THE UNIVERSITY OF IOWA

Axiom Pinpointing

Justifying consequences in Automated Reasoning

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April 27 2020

Agenda of the Talk

01 Motivation

02 Problem Formulation

03 Proposed Solutions

04 Ongoing Work

Motivation

Intelligent applications need to represent and handle knowledge effectively

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A Knowledge Representation Language, say $\mathcal{K}$, provides formal semantics and reasoning methods for driving an implicit consequence, called axiom α from explicitly represented elements.

$$\boxed{\mathcal{K} \models \alpha}$$

Motivation

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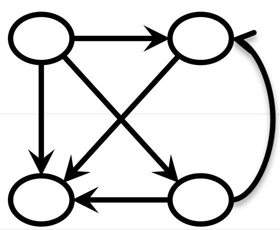
Understand the error and repair to rectify the error

Debugging Ontologies

SNOMED-CT is a medical ontology used by U.S Federal Government systems
For the electronic exchange of clinical health information

$\text{AmpOfFinger} \sqsubseteq \text{AmpOfHand}$

Amputation of finger is an Amputation of a Hand [BP08]



Ontology $\mathcal{O}$

Unexpected Behavior $\mathcal{B}$

Does $\mathcal{O}$ entails $\mathcal{B}$?

Bug Explanation and Repair

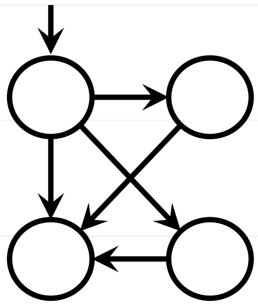
Motivation

A Knowledge Representation Language, say $\mathcal{K}$, provides formal semantics and reasoning methods for driving an implicit consequence, called axiom α from explicitly represented elements.

$$\mathcal{K} \models \alpha$$

If **Consequence** is a property of an interest then **explaining** it enhances both the correctness and acceptance of an intelligent computational system

Symbolic Model Checking



Abstract Model $\mathcal{M}$

Requirements $\mathcal{R}$

Does $\mathcal{M}$ meets $\mathcal{R}$?

Proof Explanation

Proof explanation in Symbolic Model Checking [GWG17]

Pinpointing lines in the source code responsible for an error [JM11]

Axiom Pinpointing:

Identifying the axioms in a $\mathcal{K}$ that are responsible for a given consequence α

Identified axioms are called justification

A single consequence can have multiple justifications

Finding not only one but all justification-based explanations

Problem Relevance

Axiom pinpointing is a well-studied problem across several domains

Different research communities know **justification** by different names

[RR87]

CSP

MESCs

[MM14]

DL

MUPS

MinAs

[SC03, BP07]

SAT

MUSes

[BL03,LS08]

DBs

Causes

[MGHKMS10]

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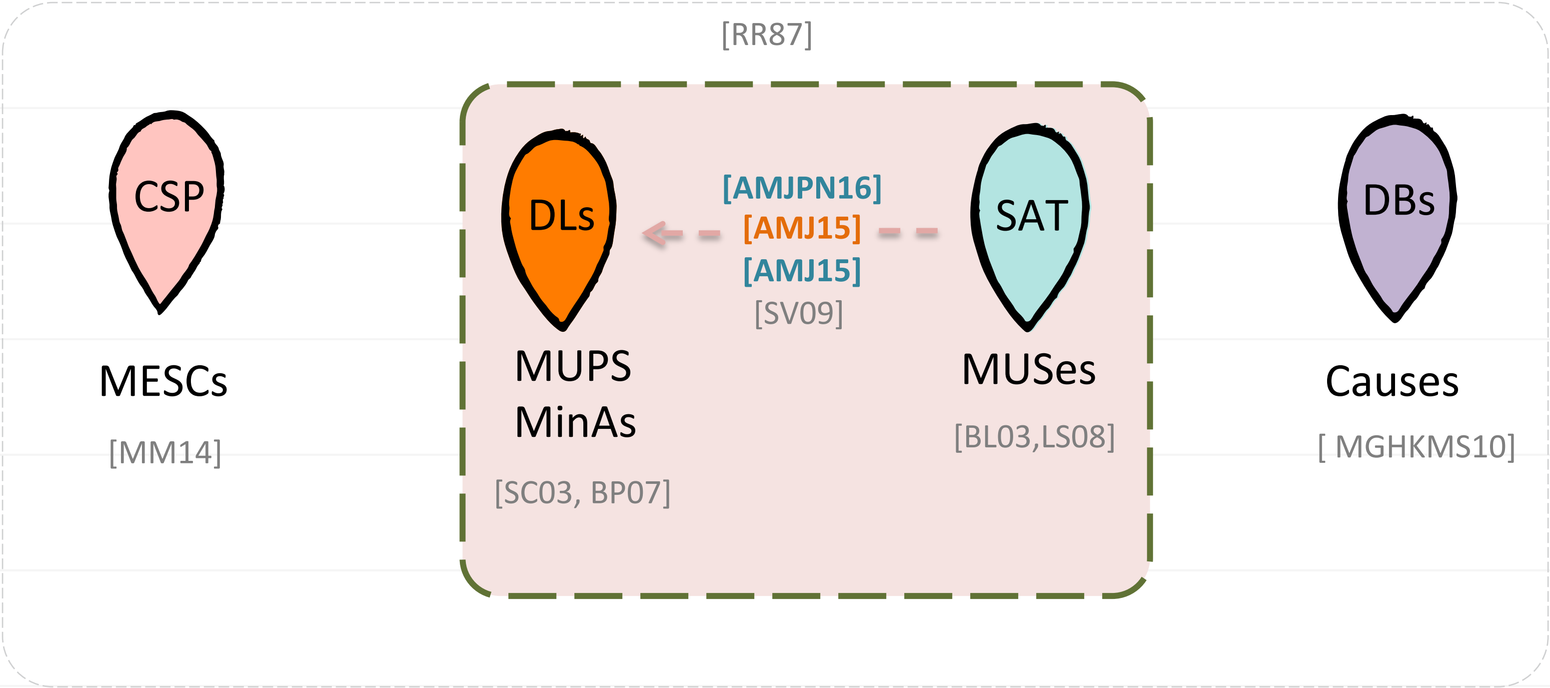
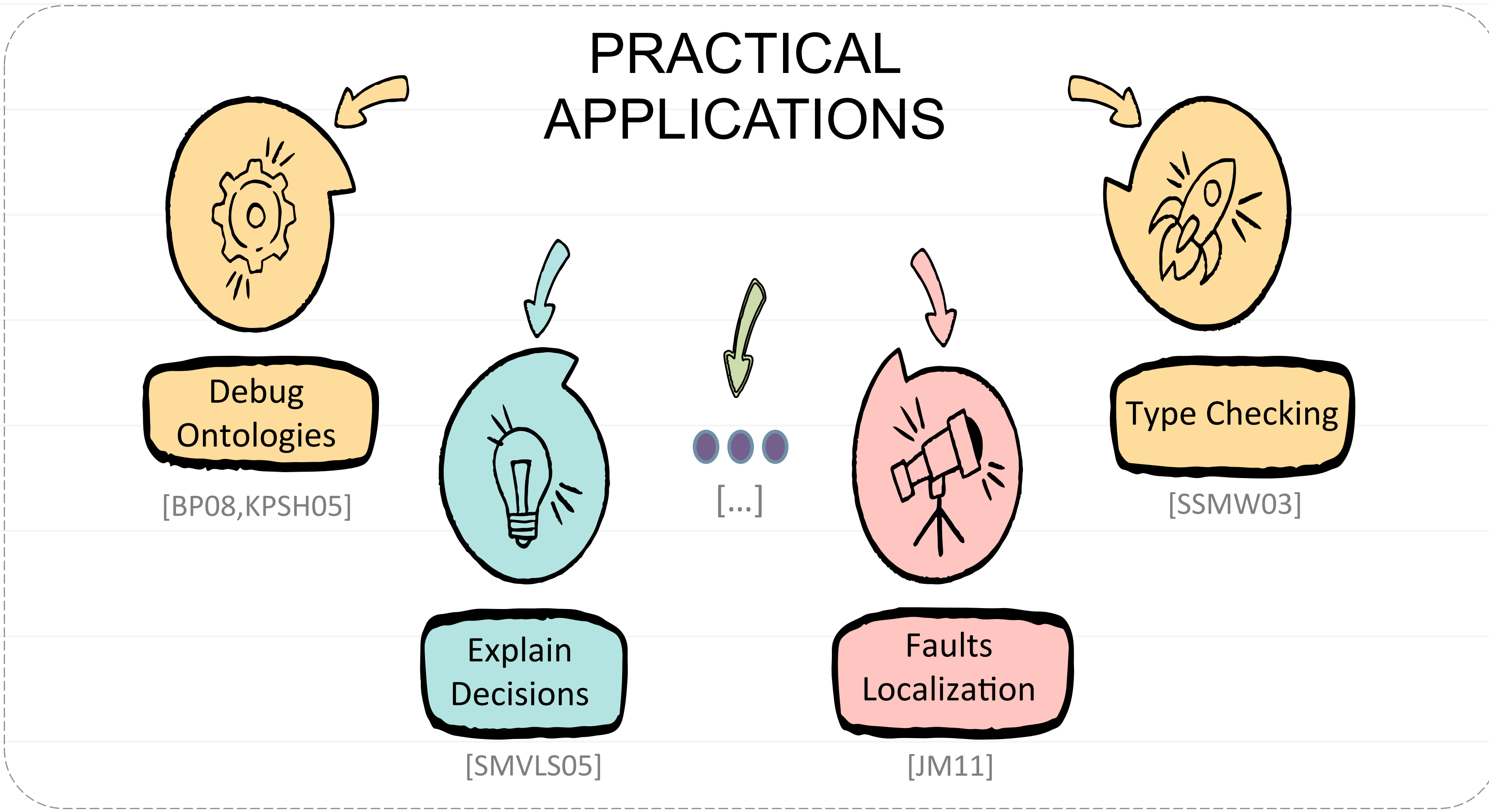


Axiom pinpointing is well-studied problem across several domains and has many interesting applications



02 Problem Formulation

A Simple Ontology Language

A simple Ontology Language $\mathcal{L}(\mathcal{O})$ that contains four components

$$\mathcal{L}(\mathcal{O}) \begin{cases} \mathcal{A} \longrightarrow \text{a class of well-formed axioms} \\ \mathcal{C} \longrightarrow \text{a class of consequences} \\ \mathscr{O} \subseteq \mathscr{P}(\mathcal{A}) \longrightarrow \text{a valid-set of ontologies} \\ \models \subseteq \mathscr{O} \times \mathcal{C} \longrightarrow \text{entailment relation} \end{cases}$$

We only consider a monotonic relation:

$$\mathcal{O}, \mathcal{O}' \in \mathscr{O} \text{ and } c \in \mathcal{C} \text{ if } \mathcal{O} \models c \text{ and } \mathcal{O} \subseteq \mathcal{O}', \text{then } \mathcal{O}' \models c$$

A Simple Ontology Language:

Given an ontology $\mathcal{O} \in \mathscr{O}$ and a consequence $c \in \mathcal{O}$

Entailment:

Decide $\mathcal{O} \models c$ holds?

Justification: $\mathcal{O} \models c$

A minimal subset Ontology $\mathcal{M} \subseteq \mathcal{O}$ that meets two conditions:

1. $\mathcal{M} \models c$
2. For every $\mathcal{M}' \subsetneq \mathcal{M}, \mathcal{M} \not\models c$

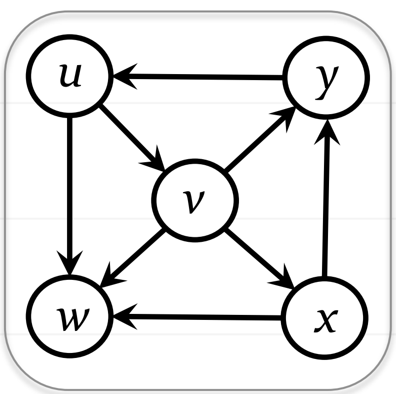
Graph Ontology Example:

$\mathcal{V}$ $\longrightarrow$ a countable set of vertices

$\mathcal{A} = \mathcal{C} = \{(v, w) | v, w \in \mathcal{V}\}$ $\longrightarrow$ edges

$\mathscr{O} \subseteq \mathscr{P}(\mathcal{A})$ $\longrightarrow$ a finite set of graphs

$\models$ $\longrightarrow$ any two reachable vertices in a graph



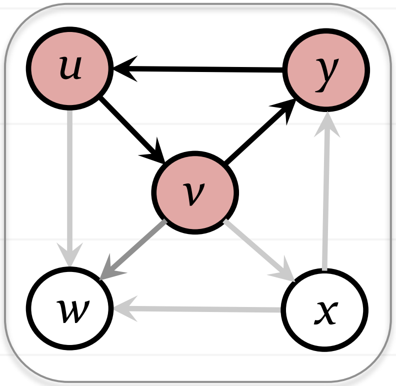
$\mathcal{G} \in \mathscr{O}$ is a finite graph, such that

$\mathcal{G} \models (u, x)$ but $\mathcal{G} \not\models (w, u)$

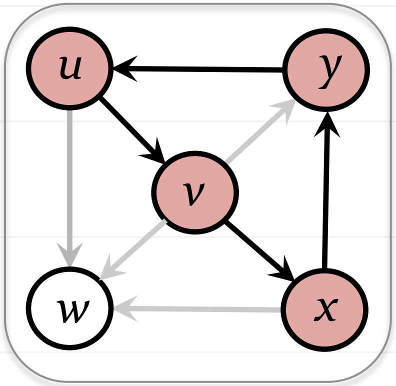
Axiom Pinpointing:

A justification is a minimal sub-ontology that still entails the consequence

Find one or more justification in ontology $\mathcal{G}$ for a consequence: u is reachable from u.



$\mathcal{G}1 \models (\mathrm{u,u})$



$\mathcal{G}2 \models (\mathrm{u,u})$

Over-constrained Formulas in SAT:

Minimal Unsatisfiable Subset [MUS]:

$\mathcal{M} \subseteq \mathcal{F}$ is minimally unsatisfiable subformula of $\mathcal{F}$ if and only if

$\mathcal{M}$ is unsatisfiable and $\forall_{\mathcal{M}' \subsetneq \mathcal{M}} \mathcal{M}'$ is satisfiable

Minimal Correction Subset [MCS]:

$\mathcal{C} \subseteq \mathcal{F}$ is minimally correction subformula of $\mathcal{F}$ if and only if

$\mathcal{F} \setminus \mathcal{C}$ is satisfiable and $\forall_{\mathcal{C}' \subsetneq \mathcal{C}} \mathcal{F} \setminus \mathcal{C}'$ is unsatisfiable

03 Proposed Solutions

Black-box Method:

Compute one justification by removing an axiom at a time

Data: Ontology $\mathcal{O}$, consequence c

Result: A justification $\mathcal{M}$ for c w.r.t. $\mathcal{O}$

$\mathcal{M} \leftarrow \mathcal{O}$

for $\alpha \in \mathcal{O}$ **do**

  if $\mathcal{M} \setminus \{\alpha\} \models c$ **then**

    $\mathcal{M} \leftarrow \mathcal{M} \setminus \{\alpha\}$

Return $\mathcal{M}$

There is a relation between the order of axioms selected for removal and the computed justification

Glass-box Method:

Modified Tableau-based reasoning method for standard reasoning can track the relevant traces but are inefficient

Tableau-based pinpointing approaches does not behave well in practice

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Modified Tableau-based reasoning method for standard reasoning can track the relevant traces but are inefficient

Tableau-based pinpointing approaches does not behave well in practice

A propositional formula encodes the execution of a consequence-based algorithm

Enumeration over Justifications

Increase our understanding about the derivation by finding more than one or all justifications

To find all Justifications, one can simply enumerate all sub-ontologies

Problem: Exponential many entailment checks, one for each subset of $\mathcal{O}$

The question is: can we do better?

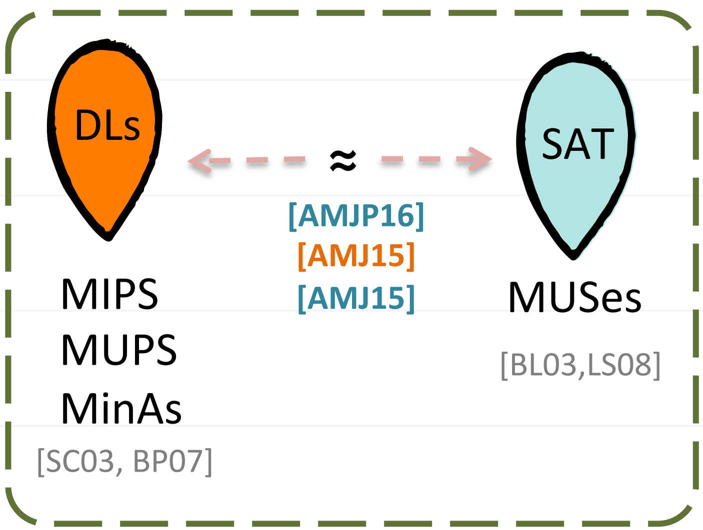
Contribution

[RR87]

Minimal Conflict Set (Justification) ⟷ Hitting set Duality ⟷ Diagnosis (Repair)

[AMJ15]

MUSes are justifications and MCSes are repairs



Optimizations:

[AMJ15]

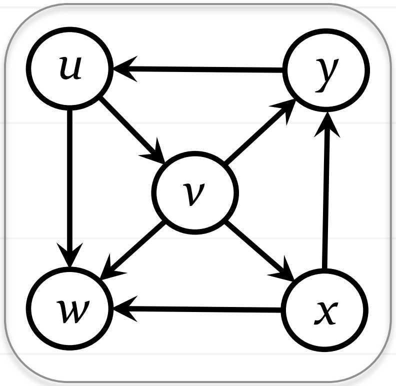
If a set $\mathcal{O}' \subseteq \mathcal{O}$ is such that $\mathcal{O}' \models c$, then ignore all strict superset of $\mathcal{O}'$ because minimality condition fails for superset

If $\mathcal{O}' \not\models c$ then ignore subset of $\mathcal{O}'$ because subsets fail to entail c

Consequence-based Axiom Pinpointing:

[SV09]

A Horn clause simulates each rule possible application and a propositional variable represents a conclusion



$$\mathcal{F}_{\mathcal{G}} = x_{(u,v)} \wedge x_{(v,y)} \wedge x_{(x,w)} \wedge x_{(y,u)}$$
$$x_{(u,w)} \wedge x_{(v,w)} \wedge x_{(x,y)} \wedge x_{(v,x)}$$

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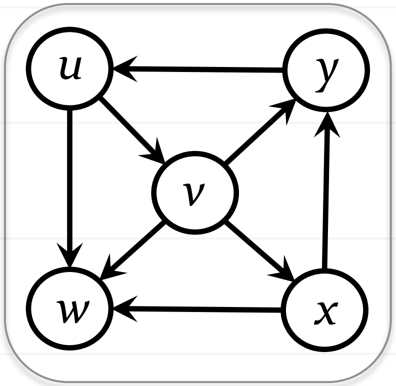


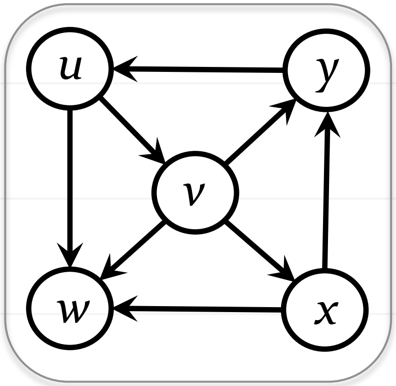
Constraints encode the edges of the example graph

Graph Ontology Example:

A consequence-based algorithm contains only one rule:

$$\{(X,Y),(Y,Z)\} \Rightarrow \{(X,Z)\}$$

$$\mathcal{F}_{\mathcal{H}} = \begin{array}{l} x_{(u,v)} \wedge x_{(v,y)} \rightarrow x_{(u,y)} \\ x_{(u,v)} \wedge x_{(v,w)} \rightarrow x_{(u,w)} \\ x_{(u,v)} \wedge x_{(v,x)} \rightarrow x_{(u,x)} \\ x_{(u,v)} \wedge x_{(v,y)} \rightarrow x_{(u,y)} \\ x_{(u,x)} \wedge x_{(x,w)} \rightarrow x_{(u,w)} \\ x_{(u,x)} \wedge x_{(x,y)} \rightarrow x_{(u,y)} \\ x_{(u,y)} \wedge x_{(y,u)} \rightarrow x_{(u,u)} \end{array}$$



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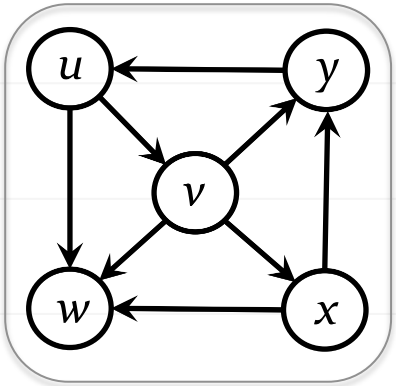

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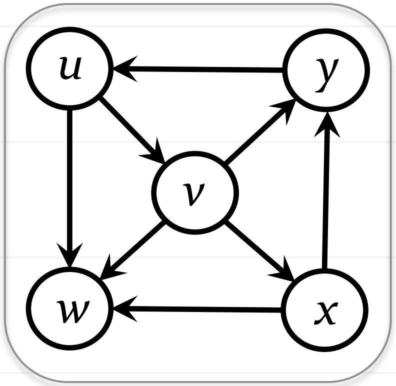

$$\mathcal{F} = \mathcal{F}_{\mathcal{H}} \wedge \mathcal{F}_{\mathcal{G}}$$

$$\mathcal{F}_{\mathcal{H}} = x_{(u,v)} \wedge x_{(v,y)} \rightarrow x_{(u,y)} \wedge$$
$$x_{(u,v)} \wedge x_{(v,w)} \rightarrow x_{(u,w)} \wedge x_{(u,x)} \wedge x_{(x,w)} \rightarrow x_{(u,w)}$$
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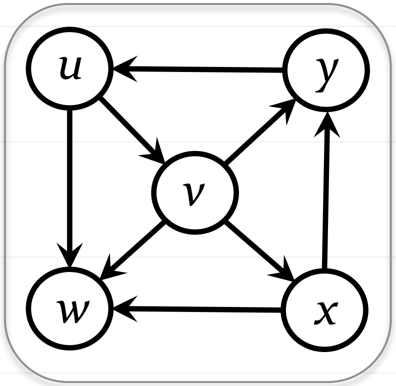

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Constraints encode the edges of the example graph

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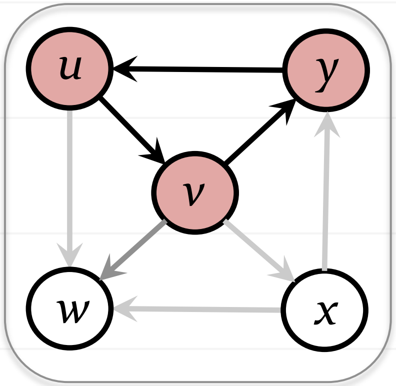

$$\mathcal{G} \models (u, u) \quad \approx \quad \mathcal{F} \models x_{(u,u)}$$

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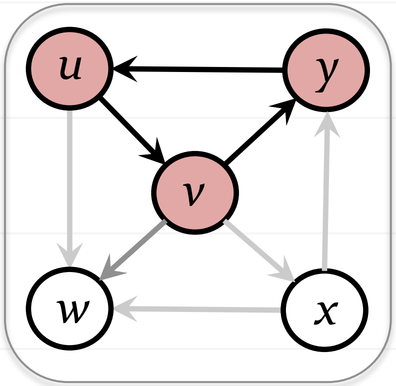
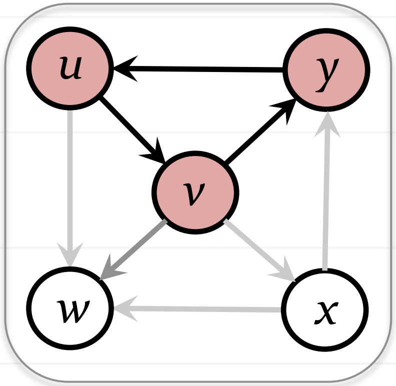


Entailment Check

Graph Ontology Example:

$$\mathcal{F} = \mathcal{F}_{\mathcal{H}} \wedge \mathcal{F}_{\mathcal{G}}$$



$$\mathcal{F}_{\mathcal{H}} \quad \begin{array}{l} x_{(u,v)} \wedge x_{(v,y)} \to x_{(u,y)} \\ x_{(u,y)} \wedge x_{(y,u)} \to x_{(u,u)} \end{array}$$

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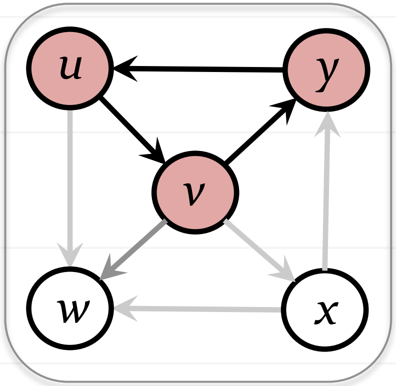

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$$\mathcal{F}_{\mathcal{G}} \quad x_{(u,v)} \quad x_{(v,y)} \quad x_{(y,u)}$$

$$\mathcal{G} \models (u, u) \quad \approx \quad \mathcal{F} \cup \{\neg x_{(u,u)}\}$$

Graph Ontology Example:

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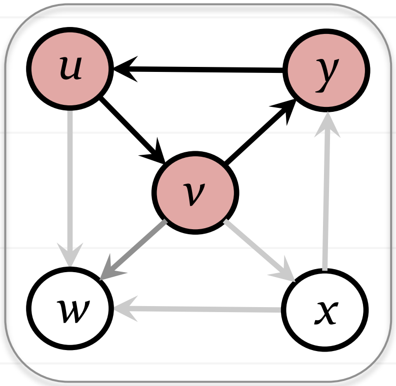

$\mathcal{F}_{\mathcal{H}}$

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$\mathcal{F}_{\mathcal{G}}$

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[AMJ15]

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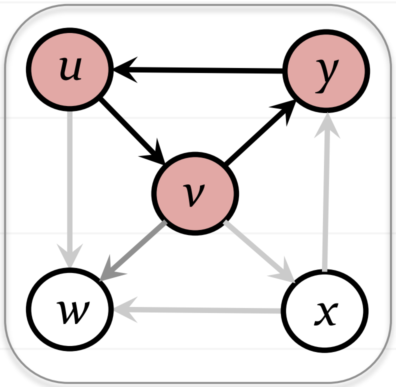

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$$\mathcal{F}_{\mathcal{G}} \quad x_{(u,v)} \quad x_{(v,y)} \quad x_{(y,u)}$$

$$\mathcal{F} = \overbrace{\mathcal{F}_{\mathcal{H}}}^{\mathcal{G}_0} \wedge \underbrace{\mathcal{F}_{\mathcal{G}}}_{\mathcal{G}_1, \ldots, \mathcal{G}_8} \wedge \neg x_{(u,u)}$$

Graph Ontology Example:

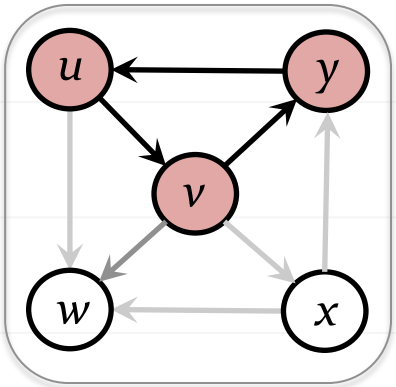
[AMJ15]

$\mathcal{F} = \mathcal{G}_0 \cup \cdots \cup \mathcal{G}_k$, a group-MUS of $\mathcal{F}$ is a set of groups

$\mathcal{G} \subseteq \{\mathcal{G}_1 \cup \cdots \cup \mathcal{G}_k\}$, such that

$\mathcal{G}_0 \cup \mathcal{G}$ is unsatisfiable and $\mathcal{G}_i \in \mathcal{G}, \mathcal{G}_0 \cup (\mathcal{G} \setminus \mathcal{G}_i)$ is satisfiable

$$\mathcal{F} = \overbrace{\mathcal{F}_{\mathcal{H}}}^{\mathcal{G}_0} \wedge \underbrace{\mathcal{F}_{\mathcal{G}}}_{\mathcal{G}_1, \ldots, \mathcal{G}_8} \wedge \neg x_{(u,u)}$$



$$\mathcal{F} \models x_{(u,u)}$$

Graph Ontology Example:

[AMJ15]

$$\mathcal{F} = \overbrace{\mathcal{F}_{\mathcal{H}}}^{\mathcal{G}_0} \wedge \underbrace{\mathcal{F}_{\mathcal{G}}}_{\mathcal{G}_1, \ldots, \mathcal{G}_8} \wedge \neg x_{(u,u)}$$

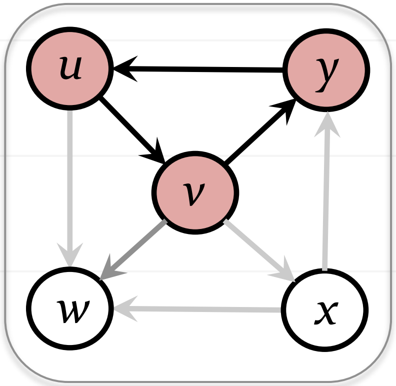
$$\boxed{\mathcal{G}_0 = \mathcal{F}_{\mathcal{H}}}$$

$$\mathcal{G}_1 = \{x_{(u,v)}\} \quad \mathcal{G}_3 = \{x(x,w)\} \quad \mathcal{G}_5 = \{x(u,w)\}$$

$$\mathcal{G}_2 = \{x(v,y)\} \quad \mathcal{G}_4 = \{x(y,u)\} \quad \mathcal{G}_6 = \{x(v,w)\}$$

$$\mathcal{G}_7 = \{x(x,y)\} \quad \mathcal{G}_8 = \{x(v,x)\}$$

Graph Ontology Example:



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$\mathcal{G}_1 = \{x_{(u,v)}\}$ $\mathcal{G}_2 \models \{x_{(x,w)}\}$ $\mathcal{G}_5 \models \{x_{(u,w)}\}$

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$$\mathcal{F} = \mathcal{G}_0 \cup \mathcal{G}_1 \cup \cdots \cup \mathcal{G}_8 \{\neg x_{(u,u)}\}$$

$$\mathcal{M} = \{\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_4\}$$

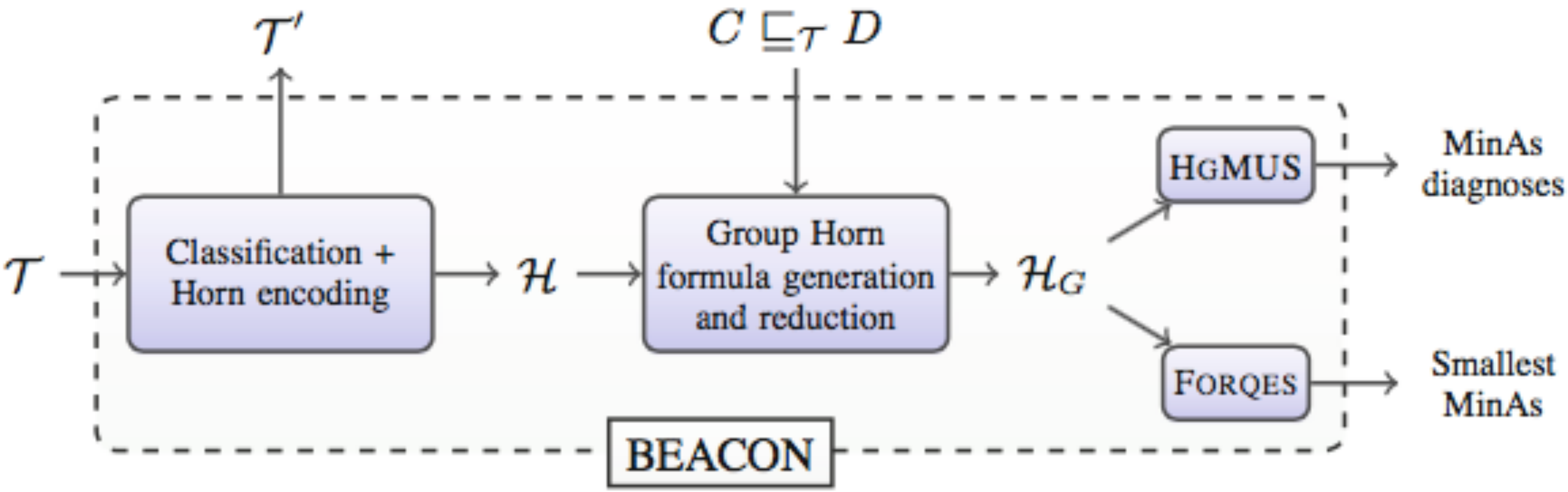
Hitting set Duality

$$\mathcal{S}_1 = \{\mathcal{G}_1\}$$
$$\mathcal{S}_2 = \{\mathcal{G}_2\}$$
$$\mathcal{S}_3 = \{\mathcal{G}_4\}$$

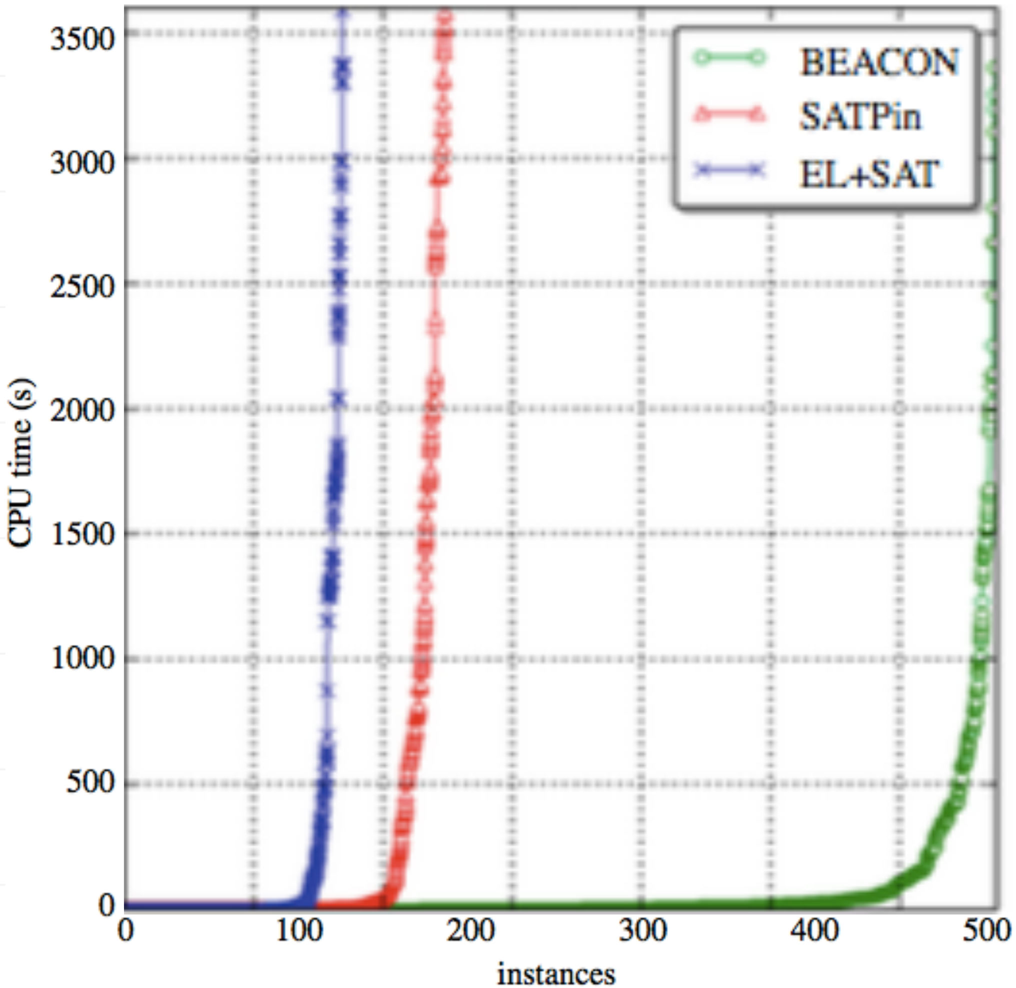
[AMJ15]

BEACON Tool:

[AMJP16]



Performance:



Medical Ontologies

- ❑ SNOMED-CT
- ❑ GENE
- ❑ GALEN
- ❑ NCI

04 Ongoing Work

List of applications

1. Debugging Medical Ontologies [FS08]
2. Requirement analysis [JB17]
3. Equivalence checking [OGNLV10]
4. Counter-example Guided Abstraction Refinement[CGJLV00]
5. Proof-based abstraction refinement [MA03]
6. Boolean function bi-decomposition [CM11]
7. Circuit error Diagnosis [HL99]
8. Type Debugging in Haskell [SSW03]
9. Proof explanation in Symbolic model checking[GWG17]

Domain of Constraints:

1. Axioms of an Ontology
2. Boolean Formulas
3. Temporal logic formula
4. Transition state Predicates

Domain agnostic MUS enumeration Tool

University of Iowa

Computational Logic Centre

M. Fareed Arif

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