How to get local First Piola at each vertex from global Deformation Gradient and Cauchy Stress in each triangle:

1. We get global deformation gradient (F) and Cauchy stress (S) at the centroid of each triangle.

2. Then, when we subdivide the mesh, we copy each original triangle's F and S to its subdivided ones.

3. After that, we calculate First Piola, P, for each Triangle using following equation:

$$\text{First Piola, } P = J * S * (F^{-1})^T, \text{ where J = det(F)}$$

4. Now that we have P for each triangle of the mesh,

5.   To get P in global frame at each vertex,

6.   Compute area weighted average of P of all triangles in it's one ring neighborhood.

6. Then, project each vertex's global P(3x3) onto its tangent plane to get local P(2x2).