$$\hat{J}(U,V) = \sum_{k=1}^{N} \sum_{i=1}^{c} u_{ik}^{q}\, d(\vec{x}_k, \theta_i) - \sum_{k=1}^{N} \lambda_k \left(\sum_{i=1}^{c} u_{ik} - 1 \right)$$

$$\frac{\partial \hat{J}(U,V)}{\partial \vec{v}_j} = \sum_{k=1}^{N} u_{ik}^{q} \boxed{\frac{\partial d(\vec{x}_k, \theta_j)}{\partial \vec{v}_j}} = \vec{0}$$

$$\sqrt{(\vec{x}_k - \vec{v}_j)^t (\vec{x}_k - \vec{v}_j)}$$

$$\frac{\partial \hat{J}(U,V)}{\partial v_{ie}} = \cdots 0$$

$$d_{jk}^2$$

$$d^2(x,v) = (\vec{x} - \vec{v})^t (\vec{x} - \vec{v})$$

$$\frac{\partial d^2}{\partial v_i^2}$$

$$\vec{x}^t \vec{x} - 2\vec{x}^t \vec{v} + \vec{v}^t \vec{v}$$

$$\frac{\partial d^2}{\partial v} = -2\vec{x} + 2\vec{v} = -2(\vec{x} - \vec{v})$$

$$\frac{\partial \hat{J}(U,V)}{\partial \vec{v}_j} = -2\sum_{k=1}^{N} u_{jk}^{Q}\left(\vec{x}_k - \vec{v}_j\right) = \vec{0}$$

$$= \underbrace{-2\sum_{k=1}^{N} u_{jk}^{Q}\,\vec{x}_k}\; + 2\,\underline{\underline{\vec{v}_j}}\sum_{k=1}^{N} u_{jk}^{Q} = \vec{0}$$

$$\cancel{2}\,\vec{v}_j \sum_{k=1}^{N} u_{jk}^{Q} = \cancel{2}\sum_{k=1}^{N} u_{jk}^{Q}\,\vec{x}_k$$

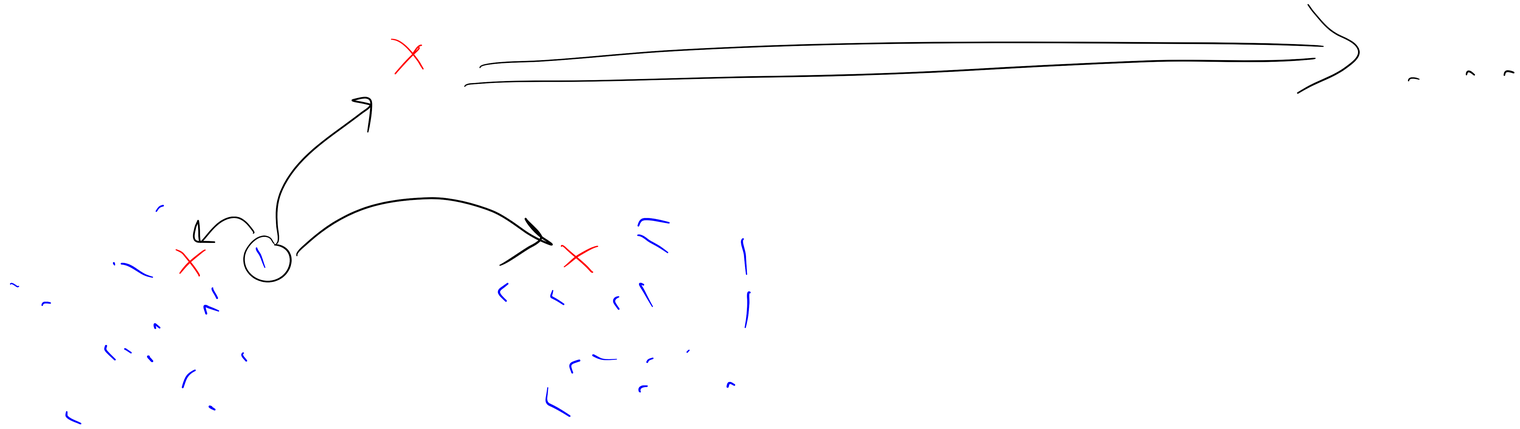
$$\boxed{\vec{v}_j} = \frac{\sum_{k=1}^{N} \boxed{u_{jk}^{Q}}\,\vec{x}_k}{\sum_{k=1}^{N} \boxed{u_{jk}^{Q}}}$$

?

$$u_{jk} \in [0,1]$$

$$\sum_{j=1}^{c} u_{jk} = 1$$

What's the FCM equation



For the reader: think about normalizer... problem??

$$u_{rs} = \frac{1}{\sum_{i=1}^{c} \left(\frac{d(\vec{x}_s, \vec{v}_r)}{d(\vec{x}_s, \vec{v}_i)} \right)^{\frac{1}{\phi - 1}}}$$

$c \times N$

$\|\cdot\|_{\boxed{A}}$ → A-norms
↳ matrix

$\|\vec{x}-\vec{v}\|_2^2$ ← squared

$$d_{ik}^2 = \|\vec{x}_k - \vec{v}_i\|_A^2 = (\vec{x}_k - \vec{v}_i)^T A_i (\vec{x}_k - \vec{v}_i)$$

$A_i \to \Sigma_i^{-1}$

Recall

$$\frac{\partial}{\partial \vec{v}} \left\{ \underbrace{(\vec{x}-\vec{v})^T A (\vec{x}-\vec{v})} \right\}$$

$$= \frac{\partial}{\partial \vec{v}} \left\{ (\underbrace{\vec{x}^T A}_{1\times d \quad d\times d} - \vec{v}^T A)^T (\underset{d\times 1}{\vec{x}-\vec{v}}) \right\}$$

$$= \frac{\partial}{\partial \vec{v}} \left\{ \underset{0}{\vec{x}^T A \vec{x}} - 2\vec{v}^T A \vec{x} + \vec{v}^T A \vec{v} \right\}$$

$\vec{v} \to \pi$

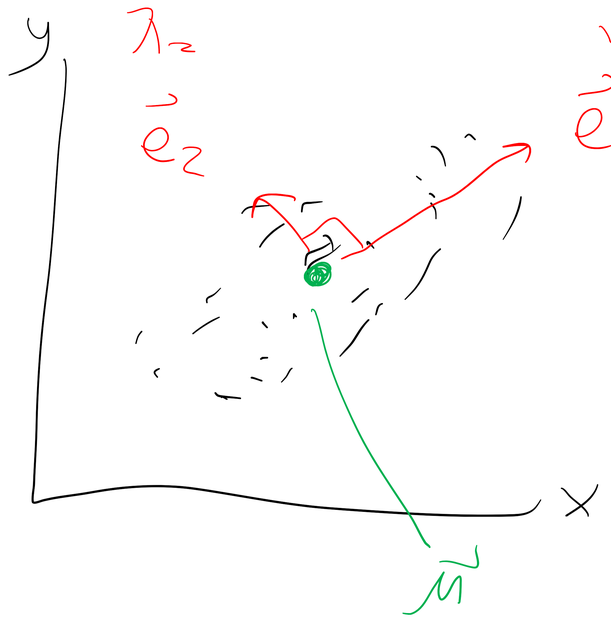
$$\boxed{-2A\vec{x} + 2A\boxed{\vec{v}} = \vec{0}}$$

$$\star \quad \sum_{k=1}^{N} u_{jk}^{\varphi} \left(-2A_j \vec{x}_k + 2A_j \vec{v}_j \right) = \vec{0}$$

$$\cancel{2A_j} \sum_{k=1}^{N} u_{jk}^{\varphi} \vec{x}_k = \cancel{2A_j} \boxed{\vec{v}_j} \underbrace{\sum_{k=1}^{N} u_{jk}^{\varphi}} \quad \left(\frac{1}{2}\right) A_j$$

$$\boxed{\vec{v}_j = \frac{\sum_{k=1}^{N} u_{jk}^{\varphi} \vec{x}_k}{\sum_{k=1}^{N} u_{jk}^{\varphi}}}$$

$\sigma_x \sigma_x$	$\sigma_y \sigma_x$
$\sigma_x \sigma_y$	$\sigma_y \sigma_y$



eigen vector

$\vec{e}_1 \rightarrow \lambda_1$

eigen value

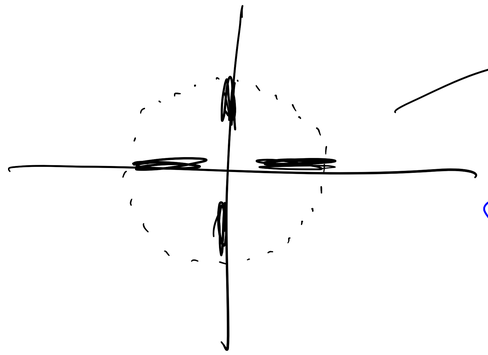
$\Rightarrow \Sigma$ covariance

$(\lambda_1, \vec{e}_1)$

$(\lambda_2, \vec{e}_2)$

eigen decomp

2	0
0	2



circular

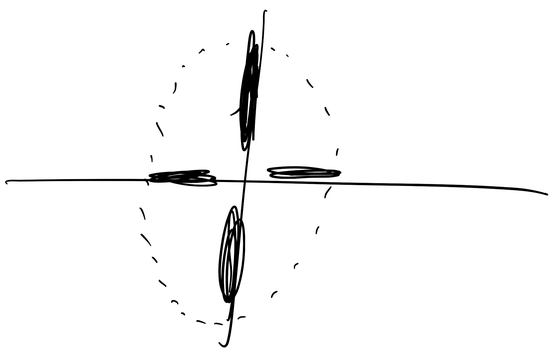
ℓ_2-norm

$(\vec{x}-\vec{v})^T \boxed{A} (\vec{x}-\vec{v})$

$A \to I$

βI

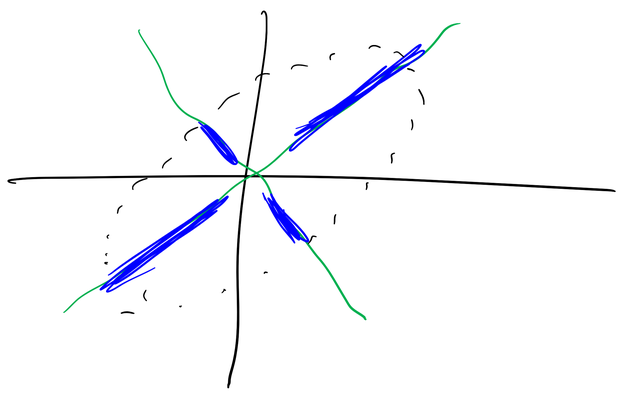
2	0
0	4

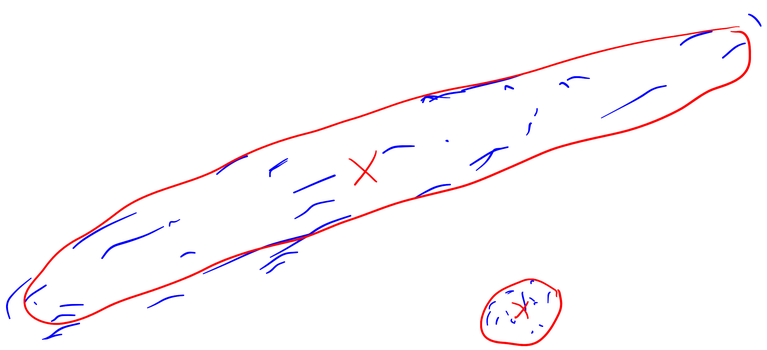


2	2
2	4

> 0

PSD





A-norm

$$\frac{\partial}{\partial A_{ij}} \left\{ x^t A \vec{x} \right\}$$

$(\vec{x} - \vec{v}_j) \to A_j$

$$\hookrightarrow \begin{bmatrix} x_1 & \cdots & x_d \end{bmatrix} \begin{bmatrix} a_{11} & \cdots & a_{1d} \\ \vdots & & \\ a_{d1} & \cdots & a_{dd} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}$$

$$= \begin{bmatrix} (x_1 a_{11} + \cdots + x_d a_{d1}) & \cdots & (x_1 a_{1d} + \cdots + x_d a_{dd}) \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^{d} x_i a_{i1} & \cdots & \sum_{i=1}^{d} x_i a_{id} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_d \end{bmatrix}$$

$$= \sum_{i=1}^{d} x_i a_{i1} x_1 + \cdots + \sum_{i=1}^{d} x_i a_{id} x_d$$

$$\frac{\partial}{\partial A_{ij}} \left(= \sum_{j=1}^{d} \sum_{i=1}^{d} \underline{\underline{a_{ij}}} x_i x_j \right) = x_i x_j$$

$$\frac{\partial}{\partial A}\left\{x^T A x\right\} = \boxed{x x^T}$$

outer product

der of $\dfrac{\partial \hat{J}(u,v)}{\partial A_{ij}}$????