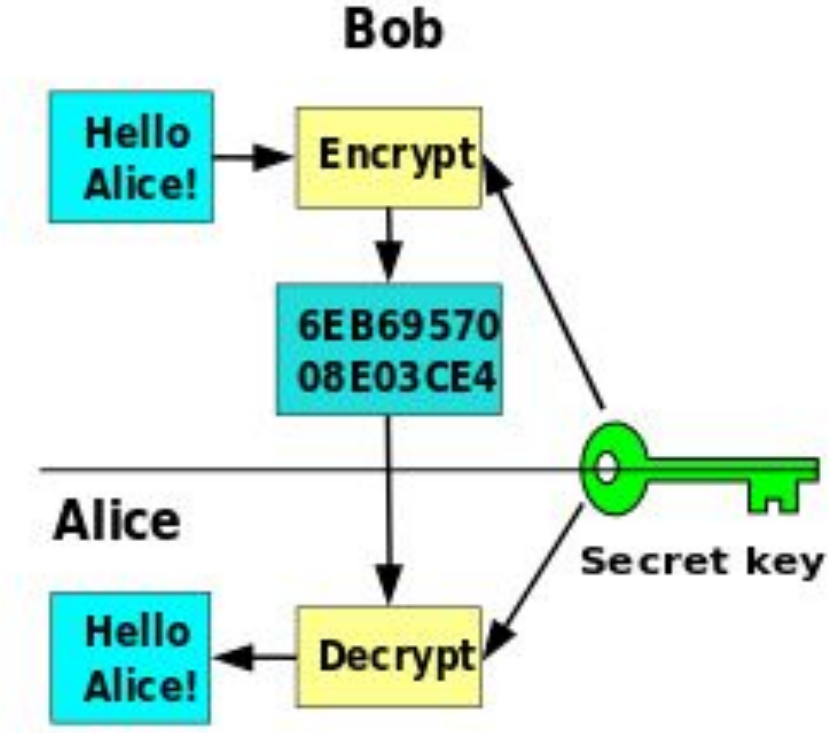
Cryptography

Cryptography

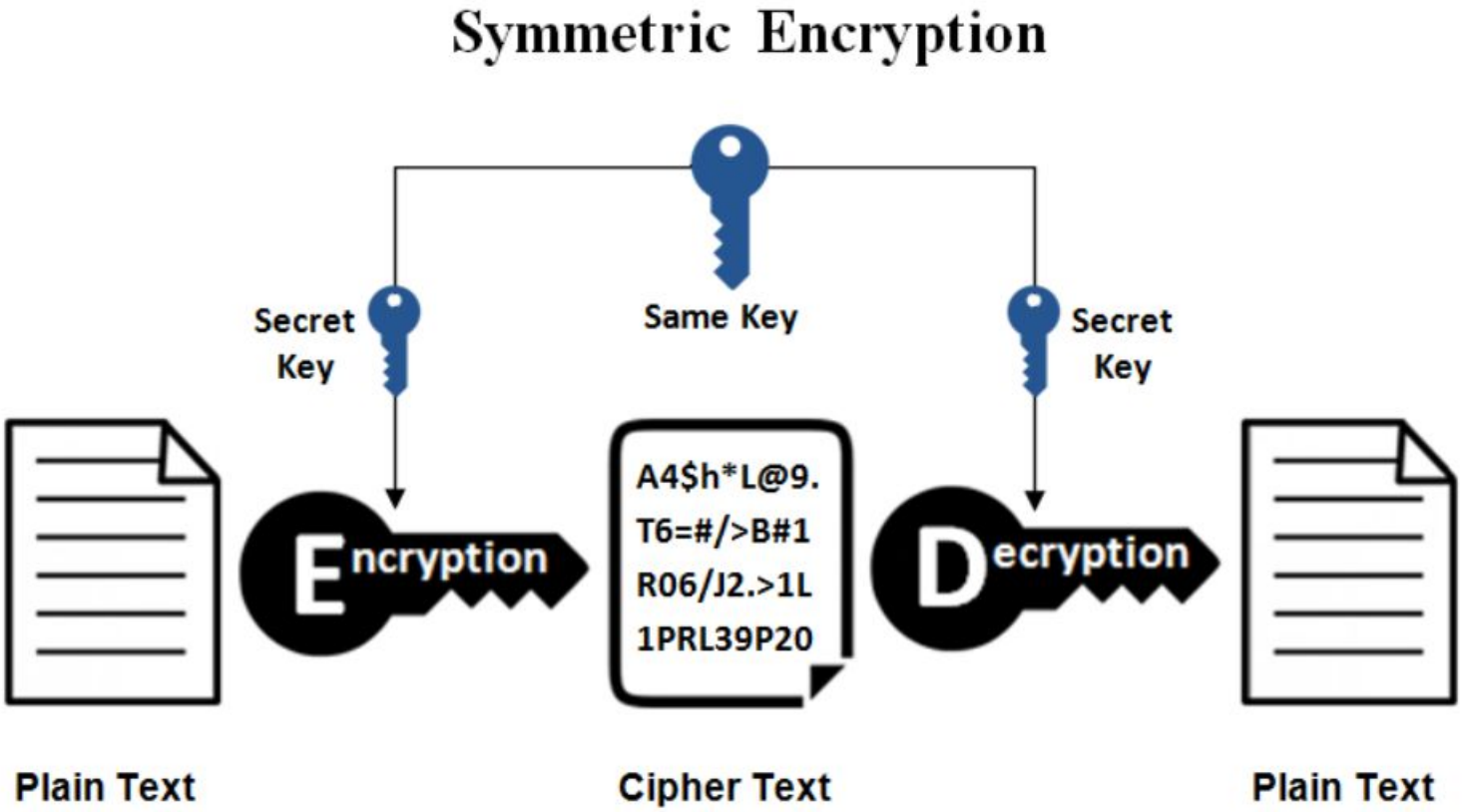
Cryptography is the study of secure communications techniques that allow only the sender and intended recipient of a message to view its contents.

ABC (meaningful message) → ZYX(cipher)

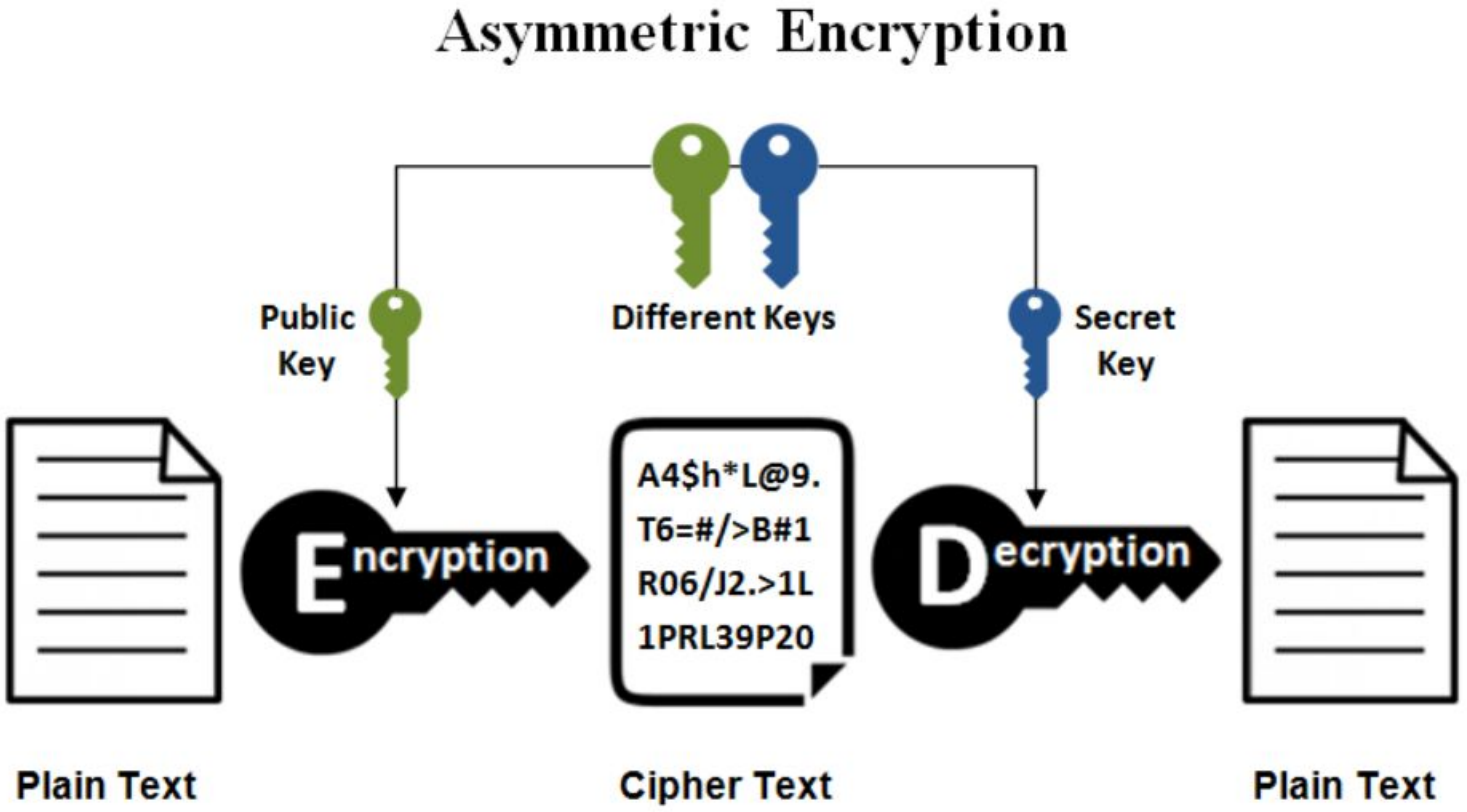
Cryptography



Symmetric vs Asymmetric Cryptography



Symmetric vs Asymmetric Cryptography



What is AES?

- AES is an encryption standard chosen by the National Institute of Standards and Technology(NIST), USA to protect classified information. It has been accepted world wide as a desirable algorithm to encrypt sensitive data.
- It is a block cipher which operates on block size of 128 bits for both encrypting as well as decrypting.
- Each Round performs similar operations.

Why AES?

- In 1990's the cracking of DES algorithm became possible.
- Around 50 hours of brute-forcing allowed to crack the message.
- NIST started searching for new feasible algorithm and proposed its requirement in 1997.
- In 2001 Rijndael algorithm designed by Rijment and Daemon of Belgium was declared as the winner of the competition.
- It met all Security, Cost and Implementation criteria.

How Does AES work?

- AES basically repeats 4 major functions to encrypt data. It takes 128 bit block of data and a key and gives a ciphertext as output.The functions are:
  - Substitute Bytes
  - Shift Rows
  - Mix Columns
  - Add Key

Steps for encryption and decryption

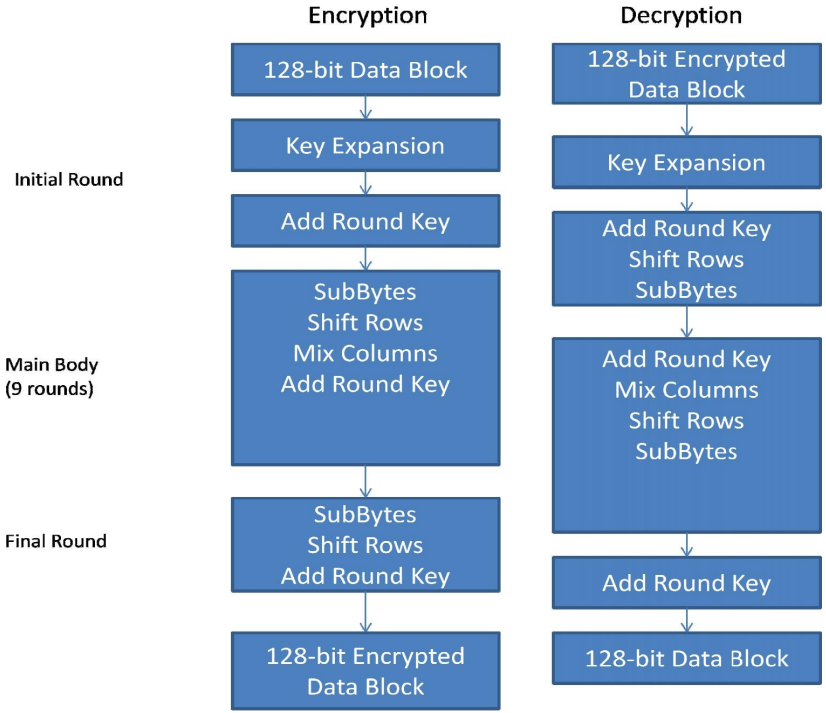
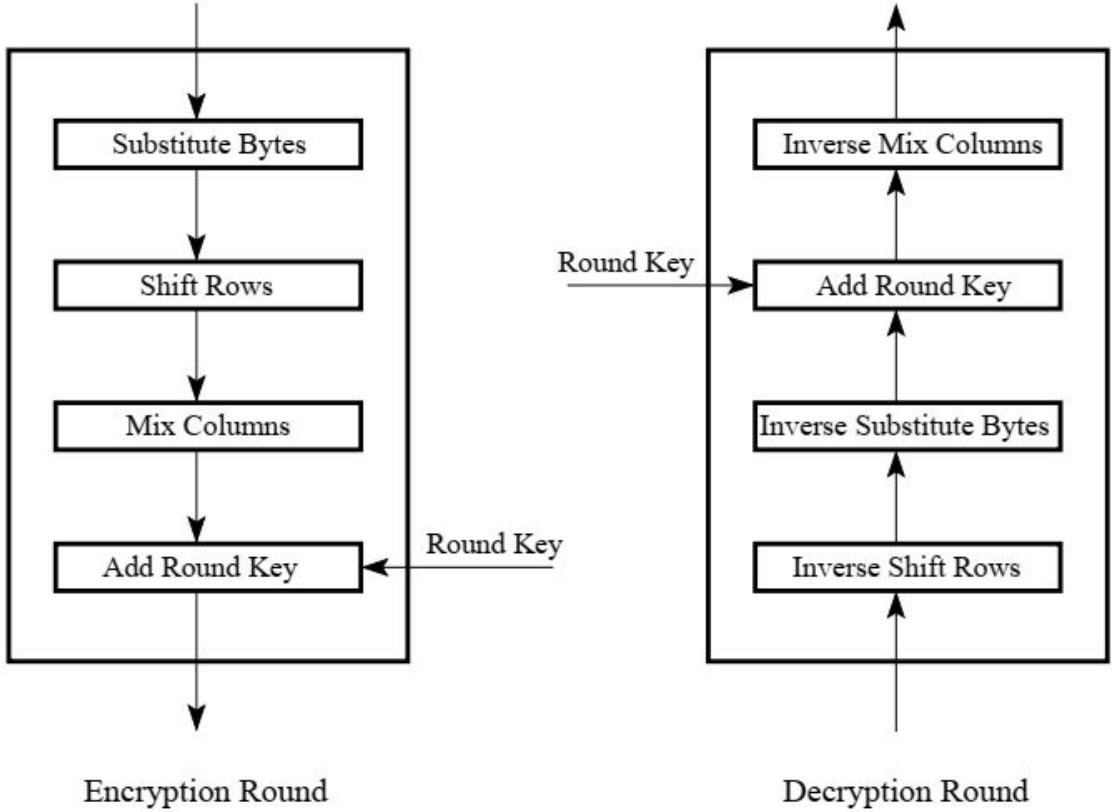


Figure 1 (Encryption on the left, Decryption on the right)

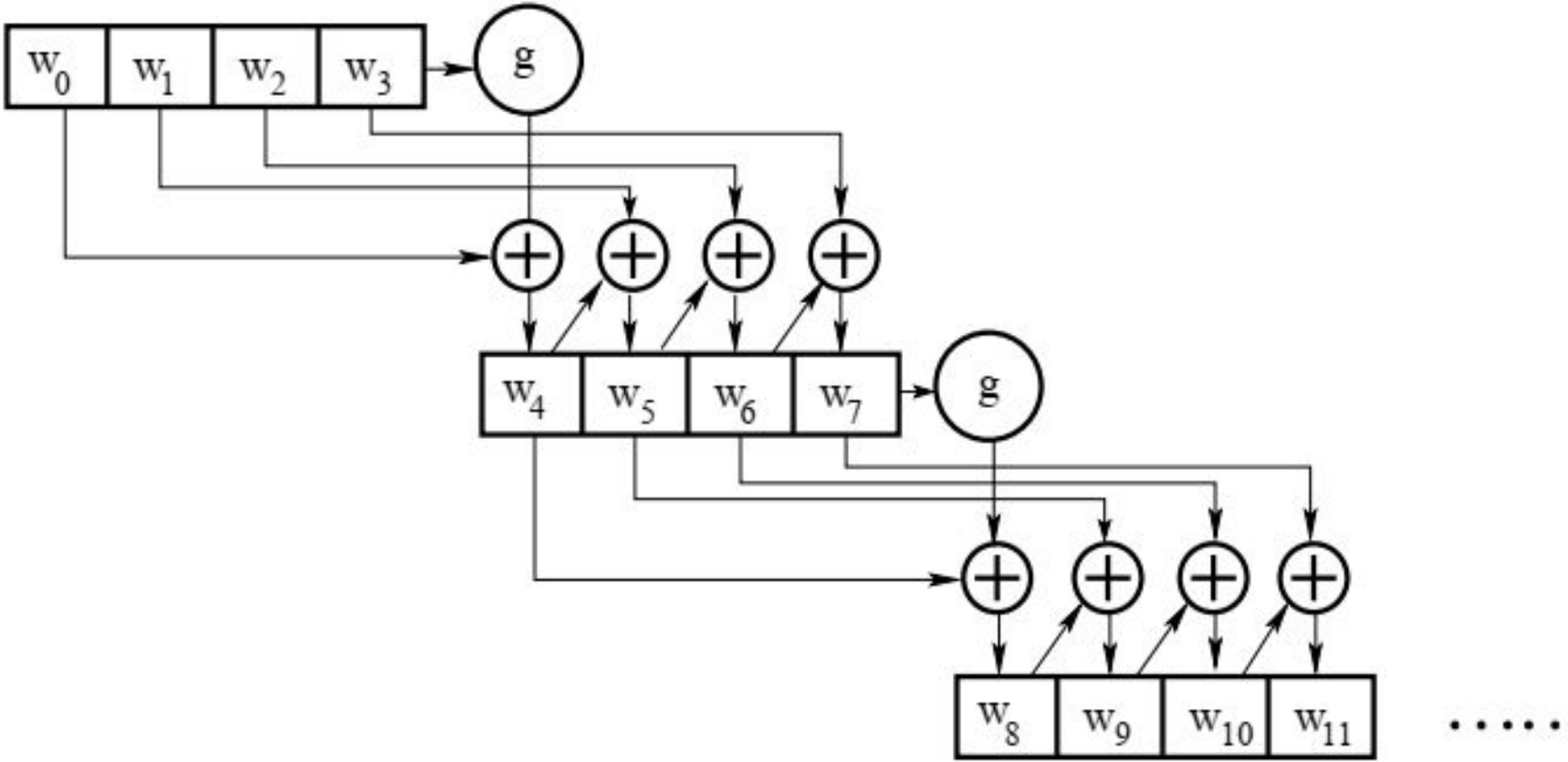
Steps for encryption and decryption



Analysis of Steps: Key Expansion

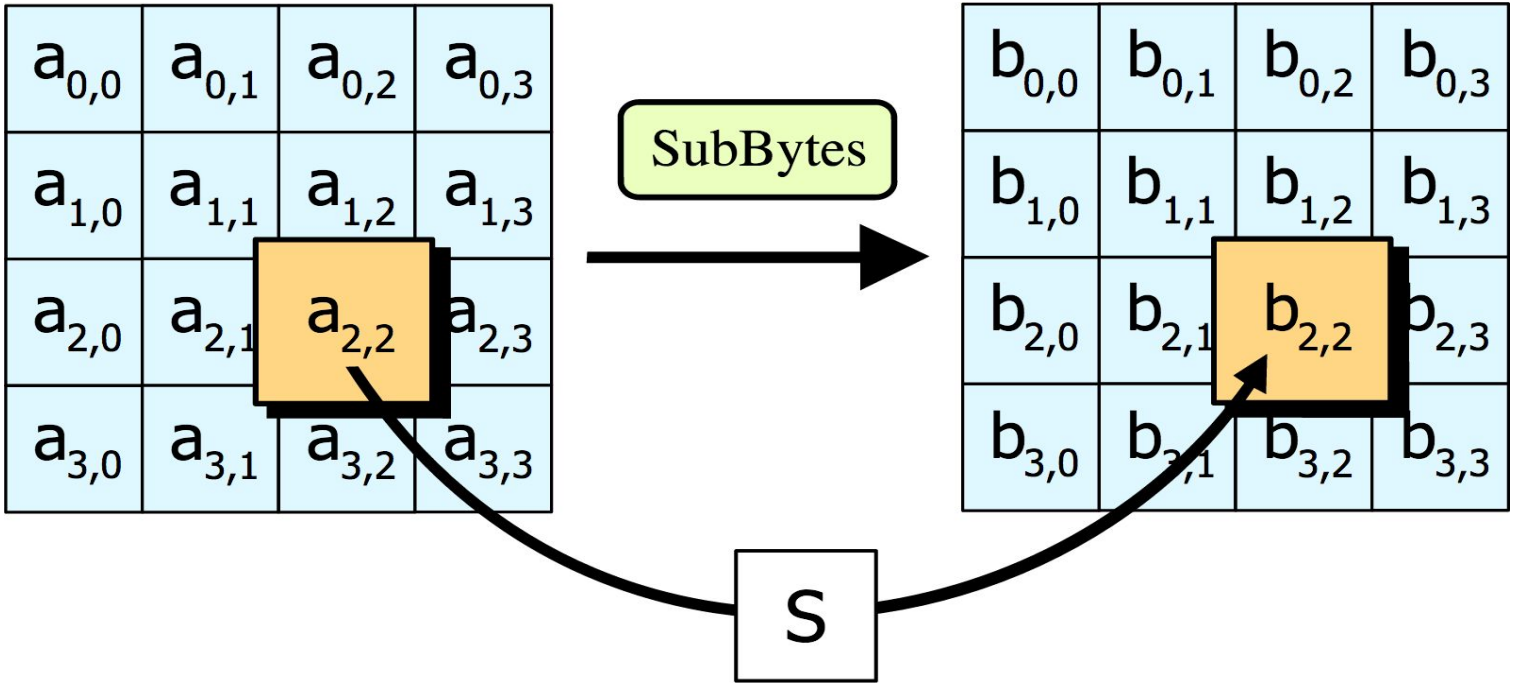
- **Key Expansion:** In the key expansion process the given 128 bits cipher key is stored in a [4]x[4] byte matrix (16*8=128 bits) and then the four column words of the key matrix is expanded into a schedule of 44 words (44*4=176 bytes) resulting in 11 round keys.
- Number of round keys = N_r + 1. Where N_r is the number of rounds (which is 10 in case of 128 bits key size). So, here, number round keys = 11.

Analysis of Steps: Key Expansion



Analysis of Steps: Substitute Bytes

SubBytes: Each element of the matrix is replaced by an element of the S-box matrix.



Analysis of Steps: Substitute Bytes

SubBytes: Each element of the matrix is replaced by an element of the S-box matrix.

- The S-box is a special lookup table which is constructed from Galois fields.
- The Generating function used in this algorithm is $GF(2^8)$.
  - i.e., 256 values are possible
- The elements of the S-box are written in hexadecimal system.

Analysis of Steps: Substitute Bytes

AES S-box

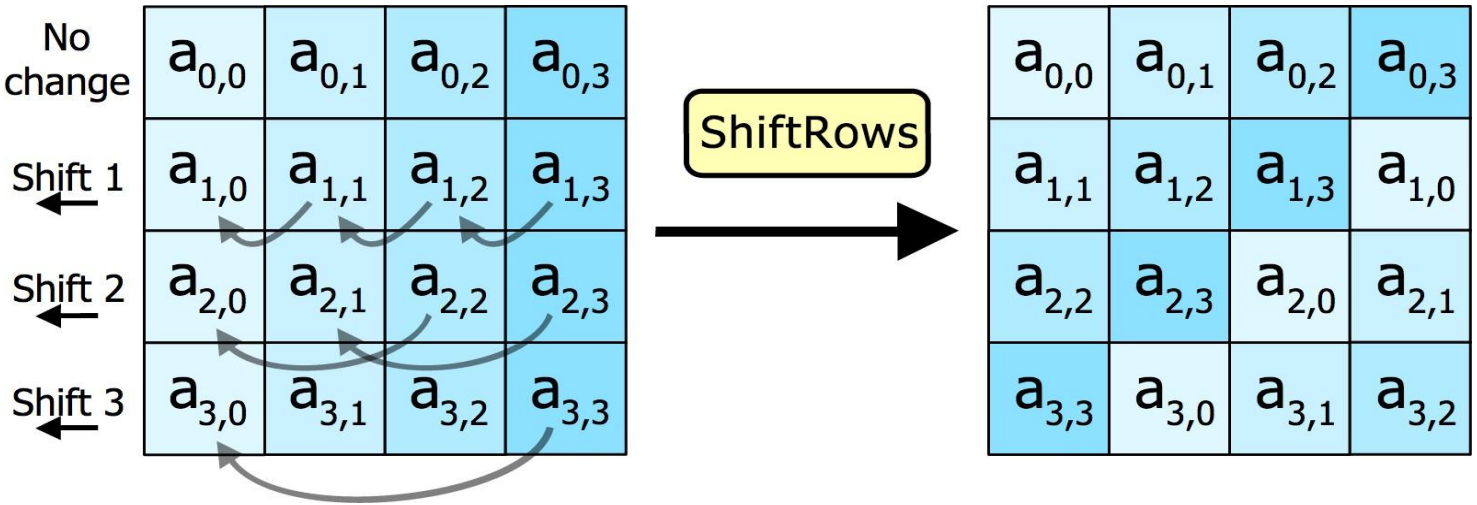
	00	01	02	03	04	05	06	07	08	09	0a	0b	0c	0d	0e	0f
00	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
10	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
20	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
30	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
40	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
50	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
60	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
70	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
80	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
90	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
a0	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
b0	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
c0	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
d0	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
e0	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
f0	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

Inverse S-box

	00	01	02	03	04	05	06	07	08	09	0a	0b	0c	0d	0e	0f
00	52	09	6a	d5	30	36	a5	38	bf	40	a3	9e	81	f3	d7	fb
10	7c	e3	39	82	9b	2f	ff	87	34	8e	43	44	c4	de	e9	cb
20	54	7b	94	32	a6	c2	23	3d	ee	4c	95	0b	42	fa	c3	4e
30	08	2e	a1	66	28	d9	24	b2	76	5b	a2	49	6d	8b	d1	25
40	72	f8	f6	64	86	68	98	16	d4	a4	5c	cc	5d	65	b6	92
50	6c	70	48	50	fd	ed	b9	da	5e	15	46	57	a7	8d	9d	84
60	90	d8	ab	00	8c	bc	d3	0a	f7	e4	58	05	b8	b3	45	06
70	d0	2c	1e	8f	ca	3f	0f	02	c1	af	bd	03	01	13	8a	6b
80	3a	91	11	41	4f	67	dc	ea	97	f2	cf	ce	f0	b4	e6	73
90	96	ac	74	22	e7	ad	35	85	e2	f9	37	e8	1c	75	df	6e
a0	47	f1	1a	71	1d	29	c5	89	6f	b7	62	0e	aa	18	be	1b
b0	fc	56	3e	4b	c6	d2	79	20	9a	db	c0	fe	78	cd	5a	f4
c0	1f	dd	a8	33	88	07	c7	31	b1	12	10	59	27	80	ec	5f
d0	60	51	7f	a9	19	b5	4a	0d	2d	e5	7a	9f	93	c9	9c	ef
e0	a0	e0	3b	4d	ae	2a	f5	b0	c8	eb	bb	3c	83	53	99	61
f0	17	2b	04	7e	ba	77	d6	26	e1	69	14	63	55	21	0c	7d

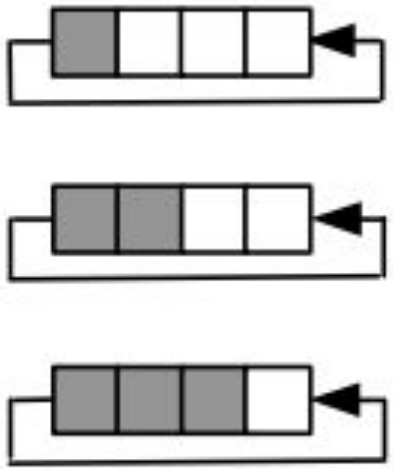
Analysis of Steps: Shift Rows

Shift Rows: In this step rows of the block are cylindrically shifted in left/right direction. The first row is untouched, the second one is shift by one, third one by two and the fourth one by three.



Analysis of Steps: Shift Rows

$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,0}$	$s_{1,1}$	$s_{1,2}$	$s_{1,3}$
$s_{2,0}$	$s_{2,1}$	$s_{2,2}$	$s_{2,3}$
$s_{3,0}$	$s_{3,1}$	$s_{3,2}$	$s_{3,3}$



$s_{0,0}$	$s_{0,1}$	$s_{0,2}$	$s_{0,3}$
$s_{1,1}$	$s_{1,2}$	$s_{1,3}$	$s_{1,0}$
$s_{2,2}$	$s_{2,3}$	$s_{2,0}$	$s_{2,1}$
$s_{3,3}$	$s_{3,0}$	$s_{3,1}$	$s_{3,2}$

Analysis of Steps: Mix columns

Mix columns: This is the most important part of the algorithm. It causes the flip of bits to spread all over the block. In this step the block is multiplied with a fixed matrix. The multiplication is a field multiplication in galois field.

For each row there are 16 multiplication, 12 XORs and a 4 byte output.

Analysis of Steps: Mix columns

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \times \begin{bmatrix} s_{0.0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1.0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2.0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3.0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0.0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1.0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2.0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3.0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

Analysis of Steps: Mix columns

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

$$\begin{aligned} s'_{0,j} &= (2 \bullet s_{0,j}) \oplus (3 \bullet s_{1,j}) \oplus s_{2,j} \oplus s_{3,j} \\ s'_{1,j} &= s_{0,j} \oplus (2 \bullet s_{1,j}) \oplus (3 \bullet s_{2,j}) \oplus s_{3,j} \\ s'_{2,j} &= s_{0,j} \oplus s_{1,j} \oplus (2 \bullet s_{2,j}) \oplus (3 \bullet s_{3,j}) \\ s'_{3,j} &= (3 \bullet s_{0,j}) \oplus s_{1,j} \oplus s_{2,j} \oplus (2 \bullet s_{3,j}) \end{aligned}$$

Predefine Matrix **State Array** **New State Array**

2	3	1	1
1	2	3	1
1	1	2	3
3	1	1	2

*

87	F2	4D	97
6E	4C	90	EC
46	E7	4A	C3
A6	8C	D8	95

$$\begin{aligned} &(\{02\} \cdot \{87\}) \oplus (\{03\} \cdot \{6E\}) \oplus \{46\} \oplus \{A6\} = \{47\} \\ &\{87\} \oplus (\{02\} \cdot \{6E\}) \oplus (\{03\} \cdot \{46\}) \oplus \{A6\} = \{37\} \\ &\{87\} \oplus \{6E\} \oplus (\{02\} \cdot \{46\}) \oplus (\{03\} \cdot \{A6\}) = \{94\} \\ &(\{03\} \cdot \{87\}) \oplus \{6E\} \oplus \{46\} \oplus (\{02\} \cdot \{A6\}) = \{ED\} \end{aligned}$$

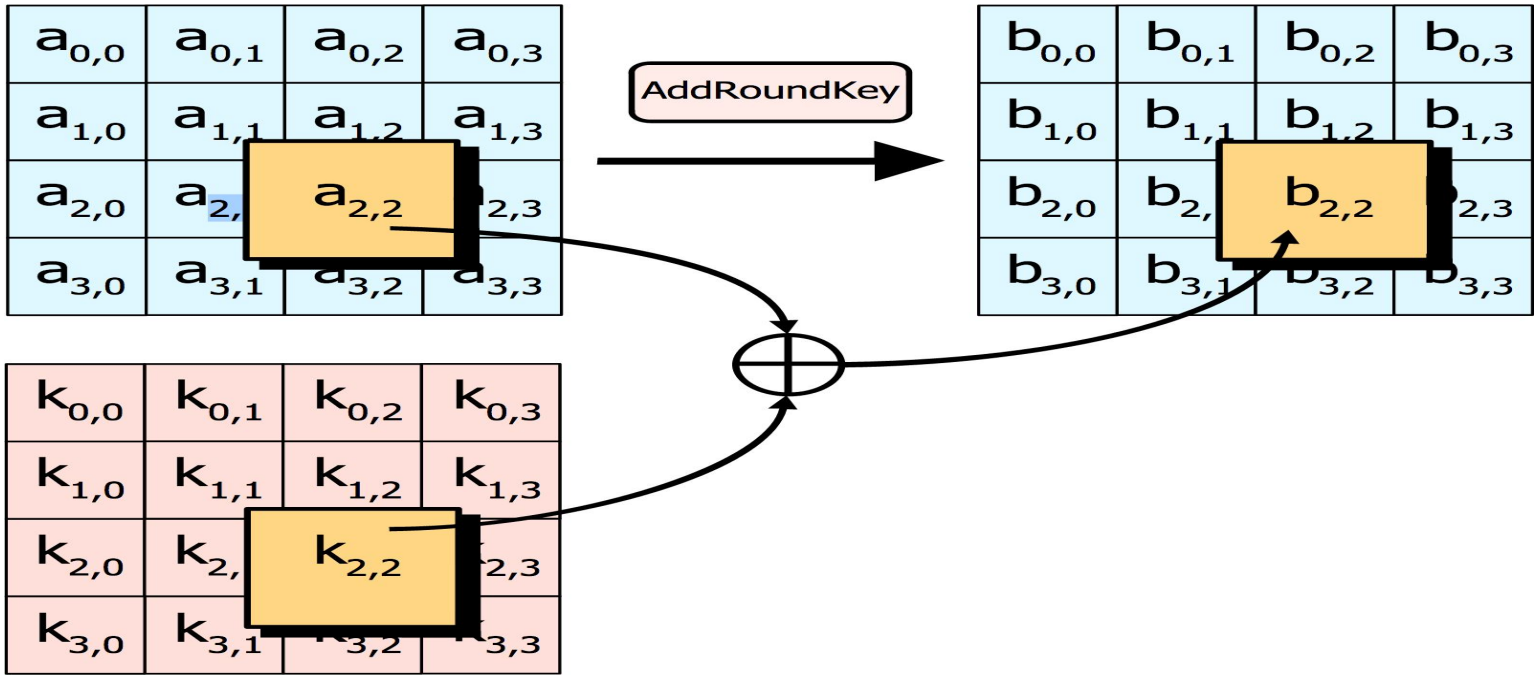
47	40	A3	4C
37	D4	70	9F
94	E4	3A	42
ED	A5	A6	BC

Analysis of Steps: Mix columns

$$\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix} \times \begin{bmatrix} s_{0.0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1.0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2.0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3.0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0.0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1.0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2.0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3.0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

Analysis of Steps: Add round key

Add round key: In this step, each byte is XOR-ed with corresponding element of the key matrix.



Diffie Hellman

Diffie Hellman method

- mathematical method of securely exchanging cryptographic keys over a public channel
- one of the earliest practical examples of public key exchange implemented within the field of cryptography
- earliest publicly known work that proposed the idea of a private key and a corresponding public key
- The security of this protocol relies on the practical difficulty of finding discrete logarithm

Diffie Hellman steps

- Alice and Bob agree on a natural number n and a generating element g in the finite cyclic group G of order n. (This is usually done long before the rest of the protocol; g is assumed to be known by all attackers.) The group G is written multiplicatively.
- Alice picks a random natural number a with $1 < a < n$, and sends the element g^a of G to Bob.
- Bob picks a random natural number b with $1 < b < n$, and sends the element g^b of G to Alice.
- Alice computes the element $(g^b)^a = g^{ba}$ of G.
- Bob computes the element $(g^a)^b = g^{ab}$ of G.

RSA

RSA

- RSA is one of the oldest asymmetric encryption algorithm.
- The acronym "RSA" comes from the surnames of Ron Rivest, Adi Shamir and Leonard Adleman.
- The security of RSA relies on the practical difficulty of factoring the product of two large prime numbers, the "factoring problem".

RSA algorithm steps: Key-Generation

- Step-1: Select two large prime numbers p and q where $p \neq q$
- Step-2: Calculate $n = p \cdot q$
- Step-3: Calculate $\lambda = \text{LCM}((p-1),(q-1))$
- Step-4: Select e such that,
  - e is relatively prime to λ, i.e. $\gcd(e, \lambda) = 1$
  - $1 < e < \lambda$
- Step-5: Calculate $d = (e^{-1}) \pmod{\lambda}$, so $e \cdot d = 1 \pmod{\lambda}$
- Step-6: Public key = {e, n}, private key = {d, n}

RSA algorithm steps: Encryption

- Let P be the plain text
- Find out the ciphertext C using the formula, $C = P^e \pmod{n}$ where, $P < n$

RSA algorithm steps: Decryption

- Let C be the ciphertext
- Find out the plain text P using the formula, $P = C^d \pmod{n}$ where, $C < n$

Thank you