MATH 101

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Learning Goals 1

(1) Approximate the area between a curve and x-axis by using **left, right, or midpoint** sums. Interpret a definite integral in terms of the area between a curve and x-axis. Compute definite integral by using the **Riemann Sum**, the definition of definite integral.

Examples,

- Estimate the area under the graph $y = \sqrt{x}$ from $x = 0$ to $x = 4$ using N approximating rectangles and right endpoints. Sketch the graph and rectangles. Is your estimate an underestimate or overestimate?
- Write an integral that is defined by the expression

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{n}\sin\left(\frac{i}{4n}\right).$$

- Use the definition of a definite integral to show that

$$\int_a^b x^2 dx = \frac{b^3 - a^3}{3}.$$

Learning Goals 2

Compute definite integrals by using the **fundamental theorem of calculus**. Be able to recognize functions that are given as definite integrals with variable upper and lower limits and find their derivatives, relate antiderivatives to definite and indefinite integrals, and the **net change** as the definite integral of a rate of change. Examples:

1. Evaluate $\int_0^4 (\frac{x^2}{4} + \sqrt{x} + e^x) dx$.
2. Differentiate $\int_{\ln x}^{e^x} \frac{1}{\sqrt{1+t^4}} dt$.

Learning Goal 3

Compute the following integrals using **substitution**.

1. $\int (2x-1)e^{x^2-x}dx$.
2. $\int tan^3(\theta)d\theta$.
3. $\int_e^{e^2} \frac{1}{x\ln x}dx$.
4. $\int_2^3 \frac{1}{e^x+e^{-x}}dx$.

Learning Goal 4

Construct an integral or a sum of integrals that can be used to find the **volume of a solid** by considering its **cross-sectional areas.** For solids that are obtained by revolving a region about an axis of rotation, find the volume by considering **cross-sectional discs or washers.** Examples:

- Let R be one of the infinitely many regions bounded by $y = 1 + \sec x$ and $y = 3$. Find the volume of the solid obtained by rotating R about $y = 1$.
- Find the volume of the solid by rotating the region bounded by $y = x^2$ and $y = x + 2$ about the line $x = 3$.
- Consider a cone with base radius of r cm and height of h cm. Use the method of cross sections to show that the volume of the cone is $\frac{1}{3}\pi r^2 h\ cm^3$.
- The base of a solid S is the triangular region with vertices $(0,0), (1,0)$ and $(0,1)$. Each cross-section of S perpendicular to the y-axis is a right isosceles triangle with hypotenuse on the xy-plane. Find the volume of S.

Learning goals 5

Construct an integral or a sum of integrals that can be used to find the total work done in moving an object either by considering the entire object moving over an infinitesimal distance or by considering an infinitesimal section of the object moving over the entire distance. Examples:

- If 6 J of work is needed to stretch a spring from 10 cm to 12 cm and another 10 J is needed to stretch it from 12 cm to 14 cm, what is the natural length of the spring?
- A 10-kg bucket containing 36 kg of water is lifted from the ground to a height of 12 m with a rope that weigh 0.8 kg/m. How much work is done?
- A spherical tank of radius 5 m is built underground with the top 3 m below ground level. If the tank is full of water, how much work is needed to pump all the water out to the ground level? Assume that the density of water is 1000 kg/m^3.

Learning Goal 6

Compute integrals of basic functions by using antiderivative formulas and techniques such as **substitution and integration by parts**.

- $\int \frac{\ln(x)}{x^7} dx$
- $\int x^2 \arctan(x) dx$
- $\int \sqrt{x} e^{\sqrt{x}} dx$
- $\int \sin(\ln(x)) dx$
- Compute $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}(\frac{2}{n}i - 1)e^{(\frac{2}{n}i-1)}$ by changing it to a definite integral.

Also, **trigonometric integrals,**

- $\int \sin^a(x) \cos^b(x) dx$
- $\int \tan^a(x) \sec^b(x) dx$

Learning Goal 7

Compute the integrals of the following functions by using trig substitution.

- $\int \frac{dx}{\sqrt{3-2x-x^2}}$
- $\int \frac{dx}{(x^2+1)^3}$
- $\int \frac{1}{(2x-3)^2\sqrt{4x^2-12x+8}}$ when $x > 2$

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$
$\sqrt{a^2 + x^2}$ or $\frac{1}{a^2+x^2}$	$x = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$