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Implementation of Quadrotor UAV Proportional-Derivative Stabilization on Quaternions with Madgwick Filter

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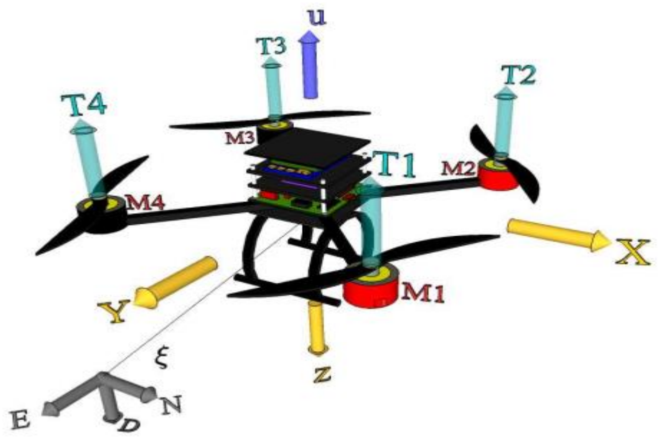
Abstrack

A quadrotor is a kind of non-coaxial multi-rotor UAV with flight attitude control can be achieved by adjusting the speed of the four rotors. In order to analyze the dynamic characteristics and PD (proportional-derivative) controller performance of the quadrotor, the first step is designing the quadrotor dynamics using Newton-Euler model. Then, based on the scheme of PD control, this report designs a controller, which aims to regulate the orientation of the quadrotor.

In physical implementation, inertial measurement unit (IMU) as orientation sensor which used on quadrotor with quaternions representation of orientation and Madgwick filter as a filter of sensor data readings. Quaternion representation can improve computational efficiency for a microcontroller, and avoiding the problems of singularities in Euler angles. Quadrotor dynamics with PD controller first simulated using Matlab / Simulink and made for implementation testing. The results shows that PD controller can stabilize the quadrotor quickly for physical implementation.

Quadrotor Dynamics and Control

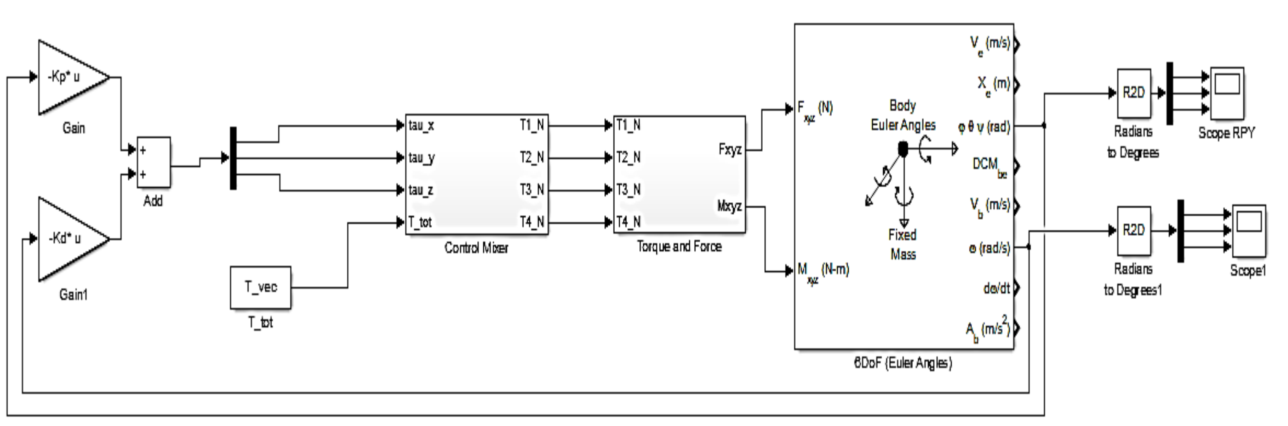


The quadrotor with respect to the inertial frame is expressed by η = $(\psi,\ \vartheta,\ \varphi)^T$, where ψ, ϑ and φ are the yaw, pitch and roll Euler angles respectively. In the "X-type" quadrotor model.The Newton-Euler equations of quadrotor attitude control can be expressed as $\mathbf{J}\dot{\omega} = \tau - \omega \times \mathbf{J}\omega$, then linearized the equation to obtain the generalized torqes for four rotors. Torque is the multiplication between $\mathbf{J}$ and $\dot{\omega}$ and for pitch and roll rotor's torque obtained by the product of moment arm and force of quadrotor.

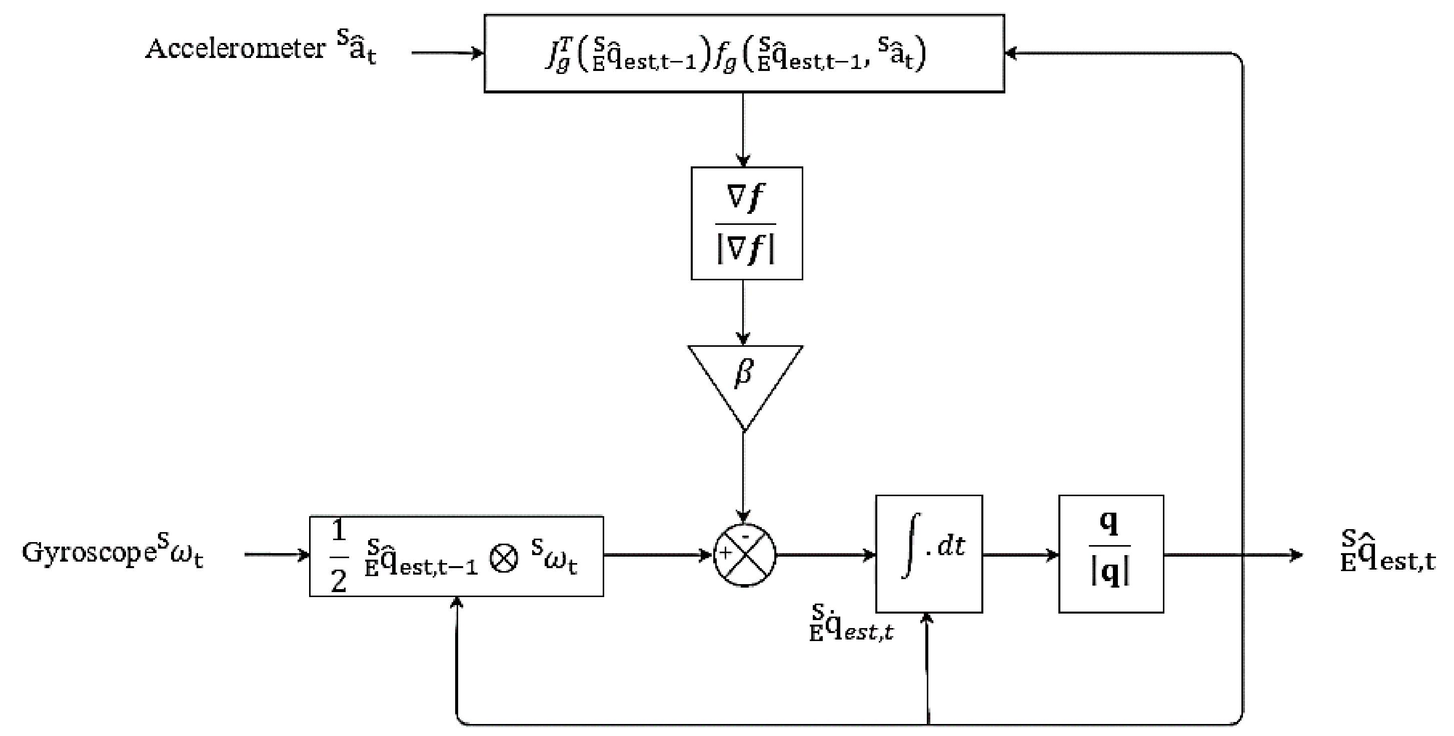
$$\begin{bmatrix} \tau_\psi \\ \tau_\theta \\ \tau_\varphi \\ T_{total} \end{bmatrix} = \begin{bmatrix} -C_M & C_M & -C_M & C_M \\ -d & -d & d & d \\ -d & d & d & -d \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}.$$

where C_M is a constant value depending on the propeller aerodynamic characteristic , d is the distance between center of mass and rotor and T_i denotes the speed of the rotor i. Stabilization of quadrotor can be achieved using PD controller which developed by Newton approach model in equation above can be stabilized by the following PD controller law based on equation and then test with simulation scheme using simulink before physical implementation testing.

$$u = K_p(\eta^{des} - \eta) + K_d(\omega^{des} - \omega)$$



Madgwick IMU Filter



Madgwick filter is IMU data fusing algorithm proposed by Sebastian O.H Madgwick that used a quaternion approach to determine a body's attitude such as pitch and roll angles from measured accelerations and rotation rates by fusing accelerometer and gyroscope readings.

The input of angular velocity in *x* axis, *y* axis and *z* axis from gyroscope, ${}^S\omega_t$, multiplied by half of quaternions to find the estimate orientation of sensor frame, ${}^S_E\dot{\mathbf{q}}_{\omega,t}$ and then the input of acceleration, ${}^S\hat{a}_t$, measured in sensor frame, used the gradient descent algorithm by optimizing the vector of sensor orientation for finding the minimum point of quaternion objective with the gradient of the solution surface defined by objective function and its Jacobian transpose,${}^S_E\hat{\mathbf{q}}_{\epsilon,t}$.

Result and Analysis

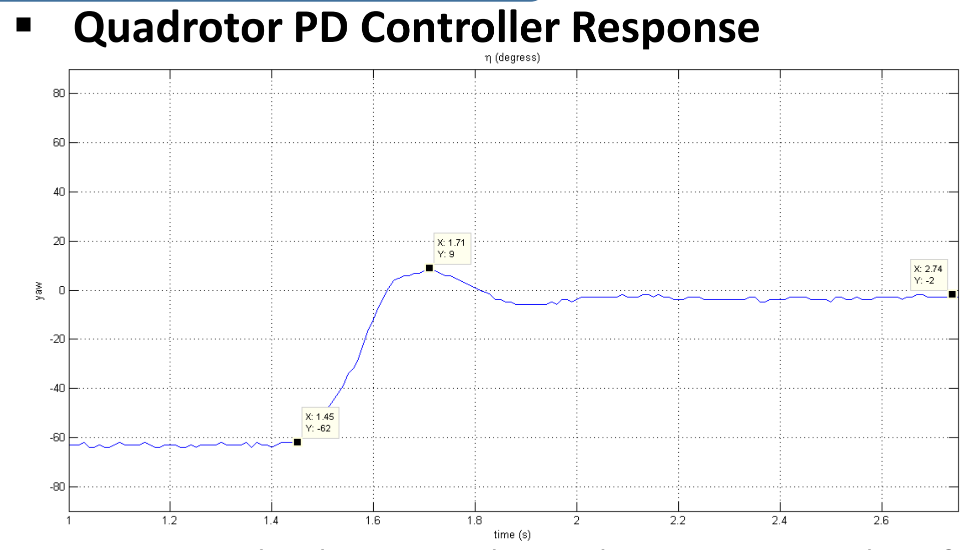
- **IMU Functional Testing**

Protractor	Sensor	Offset
Angle	Average Roll Angle	Error
30°	30.42°	0.42°
45°	45.20°	0.20°
60°	60.34°	0.34°
90°	90.29°	0.29°

Protractor	Sensor	Offset
Angle	Average Pitch Angle	Error
30°	30.27°	0.27°
45°	45.41°	0.41°
60°	60.32°	0.32°
90°	89.13°	0.87°

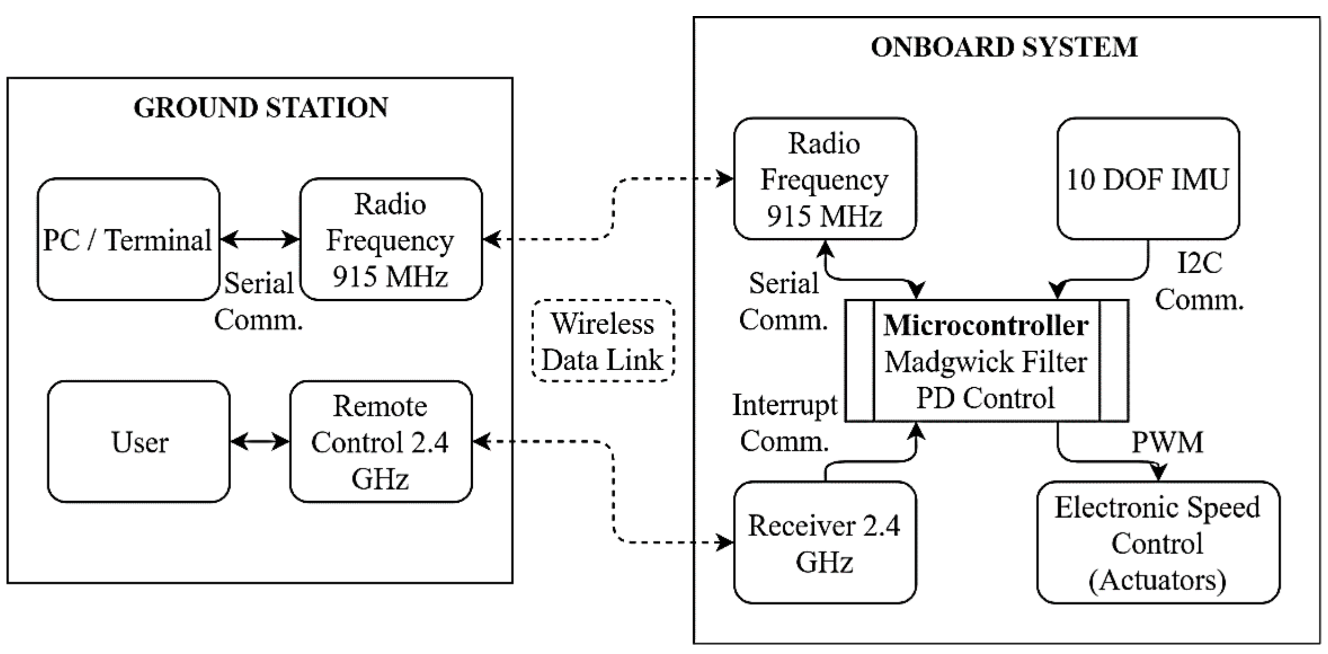
Based on both result in Table above, IMU with Madgwick filter resulting the accurate sensor readings only in integer value while in fraction number the readings are less accurate. We get the error of sensor readings are under 1° in average.

- **Quadrotor PD Controller Response**



Start with choosing the right constant value for both proportional and derivative controller and with yaw initial position is -60 °. We get the quadrotor system response like in figure above, based on the response we know the parameters of quadrotor system response that the overshoot of 14.5% or 9° , it can be said that the system have the small overshoot, rise time is still the same but fast settling time of 0.6 s untill the system stable and steady state error is almost identic from previous tuning that is -2° as for the steady state error existed because of the system is not ideal. For pitch and roll have system responses indentically to yaw.

Quadrotor Experimental Platform



Conclusion

Quadrotor UAV has been made to work in accordance with the system design of attitude controller using IMU GY-88 (Accelerometer & Gyroscope) with Madgwick filter that able to produce fairly accurate readings for pitch and roll angles with minimal error and noise. The proportional and derivative controller with proper constants has able to stabilize the quadrotor quickly for physical implementation.