I.

1. The connecting line between two 2D points would be, if the points are x and x', the line, $l = x \times x'$

$$= \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} x_2 x_3' - x_3 x_2' \\ x_3 x_1' - x_1 x_3' \\ x_1 x_2' - x_2 x_1' \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix}$$

So, the connecting line, l is $\begin{pmatrix} x_2 x_3' - x_3 x_2' \\ x_3 x_1' - x_1 x_3' \\ x_1 x_2' - x_2 x_1' \end{pmatrix}$

If the two points are identical, say x and x then $l = x \times x$

$$= \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_2 x_3 - x_3 x_2 \\ x_3 x_1 - x_1 x_3 \\ x_1 x_2 - x_2 x_1 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

So, we get a line of zero vector with zero dimension.

2. If the general line $x\cos\phi + y\sin\phi = d$ intersects the line $(0,0,1)^T$ given in homogeneous coordinates, lets say, at point X,

Then $X = I_1 \times I_2$

where, $I_1 = \begin{pmatrix} \cos\phi \\ \sin\phi \\ -d \end{pmatrix}$ and $I_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$$\therefore I_1 \times I_2 = \begin{pmatrix} \cos\phi \\ \sin\phi \\ -d \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \cos\phi \\ \sin\phi \\ -d \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} \sin\phi - (-0) \\ 0 - \cos\phi \\ 0 - 0 \end{pmatrix}$$

$$= \begin{pmatrix} \sin\phi \\ -\cos\phi \\ 0 \end{pmatrix}$$

$$\therefore X = \begin{pmatrix} \sin\phi \\ -\cos\phi \\ 0 \end{pmatrix}$$

$\therefore$ The point is, $X = \begin{pmatrix} \sin\phi \\ -\cos\phi \\ 0 \end{pmatrix}$

3. The horizon would be a straight line if three points on the horizon are always collinear.

Now, three points would be collinear if $\det[x_1\ x_2\ x_3] = 0$

where x_1, x_2, x_3 are those three points.

Lets say, if $x_1 = (u_1, v_1, 0)$

$x_2 = (u_2, v_2, 0)$

$x_3 = (u_3, v_3, 0)$

since these three points are in the horizon.

$$\therefore \det[x_1\ x_2\ x_3]$$

$$= \det\begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ 0 & 0 & 0 \end{bmatrix} = 0-0+0-0+0-0$$

$$= 0$$

$$\begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \det[x_1\ x_2\ x_3] = 0$$

$\therefore$ The horizon is a straight line since the three points are collinear.