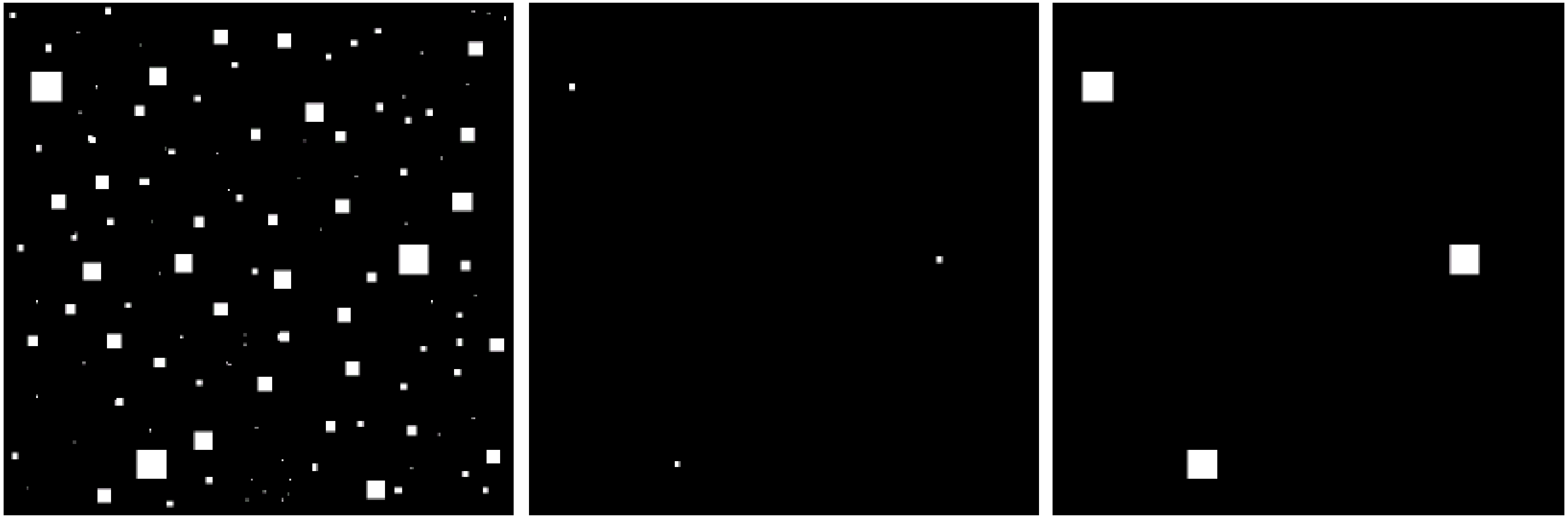
- Binary Erosion: also called Minkowski subtraction. An image *F* eroded by a SE *K* is defined as:

$$E(F,K) = F \ominus K = \bigcup_{b \in K} (\{a - b \mid a \in F\})$$

- It can be regarded as an shrinking operation

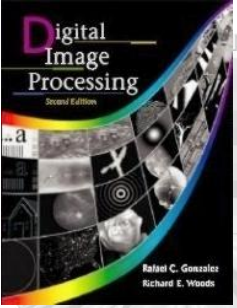


Morphological Image Processing



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.



Opening & Closing

- In essence, dilation expands an image and erosion shrinks it.
- Opening:
  - generally smoothes the contour of an image, breaks isthmuses, eliminates protrusions.
- Closing:
  - smoothes sections of contours, but it generally fuses breaks, holes, gaps, etc.

Opening & Closing

- Opening of A by structuring element B:

$$A \circ B = (A \ominus B) \oplus B$$

- Closing:

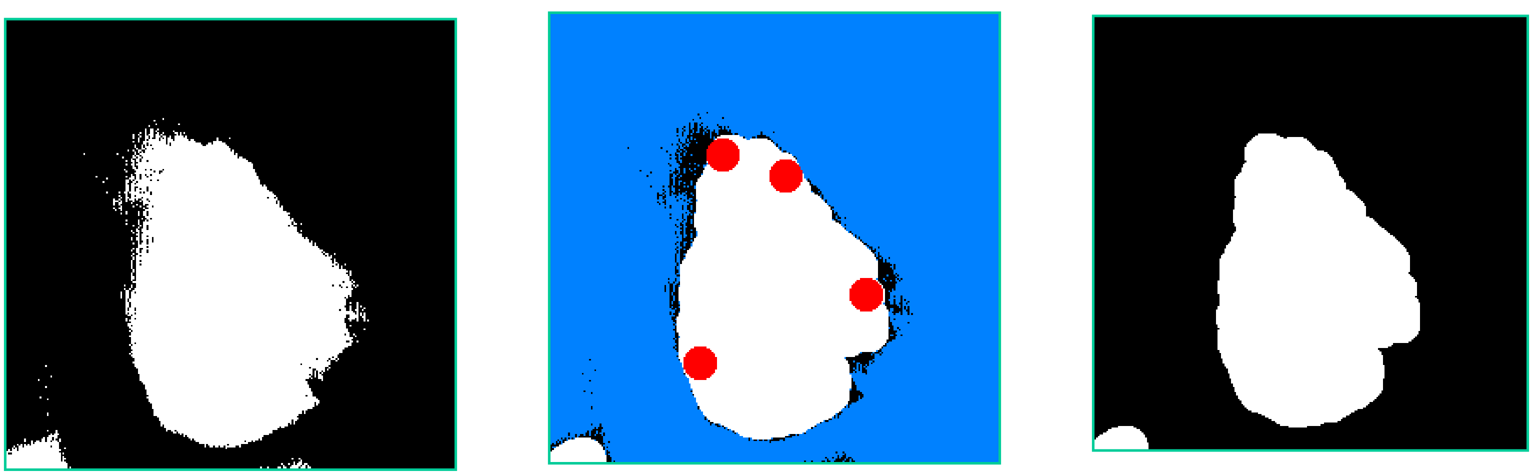
$$A \bullet B = (A \oplus B) \ominus B$$

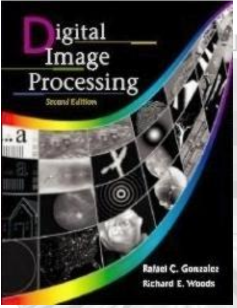
Opening

- Binary Opening: An image *F* opened by a SE *K* is defined as:

$$O(F, K) = F \circ K = (F \ominus K) \oplus K$$

- It can remove the small regions which are smaller than the structuring element



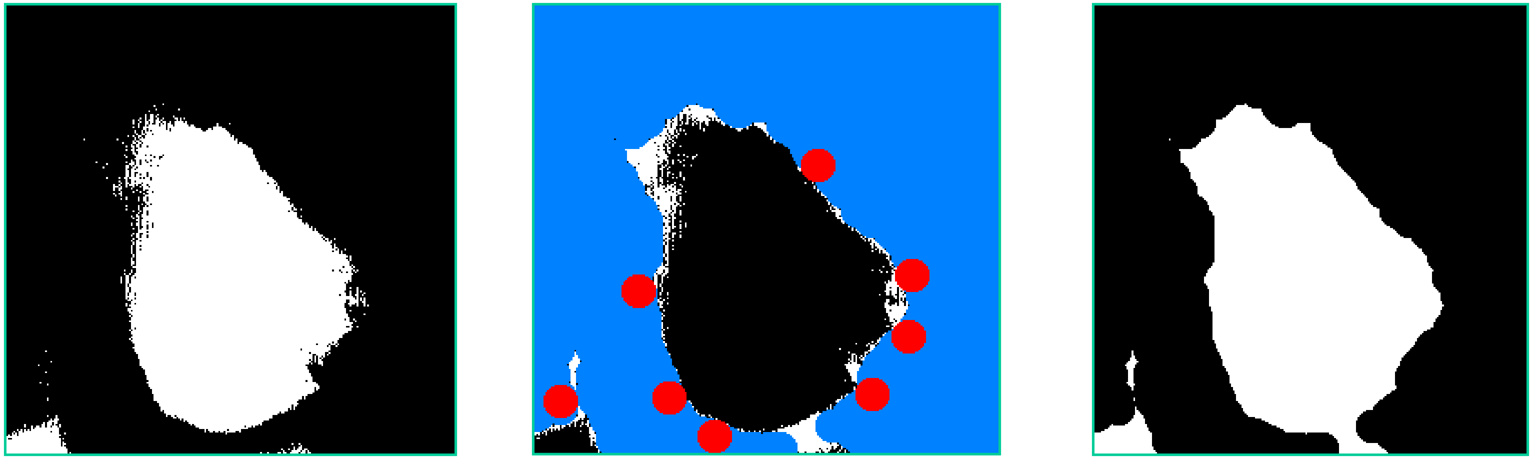


Closing

- Binary Closing: An image *F* closed by a SE *K* is defined as:

$$C(F,K) = F \bullet K = (F \oplus K) \ominus K$$

- It can fill the small holes which are smaller than the structuring element



Morphological Image Processing

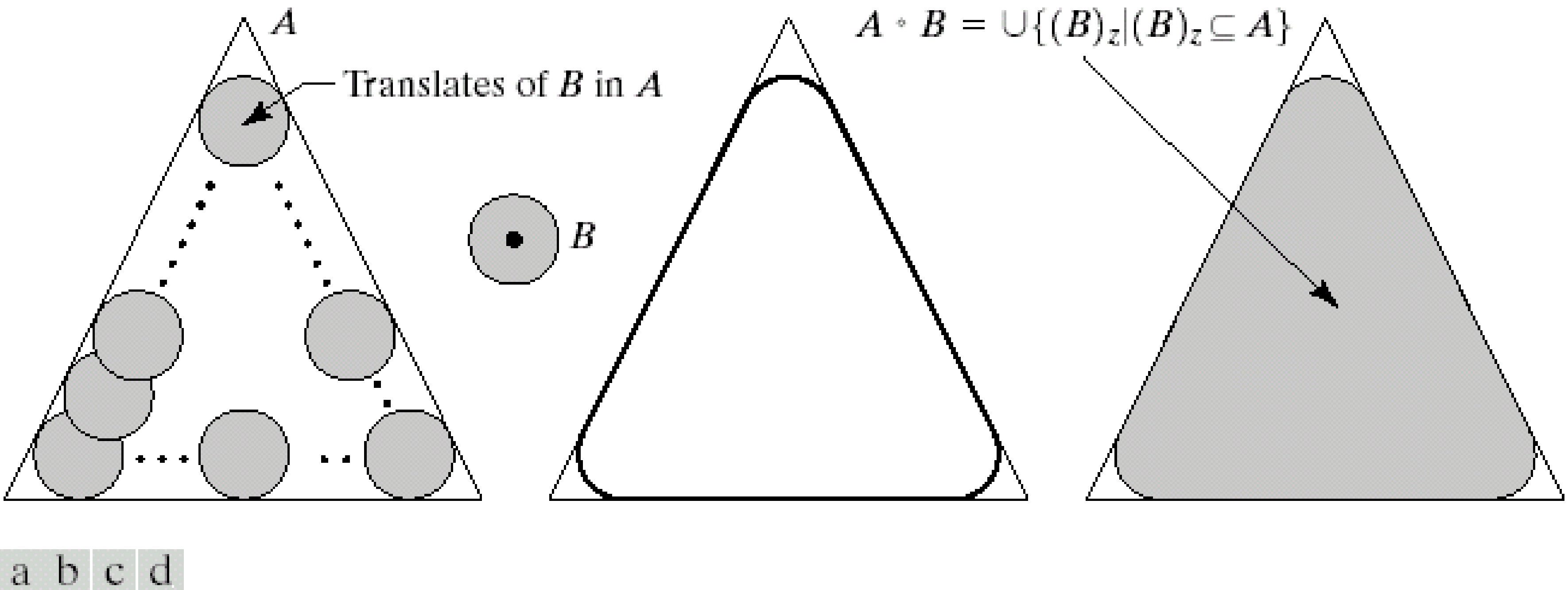
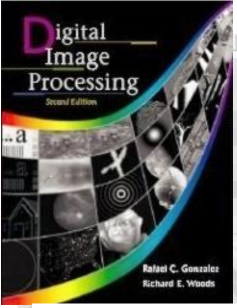
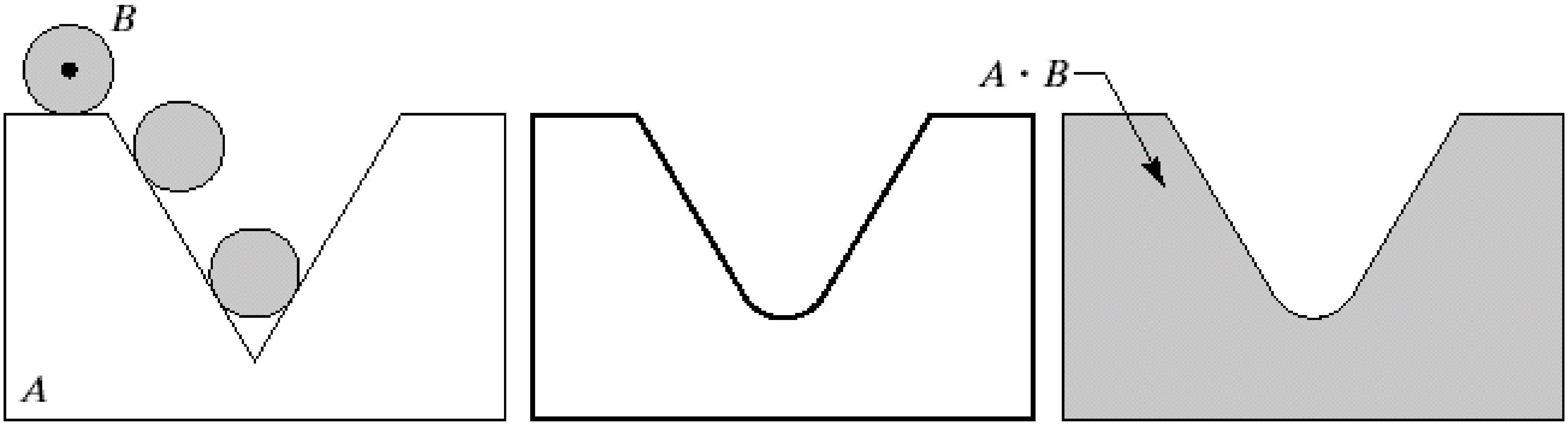


FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

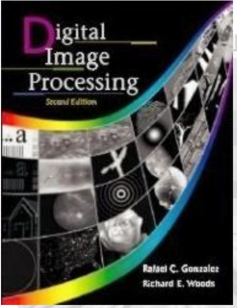


Morphological Image Processing

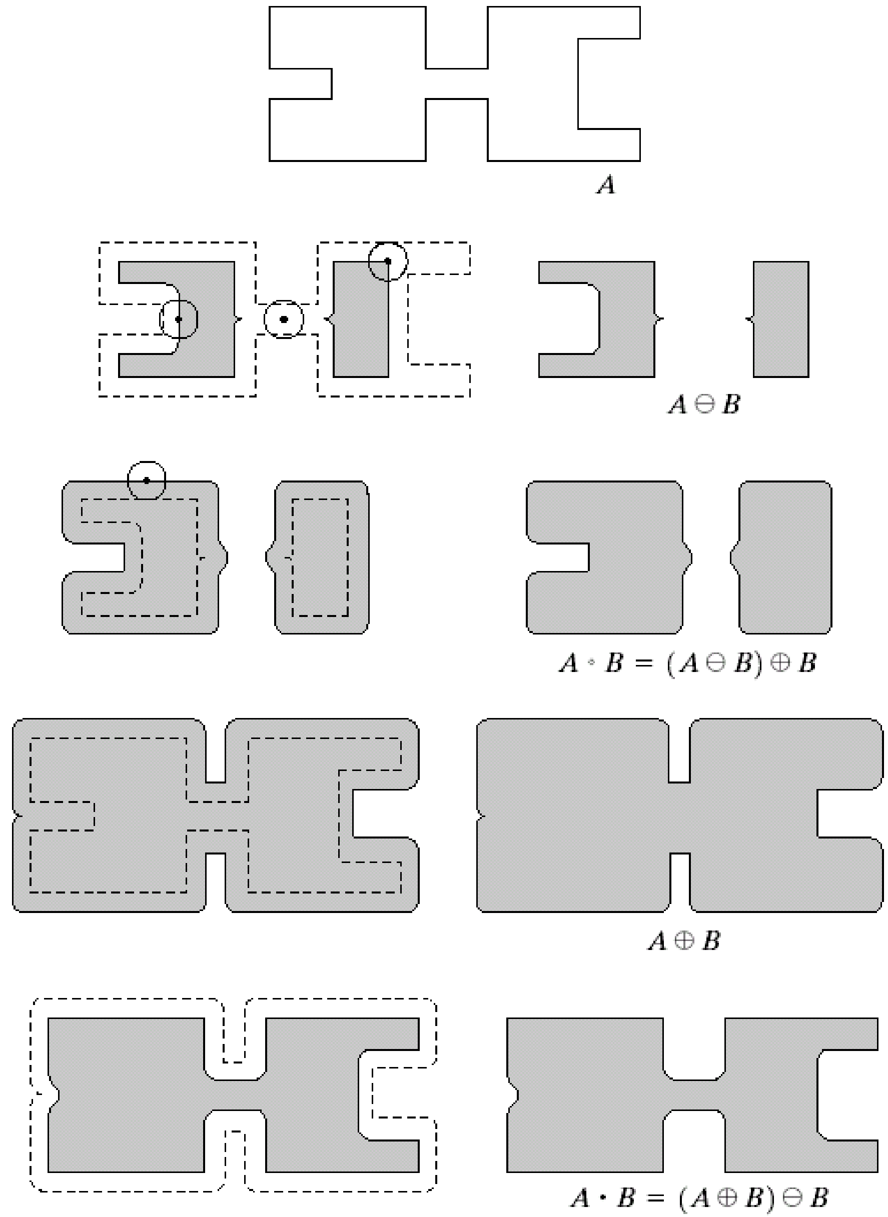


a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

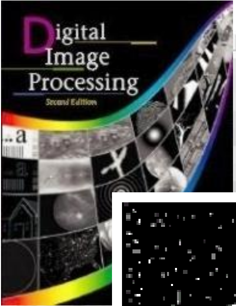


Morphological Image Processing

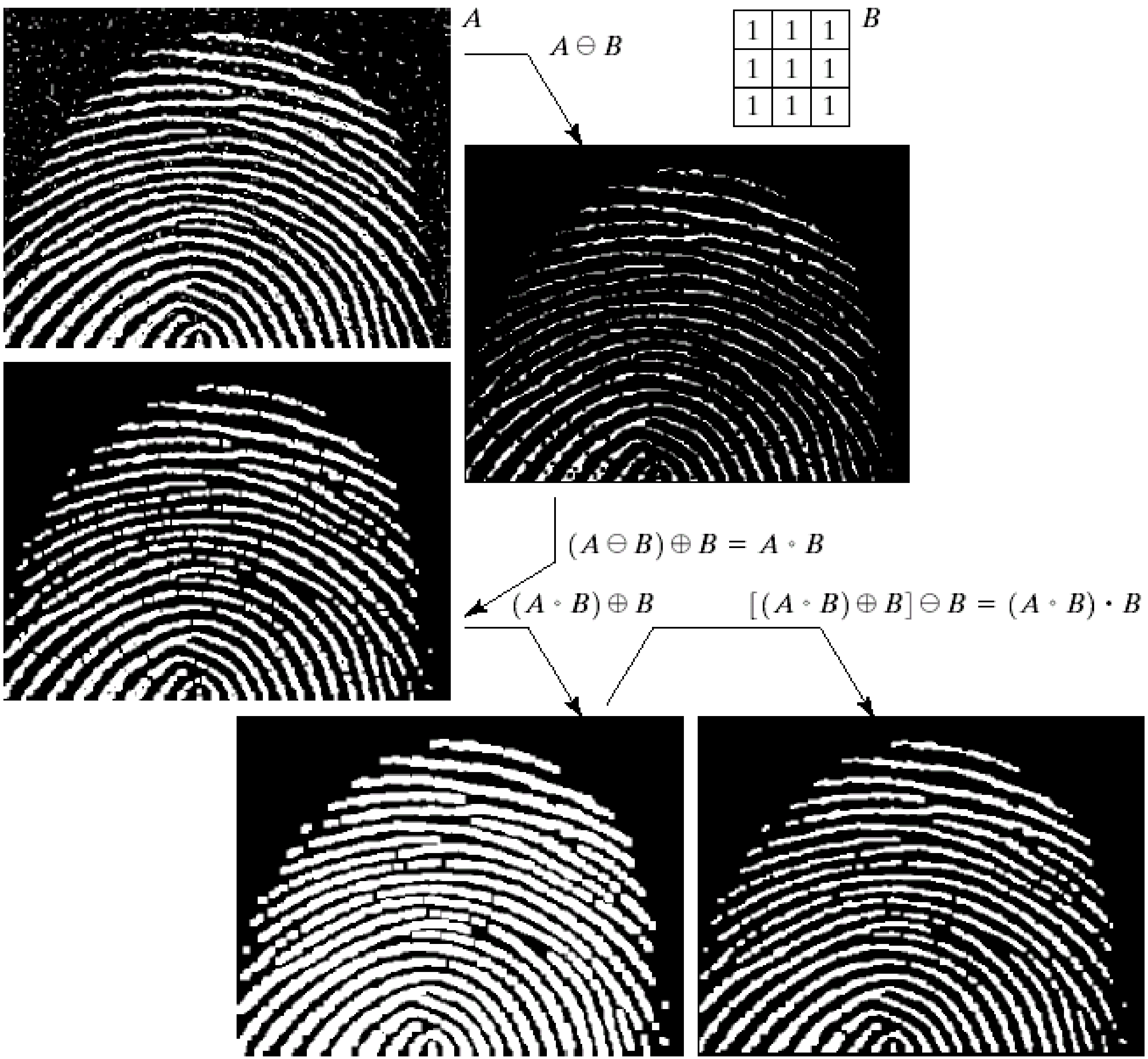


a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.



Morphological Image Processing

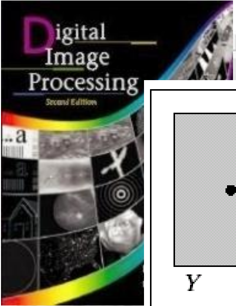


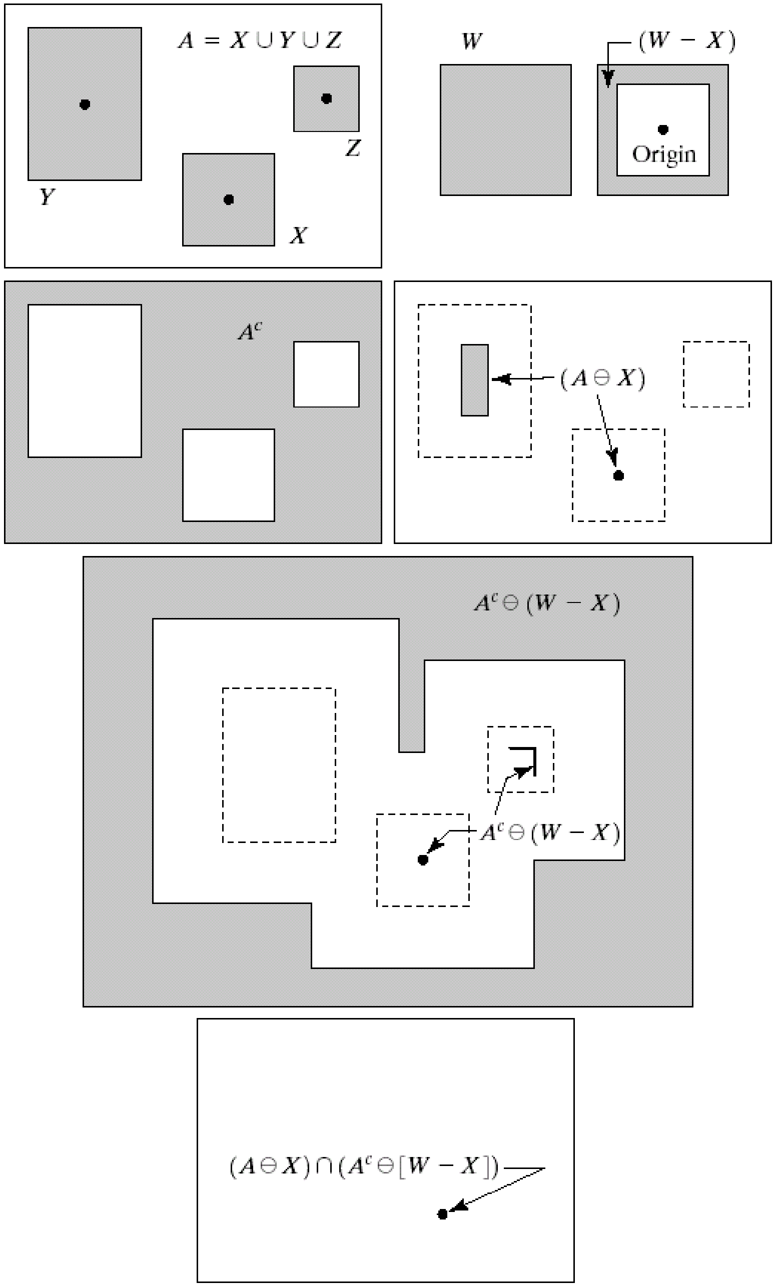
a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(c) Eroded image.
(d) Opening of A.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Hit-or-Miss Transform

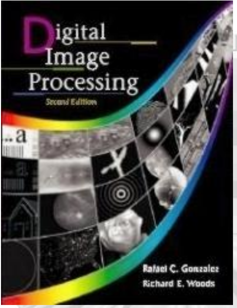
- Morphological hit-or-miss transform is a basic tool for shape detection.
- Definitions:
  - $B \rightarrow (B_1, B_2)$
    - B_1 is the set of elements of B associated with an object
    - B_2 is the set of elements of B associated with the corresponding background.





a b
c d
e
f

FIGURE 9.12
(a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$. (c) Complement of A. (d) Erosion of A by X. (e) Erosion of A^c by $(W - X)$. (f) Intersection of (d) and (e), showing the location of the origin of X, as desired.



Hit-or-Miss Transform

- $A \circledast B$
  - contains all the origin points at which, simultaneously:
    - B_1 found a match ("hit") in A and
    - B_2 found a match in A^c.

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

or

$$A \circledast B = (A \ominus B_1) - (A \oplus \hat{B}_2)$$

Example Basic Morphological Algorithms

- Purpose:
  - to extract image components that are useful in the representation and description of shape.

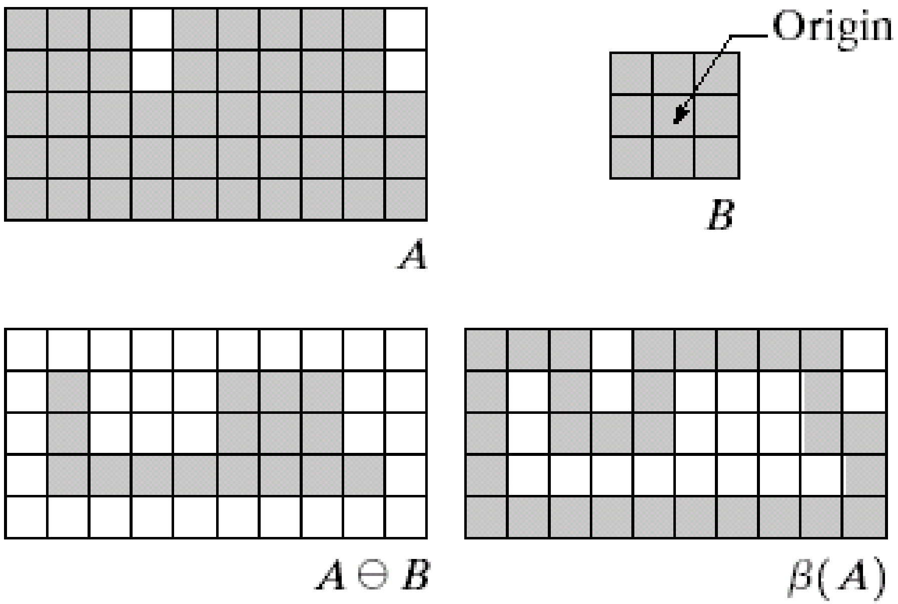
- Boundary Extraction:

$$\beta(A) = A - (A \ominus B)$$

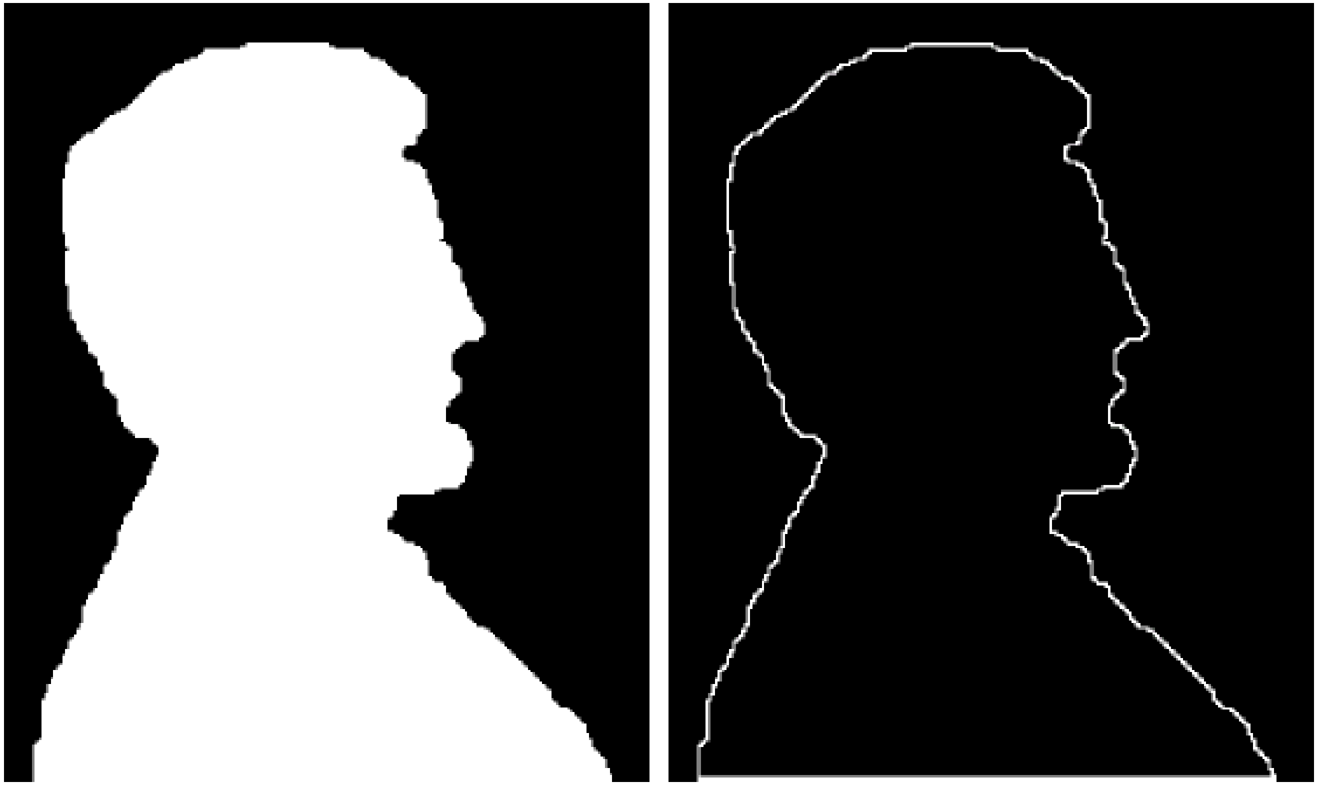
Morphological Image Processing

a	b
c	d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



Morphological Image Processing

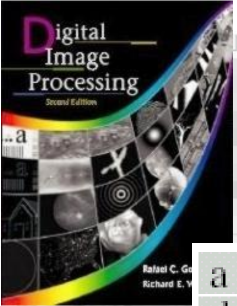


a b

FIGURE 9.14
(a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

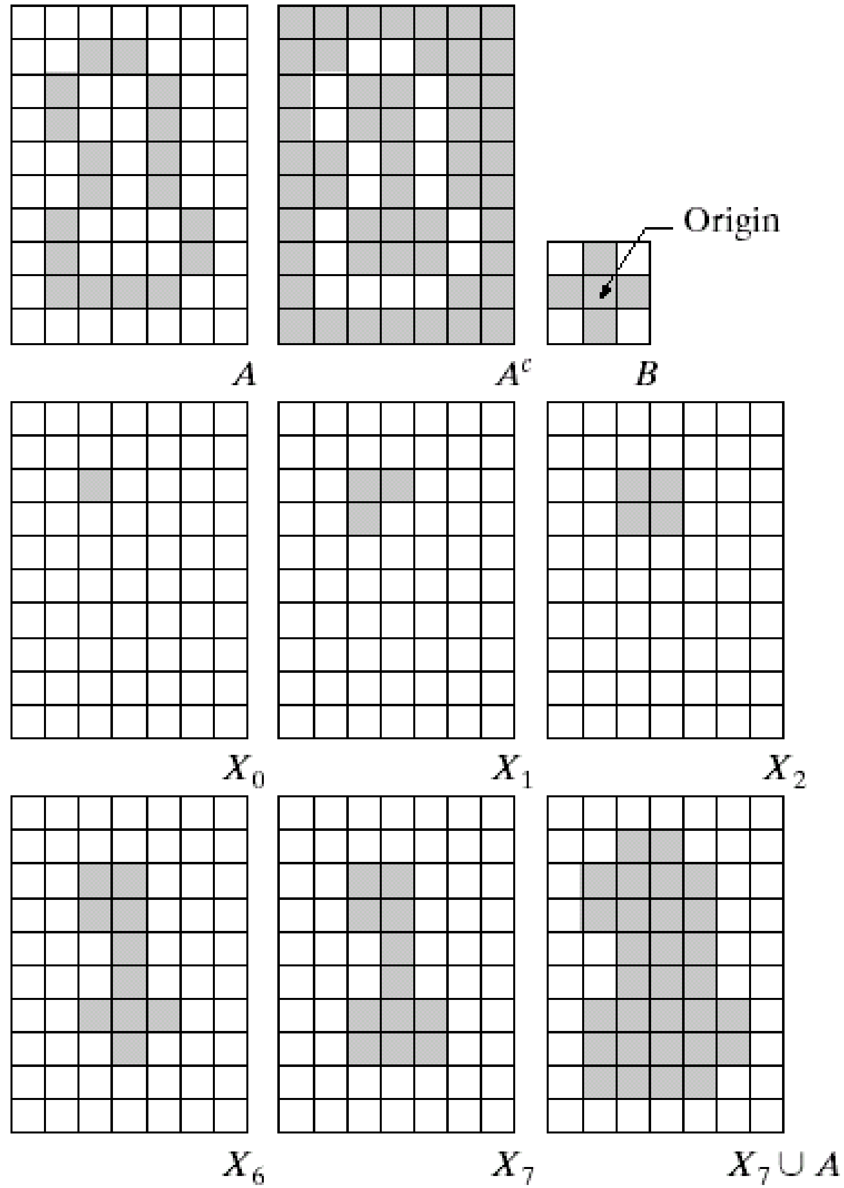
Region Filling Examples

$$X_k = (X_{k-1} \oplus B) \cap A^c$$



a b c
d e f
g h i

FIGURE 9.15 Region filling. (a) Set A. (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].



Start with a point inside

Morphological Image Processing
Region Filling

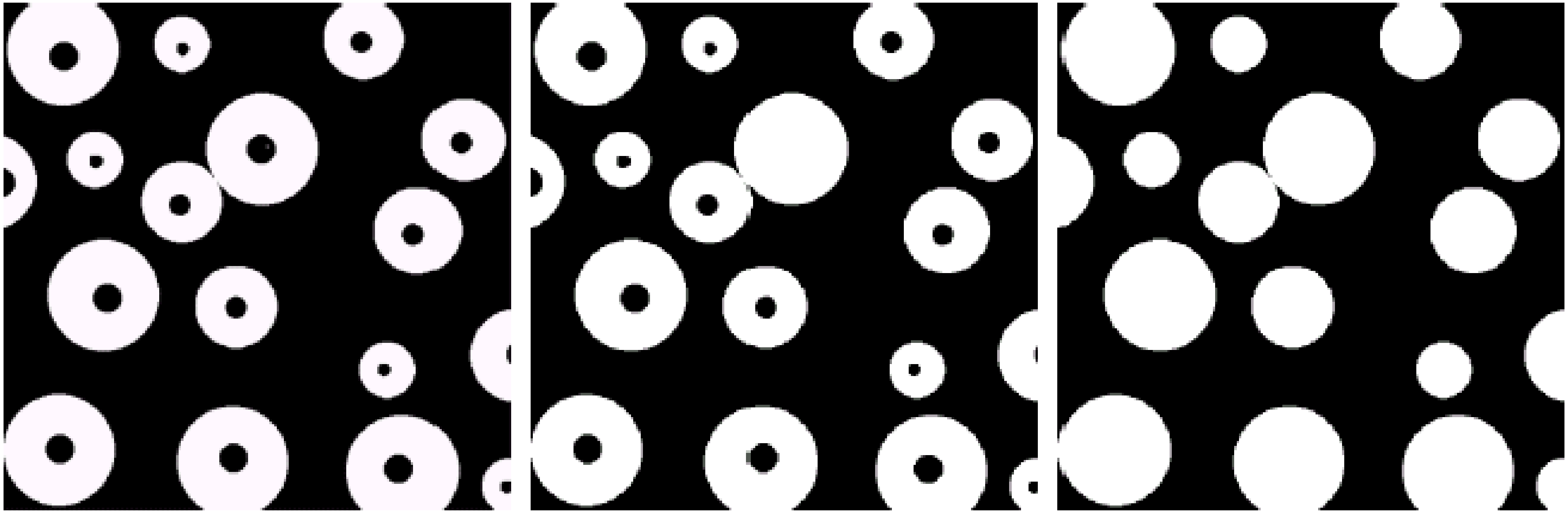


FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

Basic Morphological Algorithms

- Extraction of Connected Components:

$$X_k = (X_{k-1} \oplus B) \cap A \qquad \text{k=1,2,3,...}$$

Where X_0=p → a point p of Y is known
when X_k=X_{k-1} the algorithm has converged.

Morphological Image Processing

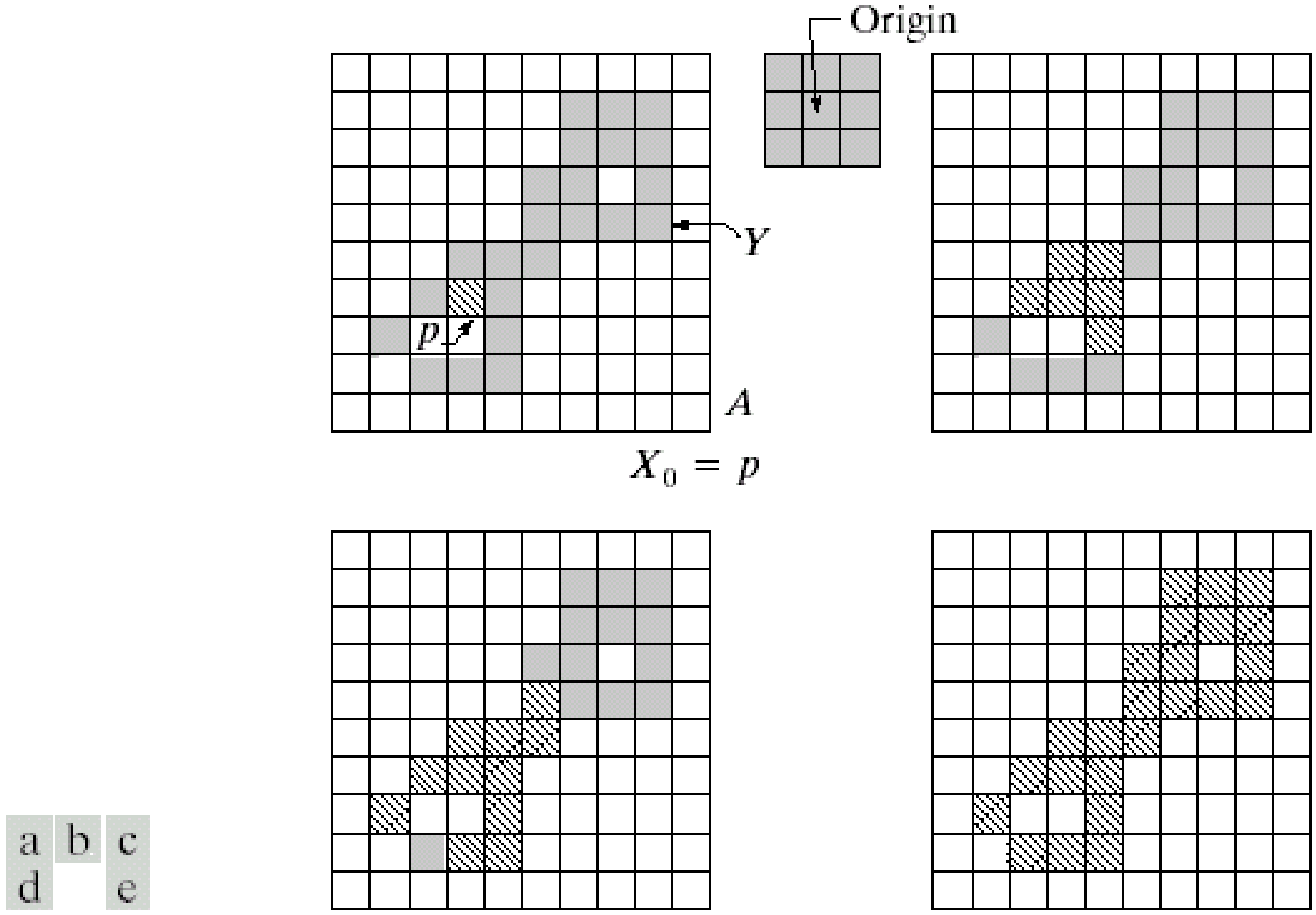
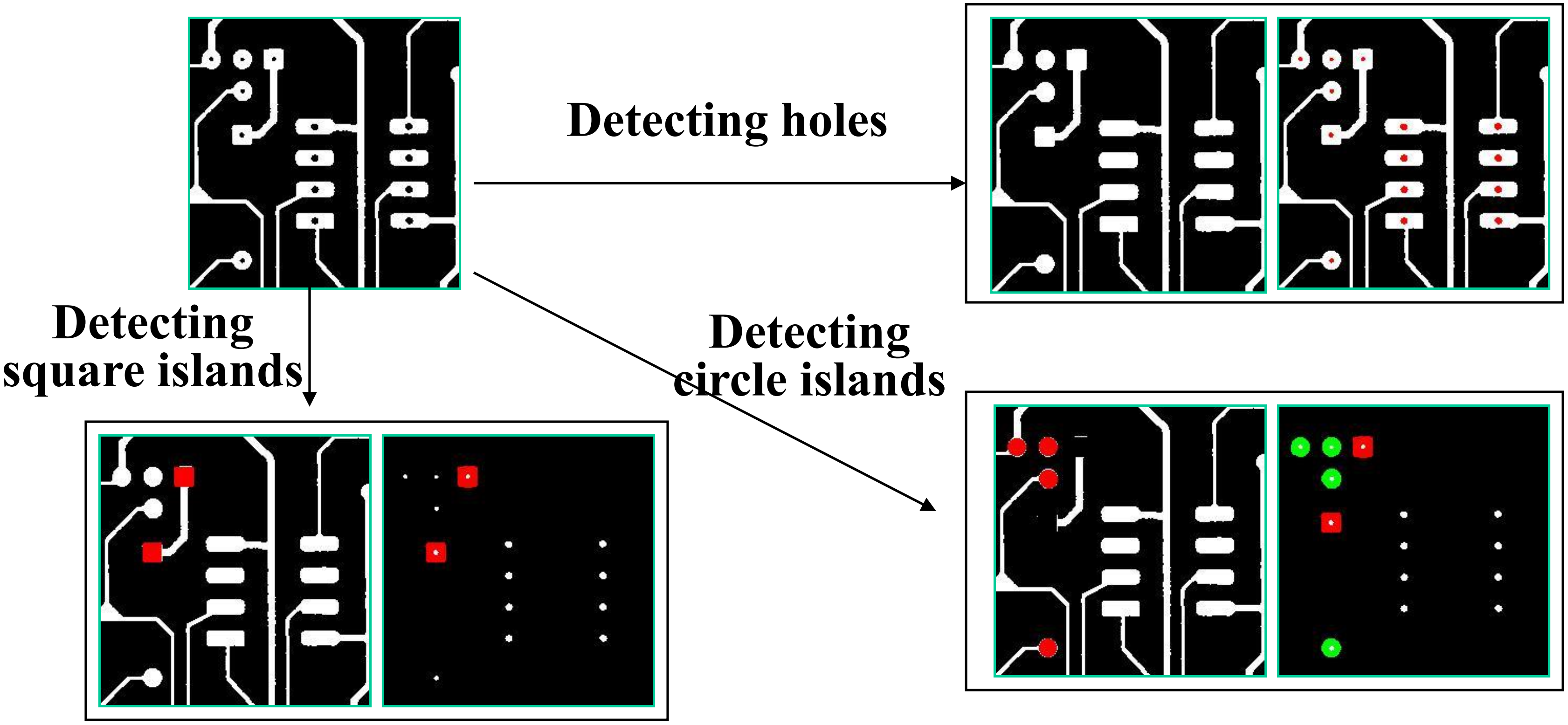
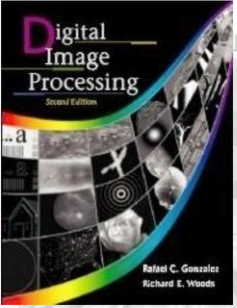


FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

Shape Feature Detection

- Example2: Decompose a printed circuit board in its main parts.





Recursive Dilation

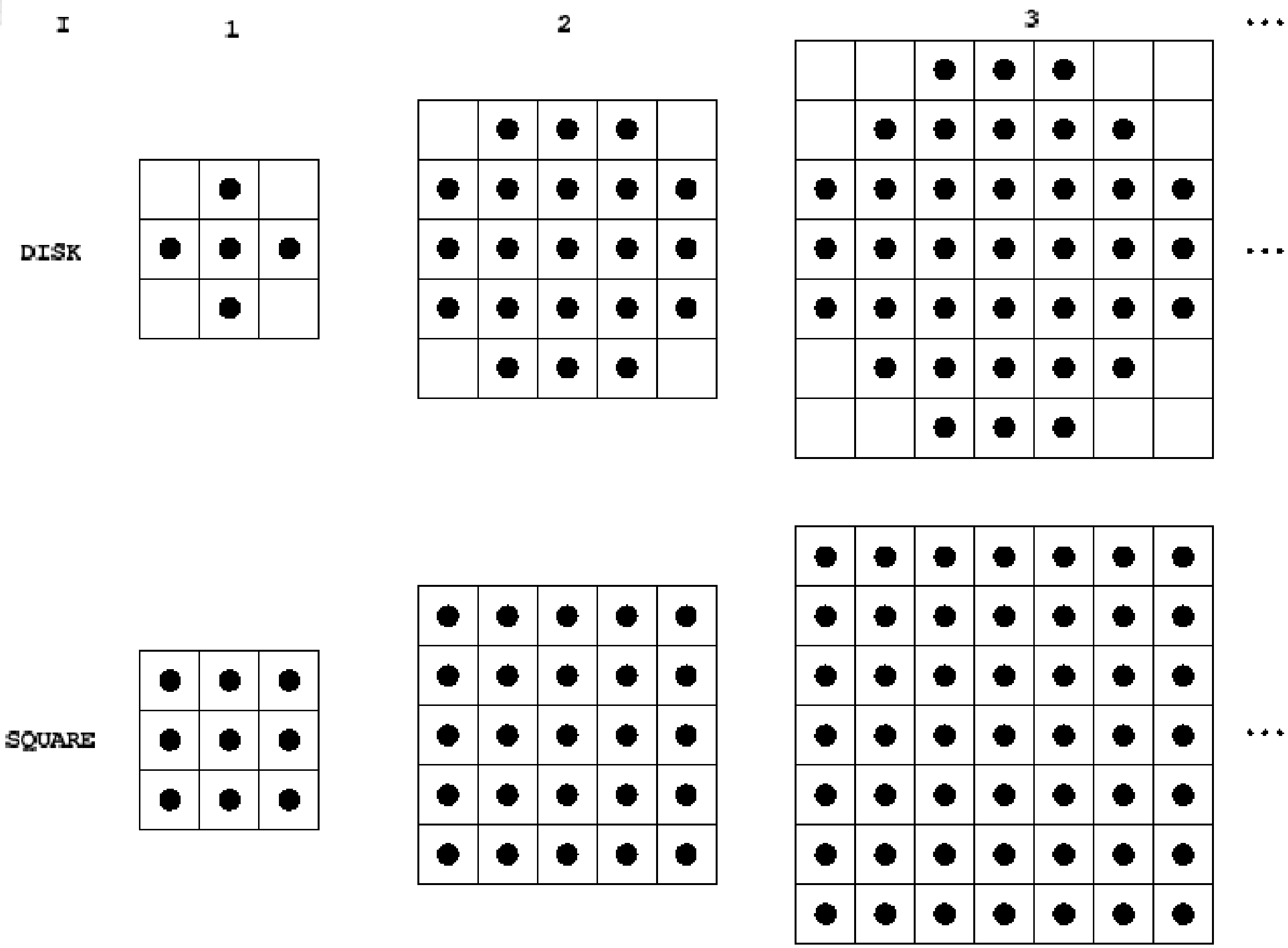
- Recursive Dilation is defined as:

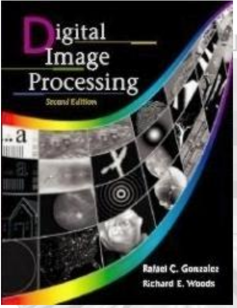
$$F^{i} \oplus K = \begin{cases} F & if \;\; i = 0 \\ (F^{i-1} \oplus K) \oplus K & if \;\; i \geq 1 \end{cases}$$

where, i is defined as scalar factor and K as its base.

- Recursive Dilation is employed to compose SE series in the same shape but different sizes.

Recursive Dilation





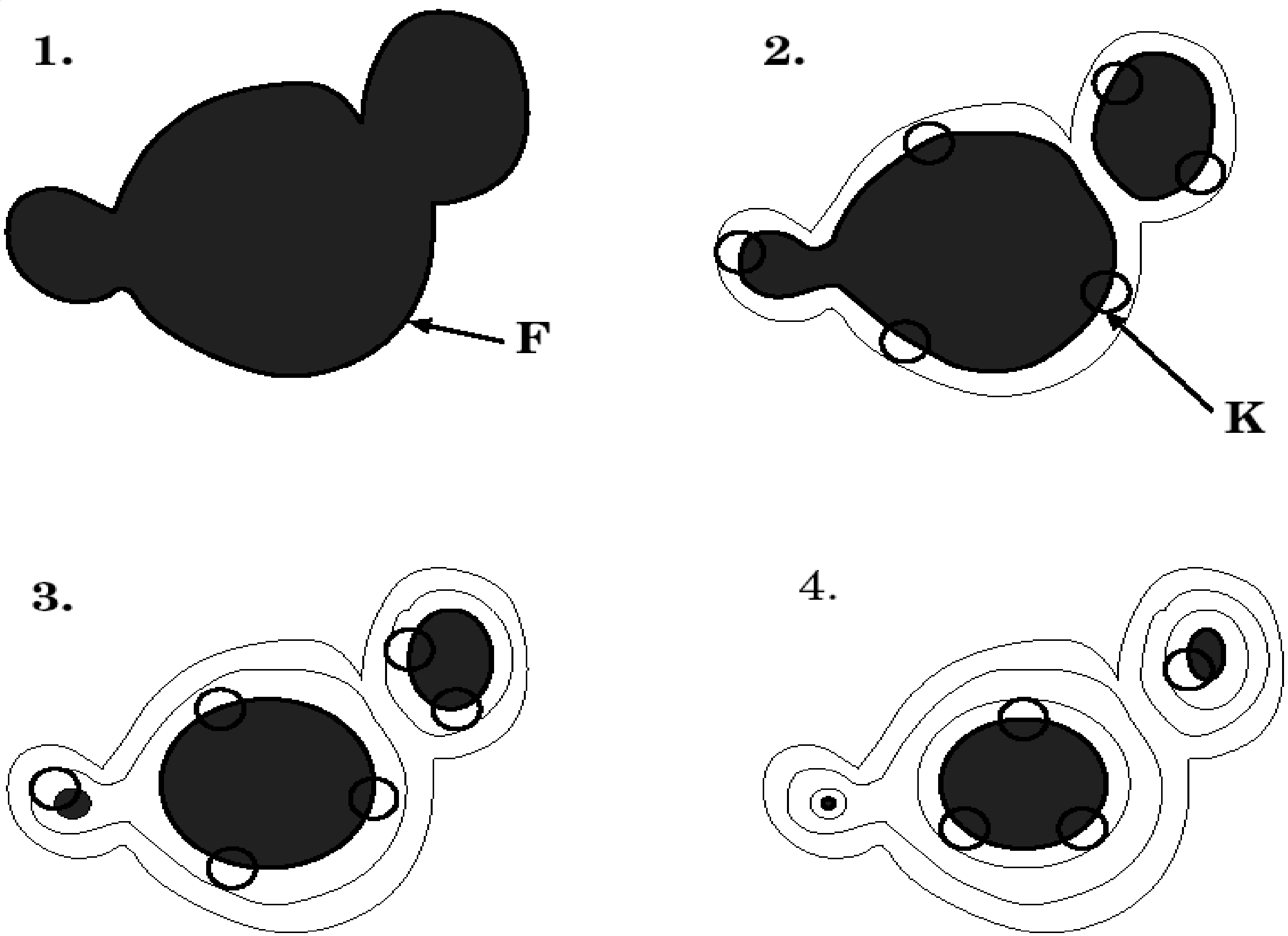
Recursive Erosion

- Recursive Erosion is also called *successive erosion which is defined as:*

$$F^{i} \ominus K = \begin{cases} F & if \;\; i = 0 \\ (F^{i-1} \ominus K) \ominus K & if \;\; i \geq 1 \end{cases}$$

- When performing recursive erosions of an object, its components are progressively shrunk until completely disappeared.
- Useful for distance transform and segmentation

Recursive Erosion



Dilation

- D_f and D_b are the domains of f and b, respectively

$$(f \oplus b)(s,t) = \max\{f(s-x, y-t) + b(x,y) \mid (s-x),(t-y) \in D_f; (x,y) \in D_b\}$$

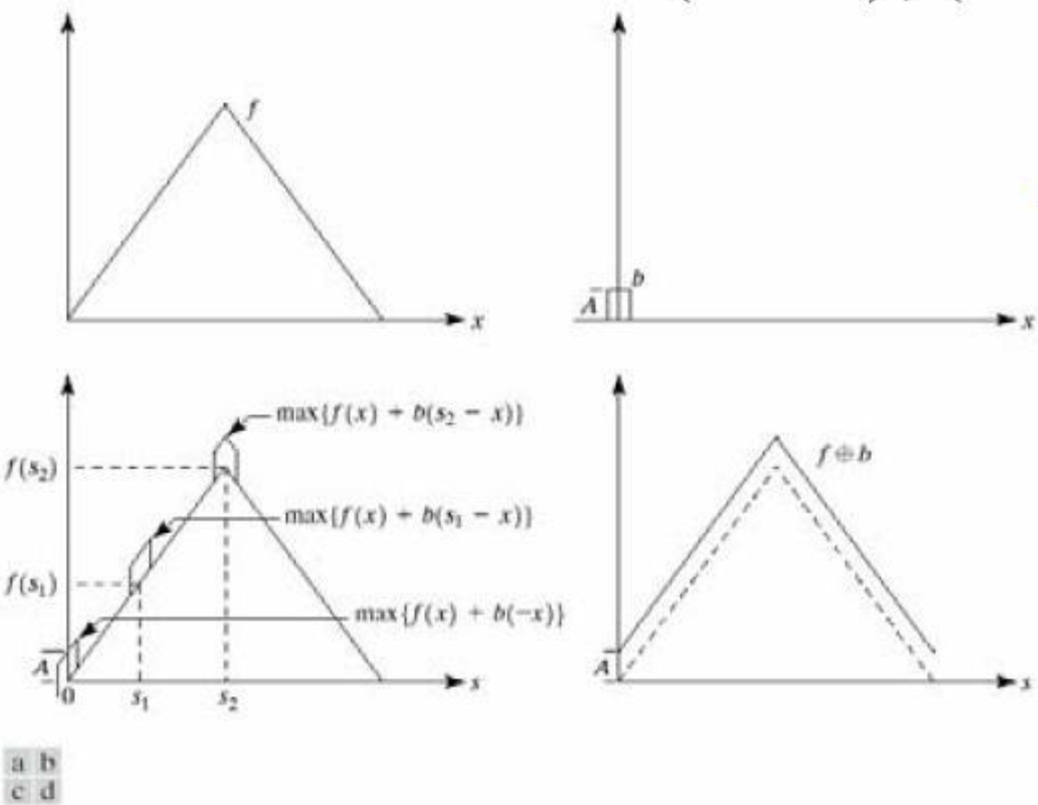


FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).

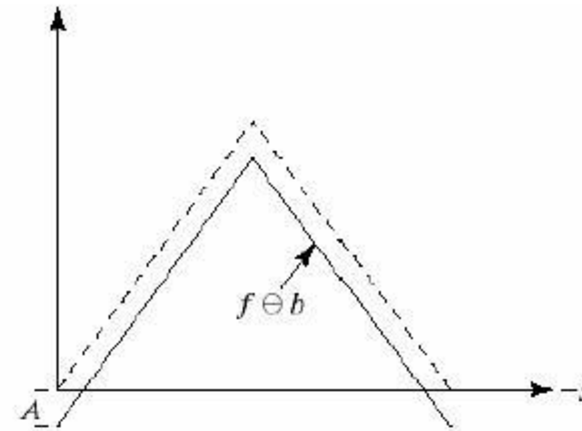
- condition (s-x) and (t-y) have to be in the domain of f and (x,y) have to be in the domain of b is similar to the condition in binary morphological dilation where the two sets have to overlap by at least one element

Dilation

- similar to 2D convolution
  - $f(s-x)$: $f(-x)$ is simply $f(x)$ mirrored with respect to the original of the x axis. the function $f(s-x)$ moves to the right for positive s, and to the left for negative s.
  - max operation replaces the sums of convolution
  - addition operation replaces with the products of convolution
- general effect
  - if all the values of the structuring element are positive, the output image tends to be brighter than the input
  - dark details either are reduced or eliminated, depending on how their values and shapes relate to the structuring element used for dilation

Erosion

FIGURE 9.28 Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).



$$(f \ominus b)(s,t) = \min\{ f(s+x, y+t) - b(x,y) \mid (s+x),(t+y) \in D_f ; (x,y) \in D_b \}$$

- condition (s+x) and (t+y) have to be in the domain of f and (x,y) have to be in the domain of b is similar to the condition in binary morphological erosion where the structuring element has to be completely contained by the set being eroded

Erosion

- similar to 2D correlation
  - $f(s+x)$ moves to the left for positive s and to the right for negative s.
- general effect
  - if all the elements of the structuring element are positive, the output image tends to be darker than the input
  - the effect of bright details in the input image that are smaller in area than the structuring element is reduced, with the degree of reduction being determined by the gray-level values surrounding the bright detail and by the shape and amplitude values of the structuring element itself

Dual property

- gray-scale dilation and erosion are duals with respect to function complementation and reflection.

$$(f \ominus b)^c(s,t) = (f^c \oplus \hat{b})(s,t)$$

where

$$f^c = -f(x,y) \text{ and } \hat{b} = b(-x,-y)$$

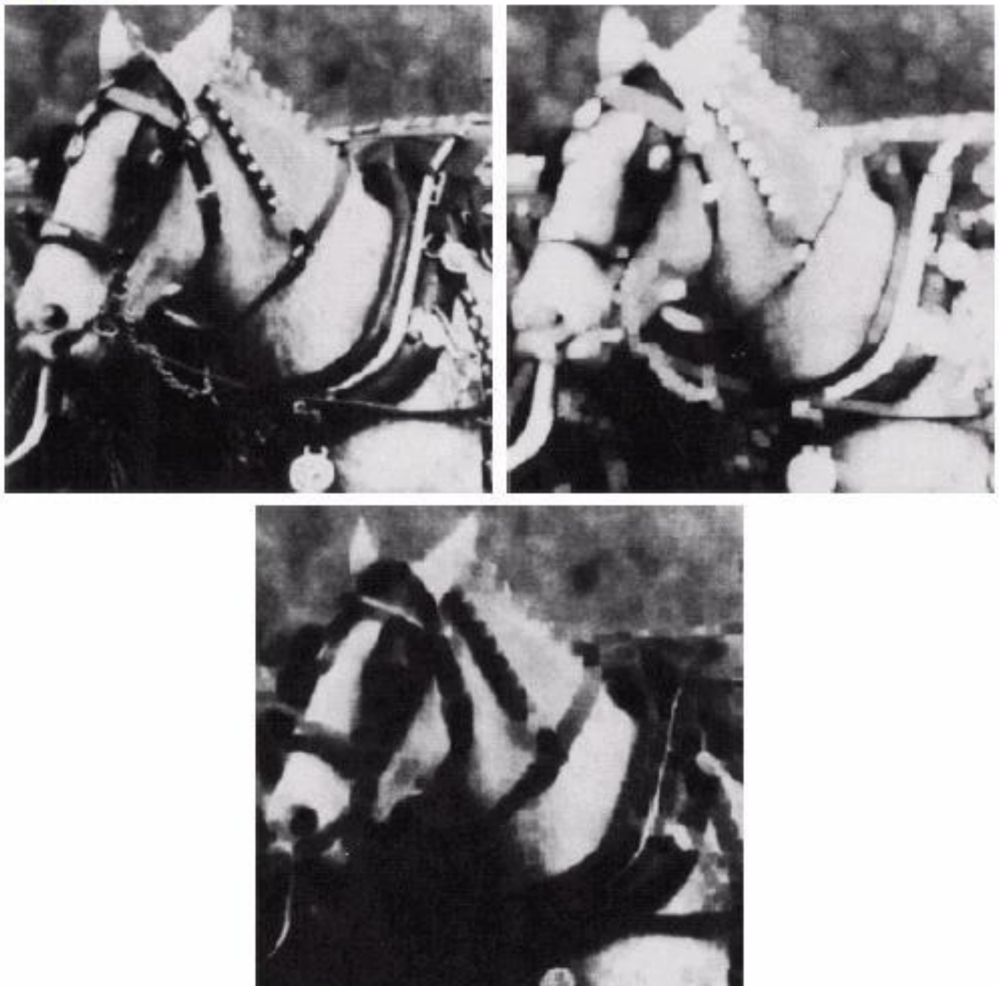
Example

a	b
c	

a) 512x512 original image

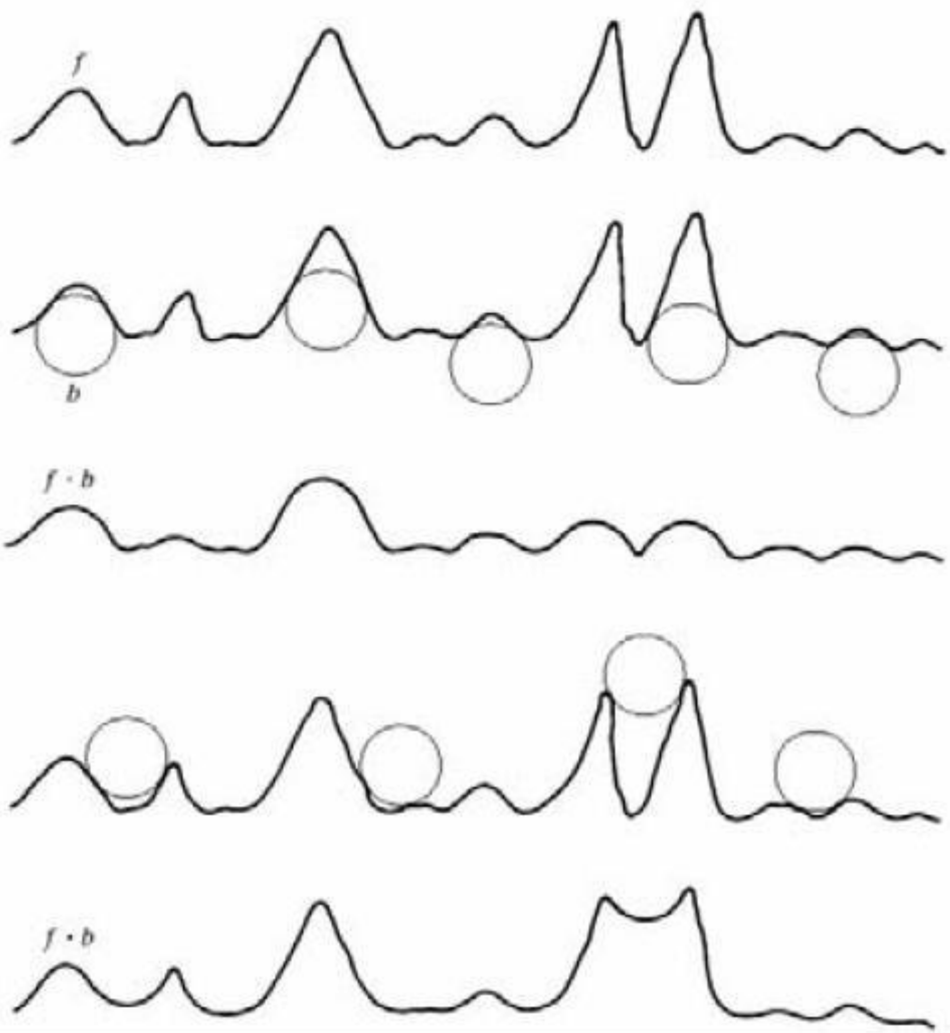
b) result of dilation with a flat-top structuring element in the shape of parallelepiped of unit height and size 5x5 pixels
note: brighter image and small, dark details are reduced

c) result of erosion
note: darker image and small, dark details are reduced



view an image function f(x,y) in 3D perspective, with the x- and y-axes and the gray-level value axis

Opening and closing



a) a gray-scale scan line
b) positions of rolling ball for opening
c) result of opening
d) positions of rolling ball for closing
e) result of closing

$$f \circ b = (f \ominus b) \oplus b$$

$$f \bullet b = (f \oplus b) \ominus b$$

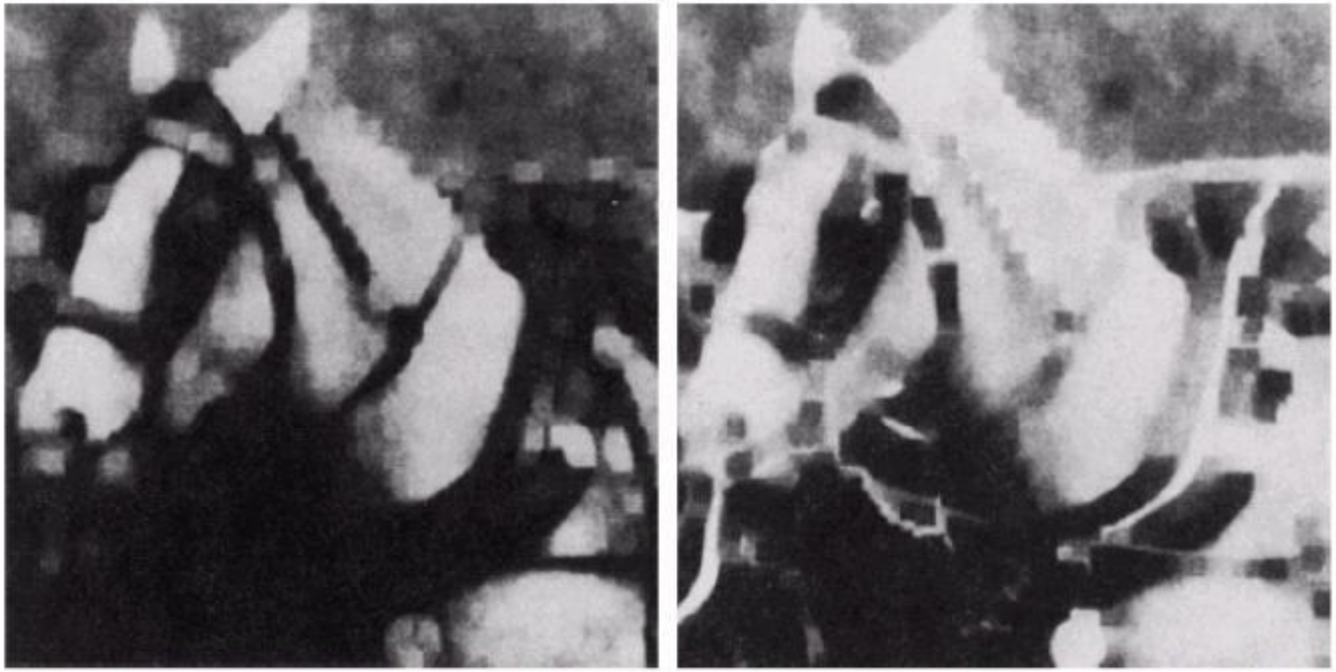
Effect of opening

- opening
  - the structuring element is rolled underside the surface of f
  - all the peaks that are narrow with respect to the diameter of the structuring element will be reduced in amplitude and sharpness
  - so, opening is used to remove small light details, while leaving the overall gray levels and larger bright features relatively undisturbed.
  - the initial erosion removes the details, but it also darkens the image.
  - the subsequent dilation again increases the overall intensity of the image without reintroducing the details totally removed by erosion

Effect of closing

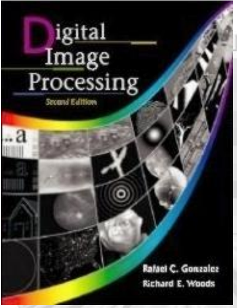
- closing
  - the structuring element is rolled on top of the surface of f
  - peaks essentially are left in their original form (assume that their separation at the narrowest points exceeds the diameter of the structuring element)
  - so, closing is used to remove small dark details, while leaving bright features relatively undisturbed.
  - the initial dilation removes the dark details and brightens the image
  - the subsequent erosion darkens the image without reintroducing the details totally removed by dilation

Examples



a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

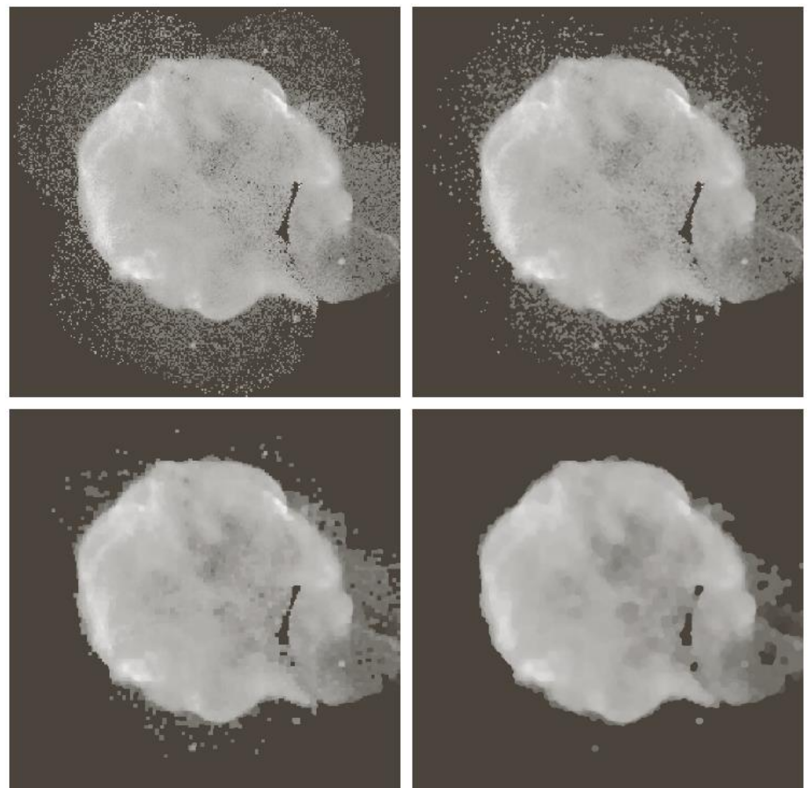


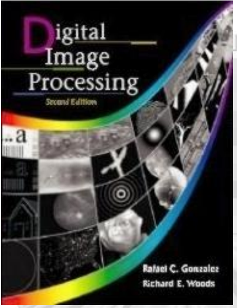
Applications: Morphological Smoothing

- Sequential application of Opening, then Closing

a b
c d

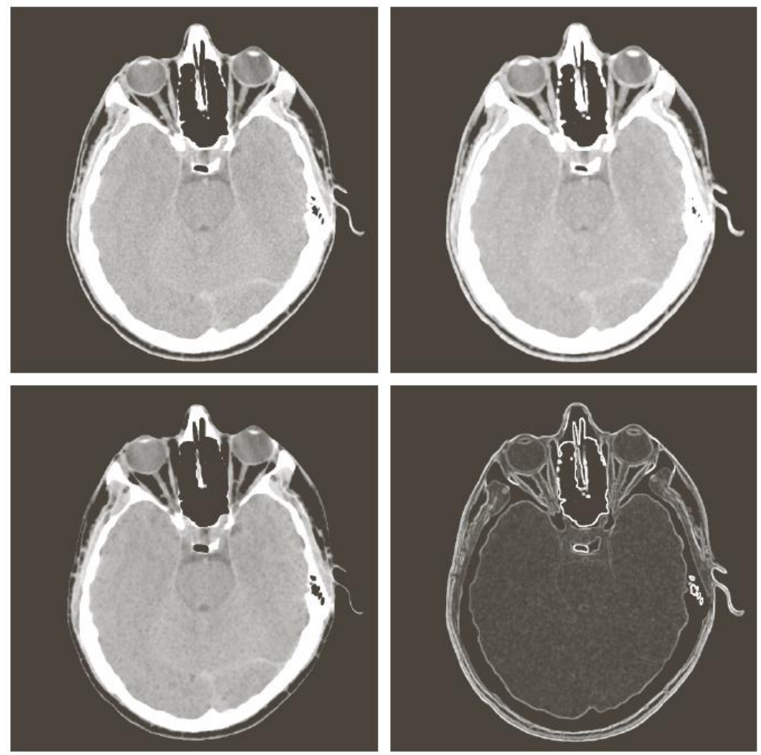
FIGURE 9.38
(a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)





Morphological Gradient

- The dilation thickens regions in an image and the erosion shrinks them.
- Their difference emphasizes the boundaries between regions.
- Homogenous areas are not affected



a b
c d

FIGURE 9.39 (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

$$g = (f \oplus b) - (f \ominus b)$$

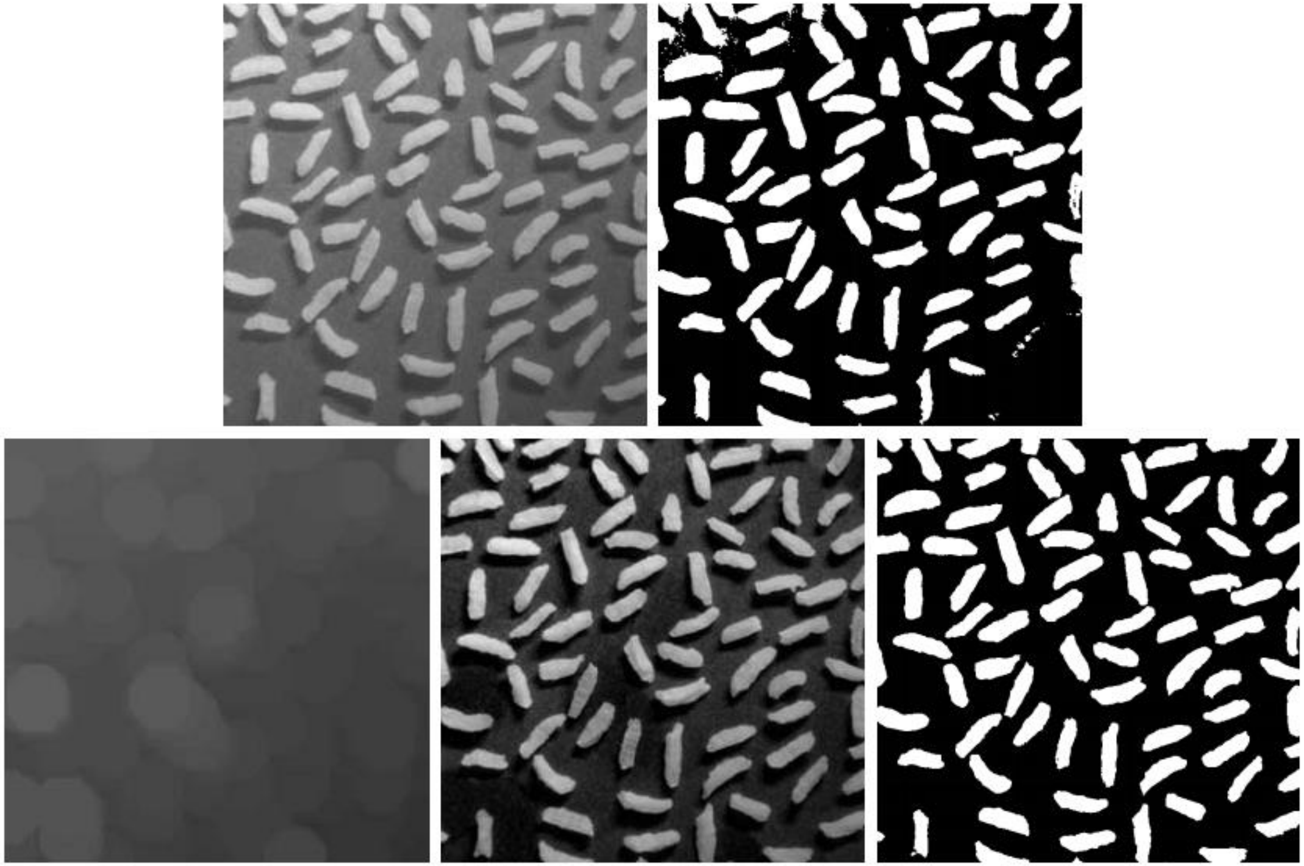
Top-Hat / Bottom-Hat Transform

- Combining image subtraction with openings and closings results in so-called *top-hat and bottom-hat* transformations.

$$T_{\text{hat}}(f) = f - (f \circ b) \qquad B_{\text{hat}}(f) = (f \bullet b) - f$$

- The top-hat transform is used for light objects on a dark background (*white top-hat*)
  - Can correct non-uniform illumination.
- the bottom-hat transform is used for the converse (*black top-hat*).

Example: Top-Hat Transform

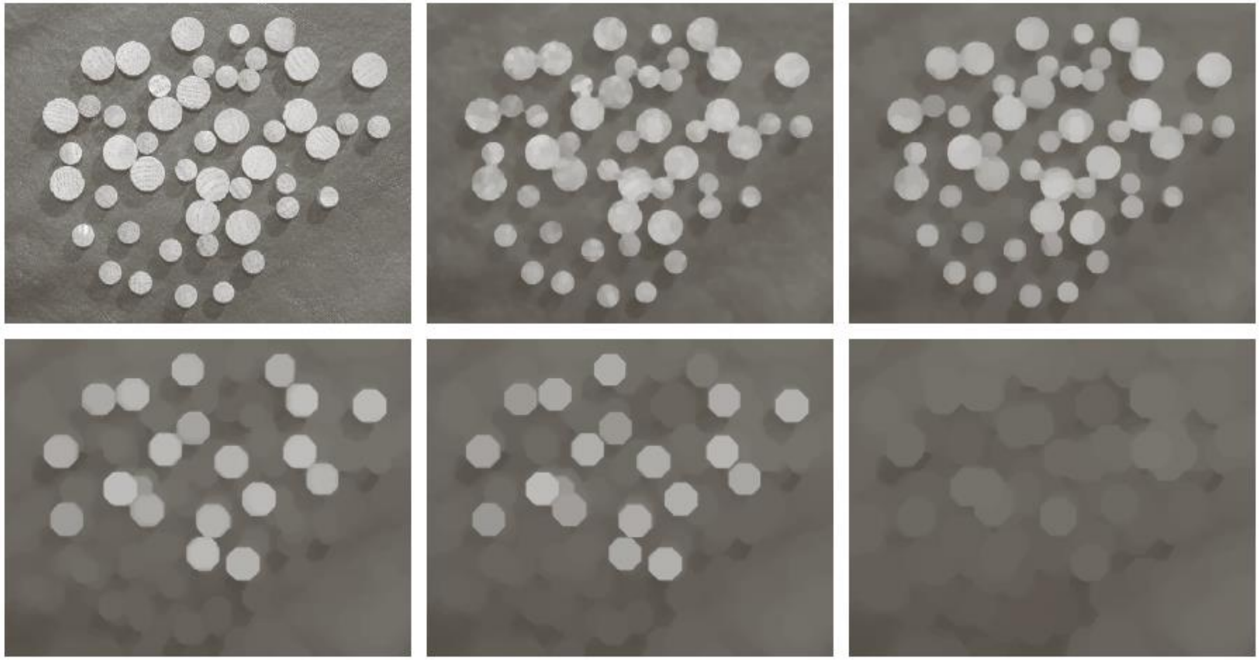


a b
c d e

FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

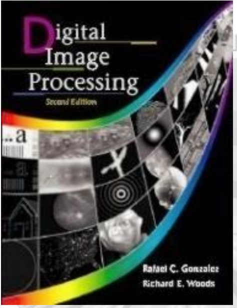
Granulometry

- Method consists of applying openings with SEs of increasing size.



a b c
d e f

FIGURE 9.41 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)



Granulometry

We expect significant differences (peaks in the plot) around radii at which the SE is large enough to encompass a set of particles of approximately the same diameter.

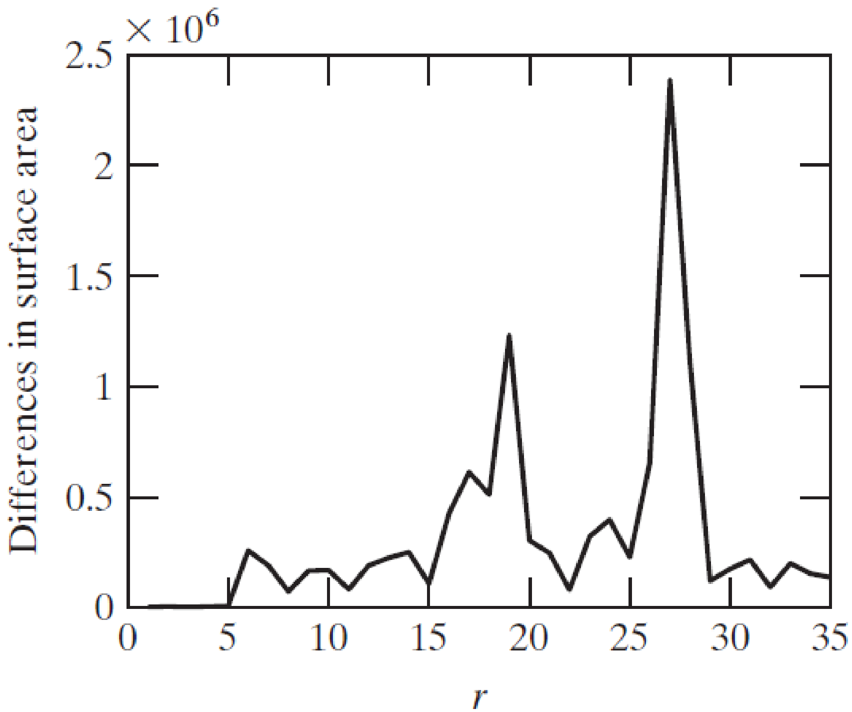
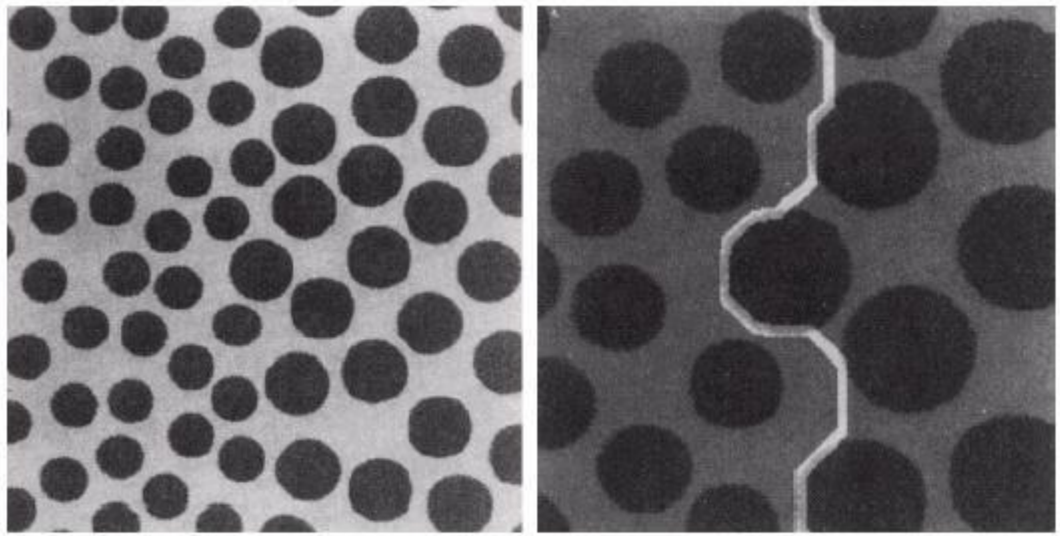


FIGURE 9.42 Differences in surface area as a function of SE disk radius, *r*. The two peaks are indicative of two dominant particle sizes in the image.

Textural segmentation



a b

FIGURE 9.35 (a) Original image. (b) Image showing boundary between regions of different texture. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

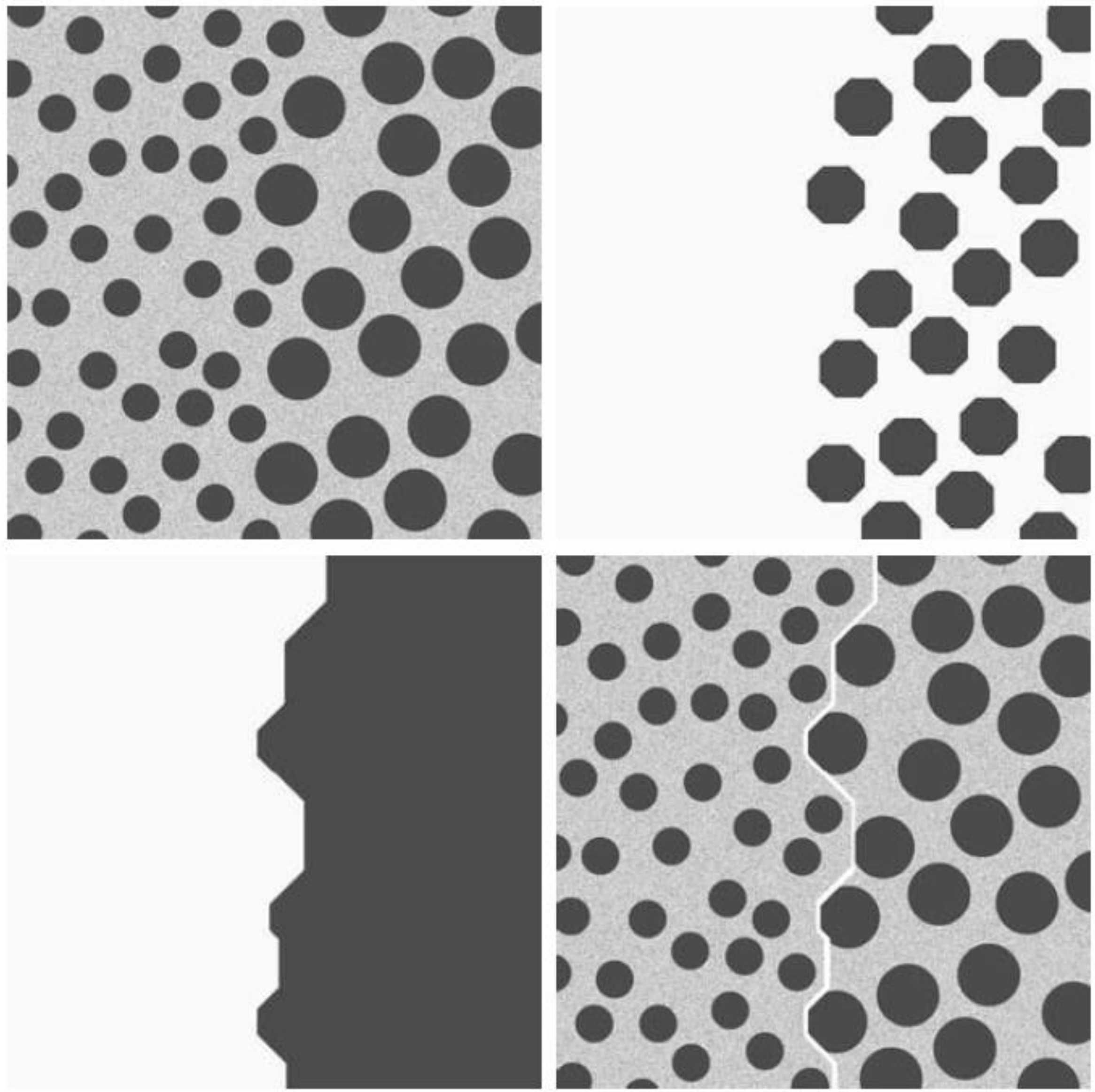
- the region the right consists of circular blobs of larger diameter than those on the left.
- the objective is to find the boundary between the two regions based on their textural content.

white	*black*

Textural segmentation

- Perform
  - closing the image by using successively larger structuring elements than small blobs
    - as closing tends to remove dark details from an image, thus the small blobs are removed from the image, leaving only a light background on the left and larger blobs on the right
  - opening with a structuring element that is large in relation to the separation between the large blobs
    - opening removes the light patches between the blobs, leaving dark region on the right consisting of the large dark blobs and now equally dark patches between these blobs.
  - by now, we have a light region on the left and a dark region on the right, so we can use a simple threshold to yield the boundary between the two textural regions.

Textural Segmentation



a b
c d

FIGURE 9.43 Textural segmentation. (a) A 600 × 600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient operation.