# Development of Fixed-Wing and Multi-Rotor sUAS

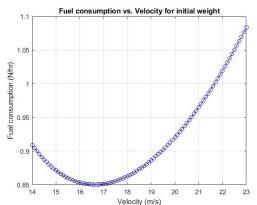
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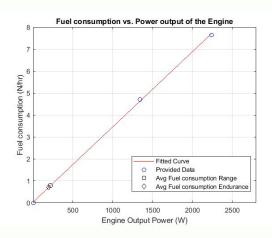
## Fixed-Wing sUAS Development (I)

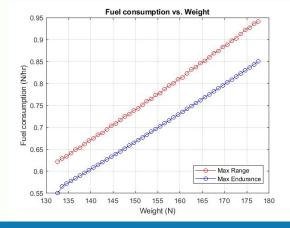
- Max range is ~4034 km and max endurance is ~65.1 hr
- Numerical integration through the changing weight
- Optimizing the velocity throughout the trip for best fuel consumption (endurance) & best fuel consumption / velocity (range)
- Due to oversized engine, fuel consumption curve breaks down estimating low values (e.g. idle = 450 W vs. power needed ~ 200W)

$$Endurance = \int_{w_{initial}}^{w_{final}} \frac{1}{fuel \, rate} dw$$

$$Range = \int_{w_{initial}}^{w_{final}} \frac{velocity}{fuel \, rate} dw$$







## Fixed-Wing sUAS Development - Dynamics Model (II)

## Longitudinal Dynamics Model for Altitude Hold

$$\dot{oldsymbol{x}} = oldsymbol{A}_{long}oldsymbol{x} + oldsymbol{B}_{long}oldsymbol{u}$$

$$\begin{pmatrix} \dot{\bar{u}} \\ \dot{\bar{w}} \\ \dot{\bar{q}} \\ \dot{\bar{\theta}} \end{pmatrix} = \begin{pmatrix} X_u & X_w & X_q & -g\cos\theta^* & 0 \\ Z_u & Z_w & Z_q & -g\sin\theta^* & 0 \\ M_u & M_w & M_q & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \sin\theta^* - \cos\theta^* & 0 & u^*\cos\theta^* + w^*\sin\theta^* & 0 \end{pmatrix} \begin{pmatrix} \bar{u} \\ \bar{w} \\ \bar{q} \\ \bar{\theta} \\ \bar{h} \end{pmatrix}$$

$$+egin{pmatrix} X_{\delta_e} & X_{\delta_t} \ Z_{\delta_e} & 0 \ M_{\delta_e} & 0 \ 0 & 0 \ 0 & 0 \end{pmatrix} egin{pmatrix} ar{\delta}_e \ ar{\delta}_t \end{pmatrix}$$

## **Lateral Dynamics Model for Coordinated Turn**

$$\dot{oldsymbol{x}} = oldsymbol{A}_{latr}oldsymbol{x} + oldsymbol{B}_{latr}oldsymbol{u}$$

$$\begin{pmatrix} \dot{\bar{v}} \\ \dot{\bar{p}} \\ \dot{\bar{r}} \\ \dot{\bar{\phi}} \\ \dot{\bar{\psi}} \end{pmatrix} = \begin{pmatrix} Y_v & Y_p & Y_r & g \cos \theta^* \cos \phi^* & 0 \\ L_v & L_p & L_r & 0 & 0 \\ N_v & N_p & N_r & 0 & 0 \\ 0 & 1 & \cos \phi^* \tan \theta^* & q^* \cos \phi^* \tan \theta^* - r^* \sin \phi^* \tan \theta^* & 0 \\ 0 & 0 & \cos \phi^* \sec \theta^* & p^* \cos \phi^* \sec \theta^* - r^* \sin \phi^* \sec \theta^* & 0 \end{pmatrix}$$

$$imes egin{pmatrix} ar{v} \ ar{p} \ ar{r} \ ar{\phi} \ ar{\psi} \end{pmatrix} + egin{pmatrix} Y_{\delta_a} & Y_{\delta_r} \ L_{\delta_a} & L_{\delta_r} \ L_{\delta_a} & L_{\delta_r} \ N_{\delta_a} & N_{\delta_r} \ 0 & 0 \ 0 & 0 \end{pmatrix} egin{pmatrix} ar{\delta}_a \ ar{\delta}_r \end{pmatrix}$$

# Fixed-Wing sUAS Development - LQR Control (III)

State-space dynamics: 
$$\left\{ \begin{array}{lcl} \dot{\boldsymbol{x}} & = & \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{u} & = & -\boldsymbol{K}\boldsymbol{x} \end{array} \right.$$

Linear Quadratic Performance Index:

$$J = \frac{1}{2} \int_0^\infty \left( \underline{\boldsymbol{x}}^T \boldsymbol{Q} \underline{\boldsymbol{x}} + \underline{\boldsymbol{u}}^T \boldsymbol{R} \underline{\boldsymbol{u}} \right) dt$$

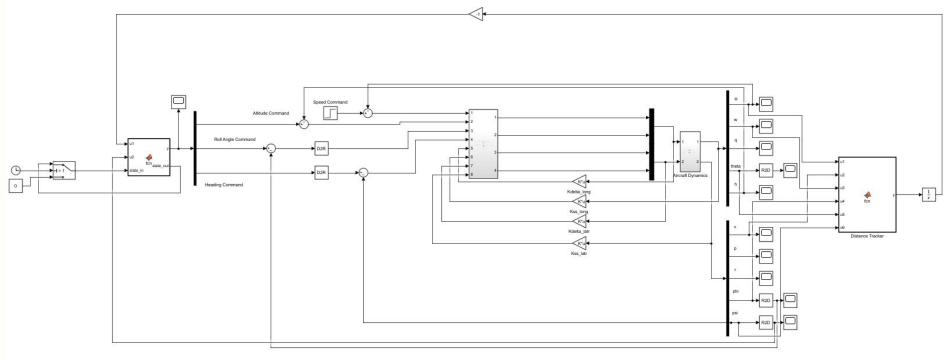
Closed-loop system:

$$A_c = A - BK$$

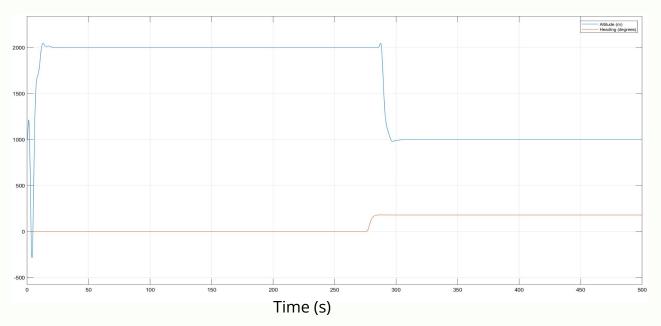
- Advantages of LQR:
  - Known to provide robust stability and optimal control
  - Computationally efficient
  - Easy to implement \_\_\_\_\_ [K,S,CLP]=lqr(SYS,Q,R,N)

## Fixed-Wing sUAS Development - Control Structure (IV)

Goal: Perform an ascent to 2000m, a 180° turn, and a descent to 1000m



## Fixed-Wing sUAS Development - Results (V)



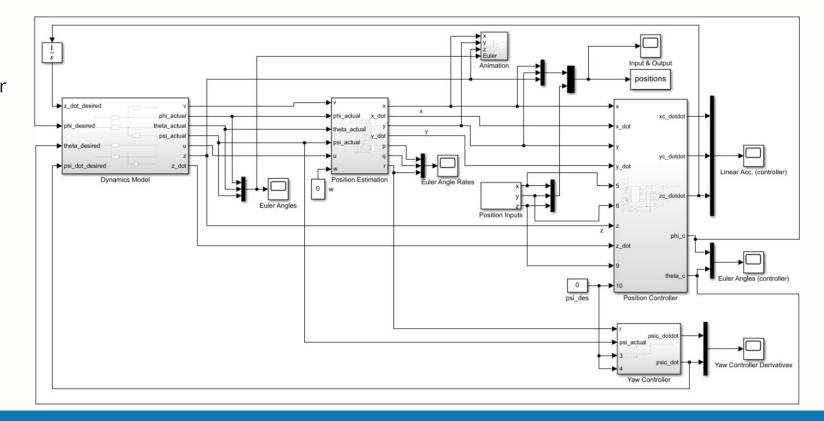
- Commands are performed, but the turn is not performed with the desired radius
- Instability at beginning of simulation for altitude, possibly due to tuning issues with Q and R parameters

## Multi-Rotor sUAS Development - Overview (I)

#### **Goal:**

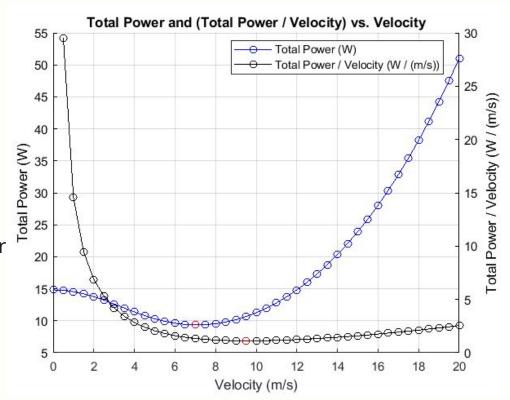
Simulate a quadcopter travelling between (0, 0, 0), (0, 0, 2), (5, 6, 4), (-5, -6, 4), (0, 0, 2),

(0, 0, 0)



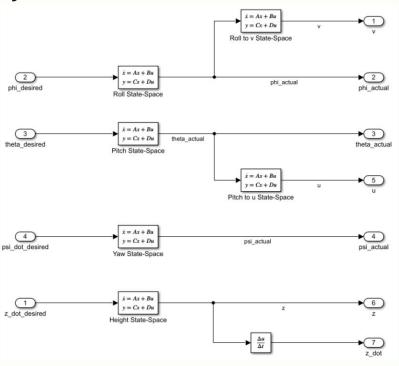
## Multi-Rotor sUAS Development - Range & Endurance (II)

- Max endurance is ~68.1 min at a speed of 7 m/s
- Max range is ~34.061 km at a speed of 9.5 m/s
- Forward flight momentum theory used for total power calculation in velocity increments of 0.5 between 0-20 m/s
- Max endurance is at minimum power max range is at minimum (power/velocity)
- Oth order battery model used (assume constant efficiencies)



## Multi-Rotor sUAS Development - Dynamics and Position Estimation (III)

#### **Dynamics Model**



#### **Position Estimation**

$$\begin{pmatrix} \dot{p}_n \\ \dot{p}_e \\ \dot{p}_d \end{pmatrix} = \begin{pmatrix} c_{\theta} c_{\psi} & s_{\phi} s_{\theta} c_{\psi} - c_{\phi} s_{\psi} & c_{\phi} s_{\theta} c_{\psi} + s_{\phi} s_{\psi} \\ c_{\theta} s_{\psi} & s_{\phi} s_{\theta} s_{\psi} + c_{\phi} c_{\psi} & c_{\phi} s_{\theta} s_{\psi} - s_{\phi} c_{\psi} \\ -s_{\theta} & s_{\phi} c_{\theta} & c_{\phi} c_{\theta} \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix},$$
 (3.1)

Inertial Frame Body Frame

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} c\theta & 0 & -c\phi s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi c\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$
Body
Frame
Inertial
Frame

## Multi-Rotor sUAS Development - Control (IV)

- Use equations from Lecture 9 Part III
- Tuning procedure:
  - o Increase Kp until no undershoot
  - Increase Kd to speeden response time
  - Iterate
- Added integral control for y to reduce steady-state error
- Tradeoff between faster response time with oscillations or slower response time without oscillations (damping)

#### **Tuned PID Gains**

	P	I	D
Х	20	-	35
у	25	3.5	20
Z	30	-	8
Ψ	10	-	0.01

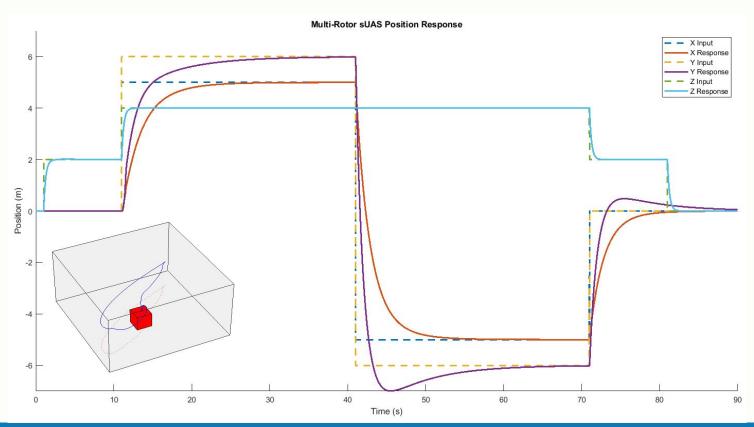
$$\begin{split} \ddot{x}_c &= \ddot{x}_{des} + k_{p,x}(x_{des} - x) + k_{d,x}(\dot{x}_{des} - \dot{x}) \\ \ddot{y}_c &= \ddot{y}_{des} + k_{p,y}(y_{des} - y) + k_{d,y}(\dot{y}_{des} - \dot{y}) + k_{i,y} \int_0^t (y_{des} - y) \, dt \\ \ddot{z}_c &= \ddot{z}_{des} + k_{p,z}(z_{des} - z) + k_{d,z}(\dot{z}_{des} - \dot{z}) \end{split}$$

$$\phi_c = (\ddot{x}_c \sin \psi_{des} - \ddot{y}_c \cos \psi_{des})/g$$

$$\theta_c = (\ddot{x}_c \cos \psi_{des} - \ddot{y}_c \sin \psi_{des})/g$$

$$\psi_c = \psi^{des}$$

## Multi-Rotor sUAS Development - Results (V)



### **Conclusion**

- Simulations were conducted on a fixed-wing Aerosonde UAV and a quadcopter
- Range and endurance estimates were developed for both. Fixed wing estimates were beyond expected values
- A linear dynamics model was developed for both UASs using Simulink
- A LQR controller was used to control the fixed-wing UAS
- A PID controller was used to control the quadrotor
- Fixed wing model still needs further optimization to reduce oscillations, while the quadcopter model generally performs as expected.
- Learned that trying to decrease settling time usually results in increased overshoot (balance is required)



