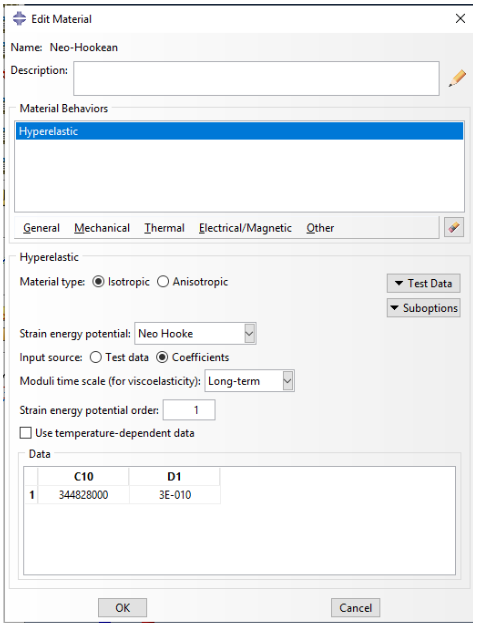
Solid Mechanics Simulation in Abaqus and First Piola Calculation

for Asymmetric Tensor Project

Material: Neo-Hookean



First Piola-Kirchhoff stress tensor computation:

A. Solid to Surface:

We get centroidal/nodal deformation gradient, F, and Cauchy stress, S, in global frame from Abaqus for each tetrahedron of a solid mesh. Then if centroidal output, for each face of the mesh, we take the average F and S of the tets attached to it. Each internal face has two tets and each boundary face has one tet attached to it.

Then, we create the surface mesh from this solid mesh with all boundary triangles. Each triangle of this surface mesh has F and S values.

i. How to get First Piola, P, on each **vertex's** tangent plane from deformation gradient, F, and Cauchy stress, S, of each triangle/ Vertex

1. Deformation gradient (F) and Cauchy stress (S) of each triangle/ vertex are in the global coordinate system.

2. First, we subdivide[1] the mesh and copy each original triangle's F and S (if centroidal) or take average of original Vertices' F and S (if nodal) to subdivided ones.

3. After that, we calculate First Piola, P, for each Triangle/Vertex using following equation:

First Piola, $P = J * S * (F^{-1})^T$, where $J = det(F)$

4. Now, if centroidal, we have P for each triangle of the mesh,

5. And, to get P at each vertex, V,

6. We compute area weighted average of P of all triangles that are incident to V.

7. P is also in global basis.

7. Then, we project P at each vertex onto vertex's tangent plane to get surface tensor there.

<u>**ii.** How to get tangent plane at vertices of our surface mesh:</u>

1. To get surface normal of each vertex, V,
2. We take average of the normals of all triangles incident to V.
3. Tangent plane at V is the plane that goes through V and perpendicular to the vertex normal at V.
4. To pick a X and Y-axis on the tangent plane,
5. Among the edges that are incident to V,
6. We find the one that's closest to V's tangent plane and
7. We take this edge's corresponding line on the tangent plane. This our X-axis.
8. We get Y-axis by taking cross product of this X-axis and vertex normal.

<u>**iii.** Projection:</u>

1. To project P at a vertex V with normal N, onto V's tangent plane(X, Y),
   a. We form a matrix M = [X ; Y ; N]
   b. Then, we calculate $P_{proj} = M^T * P * M$ and take upper left 2x2 matrix of P_proj.

[1]we need to subdivide to make sure none of the triangle has more than one angle deficit vertex.

<u>**B.** Shell:</u>

We take unique nodal Cauchy Stress (J) and unique nodal deformation gradient (F) output from abaqus for shell. Both output are 2x2 symmetric tensor. Abaqus rotates deformation gradient to make it symmetric before writing it in .dat file. To get back the asymmetric deformation gradient, we also take the unique nodal rotation output from Abaqus. Additionally, shell thickness is set to 0.0001 and shell offset, to "Top Surface" and thickness integration rule, to Simpson with 5 integration points. Both top and bottom surface outputs are written in .odb (cauchy stress, rotation) and .dat (deformation gradient) files and we take the top surface value for both Cauchy and deformation gradient as we set our shell's offset to 0 from top surface i.e. shell is right there.

<u>**i.** How to get First Piola, P, on each **vertex's** tangent plane from deformation gradient, F, and Cauchy stress, S:</u>

1. We rotate deformation gradient at each vertex by the inverse amount of rotation reported at that vertex.

2. Both Cauchy stress and deformation gradient are in vertex local frame defined in abaqus.

2. We transfer (basis transformation) them from Abaqus local frame to local frame used in our program.

3. Then, at each Vertex, we calculate First Piola, P, as follows,

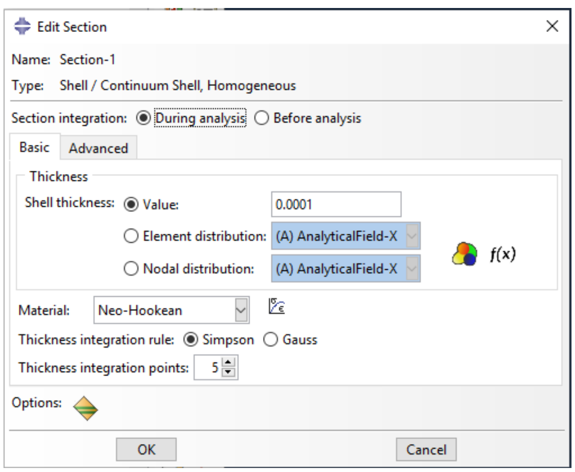
$$\text{First Piola, } P = J * S * (F^{-1})^T, \text{ where } J = det(F)$$

C. Planar mesh:

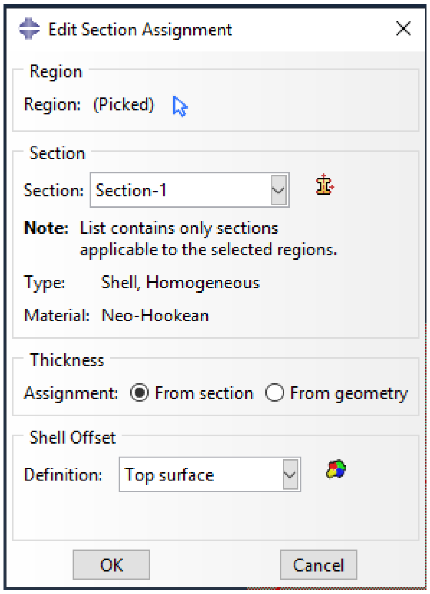
For planar (2D plane or 2D shell), getting first piola tensor is straightforward. Abaqus provides original 2x2 symmetric Cauchy tensor (J) and 2x2 asymmetric Deformation Gradient (F). We take unique nodal output for both J an F and then use following formula to calculate First Piola (P),

$$P = J * S * (F^{-1})^T, \text{ where } J = det(F)$$

Shell Section Parameters:



Shell Section Assignment:


Edit Section Assignment
Region
Region: (Picked)
Section
Section: Section-1
Note: List contains only sections applicable to the selected regions.
Type: Shell, Homogeneous
Material: Neo-Hookean
Thickness
Assignment: From section From geometry
Shell Offset
Definition: Top surface
OK
Cancel