

**Name:**

## PURPOSE

The purpose of this lab is to measure transient responses of different circuits and compare measured behavior to what we would expect from the analysis of the poles of corresponding transfer functions.

### 1. Low Pass RC Filter

Write down  $s$ -plane representation of the transfer function  $\mathbf{H}(s)$  for a low pass RC filter and find its pole. The transient response  $V(t)$  for the transfer function with a single real pole  $s=\sigma$  is an exponential function:

$$V(t) \sim e^{\sigma t}.$$

It is an exponential decay if  $\sigma$  is negative, and an exponential grow, if  $\sigma$  is positive. In both cases corresponding the constant  $\tau$  is:

$$\tau = \frac{1}{|\sigma|}$$

Show here your math. Provide expressions for  $V(t)$  and  $\tau$ .

Build a low pass filter using the same components that you used for Lab 3.1. To observe a transient response switch the function generator to a square wave of fundamental frequency not higher than  $f_c/10$ , where  $f_c$  is the corner frequency of the filter. Trigger the oscilloscope with a falling edge of the square wave. Save observed transient response, plot it extract the time constant. Compare the measured time constant to what you would expect for this circuit.

Insert here a figure that shows the observed transient response. Report the measured and estimated time constant. Discuss your results.

## 2. High Pass RC Filter

Write down s-plane representation of the transfer function  $\mathbf{H}(s)$  for a high pass RC filter. Find the pole of the transfer function. Find an expression for the transient response and the time constant.

Show here your math. Provide expressions for  $V(t)$  and  $\tau$ .

Build a high pass filter using the same components that you have used in the Lab 3.1. last lab. Save observed transient response, plot it extract the time constant. Compare the measured time constant to what you would expect for this circuit.

Insert here a figure that shows the observed transient response. Report the measured and estimated time constant. Discuss your results.

## 2. Tank Circuit Band Pass Filter.

A band pass filter with a tank circuit is shown on the figure 1a.

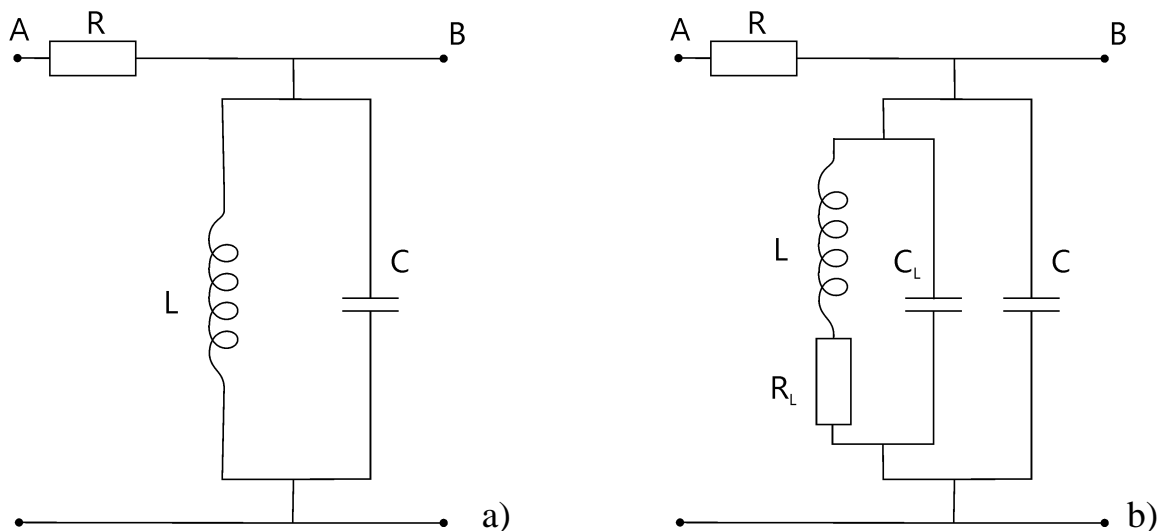


Figure 1. a) Band pass filter with tank circuit; b) Band pass filter with inductor replaced with its equivalent circuit.

The analysis of this circuit is complicated by the less than ideal nature of the typical inductive component (See figure 1b). During the Lab 2.1 you have learned that an inductor of the type used in this lab could be represented by the combination of ideal

elements shown on the figure. We will take some steps minimize the effect of the  $R_L$  and  $C_L$  elements:

- To minimize the effect of  $C_L$  we will use  $C \gg C_L$  so  $C_L$  could be neglected;
- To minimize the effect of  $R_L$  we will ... cool it down with a liquid nitrogen.

Complete the circuit by adding a Thevenin's equivalent representation of the function generator and dropping all elements that could be neglected.

Insert here the final circuit diagram.

Write the transfer function for the complete circuit. Find an expression for the resonant frequency  $\omega_r$  of the circuit.

Show here your math. Provide expression for  $\omega_r$ .

Find an expression for two poles  $s_{1,2}$  of this transfer function:

Show here your math. Provide expression for  $s_{1,2}$ .

Find the value  $R_{cd}$  of the resistor  $R$  which makes the circuit to be critically damped. At this value the discriminant in the expression for poles  $s_{1,2}$  is equal to zero.

Show here your math. Provide expression for  $R_{cd}$ .

When  $R$  is smaller than  $R_{cd}$ , both poles are real, the circuit is overdamped and does not show any resonance behavior. The transient  $V(t)$  response for the circuit with two real poles is more complex than in the case of a single real pole, but at its tail it approaches a single exponential term behavior:

$$V(t) \sim e^{\sigma t},$$

where  $\sigma$  corresponds to the longer time constant in case of exponential decay, and to a shortest time constant in case of exponential grow. Find an expression for the transient response and the time constant for the overdamped circuit.

Show here your math. Provide expression for  $V(t)$  and  $\tau$  for overdamped circuit.

When  $R$  is larger than  $R_{cd}$ , the poles  $s_{1,2}$  are complex conjugated numbers

$$s_{1,2} = \sigma_0 \pm j\omega_0,$$

the circuit is underdamped and has a resonance. The transient response  $V(t)$  in this case is a decaying sine wave:

$$V(t) \sim e^{\sigma_0 t} \sin(\omega_0 t)$$

Find an expression for the transient response and the time constant for the underdamped circuit.

Show here your math. Include results for  $V(t)$  and  $\tau$  for underdamped circuit.

Build the circuit using the same inductor that you have used in Lab 2.1 with two wires soldered to its legs. Use 0.01 mF for  $C$ . For  $R$  take a resistor approximately 10 times larger than  $R_{cd}$ . Open 'Transfer function.vi'. Place the inductor into Styrofoam container with a liquid nitrogen. Ask instructor for help. Measure the transfer function of the circuit and extract the resonance frequency. Compare the measured resonance frequency to what you expect for this circuit. Report your results.

Insert here a figure with the measured transfer function. Show your calculations of the resonant frequency and compare calculated frequency to the measured.

Refer to Equation 2.99 and 2.100 in your text, make the necessary measurements (points where  $|\mathbf{H}|$  falls to 0.7 of its peak value), and determine the quality factor  $Q$  of the underdamped circuit. Report you results.

Report here your results for the quality factor.

Measure a transient response of the circuit. How many peaks can you distinguish before the ringing comes down? Measure time interval between the peaks of the decaying signal and calculate its frequency. Compare to what you expect for this circuit.

Repeat measurements of the transient response for circuits with  $R=2R_{dc}$  (slightly underdamped),  $R=R_{dc}$  (critically underdamped) and  $R=0.5R_{dc}$  (overdamped).

Plot all transient responses on the same graph. Compare them to each other. Discuss in terms of decay rate, number of oscillations, etc.

Insert here a figure with the measured transient responses. Discuss your results.