

# Implementation of Logistic Regression for Data Point Classifier and EM Algorithm for MNIST Digit Recognition

## 1. Logistic regression

- Input:
  1.  $N$  (number of data points)
  2.  $mx_1, vx_1, my_1, vy_1, mx_2, vx_2, my_2, vy_2$  ( $m$ : mean,  $v$ : variance)
- Function:
  1. Generate  $n$  data point:  $D1 = (x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ , where  $x$  and  $y$  are independently sampled from  $N(mx_1, vx_1)$  and  $N(my_1, vy_1)$  respectively.
  2. Generate  $n$  data point:  $D2 = (x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ , where  $x$  and  $y$  are independently sampled from  $N(mx_2, vx_2)$  and  $N(my_2, vy_2)$  respectively.
  3. Use Logistic regression to separate  $D1$  and  $D2$ . You should implement both Newton's and steepest gradient descent method during optimization.
    - In other words, when the Hessian is singular, use steepest descent for instead. You should come up with a reasonable rule to determine convergence.(a simple run out of the loop should be used as the ultimatum)
- Output:
  1. The confusion matrix and the **sensitivity** and **specificity** of the logistic regression applied to the training data  $D$ .
  2. Visualization
    - Plot the ground truth
    - Plot the predict result
      - Gradient descent
      - Newton's method

Use the Gaussian random number generator in homework 3.

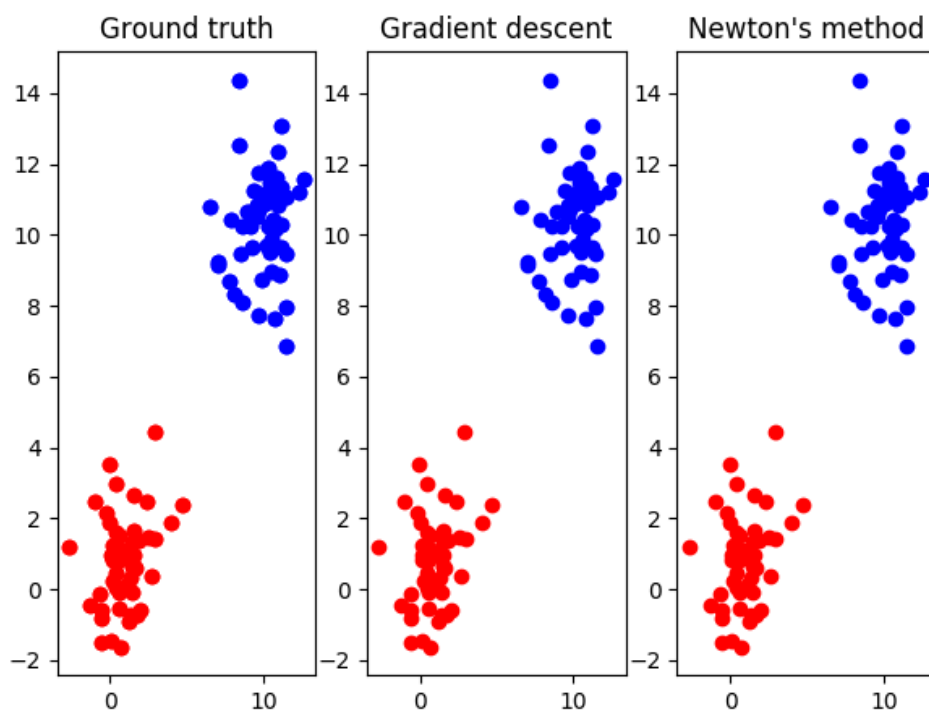
- Sample input & output (for reference only)
  - Case 1:  $N = 50, mx_1 = my_1 = 1, mx_2 = my_2 = 10, vx_1 = vy_1 = vx_2 = vy_2 = 2$

```
1 Gradient descent:
2
3 w:
```

```

4  -78.1766393662
5    6.7233419236
6   11.2430677919
7
8  Confusion Matrix:
9          Predict cluster 1 Predict cluster 2
10 Is cluster 1          50          0
11 Is cluster 2          0          50
12
13 Sensitivity (Successfully predict cluster 1): 1.00000
14 Specificity (Successfully predict cluster 2): 1.00000
15
16 -----
17 Newton's method:
18
19 w:
20 -118.3601516394
21   8.7747332848
22  10.1954120077
23
24 Confusion Matrix:
25          Predict cluster 1 Predict cluster 2
26 Is cluster 1          50          0
27 Is cluster 2          0          50
28
29 Sensitivity (Successfully predict cluster 1): 1.00000
30 Specificity (Successfully predict cluster 2): 1.00000

```

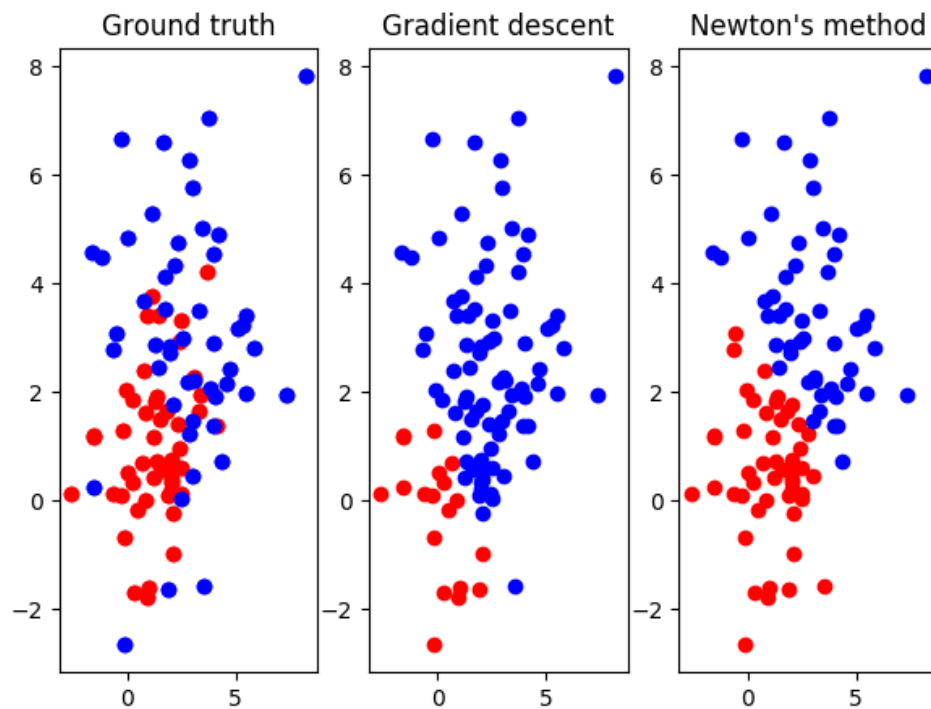


- Case 2:  $N = 50, mx_1 = my_1 = 1, mx_2 = my_2 = 3, vx_1 = vy_1 = 2, vx_2 = vy_2 = 4$

```

1 Gradient descent:
2
3 w:
4 -71.1902536008
5 46.0123814025
6 54.6803199701
7
8 Confusion Matrix:
9           Predict cluster 1 Predict cluster 2
10 Is cluster 1          16          34
11 Is cluster 2           3          47
12
13 Sensitivity (Successfully predict cluster 1): 0.32000
14 Specificity (Successfully predict cluster 2): 0.94000
15
16 -----
17 Newton's method:
18
19 w:
20 -1.9045831451
21 0.3940876974
22 0.5695243849
23
24 Confusion Matrix:
25           Predict cluster 1 Predict cluster 2
26 Is cluster 1          40          10
27 Is cluster 2          10          40
28
29 Sensitivity (Successfully predict cluster 1): 0.80000
30 Specificity (Successfully predict cluster 2): 0.80000

```



## 2. EM algorithm

- Input: [MNIST training](#) data and label sets.
- Function:
  1. Binning the gray level value into **two bins**. Treating all pixels as random variables following Bernoulli distributions. Note that each pixel follows a different Binomial distribution independent to others.
  2. Use EM algorithm to cluster each image into ten groups. You should come up with a reasonable rule to determine convergence. (a simple run out of the loop should be used as the ultimatum)
- Output:
  1. For each digit, output a confusion matrix and the **sensitivity** and **specificity** of the clustering applied to the training data.
  2. Print out the imagination of numbers in your classifier
- Hint: The algorithm is a kind of unsupervised learning, so the labels are not used during training. But you can use these labels to help you to figure out which class belongs to which number.  
 In other words, you should find a way to assign label to each class which you classified **before you compute the confusion matrix**
- Sample input & output (**for reference only**)

```

1  class 0:
2  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
3  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

```

[illegible]

```
53 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
54 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
55 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
56 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
57 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
58 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
59 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
60
61 ... all other unlabeled imagination of numbers goes here ...
62
63 class 9:
64 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
65 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
66 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
67 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
68 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0
69 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0
70 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0 0 0 0 0
71 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0
72 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
73 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
74 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 0 0
75 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0 0 0 0
76 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 0 0
77 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
78 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
79 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
80 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
81 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0
82 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0
83 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0
84 0 0 0 0 0 0 0 0 1 1 1 0 0 0 1 0 0 1 1 0 0 0 0 0 0 0 0
85 0 0 0 0 0 0 0 0 1 1 1 1 1 0 1 1 0 0 0 0 0 0 0 0 0 0 0
86 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0
87 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
88 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
89 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
90 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
91 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
92
93 No. of Iteration: 1, Difference: 3176.579389514846
94
95 -----
96
97 class 0:
98 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
99 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
100 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
101 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

[illegible]





```
200 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
201 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
202 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0
203 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0
204 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0
205 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0
206 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0
207 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0
208 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0
209 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0
210 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0
211 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0
212 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0
213 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0
214 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0
215 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0
216 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0
217 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
218 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
219 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
220 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
221 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
222 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
223
224 ... all other labeled imagination of numbers goes here ...
225
226 labeled class 9:
227 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
228 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
229 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
230 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
231 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
232 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
233 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
234 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
235 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0
236 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0
237 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
238 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
239 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
240 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0
241 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0
242 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0
243 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0
244 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0
245 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
246 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
247 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
248 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
```

```

249 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
250 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
251 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
252 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
253 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
254 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
255
256 -----
257
258 Confusion Matrix 0:
259             Predict number 0 Predict not number 0
260 Is number 0           3023           2900
261 Isn't number 0        113           53964
262
263 Sensitivity (Successfully predict number 0)      : 0.51038
264 Specificity (Successfully predict not number 0): 0.99791
265
266 -----
267
268 Confusion Matrix 1:
269             Predict number 1 Predict not number 1
270 Is number 1           5986           756
271 Isn't number 1        800           52458
272
273 Sensitivity (Successfully predict number 1)      : 0.88787
274 Specificity (Successfully predict not number 1): 0.98498
275
276 -----
277
278 ... all other confusion matrix goes here ...
279
280 -----
281
282 Confusion Matrix 9:
283             Predict number 9 Predict not number 9
284 Is number 9           2718           3231
285 Isn't number 9        5147           48904
286
287 Sensitivity (Successfully predict number 9)      : 0.45688
288 Specificity (Successfully predict not number 9): 0.90478
289
290 Total iteration to converge: 10
291 Total error rate: 0.5081666666666667

```