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Muhammad Arham (2312289) : 3-E

DMS - Assignment

Question No. (01)

Part (a): $\sim(P \vee \sim(P \wedge Q))$

$$
\begin{aligned}
&\Rightarrow \quad \sim P \wedge \sim[\sim(P \wedge Q)] \\
&\Rightarrow \quad \sim P \wedge P \wedge Q \\
&\Rightarrow \quad (\sim P \wedge P) \wedge Q \\
&\Rightarrow \quad C \wedge Q \\
&\Rightarrow \quad C
\end{aligned}
$$

Part (b): $[P \wedge (\sim(\sim P \vee Q))] \vee (P \wedge Q) = P$

$$
\begin{aligned}
&\Rightarrow \quad [P \wedge (\sim(\sim P) \wedge \sim Q)] \vee (P \wedge Q) = P \\
&\Rightarrow \quad (P \wedge P) \wedge \sim Q \vee (P \wedge Q) = P \\
&\Rightarrow \quad (P \wedge \sim Q) \vee (P \wedge Q) = P \\
&\Rightarrow \quad P \wedge (\sim Q \vee Q) = P \\
&\Rightarrow \quad P \wedge t = P \\
&\Rightarrow \quad P = P
\end{aligned}
$$

Question No. (02)

(i.) Determine whether each of these statements is true or false.

(a) $O \in \emptyset$ — false

(b) $\{\emptyset\} \subseteq \{\emptyset\}$ — True

(c) $\{x\} \in \{\{x\}\}$ — True

(d) $\emptyset \in \{x\}$ — True

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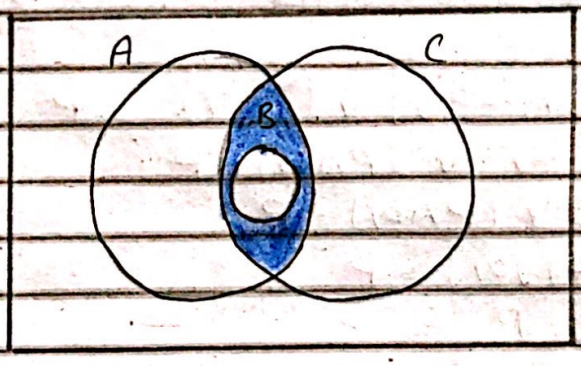
ii) Let $A = Z^+$, $B = \{n \in Z \mid 0 \leq n \leq 100\}$, and $C = \{100, 200, 300, 400, 500\}$. Evaluate the truth and falsity of each of the following statements.

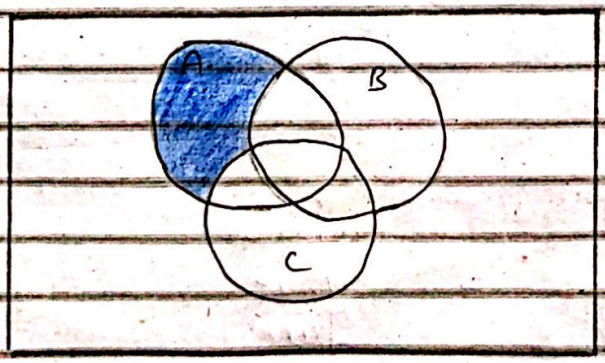
a) $B \subseteq A$ — False
b) A is a proper subset of C — False
c) C and B have at least one element in common — True
d) $B \subseteq C$ — False

QUESTION NO. 03

PART (a): Shade the appropriate region in Venn diagram for $((A \cap C) - B)$



PART (b): Shade the appropriate region in Venn diagram for $A - (B \cup C)$



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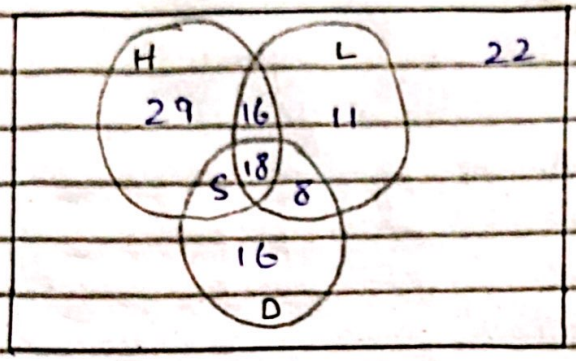
QUESTION NO (04)

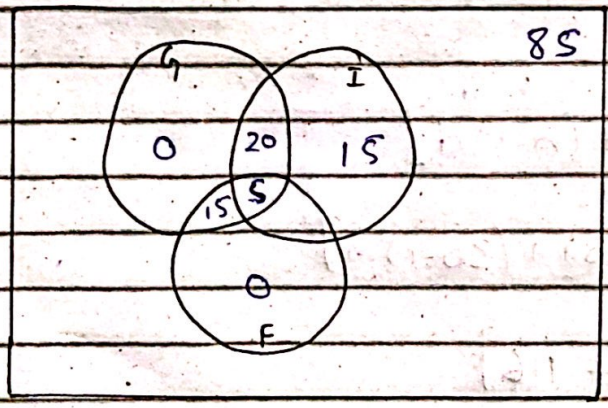
PART a): Create a Venn diagram to summarize the information.



b) 22
c) 29
d) 8
e) 11+8+16 = 35
f) 56

QUESTION NO. (05)

a)



b) 150-(20+5+15+10+15) = 85
c) 65
d) 15
e) 45

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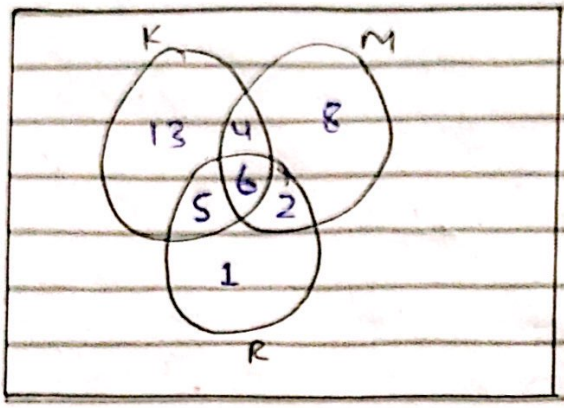
QUESTION NO. (06)

a)



b) 13

c) 12

d) 6

e) 4

QUESTION NO. (07)

SOL:-

$a = 20$

$d = 4$

$n = 30$

$\because S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{30} = \frac{30}{2}[2(20) + (30-1)4]$

$S_{30} = 15[40 + 116]$

$S_{30} = 15(156)$

$\boxed{S_{30} = 2340}$

There are 2340 seats on the Theater

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QUESTION NO. (08)

$$5 + 20 + 80 + \ldots + ar^{n-1}$$

$a = 5$

$r = 4$

$n = 10$

$$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$$

$$S_{10} = 5\left(\frac{4^{10} - 1}{4 - 1}\right)$$

$$\boxed{S_{10} = 1746825}$$

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Question No. (09)

Construct a truth tables for the statement form

a. $(\sim A \vee B) \rightarrow (\sim C \wedge D)$

A	B	C	D	~A	~C	(~A∨B)	(~C∧D)	(~A∨B)→(~C∧D)
T	T	T	T	F	F	T	F	F
T	T	T	F	F	F	T	F	F
T	T	F	T	F	T	T	T	T
T	T	F	F	F	T	T	F	F
T	F	T	T	F	F	F	F	T
T	F	T	F	F	F	F	F	T
T	F	F	T	F	T	F	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	F	T	F	F
F	T	T	F	T	F	T	F	F
F	T	F	T	T	T	T	T	T
F	T	F	F	T	T	T	F	F
F	F	T	T	T	F	T	F	F
F	F	T	F	T	F	T	F	F
F	F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	F	F

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b) $(A \vee \sim B) \leftrightarrow (C \wedge \sim D)$

A	B	C	D	~B	~D	(A∨~B)	(C∧~D)	(A∨~B) ↔ (C∧~D)
T	T	T	T	F	F	T	F	F
T	T	T	F	F	T	T	T	T
T	T	F	T	F	F	T	F	F
T	T	F	F	F	T	T	F	F
T	F	T	T	T	F	T	F	F
T	F	T	F	T	T	T	T	T
T	F	F	T	T	F	T	F	F
T	F	F	F	T	T	T	F	F
F	T	T	T	F	F	F	F	T
F	T	T	F	F	T	F	T	F
F	T	F	T	F	F	F	F	T
F	T	F	F	F	T	F	F	T
F	F	T	T	T	F	T	F	F
F	F	T	F	T	T	T	T	T
F	F	F	T	T	F	T	F	F
F	F	F	F	T	T	T	F	F

QUESTION NO. (10)

PART (a)

$f(x) = ax + b$, $f(g(x)) = ?$

$g(x) = \frac{3}{x-2}$, $g(f(x)) = ?$

$$f(g(x)) = a\left(\frac{3}{x-2}\right) + b = \boxed{\frac{3a}{x-2} + b}$$

$$g(f(x)) = \frac{3}{(ax+b)-2} = \boxed{\frac{3}{ax+b-2}}$$

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PART (b):-

$f(g(5)) = 5$, $g^{-1}(2) = f(2)$, $a = ?$ $b = ?$

$\because g(x) = \frac{3}{x-2}$

$g(5) = \frac{3}{5-2} = 1$

hence

$f(1) = 5$

$f(1) = a(1) + b$

$\boxed{a+b = 5}$ — eq (i)

for $g^{-1}(2) = f(2)$

$f(2) = a(2) + b = 2a + b$

$g^{-1}(2) = \frac{3}{2} + 2 = \frac{7}{2}$

$2a + b = \frac{7}{2}$

$4a + 2b = 7$ — eq (ii)

from eq (i) and (ii)

$a = 5 - b$

$\Rightarrow 4(5-b) + 2b = 7$

$\Rightarrow 20 - 4b + 2b = 7$

$\Rightarrow 20 - 7 = 2b$

$\Rightarrow \boxed{\frac{13}{2} = b}$

eq i $\Rightarrow a + b = 5$

$a + \frac{13}{2} = 5$

$\boxed{a = -\frac{3}{2}}$

for $g^{-1}(x)$

$g(x) = \frac{3}{x-2}$

$y = \frac{3}{x-2}$

$x = \frac{3}{y} + 2$

$\boxed{g^{-1}(x) = \frac{3}{x} + 2}$

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QUESTION NO. (11)

Two functions f and g are defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 1$

PART a): Express in similar form the functions gf, fg and f^2

$$gf = g(2x+1)^2 - 1$$
$$= 4x^2 + 2(2x)(1) + 1^2 - 1$$
$$\boxed{gf = 4x^2 + 4x}$$

$$fg = 2(x^2 - 1) + 1$$
$$= 2x^2 - 2 + 1$$
$$\boxed{fg = 2x^2 - 1}$$

$$f = 2(2x+1) + 1$$
$$f^2 = 4x + 2 + 1$$
$$\boxed{f^2 = 4x + 3}$$

PART b): Find the values of x for which $fg(x) = 7$

$$fg(x) = 2x^2 - 1$$
$$7 = 2x^2 - 1$$
$$8 = 2x^2$$
$$4 = x^2$$
$$\boxed{x = \pm 2}$$

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QUESTION NO. (12)

$$\sum_{j=0}^{2}\sum_{i=1}^{3}(2ij+j+6)$$

$$\sum_{j=0}^{2}(2(1)+j+6)+(2(2)+j+6)+(2(3)+j+6)$$

$$\sum_{j=0}^{2}(2+j+6)+(4+j+6)+(6+j+6)$$

$$\sum_{j=0}^{2}30+3j$$

$$=(30+3(0))+(30+3(1))+(30+3(2))$$

$$=\boxed{99}$$

QUESTION NO. (13)

For Total sum

$\because T_n = a+(n-1)d$

$100 = 1+(n-1)1$

$\boxed{n=100}$

$\because S_n = \frac{n}{2}(2a+(n-1)d)$

$S_{100} = \frac{100}{2}(2(1)+(100-1)1)$

$\boxed{S_{100} = 5050}$

For divisible by 3

$T_n = a+(n-1)d$

$99 = 3+(n-1)3$

$96 = 3n-3$

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$99 = 3n$

$\boxed{n = 33}$

$S_{33} = \frac{33}{2}(2(5) + (33-1)(3))$

$\boxed{S_{33} = 1683}$

For divisible by 7

$T_n = a + (n-1)d$

$98 = 7 + (n-1)7$

$92 = 7n - 7$

$\boxed{n = 14}$

$S_{14} = \frac{14}{2}(2(7) + (14-1)(7))$

$\boxed{S_{14} = 735}$

For 7 and 3

$T_n = a + (n-1)d$

$T_{7 \text{ and } 3} = 21 + (n-1)21$

$84 = 21 + 21n - 21$

$\boxed{n = 4}$

$S_4 = \frac{4}{2}(2(21) + (4-1).21)$

$\boxed{S_4 = 210}$

$S_n = (5050 - 1683 - 735) + 210$

$\boxed{S_n = 2842}$

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QUESTION NO (14)

1, 3, 5, ...

$a = 1$

$d = 2$

$n = ?$

Sum = 100

$\because S_n = \frac{n}{2}(2a + (n-1)d)$

$100 = \frac{n}{2}(2(1) + (n-1)2)$

$\Rightarrow 2 \times 100 = n(2 + 2n - 2)$

$\Rightarrow 200 = n(2n)$

$\Rightarrow 200 = 2n^2$

$\Rightarrow 100 = n^2$

$\Rightarrow \boxed{n = 10}$

Jamila can make 10 rows with 100 blocks

QUESTION NO (15)

~~Reqpea~~ Repeated question

Same as question # 8