MATH 101

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Learning Goals 1

(1) Approximate the area between a curve and x-axis by using **left, right, or midpoint** sums. Interpret a definite integral in terms of the area between a curve and x-axis. Compute definite integral by using the **Riemann Sum**, the definition of definite integral.

Examples,

- Estimate the area under the graph $y = \sqrt{x}$ from $x = 0$ to $x = 4$ using N approximating rectangles and right endpoints. Sketch the graph and rectangles. Is your estimate an underestimate or overestimate?
- Write an integral that is defined by the expression

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{n} \sin\left(\frac{i}{4n}\right).$$

- Use the definition of a definite integral to show that

$$\int_a^b x^2 dx = \frac{b^3 - a^3}{3}.$$

Learning Goals 2

Compute definite integrals by using the **fundamental theorem of calculus**. Be able to recognize functions that are given as definite integrals with variable upper and lower limits and find their derivatives, relate antiderivatives to definite and indefinite integrals, and the **net change** as the definite integral of a rate of change. Examples:

1. Evaluate $\int_0^4 (\frac{x^2}{4} + \sqrt{x} + e^x) dx$.
2. Differentiate $\int_{\ln x}^{e^x} \frac{1}{\sqrt{1+t^4}} dt$.

Learning Goal 3

Compute the following integrals using **substitution**.

1. $\int (2x-1)e^{x^2-x}dx$.
2. $\int tan^3(\theta)d\theta$.
3. $\int_e^{e^2} \frac{1}{x\ln x}dx$.
4. $\int_2^3 \frac{1}{e^x+e^{-x}}dx$.

Learning Goal 4

Construct an integral or a sum of integrals that can be used to find the **volume of a solid** by considering its **cross-sectional areas.** For solids that are obtained by revolving a region about an axis of rotation, find the volume by considering **cross-sectional discs or washers.** Examples:

- Let R be one of the infinitely many regions bounded by $y = 1 + \sec x$ and $y = 3$. Find the volume of the solid obtained by rotating R about $y = 1$.
- Find the volume of the solid by rotating the region bounded by $y = x^2$ and $y = x + 2$ about the line $x = 3$.
- Consider a cone with base radius of r cm and height of h cm. Use the method of cross sections to show that the volume of the cone is $\frac{1}{3}\pi r^2 h\ cm^3$.
- The base of a solid S is the triangular region with vertices $(0,0), (1,0)$ and $(0,1)$. Each cross-section of S perpendicular to the y-axis is a right isosceles triangle with hypotenuse on the xy-plane. Find the volume of S.