Book: Hamdy A.Taha – Operations Research: An Introduction

1) Example 2.4-4 (Multiperiod Production Smoothing Model) Using Python Pulp

Code:

```
#pip install pulp
from pulp import LpProblem, LpMinimize, LpVariable, lpSum

model = LpProblem(name="Multiperiod Production Smoothing Model",
sense=LpMinimize)

x1 = LpVariable("x1", lowBound=0)
x2 = LpVariable("x2", lowBound=0)
x3 = LpVariable("x3", lowBound=0)
x4 = LpVariable("x4", lowBound=0)

S1_minus = LpVariable("S1_minus", lowBound=0)
S2_minus = LpVariable("S2_minus", lowBound=0)
S3_minus = LpVariable("S3_minus", lowBound=0)
S4_minus = LpVariable("S4_minus", lowBound=0)

S1_plus = LpVariable("S1_plus", lowBound=0)
S2_plus = LpVariable("S2_plus", lowBound=0)
S3_plus = LpVariable("S3_plus", lowBound=0)
S4_plus = LpVariable("S4_plus", lowBound=0)

I1 = LpVariable("I1", lowBound=0)
I2 = LpVariable("I2", lowBound=0)
I3 = LpVariable("I3", lowBound=0)

model += 50 * (I1 + I2 + I3) + 200 * (S1_minus + S2_minus + S3_minus +
S4_minus) + 400 * (S1_plus + S2_plus + S3_plus + S4_plus)

model += 10 * x1 == 400 + I1
model += I1 + 10 * x2 == 600 + I2
model += I2 + 10 * x3 == 400 + I3
model += I3 + 10 * x4 == 500

model += x1 == S1_minus - S1_plus
model += x2 == x1 + S2_minus - S2_plus
model += x3 == x2 + S3_minus - S3_plus
model += x4 == x3 + S4_minus - S4_plus
```

```
model.solve()
print(f"Optimal Solution:")
print(f"Objective Value (Z): ${model.objective.value():,.2f}")

for var in model.variables():
    print(f"{var.name}: {var.value()}")
```

Output:

```
Optimal Solution:
Objective Value (Z): $19,500.00
I1: 100.0, I2: 0.0, I3: 50.0
S1_minus: 50.0, S1_plus: 0.0
S2_minus: 0.0, S2_plus: 0.0
S3_minus: 0.0, S3_plus: 5.0
S4_minus: 0.0, S4_plus: 0.0
x1: 50.0, x2: 50.0, x3: 45.0, x4: 45.0
```

2) Example 2.4-5 (Bus Scheduling Model) Using Python Pulp

Code:

```
from pulp import LpProblem, LpMinimize, LpVariable, lpSum, LpStatus

model = LpProblem(name="Bus_Scheduling", sense=LpMinimize)

x1 = LpVariable(name="x1", lowBound=0, cat="Integer")
x2 = LpVariable(name="x2", lowBound=0, cat="Integer")
x3 = LpVariable(name="x3", lowBound=0, cat="Integer")
x4 = LpVariable(name="x4", lowBound=0, cat="Integer")
x5 = LpVariable(name="x5", lowBound=0, cat="Integer")
x6 = LpVariable(name="x6", lowBound=0, cat="Integer")

model += lpSum([x1, x2, x3, x4, x5, x6]), "Total_Buses"

model += x1 + x6 >= 4, "12:01 a.m.-4:00 a.m."
model += x1 + x2 >= 8, "4:01 a.m.-8:00 a.m."
model += x2 + x3 >= 10, "8:01 a.m.-12:00 noon"
model += x3 + x4 >= 7, "12:01 p.m.-4:00 p.m."
model += x4 + x5 >= 12, "4:01 p.m.-8:00 p.m."
model += x5 + x6 >= 4, "8:01 p.m.-12:00 p.m."

model.solve()

print(f"Status: {LpStatus[model.status]}")
print(f"Total Buses: {model.objective.value()}")
print("Optimal Schedule:")
print(f"x1 = {x1.value()}, buses starting at 12:01 a.m.")
print(f"x2 = {x2.value()}, buses starting at 4:01 a.m.")
print(f"x3 = {x3.value()}, buses starting at 8:01 a.m.")
print(f"x4 = {x4.value()}, buses starting at 12:01 p.m.")
print(f"x5 = {x5.value()}, buses starting at 4:01 p.m.")
print(f"x6 = {x6.value()}, buses starting at 8:01 p.m.")
```

Output:

```
Status: Optimal
Total Buses: 26.0
Optimal Schedule:
x1 = 0.0, buses starting at 12:01 a.m.
x2 = 10.0, buses starting at 4:01 a.m.
x3 = 0.0, buses starting at 8:01 a.m.
x4 = 12.0, buses starting at 12:01 p.m.
x5 = 0.0, buses starting at 4:01 p.m.
x6 = 4.0, buses starting at 8:01 p.m.
```

3) Example 5.3-1 (SunRay Transport) Using Python Pulp

Code:

```
from pulp import LpProblem, LpMinimize, LpVariable, lpSum, value

supply = [15, 25, 10]  # Supply at each source (Silo)
demand = [5, 15, 15, 15]  # Demand at each destination (Mill)

costs = [
    [10, 2, 20, 11],
    [12, 7, 9, 20],
    [4, 14, 16, 18]
]

model = LpProblem(name="SunRayTransportation", sense=LpMinimize)

num_sources = len(supply)
num_destinations = len(demand)
x = [[LpVariable(f"x{i + 1}{j + 1}", lowBound=0) for j in
range(num_destinations)] for i in range(num_sources)]

model += lpSum(x[i][j] * costs[i][j] for i in range(num_sources) for j
in range(num_destinations))

for i in range(num_sources):
    model += lpSum(x[i][j] for j in range(num_destinations)) ==
supply[i]

for j in range(num_destinations):
    model += lpSum(x[i][j] for i in range(num_sources)) == demand[j]

for i in range(num_sources):
    for j in range(num_destinations):
        quantity = min(supply[i], demand[j])
        model += x[i][j] == quantity
        supply[i] -= quantity
        demand[j] -= quantity

model.solve()

print("Status:", pulp.LpStatus[model.status])
print("Objective Value (Total Cost):", value(model.objective))

for i in range(num_sources):
    for j in range(num_destinations):
        if value(x[i][j]) != 0:
            print(f"Ship {x[i][j].name} = {value(x[i][j])}")
```

Output:

```
Status: Optimal
Objective Value (Total Cost): 520.0
Ship x11 = 5.0
Ship x12 = 10.0
Ship x22 = 5.0
Ship x23 = 15.0
Ship x24 = 5.0
Ship x34 = 10.0
```

4) Example 6.3-6 Using Excel Solver

From	To	DVs	Distance
1	2	0	100
1	3	1	30
2	3	0	20
3	4	1	10
3	5	0	60
4	2	1	15
4	5	0	50

Nodes	LHS	Sign	RHS
1	1	=	1
2	-1	=	-1
3	0	=	0
4	0	=	0
5	0	=	0

Distance	55

Answer Report:

Microsoft Excel 16.0 Answer Report
Worksheet: [Book1]Sheet1
Report Created: 02/12/2023 3:43:57 pm
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.016 Seconds.
Iterations: 5 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$G$1	Distance Nodes	55	55

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$D$3	DVs	0	0	Contin
$D$4	DVs	1	1	Contin
$D$5	DVs	0	0	Contin
$D$6	DVs	1	1	Contin
$D$7	DVs	0	0	Contin
$D$8	DVs	1	1	Contin
$D$9	DVs	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$H$3	LHS	1	$H$3=$J$3	Binding	0
$H$4	LHS	-1	$H$4=$J$4	Binding	0
$H$5	LHS	0	$H$5=$J$5	Binding	0
$H$6	LHS	0	$H$6=$J$6	Binding	0
$H$7	LHS	0	$H$7=$J$7	Binding	0

Answer Report 1 | Sensitivity Report 1 | Sheet1

Sensitivity Report:

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Book1]Sheet1
Report Created: 02/12/2023 3:43:57 pm

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$D$3	DVs	0	45	100	1E+30	45
$D$4	DVs	1	0	30	45	1E+30
$D$5	DVs	0	45	20	1E+30	45
$D$6	DVs	1	0	10	0	45
$D$7	DVs	0	0	60	1E+30	0
$D$8	DVs	1	0	15	45	45
$D$9	DVs	0	0	50	0	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$H$3	LHS	1	90	1	1E+30	0
$H$4	LHS	-1	35	-1	1	0
$H$5	LHS	0	60	0	1E+30	0
$H$6	LHS	0	50	0	1E+30	0
$H$7	LHS	0	0	0	0	1E+30

Answer Report 1 | Sensitivity Report 1 | Sheet1

5) Example 9.1-2 (Installing Security Telephones)

Using Python Pulp Code:

```
import pulp

model = pulp.LpProblem("Minimize_Telephones_Installation",
pulp.LpMinimize)

x = pulp.LpVariable.dicts("Telephone_Installed", range(1, 9), 0, 1,
pulp.LpBinary)

model += pulp.lpSum(x[j] for j in range(1, 9)),
"Total_Telephones_Installed"

model += x[1] + x[2] >= 1, "Street_A"
model += x[2] + x[3] >= 1, "Street_B"
model += x[4] + x[5] >= 1, "Street_C"
model += x[7] + x[8] >= 1, "Street_D"
model += x[6] + x[7] >= 1, "Street_E"
model += x[2] + x[6] >= 1, "Street_F"
model += x[1] + x[6] >= 1, "Street_G"
model += x[4] + x[7] >= 1, "Street_H"
model += x[2] + x[4] >= 1, "Street_I"
model += x[5] + x[8] >= 1, "Street_J"
model += x[3] + x[5] >= 1, "Street_K"

model.solve()


print("Status:", pulp.LpStatus[model.status])
print("Optimal Solution:")
for j in range(1, 9):
    installation_status = "1.0 (Telephone Installed)" if
pulp.value(x[j]) == 1.0 else f"{pulp.value(x[j]):.1f}"
    print(f"Intersection {j}: {installation_status}")

print("Total Telephones Installed:", pulp.value(model.objective))
```

Output:

```
Status: Optimal
Optimal Solution:
Intersection 1: 1.0 (Telephone Installed)
Intersection 2: 1.0 (Telephone Installed)
Intersection 3: 0.0
Intersection 4: 0.0
Intersection 5: 1.0 (Telephone Installed)
Intersection 6: 0.0
Intersection 7: 1.0 (Telephone Installed)
Intersection 8: 0.0
Total Telephones Installed: 4.0
```

Using Excel Solver:

Decision Variables		x1	x2	x3	x4	x5	x6	x7	x8			
Quantity Produced		1	1	0	0	1	0	1	0			
										Total	Sign	RHS
Cost		1	1	1	1	1	1	1	1	4		
Street A		1	1	1	0	0	0	0	0	2	>=	1
Street B		0	1	0	0	0	0	0	0	1	>=	1
Street C		0	0	1	1	1	0	0	0	1	>=	1
Street D		0	0	0	0	0	0	1	1	1	>=	1
Street E		0	0	0	0	0	1	1	0	1	>=	1
Street F		0	1	0	0	0	1	0	0	1	>=	1
Street G		1	0	0	0	0	1	0	0	1	>=	1
Street H		0	0	0	1	0	0	1	0	1	>=	1
Street I		0	1	0	1	0	0	0	0	1	>=	1
Street J		0	0	0	0	1	0	0	1	1	>=	1
Street K		0	0	1	0	1	0	0	0	1	>=	1

Answer Report:

Microsoft Excel 16.0 Answer Report
Worksheet: [9.1-2.xlsx]Sheet1
Report Created: 02/12/2023 6:55:42 pm
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.016 Seconds.
Iterations: 13 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 0%, Assume NonNegativ

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$L$6	Cost Total	4	4

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$D$4	Quantity Produced x1	1	1	Contin
$E$4	Quantity Produced x2	1	1	Contin
$F$4	Quantity Produced x3	0	0	Contin
$G$4	Quantity Produced x4	0	0	Contin
$H$4	Quantity Produced x5	1	1	Contin
$I$4	Quantity Produced x6	0	0	Contin
$J$4	Quantity Produced x7	1	1	Contin
$K$4	Quantity Produced x8	0	0	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$L$7	Street A Total	2	$L$7>=$N$7	Not Binding	1
$L$8	Street B Total	1	$L$8>=$N$8	Binding	0
$L$9	Street C Total	1	$L$9>=$N$9	Binding	0
$L$10	Street D Total	1	$L$10>=$N$10	Binding	0
$L$11	Street E Total	1	$L$11>=$N$11	Binding	0
$L$12	Street F Total	1	$L$12>=$N$12	Binding	0
$L$13	Street G Total	1	$L$13>=$N$13	Binding	0
$L$14	Street H Total	1	$L$14>=$N$14	Binding	0
$L$15	Street I Total	1	$L$15>=$N$15	Binding	0
$L$16	Street J Total	1	$L$16>=$N$16	Binding	0
$L$17	Street K Total	1	$L$17>=$N$17	Binding	0

Sensitivity Report:

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [9.1-2.xlsx]Sheet1
Report Created: 02/12/2023 6:55:42 pm

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$D$4	Quantity Produced x1	1	0	1	0	1
$E$4	Quantity Produced x2	1	0	1	0	1
$F$4	Quantity Produced x3	0	0	1	0	0
$G$4	Quantity Produced x4	0	0	1	1	0
$H$4	Quantity Produced x5	1	0	1	0	0
$I$4	Quantity Produced x6	0	0	1	1	0
$J$4	Quantity Produced x7	1	0	1	0	1
$K$4	Quantity Produced x8	0	0	1	1E+30	0

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$L$7	Street A Total	2	0	1	1	1E+30
$L$8	Street B Total	1	0	1	0	1E+30
$L$9	Street C Total	1	0	1	0	0
$L$10	Street D Total	1	1	1	1E+30	0
$L$11	Street E Total	1	0	1	0	1E+30
$L$12	Street F Total	1	0	1	1	0
$L$13	Street G Total	1	1	1	1E+30	1
$L$14	Street H Total	1	0	1	0	1E+30
$L$15	Street I Total	1	1	1	0	0
$L$16	Street J Total	1	0	1	0	1
$L$17	Street K Total	1	1	1	0	0

6) Example 9.1-3 (Choosing a Telephone company)

Using Python Pulp Code:

```
from pulp import LpProblem, LpMinimize, LpVariable, lpSum, LpStatus,
value

prob = LpProblem("TelephoneCompany", LpMinimize)
x1 = LpVariable("x1", lowBound=0, cat="Continuous")
x2 = LpVariable("x2", lowBound=0, cat="Continuous")
x3 = LpVariable("x3", lowBound=0, cat="Continuous")

y1 = LpVariable("y1", cat="Binary")
y2 = LpVariable("y2", cat="Binary")
y3 = LpVariable("y3", cat="Binary")

prob += 0.25 * x1 + 0.21 * x2 + 0.22 * x3 + 16 * y1 + 25 * y2 + 18 *
y3, "TotalCost"

prob += x1 + x2 + x3 == 200, "TotalMinutes"
prob += x1 <= 200 * y1, "MaBellConstraint"
prob += x2 <= 200 * y2, "PaBellConstraint"
prob += x3 <= 200 * y3, "BabyBellConstraint"

prob.solve()

print("Status:", LpStatus[prob.status])
print("Optimal Solution:")
print("x1 =", value(x1))
print("x2 =", value(x2))
print("x3 =", value(x3))
print("y1 =", value(y1))
print("y2 =", value(y2))
print("y3 =", value(y3))
print("Total Cost =", value(prob.objective))
```

Output:

```
Status: Optimal
Optimal Solution:
x1 = 0.0
x2 = 0.0
x3 = 200.0
y1 = 0.0
y2 = 0.0
y3 = 1.0
Total Cost = 62.0
```

Using Excel Solver:

Decision Variables		x1	x2	x3	y1	y2	y3			
Quantity Produced		0	0	200	0	0	1			
								Total	Sign	RHS
Cost		0.25	0.21	0.22	16	25	18	62		
		1	1	1	0	0	0	200	=	200
		1	0	0	0	0	0	0	<=	0
		0	1	0	0	0	0	0	<=	0
		0	0	1	0	0	0	200	<=	200

Answer Report:

Microsoft Excel 16.0 Answer Report
Worksheet: [9.1-3.xlsx]Sheet1
Report Created: 02/12/2023 8:14:19 pm
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.015 Seconds.
Iterations: 5 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$J$6	Cost Total	62	62

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$D$4	Quantity Produced x1	0	0	Contin
$E$4	Quantity Produced x2	0	0	Contin
$F$4	Quantity Produced x3	200	200	Contin
$G$4	Quantity Produced y1	0	0	Binary
$H$4	Quantity Produced y2	0	0	Binary
$I$4	Quantity Produced y3	1	1	Binary

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$J$7	Total	200	$J$7=$L$7	Binding	0
$J$8	Total	0	$J$8<=$L$8	Binding	0
$J$9	Total	0	$J$9<=$L$9	Binding	0
$J$10	Total	200	$J$10<=$L$10	Binding	0
$G$4:$I$4=Binary					

Sensitivity Report:

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$2	Quantity Produced x1	0	0	0.25	0.06	0.02
$D$2	Quantity Produced x2	0	0	0.21	0.1	0.025
$E$2	Quantity Produced x3	200	0	0.22	0.02	0.06
$F$2	Quantity Produced y1	0	4	16	1E+30	4
$G$2	Quantity Produced y2	0	5	25	1E+30	5
$H$2	Quantity Produced y3	1	0	18	4	12

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$I$5	TOTAL	200	0.31	200	1E+30	200
$I$6	TOTAL	0	-0.06	0	200	0
$I$7	TOTAL	0	-0.1	0	200	0
$I$8	TOTAL	200	-0.09	0	200	1E+30