# Project 1
**(Marks 15)**

In this project you will construct a Monte Carlo algorithm and develop computer codes to study a $d$ dimensional scalar field theory on the lattice. In the next project you will use these codes to study the theory in $d = 2, 3$ and $4$ dimensions.
The theory you will study is defined by the lattice action

$$S_E^\ell = \sum_x \left\{ \frac{1}{2}\chi_x^2 - \kappa \sum_\mu \chi_x \chi_{x+\hat{\mu}} + g\chi_x^4 \right\},$$

where $\kappa$ and $g$ are called "bare" couplings. We will define $\kappa^{-1} = 2d + (m_0 a)^2$, where $m_0$ is defined as the bare lattice mass term and $a$ is the lattice spacing. When $g = 0$ we can verify that the physical mass of the scalar particle is $M_{\rm phys} = m_0$, however once we turn on interactions this connection between $M_{\rm phys}$ and $m_0$ is no longer true and we will treat $\alpha \equiv (m_0 a)^2$ as a parameter that defines $\kappa$.
The label $x = (\mathbf{n}, \tau)$ represents the site on a square, cubic, or hyper-cubic lattice depending on $d$ with periodic boundary conditions. Let $L$ be the number of lattice sites in each direction. Your Monte Carlo algorithm should help you compute three quantities

$$\sigma = \frac{1}{L^d} \sum_x \langle \chi_x^2 \rangle, \quad \chi = \frac{1}{L^d} \sum_{x,y} \langle \chi_x \, \chi_y \rangle, \quad F = \frac{1}{L^d} \sum_{x,y} \langle \chi_x \, \chi_y \rangle \cos(2\pi \Delta\tau / L),$$

for a fixed value of $L$, $\kappa$ and $g$. In the definition of $F$ the quantity $\Delta\tau$ is the temporal separation in lattice units between $x$ and $y$. Using $\chi$ and $F$ let us define a finite size mass $M(L)$ using the formula

$$M(L) = \frac{2\sin(\pi/L)}{\sqrt{\Big((\chi/F) - 1\Big)}}.$$

In the next project you will show that in the massive symmetric phase $M(L \to \infty) = M_{\rm phys} a)$. We will use this to compute the physical mass of the scalar particle in the Monte Carlo method.

1. For $g = 0$ write a computer program to exactly compute the three observables. Verify that $M(L) = m_0 a$ independent of $L$ when $g = 0$. This implies $M_{\rm phys} = m_0$ in the free theory as expected. Also have a computer code handy to compute the observables when $\kappa = 0$. In your report tabulate exact results for $L = 16$, $g = 0$, and $\alpha = 0.25$ and $L = 16$, $g = 0.1$, and $\kappa = 0$ in $d = 2, 3, 4$.

2. Using online lectures provided on the webpage or otherwise construct a Monte Carlo algorithm and develop computer codes to study the above $d$ dimensional lattice field theory. Verify that your Monte Carlo is producing the right results when $g = 0$ but $\kappa \neq 0$ and when $\kappa = 0$ but $g \neq 0$. You can use the exact results from above for this purpose.

3. Using your Monte Carlo method compute the three observables and $M(L)$ for $L = 8$, $(ma)^2 = -1.5$ and $g = 0.1$ in $d = 2, 3$ and $4$. Tabulate your results in the form shown below.

| $d$ | $\sigma$ | $\chi$ | $F$ | $M(L)$ |
|---|---|---|---|---|
| | | | | |

Make sure the errors in your results are around $0.2\%$.