31/05/21

Lecture 09 4:15 PM

Header file

rand()

Matlab

Origin Pro

~~randU~~

Logrand()

LCG → random numbers

rand()

$\bar{N}$

time avg # of customers

$Q = N -$ in the queue

$N$ in the system

server utilization

$$\boxed{\bar{Q} = \bar{N} - \rho}$$

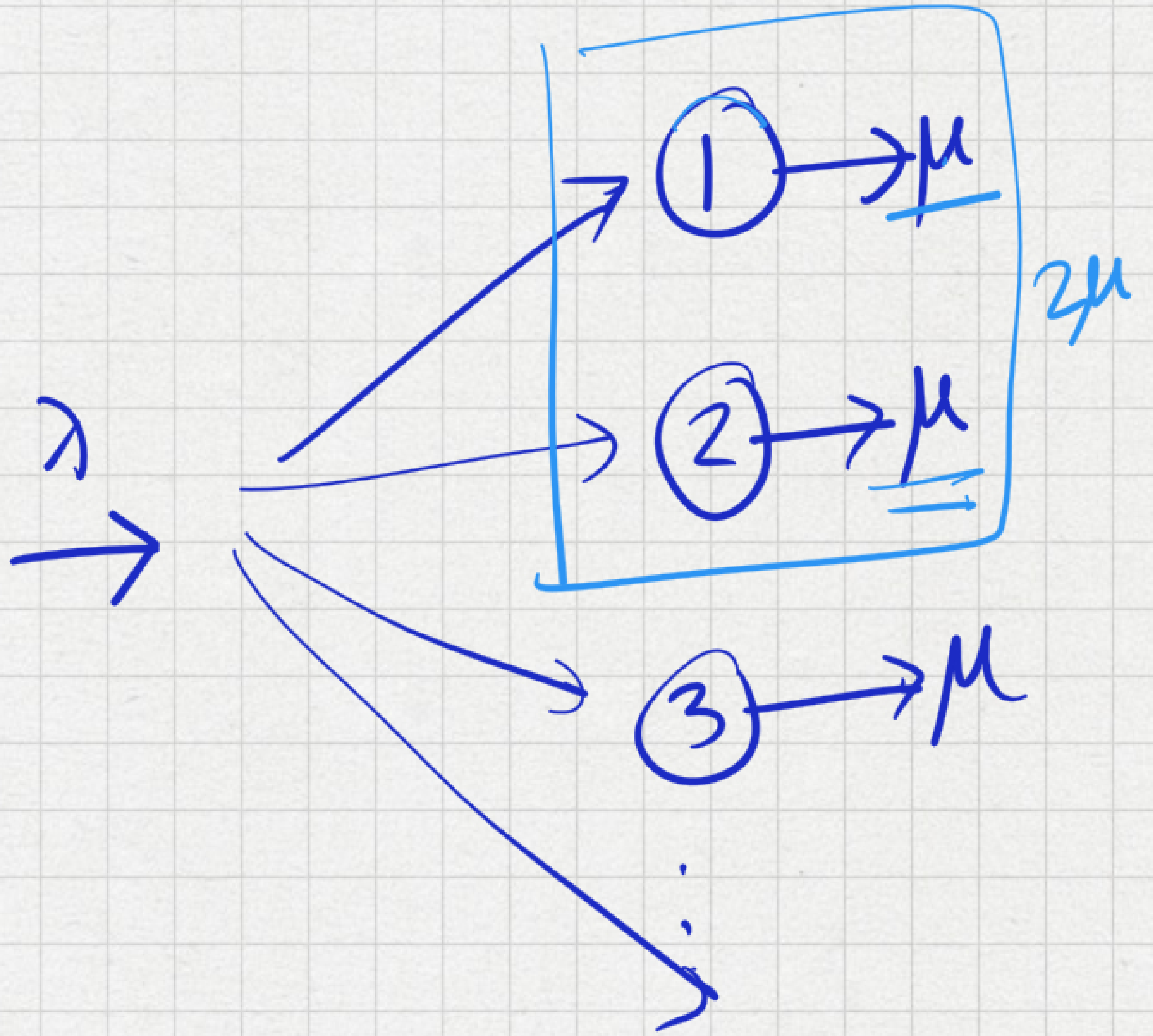
M/M/1

M/M/∞ → infinity


$\lambda_n = \lambda$

$n = 0, 1, \ldots$

$\mu_n = n\mu$




$P_0, P_1, \ldots$

Steady-state Prob.

$P_0 \,/\, P_n$

$$P_n = P_0 \, \frac{\lambda_{n-1}\lambda_{n-2}\cdots\lambda_1\lambda_0}{\mu_n \mu_{n-1} \cdots \mu_2 \mu_1}$$

$(n\mu)\ (n-1)\mu \quad \cdots \quad (2\mu)\ \mu_1$

$$P_0 = \frac{1}{1 + \sum_{n=1}^{\infty} \frac{\lambda_{n-1}\lambda_{n-2}\cdots\lambda_1\lambda_0}{\mu_n \mu_{n-1} \cdots \mu_2 \mu_1}}$$

N

$$P_n = \boxed{P_0 \frac{\lambda \quad \lambda \quad \cdots \quad \lambda}{(n\mu)\,(n-1)\mu \cdots (2\mu)(\mu)}}$$

$$= P_0 \prod_{i=0}^{n-1} \frac{\lambda}{(i+1)\mu}$$

$$\boxed{= P_0 \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!}}$$

$$= e^{-\frac{\lambda}{\mu}} \cdot \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!}$$

$$P_0 = \frac{1}{1 + \sum_{n=1}^{\infty} \frac{\lambda^n}{\mu^n n!}}$$

$$= \frac{1}{\sum_{n=0}^{\infty} \frac{\lambda^n}{\mu^n} n!} =$$

$$= \frac{1}{e^{\lambda/\mu}} = e^{-\lambda/\mu} = e^{-\rho}$$

$$P_0 \boxed{\frac{\lambda}{\mu}} \boxed{\frac{\lambda}{2\mu}}$$

$$(i+1) \Longrightarrow$$

$1 \times 2 \cdots n$

$$\sum 1 + 1!\,\rho + 2!\,\rho^2 \cdots$$

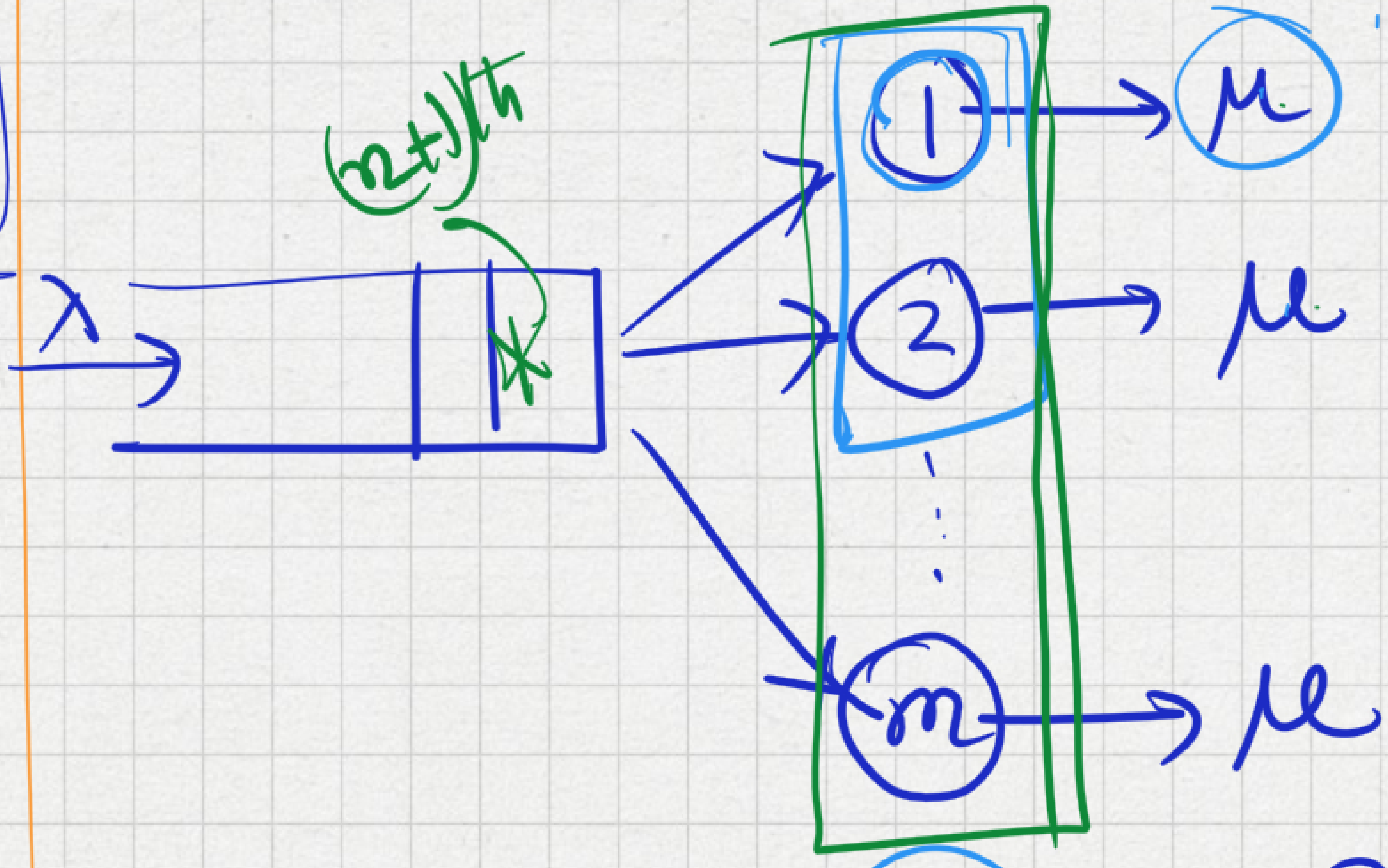
$$P_n = \left(\lambda/\mu\right)^n \cdot \frac{e^{-\lambda/\mu}}{n!}$$

Poisson distribution

$$\boxed{\bar{N} = \frac{\lambda}{\mu}}$$

$$\boxed{T = \frac{1}{\mu}}$$

M/M/m → server = m




n = 1

n = 2

$\mu_2 = 2\mu$

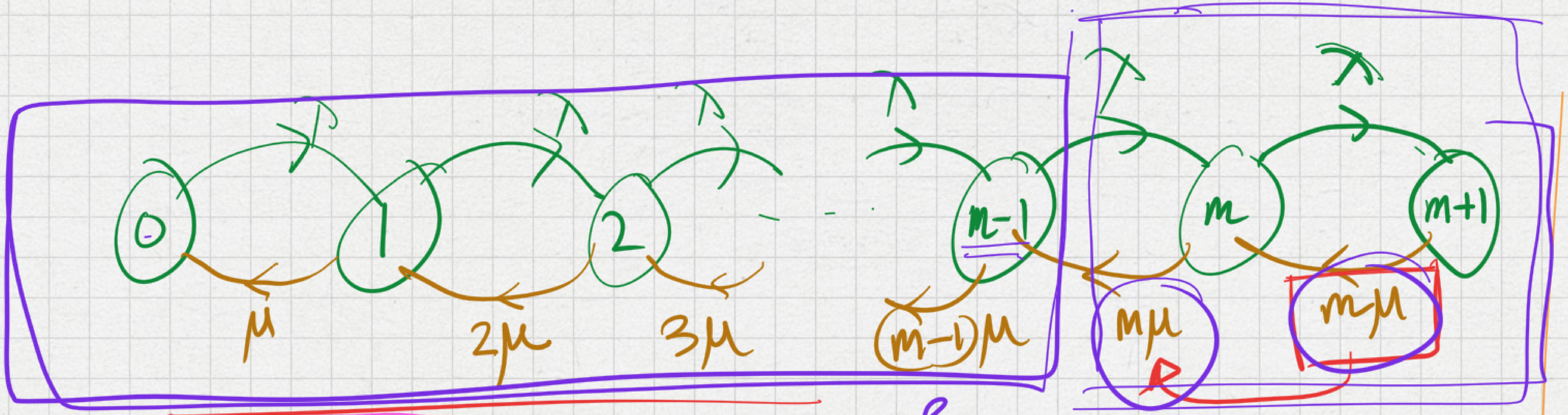
$\mu_m = m\mu$

n = m+1

$\mu_{m+1} = m\mu$

$$\lambda_n = \lambda \qquad n = 0, 1, 2, \ldots$$

$$\mu_n = \min(n\mu, m\mu) = \begin{cases} n\mu, & n < m \\ m\mu, & n \geq m \end{cases}$$




nmμ

$n \geq m$

$n < m$

nμ

$n < m$

$$P_n = P_0 \prod_{i=0}^{n-1} \frac{\lambda}{(i+1)\mu}$$

$$= P_0 \left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!}$$

$n \geq m$

$$P_n = P_0 \prod_{i=0}^{m-1} \frac{\lambda}{(i+1)\mu} \cdot \frac{1}{m!} \prod_{j=m}^{n-1} \frac{\lambda}{m\mu}$$

rough

$$\frac{\lambda_{n-1}\ \lambda_{n-2} \cdots \lambda_m\ \lambda_{m-1} \cdots \lambda_1\ \lambda_0}{\mu_n\ \mu_{n-1}\ \ \mu_m\ \mu_{m-1}\ \ \mu_2\ \mu_1}$$

$n=7,\ m=5$

$$\frac{\lambda_6\ \lambda_5\ \lambda_4\ \lambda_3\ \lambda_2\ \lambda_1\ \lambda_0}{\mu_7\ \mu_6\ \mu_5\ \mu_4\ \mu_3\ \mu_2\ \mu_1} = \frac{\lambda^7}{(5\mu)(5\mu)(5\mu)(4\mu)(3\mu)(2\mu)\mu}$$

$$= P_0 \left(\frac{\lambda}{\mu}\right)^n \frac{1}{m!\, m^{n-m}}$$

$$P_n = \begin{cases} P_0 \dfrac{(m\rho)^n}{n!} & n < m \\ P_0 \dfrac{m^m \rho^n}{m!} & n \geq m \end{cases}$$

$$\boxed{\rho = \frac{\lambda}{m\mu}} < 1$$

$$P_0 = \frac{1}{\sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!} + \frac{(m\rho)^m}{m!} \cdot \frac{1}{1-\rho}}$$