PART II A: Optimisation and Machine Learning in Finance – Software

Q1) Considering the given maximization problem of investment scenarios to determine the portfolio of stocks, bonds, and options in **Part IIA**, we need to define the approach to solve it.

Approach: Firstly, we must identify the decision variables, then present the model depending on them and then the objective function, and finally the constraints need to be defined.

Consider the following variables:

Let,

X : the number of shares of stock XYZ that should be purchased.

Y : the number of call options that should be purchased.

Z : the quantity of call options that should be sold.

B : the face value of the zero-coupon bond that needs to be purchased.

Objective Function:

We need to maximize the expected profit, which is determined by the sum of the profits in each of the three scenarios, weighted by their probabilities:

- If the price of the stock stays the same:
  20x + 1000y - 1000z
- If the price of the stock goes up to £40:
  40x - 20x + 1000y- 1000z
- If the price of the stock drops to £12:
  12x - 20x + 1000y - 1000z
- Profit from bond:
  10b

Profit expected can be calculated as follows:

Stock price remains same:

(1/3) * (Profit from stock + Profit from call option + Profit from bond)

(1/3) *(20x + 1000y - 1000z + 10b)

Stock price goes up to £40:

(1/3) * (Profit from stock + Profit from call option + Profit from bond)

(1/3) *(40x - 20x + 1000y- 1000z +10b)

Stock price drops to £12:

(1/3) * (Profit from stock + Profit from call option + Profit from bond)

(1/3) *(12x - 20x + 1000y - 1000z +10b)

Profit in using call option where 100 share of £15 can be bought or sell for £1000 : £15*100 = 1500 -1000 = 500

Profit in selling zero coupon bond with £100 face value in £90 = 100-90 => £10

Maximize: (1/3)(40x - 20x + 500y - 500z) + (1/3)(20x - 20x + 500y - 500z) + (1/3)(12x - 20x + 500y - 500z) - 90b – 20000

Simplify: 12/3x + 500/3y - 500/3z - 10b

Constraints:

The constraints extracted from the scenario are:

- We can't purchase or sell more than 50 call options, so y + z <= 50.
- We can't purchase a bond with a face value exceeding £1000, so b <= 1000.
- All of our investments cannot exceed £20,000, so 20x + 1000y - 1000z + 90b <= £20,000.
- x + 1000y - 1000z <= 5000, This is the maximum amount of call options that can be bought or sold.
- x + b <= 1000, This is the maximum amount of the bond's face value that can be purchased without exceeding the investment amount 20,000.
- The number of shares of XYZ stock purchased must exceed or equal the number of call options purchased. x + y >= 50

Now we consider the bounds of decision variables:

x >= 0

y >= 0

z >= 0

Maximize v 12/3x + 500/3y - 500/3z - 10b →Profit Equation

Subject to

20x + 1000y - 1000z + 90b <= 20,000

x + 1000y - 1000z <= 5000

y + z <= 50

x + b <= 1000

x, y, z, b >= 0

Remarks Obtained: With the given approach we use the optimal solution is to acquire 1000 shares of XYZ stock, sell 25 call options, and buy 25 call option also acquire a risk-free zero-coupon bond with a face value of £111.11 for a duration of six months. The expected return is £4000.0.

Utilizing the three possible stock price scenarios, this strategy aims to maximise the anticipated profit. The linear programme is expressed in terms of decision variables, objective function, and constraints. Python linprog function is used to solve the linear programme and determine its optimal solution. To maximise expected profit, the optimal solution indicates that the investor should purchase more shares of stock XYZ and buy and sell call options. To reduce risk, the investor may also purchase a 6-month risk-free zero-coupon bond.

Optimal solution:

x = 1000.0

y = 25.0

z = 25.0

b = 111.11

Expected profit: £ 4000.0.

Q2) Considering the given maximization problem of investment scenarios to determine the e investor's expected profit under this additional constraint in **Part IIA**, we need to define the approach to solve it.

Approach: In Q2, we added an additional constraint to the optimisation problem, requiring the investor to earn a minimum of £2,000 in any of the three scenarios for the price of XYZ six months from now. To accomplish this, we added three new constraints, one for each scenario, that required the profit to be at least £2,000 in each case.

Suppose the investor desires a minimum return of £2,000 in any of the three price scenarios for XYZ stock for six months from now.

Objective Function will remain same as Q1

Decision Variable and Constraints:

- Consider the decision variables and constraints remain the same as in Q1.
- We add a new constraint that the expected profit in any of the three scenarios should be at least £2,000.

(40x - 20x - 1000y + 1000z + 90b) >= 2000

(12x - 20x + 1000z - 1000y + 90b) >= 2000

(20x - 1000y + 1000z + 90b) >= 2000

Remarks Obtained:

The problem was solved with the Python linprog function, which yielded the optimal values for the decision variables and the optimal total profit. The optimal portfolio consisted of purchasing 600 shares of XYZ stock, purchasing 50 call options, selling 50 call options, and no zero-coupon bond was purchased. The optimal profit total was £ 2111.11.

Optimal solution:

x = 333.33

y = 27.33

z = 22.67

b = 0.0

Expected profit: £ 2111.11