Bosonic Codes in 5 Minutes

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Bosonic Codes

- Encode information in the space corresponding to the occupation (photon) number of a harmonic oscillator
- Characterize by
  1. Fock/number states $\{|n\rangle\}_{n=0}^{\infty}$
  2. position and momentum eigenstates $\{|x\rangle\}_{x\in\mathbb{R}}, \{|p\rangle\}_{p\in\mathbb{R}}$
  3. coherent states $\{|\alpha\rangle\}_{\alpha\in S}$ for some S
- Photons are prone to "loss" (action by a). So, we can focus on correcting these errors.
- Photon-photon interactions are extremely weak

Harmonic Oscillator Review

- non-Hermitian creation/annihilation operators: $a^\dagger, a$
- $a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle, \qquad a|n\rangle \quad = \sqrt{n}|n-1\rangle$
- $b = a^\dagger a, \qquad n\,|n\rangle = n\,|n\rangle$
- $[a, a^\dagger] = 1, \qquad [n, a^\dagger] = a^\dagger, \qquad [n, a] = -a,$

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- $[a, a^\dagger] = 1, \qquad [n, a^\dagger] = a^\dagger, \qquad [n, a] = -a,$
- Coherent states: eigenfunctions of annihilation operator

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle = e^{-\frac{|\alpha|^2}{2}} e^{\alpha \hat{a}^\dagger} |0\rangle \ ,$$

- Note: notation $|\alpha\rangle$ does not refer to a Fock/number state.
- Expression $|\alpha\rangle$ with $\alpha = 2$ represents a Poisson distribution of number states $|n\rangle$ with a mean photon number of two. Think Poisson arrival process, but with photons.

Hopeful approach: Apply Ken's course directly

- Simple encoding of M qubits: 2^M Fock states cover photon numbers $0, 1, ..., (2^{M-1})$.
- Use binary representation: $|n\rangle = |b_{M-1}b_{M-2}\cdots b_0\rangle$
- The jth binary digit represents the eigenvalue $(1 + Z_j)/2$ for the corresponding physical qubit
- E.g., $n = 8$: $|1000\rangle$

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- E.g., $n = 8$: $|1000\rangle$
- Photon loss occurs, $a : |1000\rangle \mapsto |0111\rangle$
- QEC schemes based on models of independent single qubit errors cannot be easily transferred to this problem
- Luckily, the stabilizer formalism provides useful intuition for codes we'll discuss

Knill-Laflamme conditions

- Quantum Error Correction criteria
- Find two logical code words $|W_\sigma\rangle$, where $\sigma = \uparrow, \downarrow$ s.t.

$$\langle W_\sigma | E_l^\dagger E_k | W_\sigma \rangle = \alpha_{l,k}\delta_{\sigma,\sigma'}$$

  for all single, independent errors $E_{l,k} \in \mathcal{E}$
- Also, require $\alpha_{l,k}$ are entries of a Hermitian matrix and independent of the logical words.

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- Protect against $\mathcal{E} = \{I, a\}$
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- Generalize
  - Greater spacing between states: can detect higher order loss errors or alternatively gain errors
  - Action by n "dephases" (see how it can shift relative phases?). This leads to a superposition of codewords and error words. Project onto word basis to recover (efficient).

Binomial Codes

- Protect against $\mathcal{E} = \{I, a, a^2, \cdots, a^L, a^\dagger, \cdots, (a^\dagger)^G, N, \cdots, N^D\}$
- Consider

$$|W_{\uparrow/\downarrow}\rangle = \frac{1}{\sqrt{2^N}} \sum_{p \text{ even/odd}}^{[0,N+1]} \sqrt{\binom{N+1}{p}} |p(S+1)\rangle$$

  with $S = L + G$, $N = \max\{L, G, 2D\}$.
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- Mean photon numbers equal (no deformation) and QEC condition holds
- Can be shown by writing difference in lth moment of photon number of codewords as lth derivative of $(1+x)^{N+1}|_{x=-1}$ with $l \leq \max\{L, G\}$ up to a factor
- Measure photon number mod $S + 1$

Cat Codes

- Superposition of well-separated coherent states ("legs")
- $2(L+1)$ legs protects L photon losses. Compare to binomial code with $S = L$
- E.g. $L = 1$

$$\left| C^{\alpha}_{\uparrow/\downarrow} \right\rangle = |\alpha\rangle \pm |i\alpha\rangle + |-\alpha\rangle \pm |-i\alpha\rangle$$

  up to a normalization factor.
- As $\alpha \to \infty$, $\left\langle C^{\alpha}_{\uparrow} \middle| N^p \middle| C^{\alpha}_{\uparrow} \right\rangle = \left\langle C^{\alpha}_{\downarrow} \middle| N^p \middle| C^{\alpha}_{\downarrow} \right\rangle$ so potentially immune from unlimited order dephasing
- Remember: distributed as Poisson and for large N, Binomial and Poisson approach normal distribution
- Loss takes coherent states to coherent states! Can measure mod $S+1$ again to determine whether jump occurred. Do we do anything, if not?

GKP Codes

- Use the continuous basis of non-normalizable eigenstates of the position operator x
- Not enough time!
- Quantum analoge of frequency combs
- Key: protect against displacement errors $D_\alpha = \exp\left\{\alpha a^\dagger - \alpha * a\right\}$, not loss errors explicitly

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Error Models

- Some reminders
- Photons are prone to loss
- Photon-photon interactions are extremely weak
- Hence, bosonic QEC codes focus on correcting photon-loss errors using very limited forms of photon-photon interactions

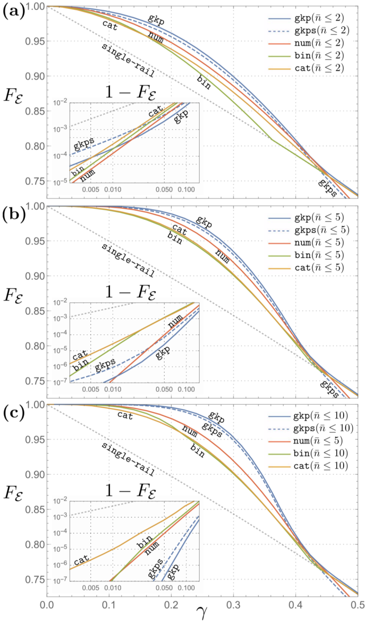
Pure Loss Bosonic Channel

- Assume encoding, recovery, and decoding are perfect
- Lossy channel $N = \exp(\kappa t D)$ with superoperator $D(\cdot) = aa^{\dagger} - 1/2\{n, \cdot\}$ with κ as the excitation loss rate and time interval t
- Kraus operators

$$E_l = \left(\frac{\gamma}{1-\gamma}\right)^{l/2} \frac{a^l}{\sqrt{l!}} (1-\gamma)^{n/2}$$

  with $\gamma = 1 - \exp(-\kappa t)$
- Channel does not contain identity as a Kraus operator for $\gamma \neq 0$ due to aforementioned damping/back-action
- VV Albert considers "channel fidelity", $F_{\mathcal{E}}$, which is the overlap between the initial state and the final state when considering an initial Bell state such that only the first qubit is acted on by the channel
- Optimal recovery for each code is computable via a semi-definite program

Channel Fidelity



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Multi-Mode Extensions

- Pair-cat codes
- $\chi^{(2)}$ codes
- noon codes
- Experimentally relevant advantages over single-mode codes
- Correct more errors

Multi-Mode Extensions: Binomial Codes

- To create more useful quantum superpositions of Fock states which can store quantum information, it is necessary to couple the bosonic mode to a non-linear element, e.g., a superconducting qubit, a trapped ion, or a Rydberg atom
- $\chi(2)$ code uses $O(n)$ instead of $O(n^2)$ qubits that previous two mode codes had used to correct m loss/gain/dephasing errors (Niu)
- Inspired by cat code, but uses lower order non-linearity

Multi-Mode Extensions: Cat Codes

- Pair-cat codes (VV Albert)
- Reduction in order of nonlinearity required for physical realization (as with $\chi(2)$)
- With ordinary cat codes and current technology, the number parity syndrome makes it difficult to realize simultaneous discrete (usually for loss) and continuous error correction (usually for dephasing)