Module 8: Lecture 2 of Process Simulation

Lecture 2: Stochastic vs. Deterministic Simulation

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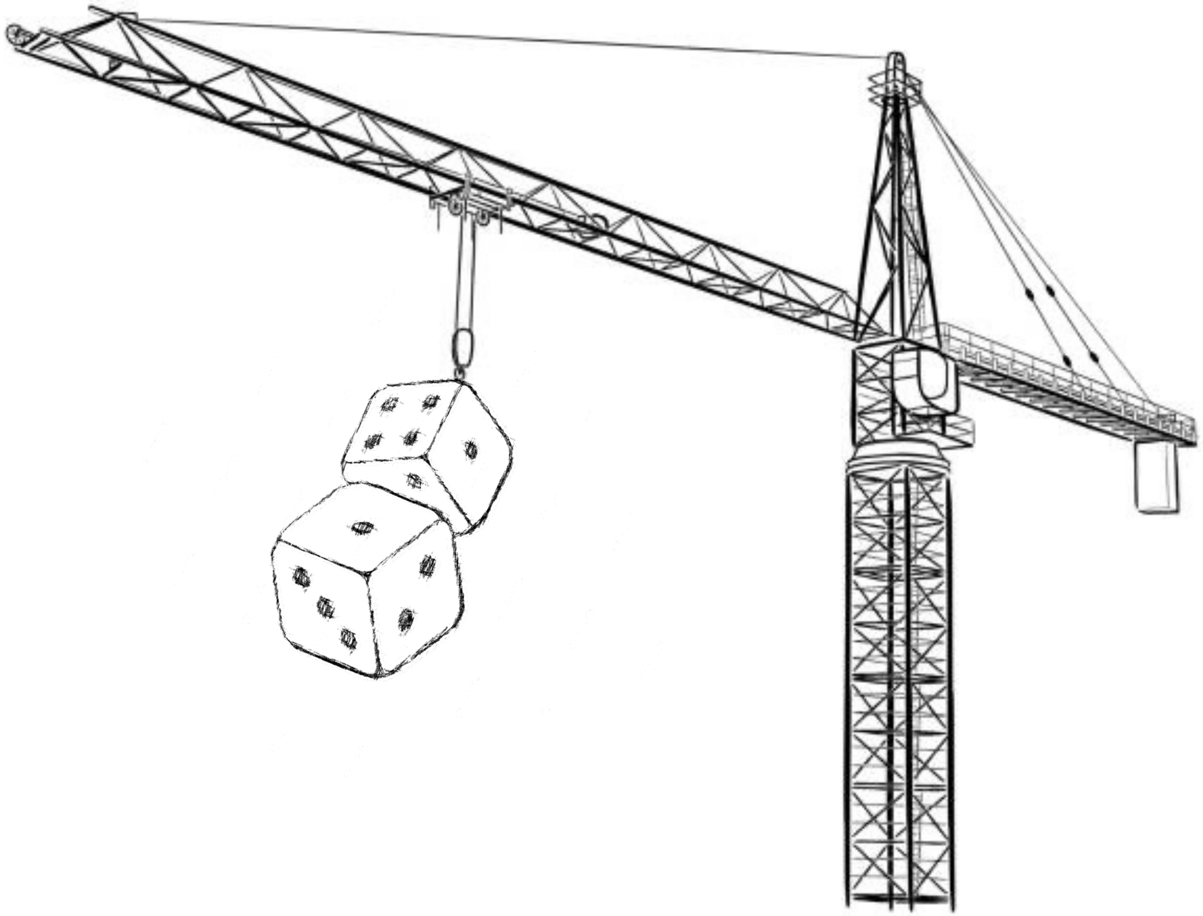


BEN SHERMAN
BURBERRY
BURBERRY
Desigual
Bench
Bench.
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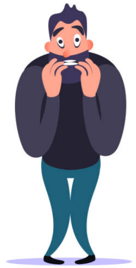
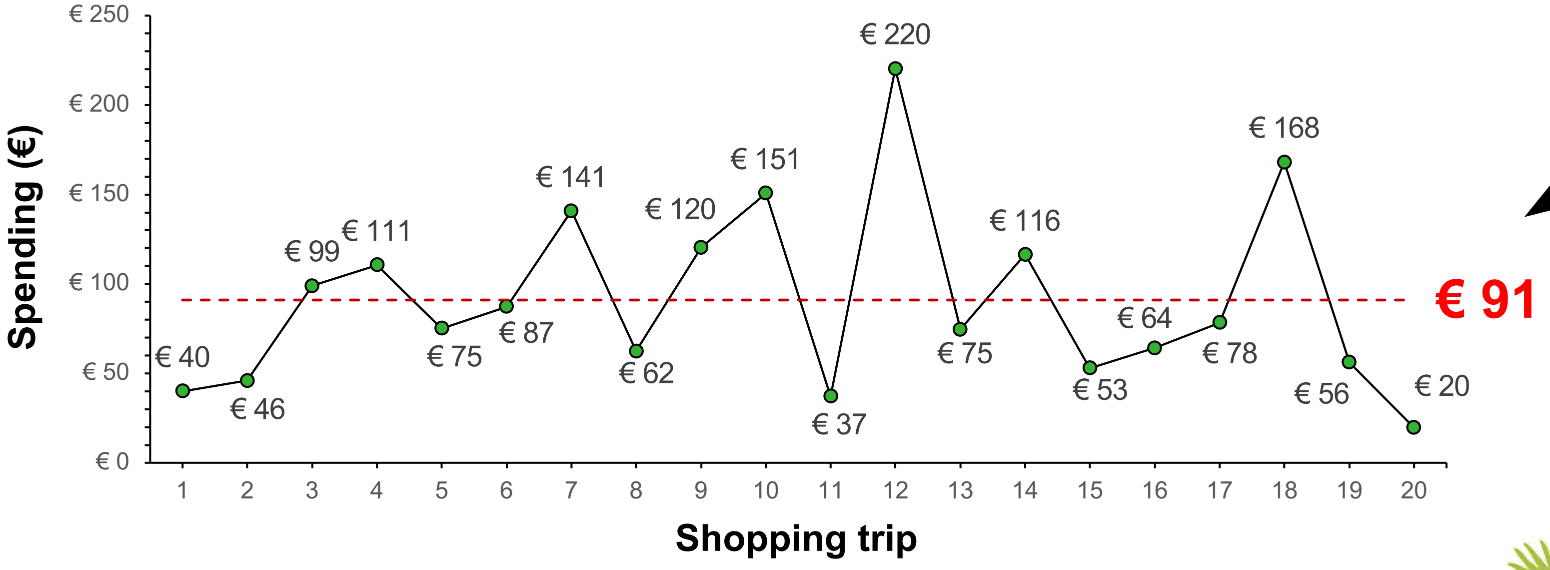
BURBERRY

How much is my wife going to spend?



Shopping history



Based on historical evidence, in average, each shopping trip costs €91

Can I use this statement?

Seirgei said it is a women’s candy store!!!!

Roermond

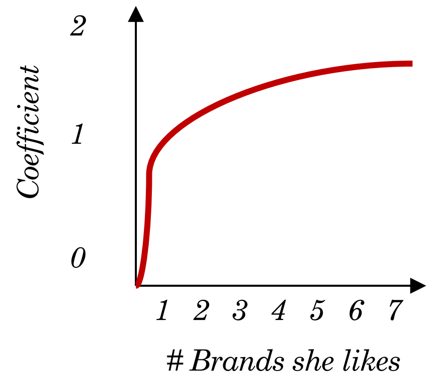
V.S

Enschede

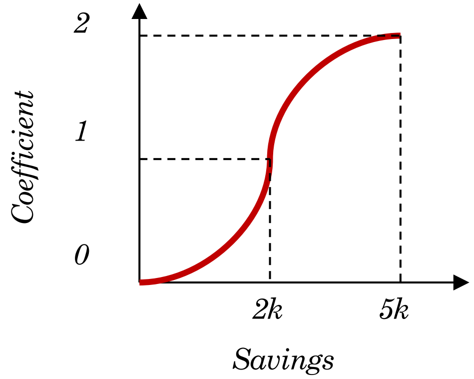
Is this a fair comparison?

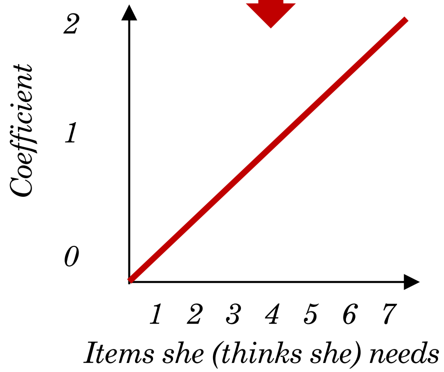
Let’s Shamy-tize it!
Shall we?

Modeling with what is known



$$C = €91 \times (\text{Savings} \times \text{ Range of choices} \times \text{Needs})$$





Deterministic Modeling

$$\mathrm{C} = €91 \times (\mathrm{Savings} \times \ \mathrm{Time} \times \mathrm{Range\ of\ choices})$$

Let' consider a scenario:

- *We have about 3k in our savings account (Coefficient 1.2)*
- *They have 6 brands that she likes (Coefficient 1.8)*
- *She needs 4 items (Coefficient 1)*

***Results**:*

$$€91 \times (1.2 \times 1.8 \times 1) = € \ \mathbf{196.56}$$

This is called **Deterministic Modeling**

- ❑ It allows you predict the future with certainty (i.e., no uncertainties)
- ❑ *You have all the data necessary for the model to make the prediction*
- ❑ *The same input (initial condition) always generate the same output*

Deterministic Modeling

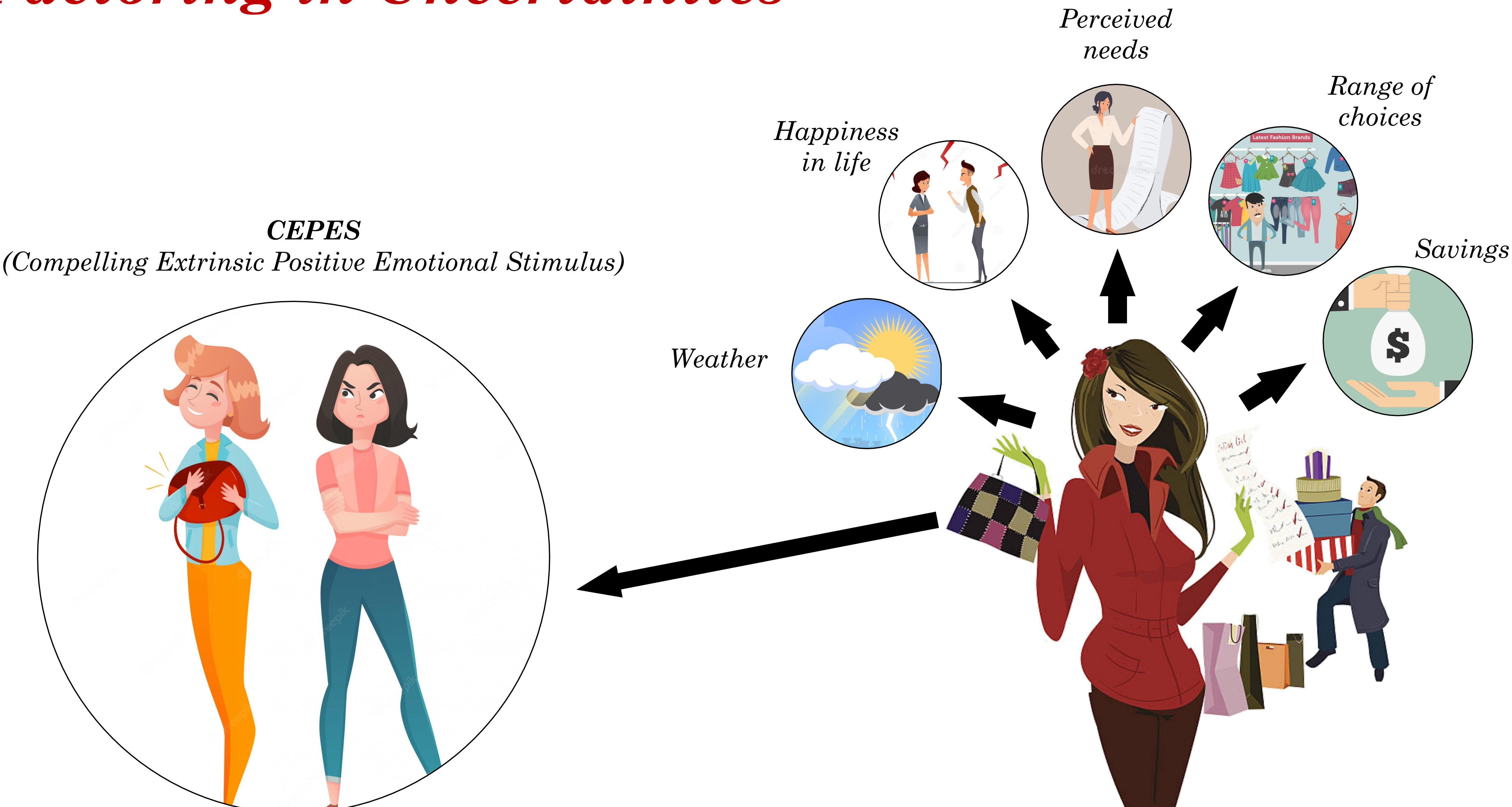
Pros

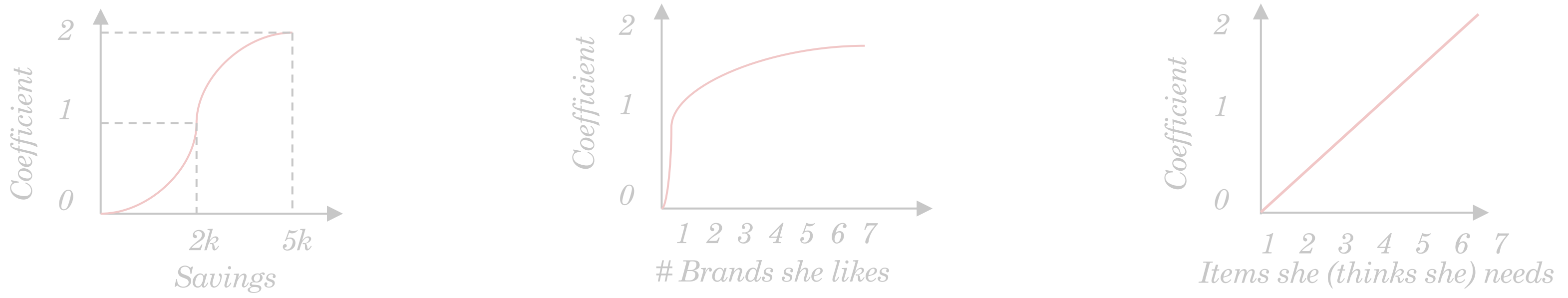
- *It is simple to build, understand and use!*
- *Since every input set generate a fix output, the model has little noise and can be used well for optimization*

Cons

- *It is ignoring uncertainties inherent in the problem*
- *It does not consider different possible scenarios*

Factoring in Uncertainties

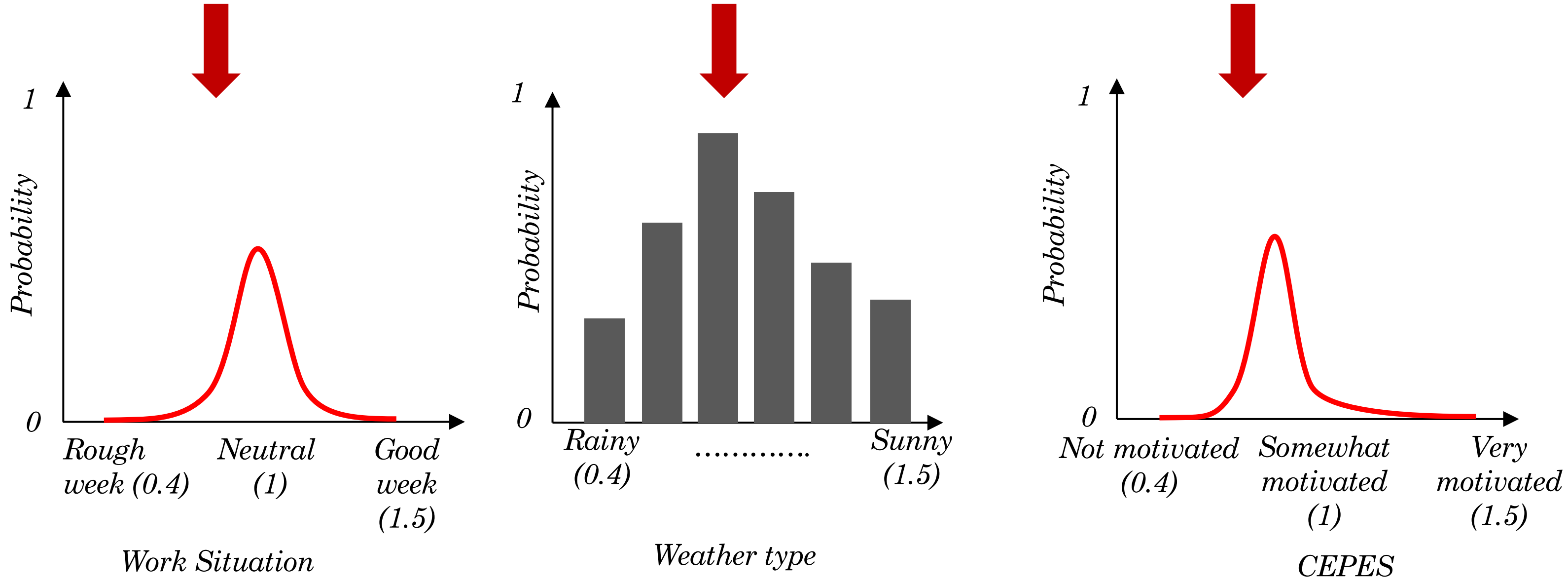




$$C = €91 \times \underbrace{(\text{Savings} \times \text{Range of choices} \times \text{Needs})}_{\textit{Known Variables}}$$

$$\times$$

$$\overbrace{(\text{Happiness} \times \text{Weather} \times \text{CEPES})}^{\textit{Unknown Variables (uncertainties)}}$$



Stochastic Modeling

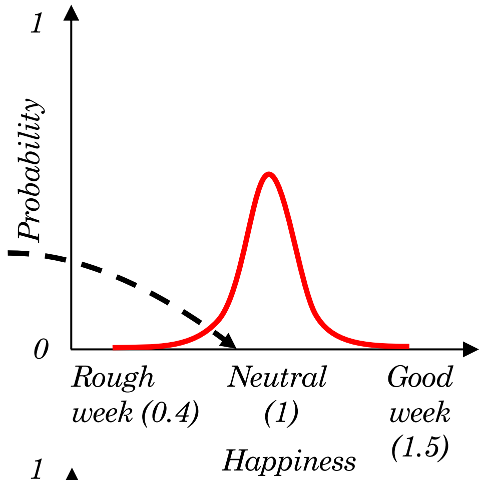
$$C = €91 \times \overbrace{(\text{Savings} \times \text{Range of choices} \times \text{Needs})}^{\textit{Known Variables}} \times \overbrace{(\text{Happiness} \times \text{Weather} \times \text{CEPES})}^{\textit{Unknown Variables (uncertainties)}}$$

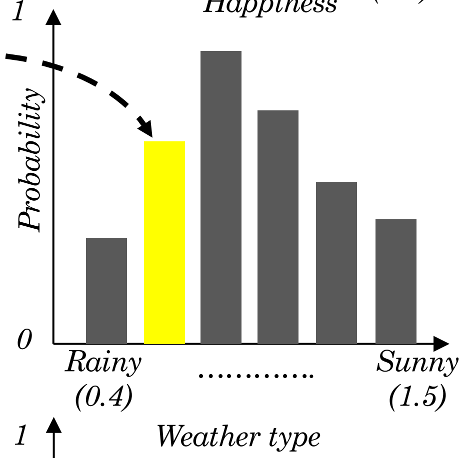
Let' consider a scenario:

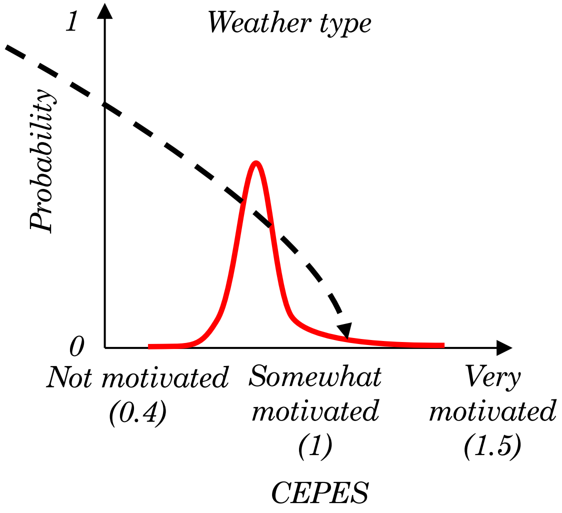
- *We have about 3k in our savings account (Coefficient 1.2)*
- *They have 6 brands that she likes (Coefficient 1.8)*
- *She needs 4 items (Coefficient 1)*

***Results**:*

$$€91 \times (1.2 \times 1.8 \times 1) \times (0.9 \times 1.1 \times 0.8) = €\,155.67$$







This is called **Stochastic Modeling**

- ❑ It captures uncertainties in natural phenomenon through probability distributions
- ❑ *Uses random number to generate a possible outcome*
- ❑ *Thus, the same input (initial condition) can generate different outputs*

Stochastic Modeling

$$C = €91 \times \underbrace{(\text{Savings} \times \text{Range of choices} \times \text{Needs})}_{\textit{Known Variables}} \times \underbrace{(\text{Happiness} \times \text{Weather} \times \text{CEPES})}_{\textit{Unknown Variables (uncertainties)}}$$

Let' consider a scenario:

- *We have about 3k in our savings account (Coefficient 1.2)*
- *They have 6 brands that she likes (Coefficient 1.8)*
- *She needs 4 items (Coefficient 1)*

$$€91 \times (1.2 \times 1.8 \times 1) \times (0.9 \times 1.1 \times 0.8) = €\ \mathbf{155.67}$$

This is called **Stochastic Modeling**

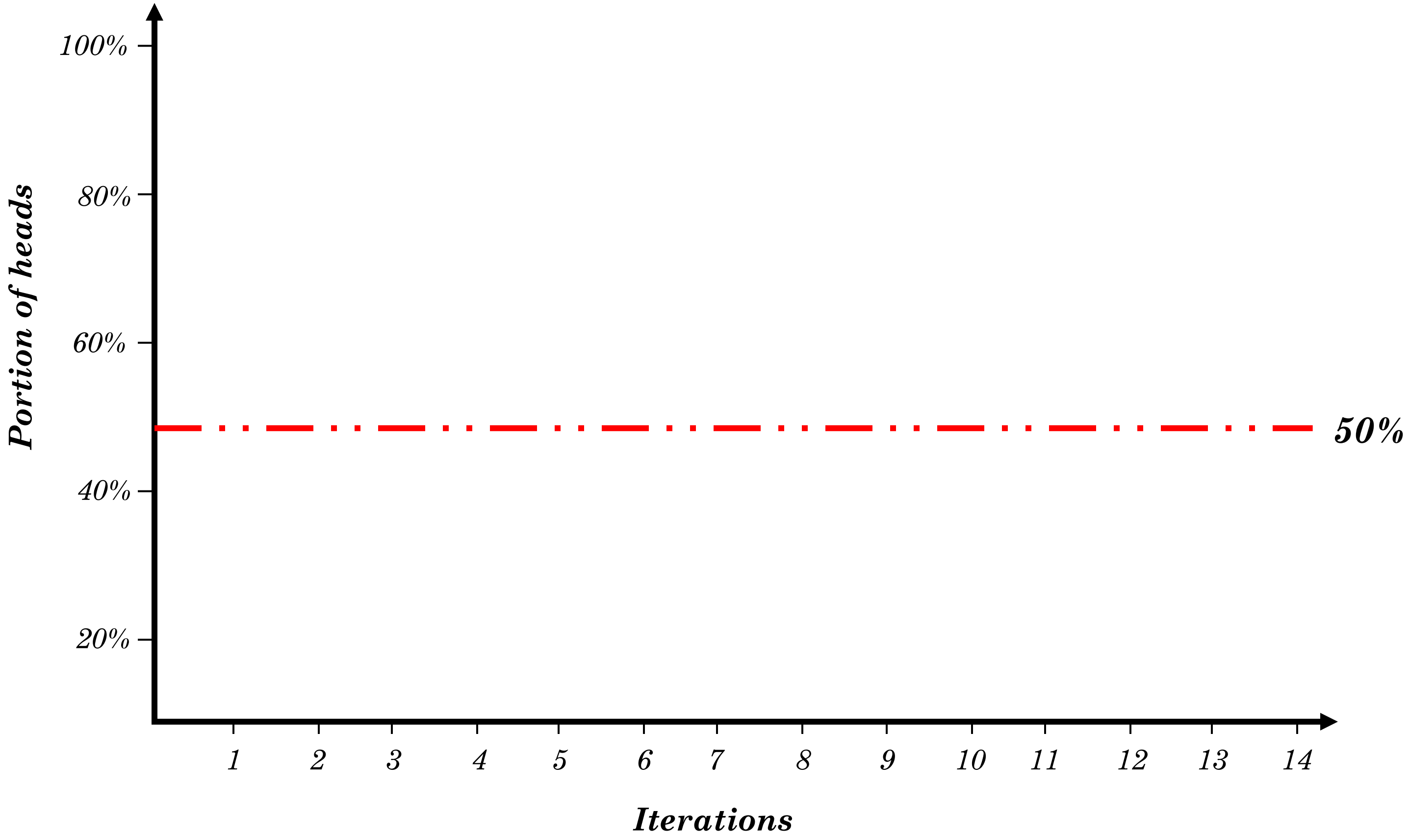
- ❑ It captures uncertainties in natural phenomenon through probability distributions
- ❑ *Uses random number to generate a*
- ❑ *Thus, the same input (initial condi*

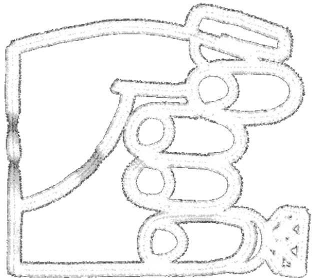
So what? How can I actually use this model?

Let’s play a game

Law of Large Numbers

Iteration	Head	Proportion
1		
2		
3		
4		
5		
6		





Monte Carlo Simulation

$$\boxed{C = €91 \times (\text{Savings} \times \text{Time} \times \text{Range of choices}) \times (\text{Work} \times \text{Family} \times \text{Weather} \times \text{CEPES})}$$

What I know:

- *We have about 3k in our savings account (Coefficient 1.2), it is June (Coefficient 0.9), they have 10 brands that she likes (Coefficient 1.8), and she needs 4 items (Coefficient 1)*

What I don't know:

- *Weather, work and family/life, and CEPES*

$$\textbf{\textit{Run 1}}\text{: } €91\times \overbrace{(1.2\times 1.8 \times 1)}^{\text{known Variables}} \times \overbrace{(0.9\times 1.1\times 0.8)}^{\text{Randomly generated variables}} = €155.67$$

$$\textbf{\textit{Run 2}}\text{: } €91\times (1.2\times 1.8 \times 1) \times (1.3 \times 0.9\times 1.4) = €321.96$$

$$\textbf{\textit{Run 3}}\text{: } €91\times (1.2 \times 1.8 \times 1) \times (0.85\times 1.5\times 0.9) = €225.55$$

.

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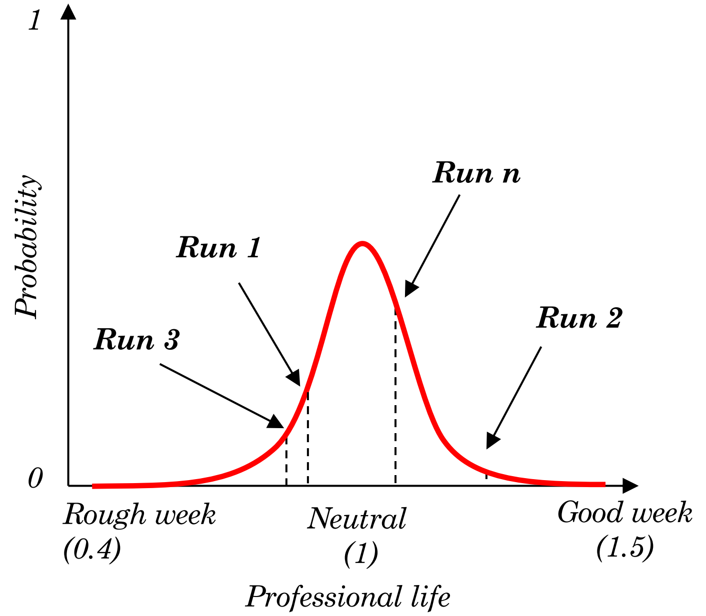
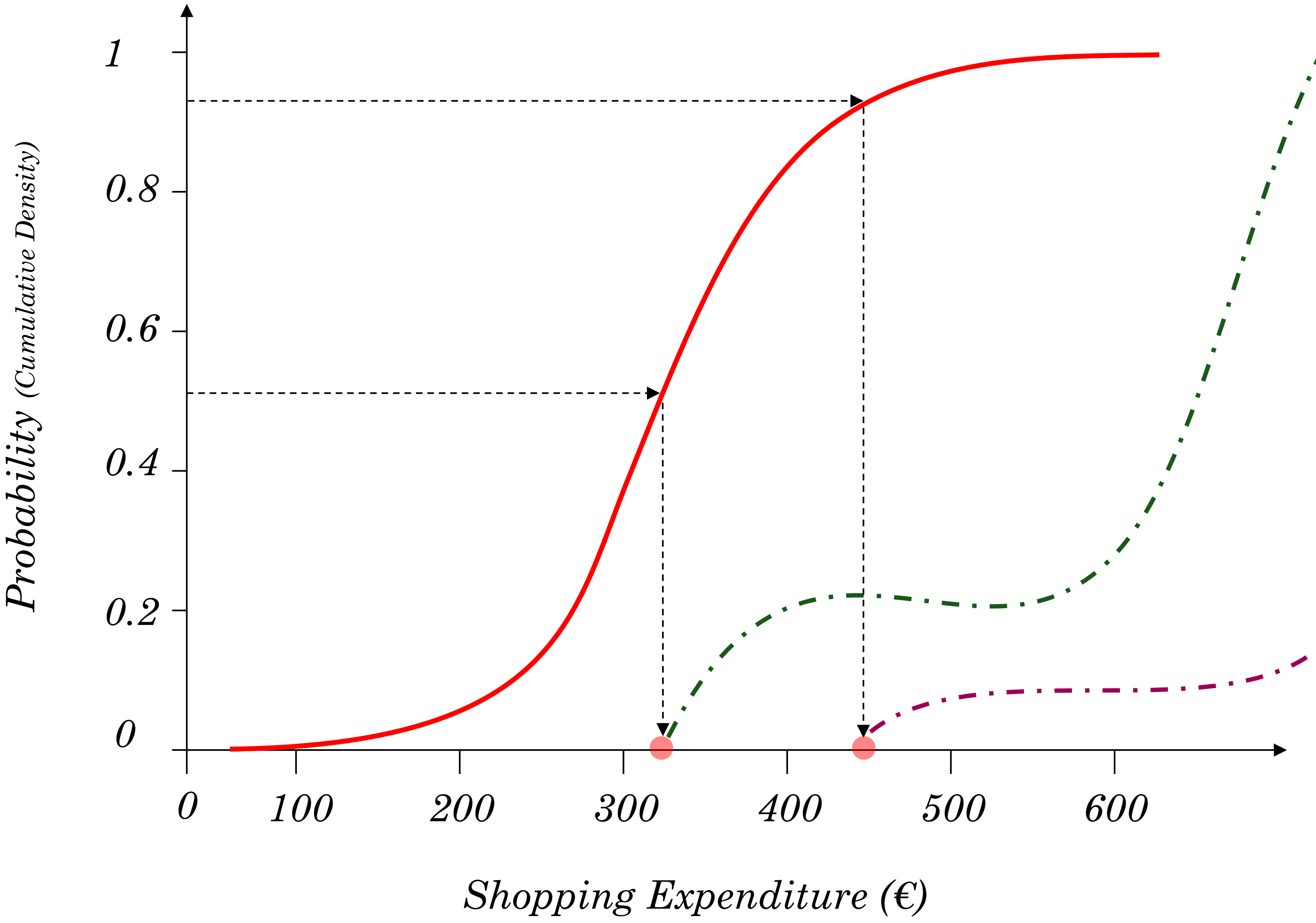
$$\textbf{\textit{Run n}}\text{: } €91\times (1.2 \times 1.8 \times 1) \times (1.1\times 0.6\times 0.9) = €116.76$$



So What?



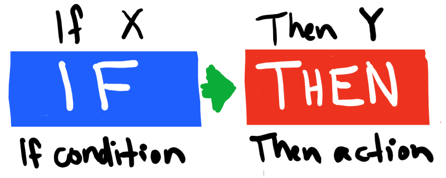
There is only 50% chance that she will spend <€320

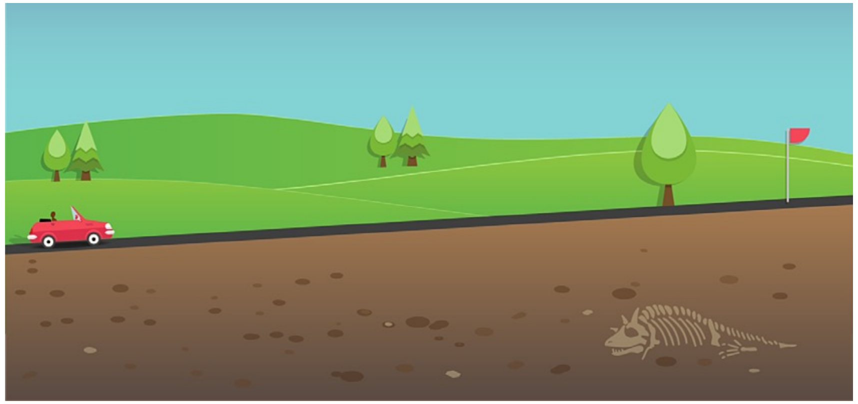
But, with 95% certainty I can say she will spend < € 450

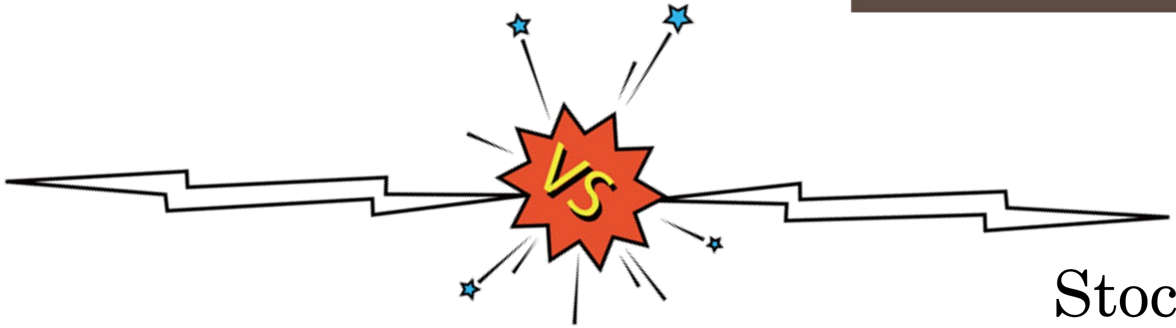
Deterministic Vs. Stochastic Modeling

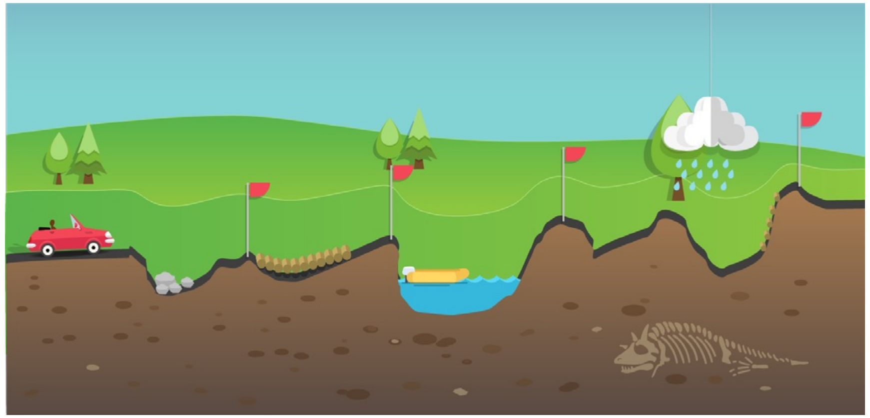
Deterministic Models

- ❑ Output is determined with certainty
- ❑ Same set of inputs result in same output every single time









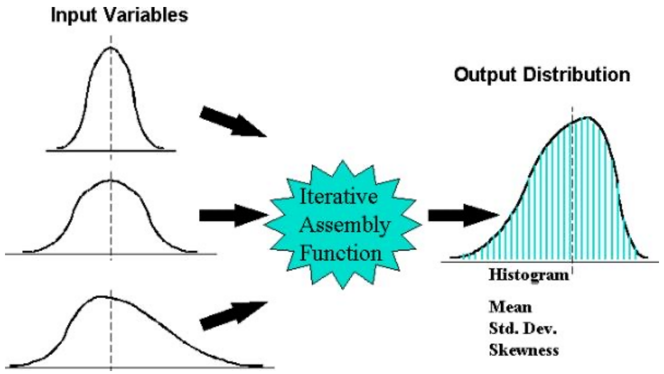
Stochastic Models

- ❑ The model includes unknown variables that need to be **randomly** assigned in each iteration
- ❑ So, the same set of inputs may result in different outputs

Monte Carlo Simulation

- It is a method for estimating the probability of different outcomes of a stochastic model

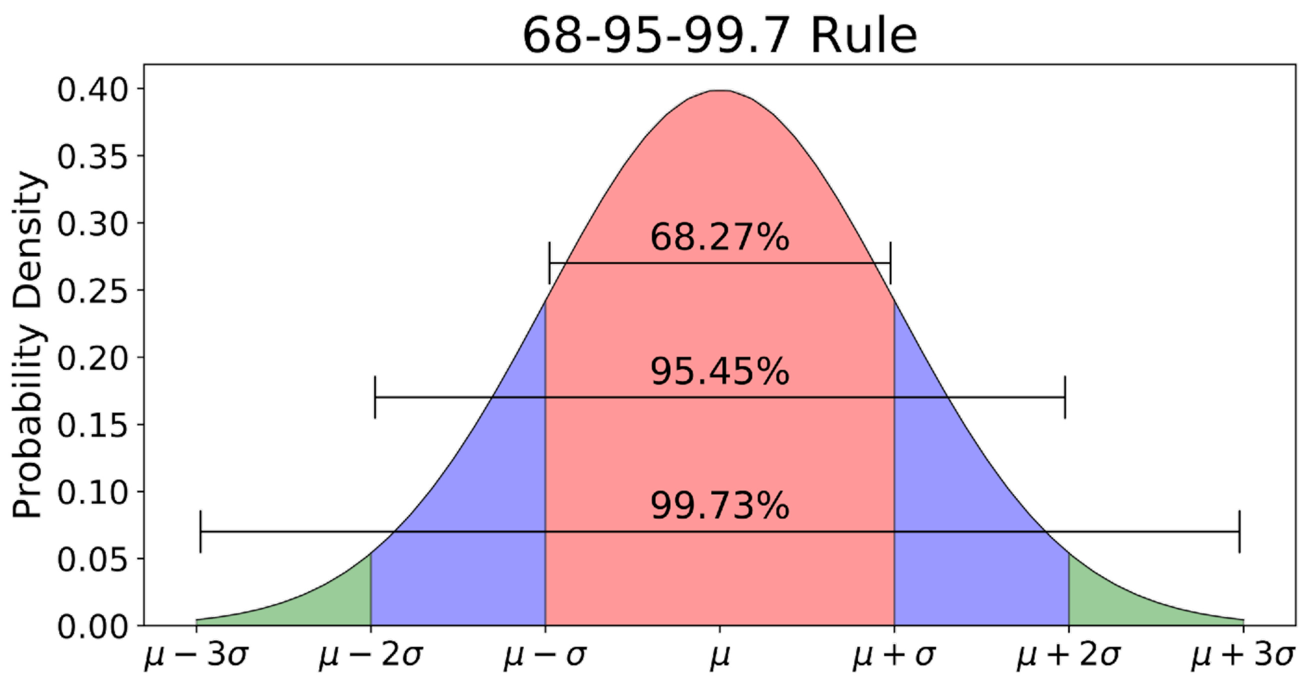
- It uses random numbers to draw from distribution function of input variables and generate different scenarios



Stochasticity, Random Number and Monte Carlo

Most of the natural and human processes are stochastic. So, modeling the stochasticity of processes is very desirable.

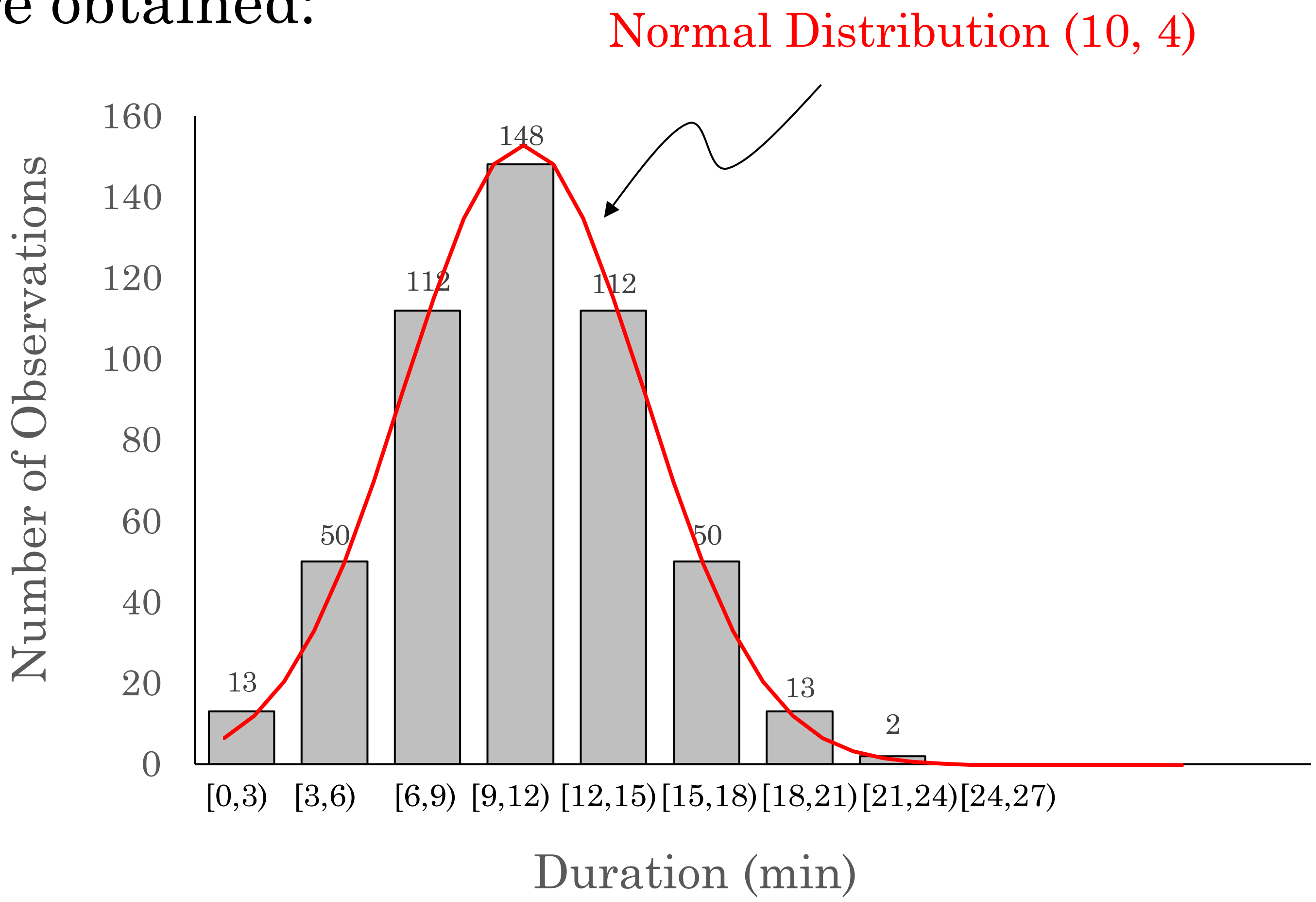
How can we measure stochasticity?



Stochasticity, Random Number and Monte Carlo

- Let's assume we measured the travel time of the trucks in our example for 500 times and the below values are obtained:

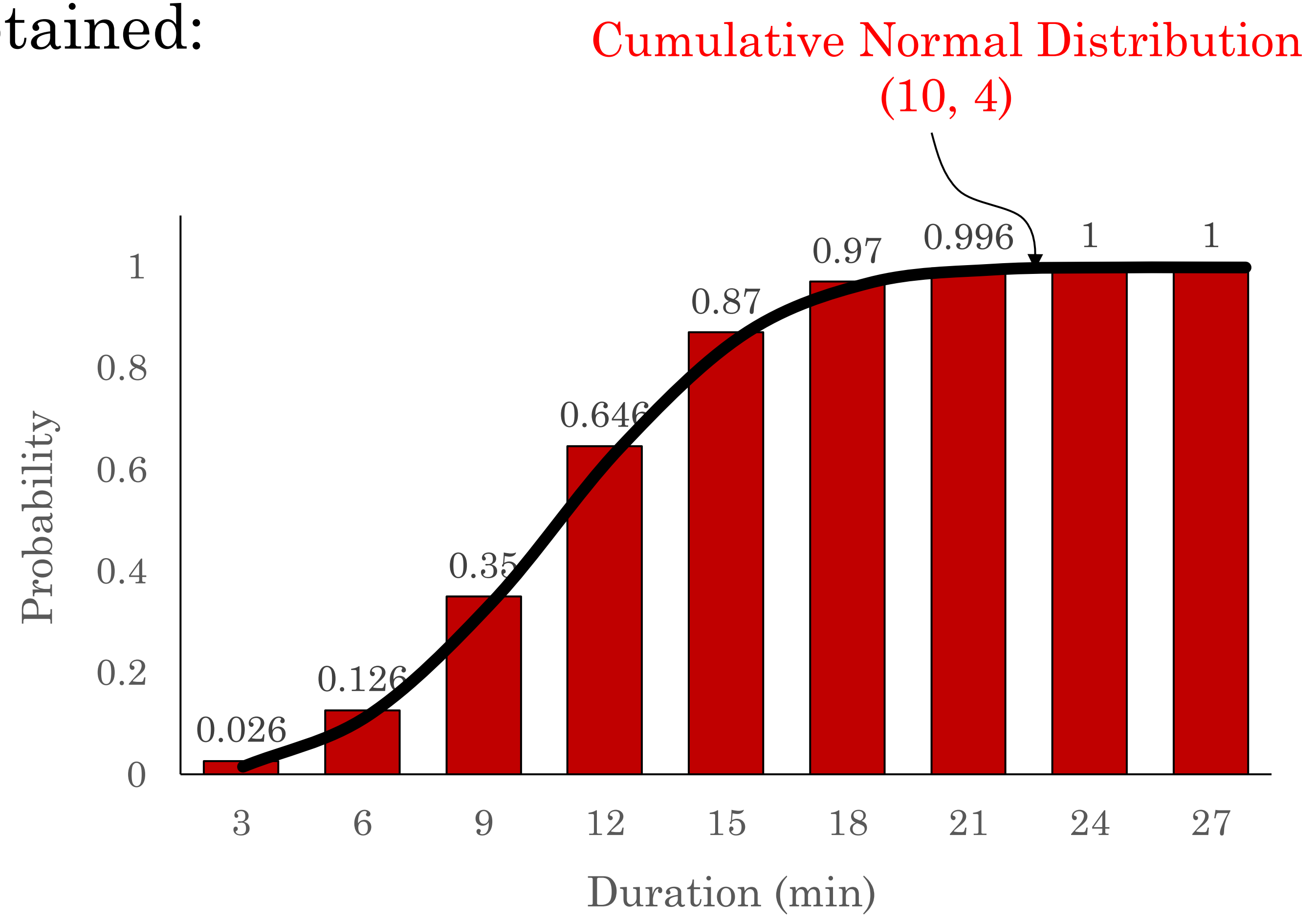
Duration	Number of observations
[0,3)	13
[3,6)	50
[6,9)	112
[9,12)	148
[12,15)	112
[15,18)	50
[18, 21)	13
[21, 24)	2
[24, 27)	0



Stochasticity, Random Number and Monte Carlo

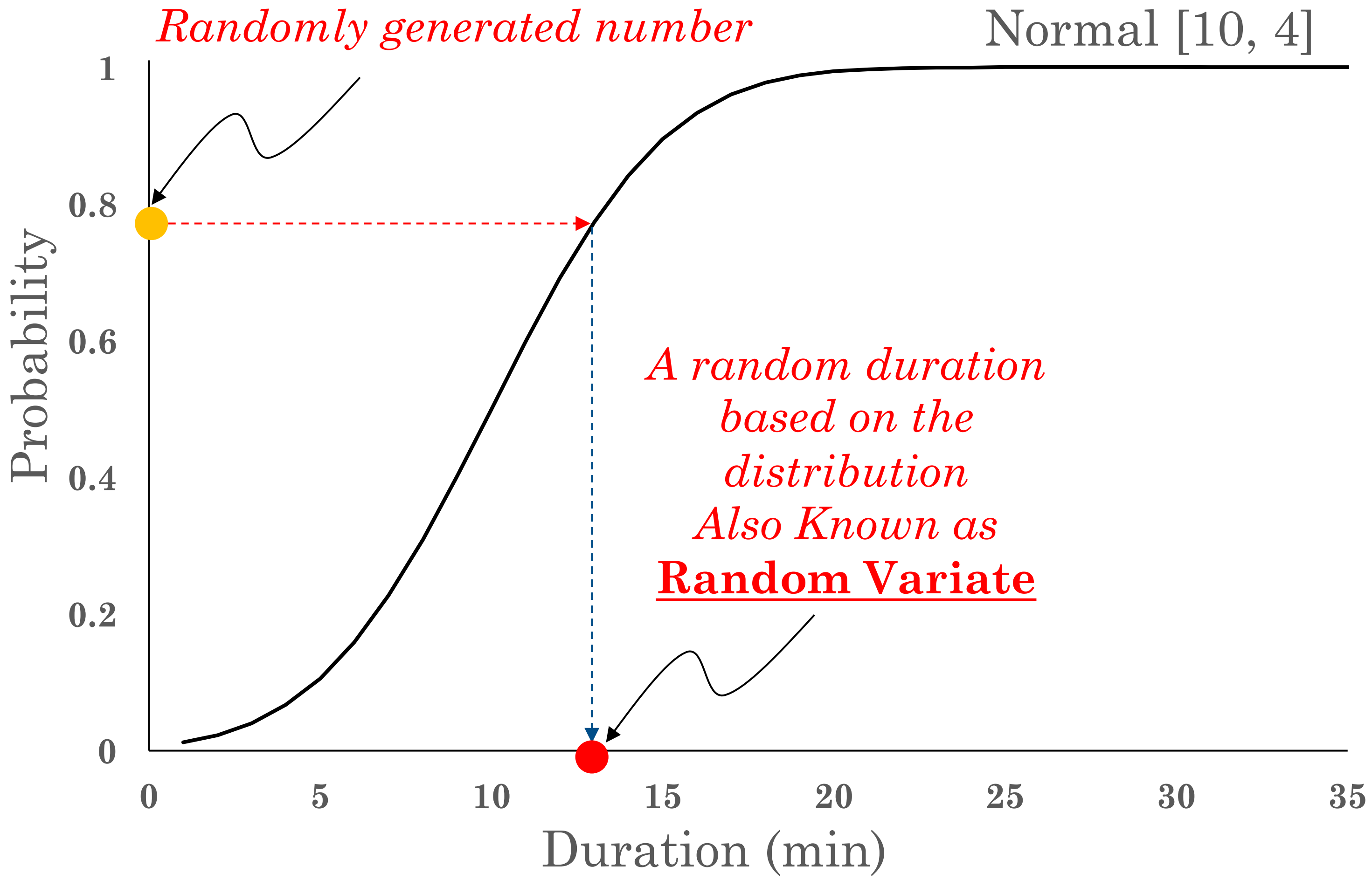
- Let's assume we measured the travel time of the trucks in our example for 500 times and the below values are obtained:

Duration	Number of observations	Ratio (# obsr. / 500)
<3	13	0,026
<6	63	0,126
<9	175	0,35
<12	323	0,646
<15	435	0,87
<18	485	0,97
<21	498	0,996
<24	500	1
<27	500	1



Stochasticity, Random Number and Monte Carlo

- ❑ How can we use this distribution in simulation?

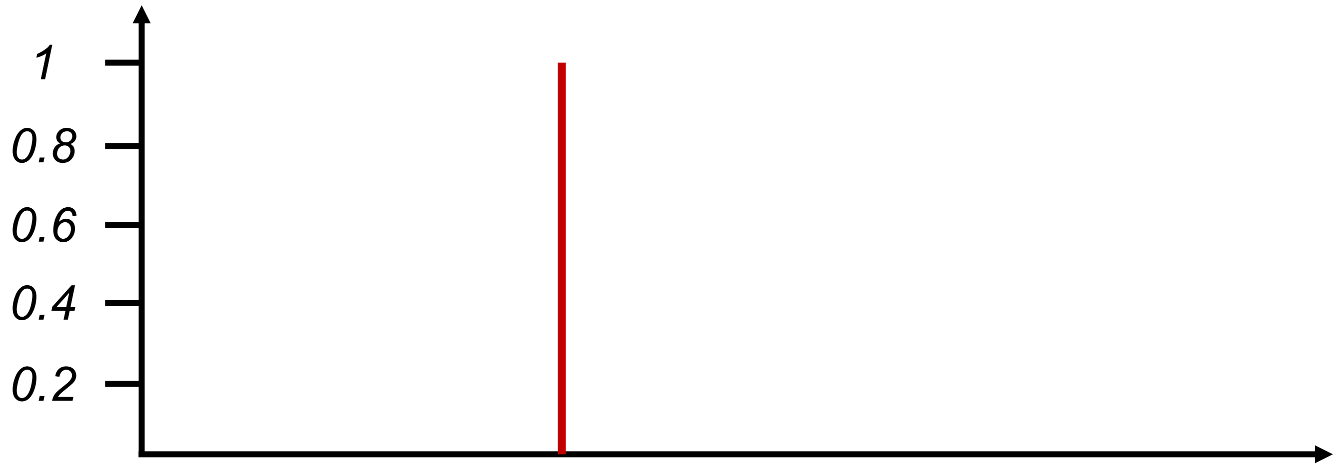


When we repeat this processes many times, we can mimic the stochastic behavior of a phenomenon!

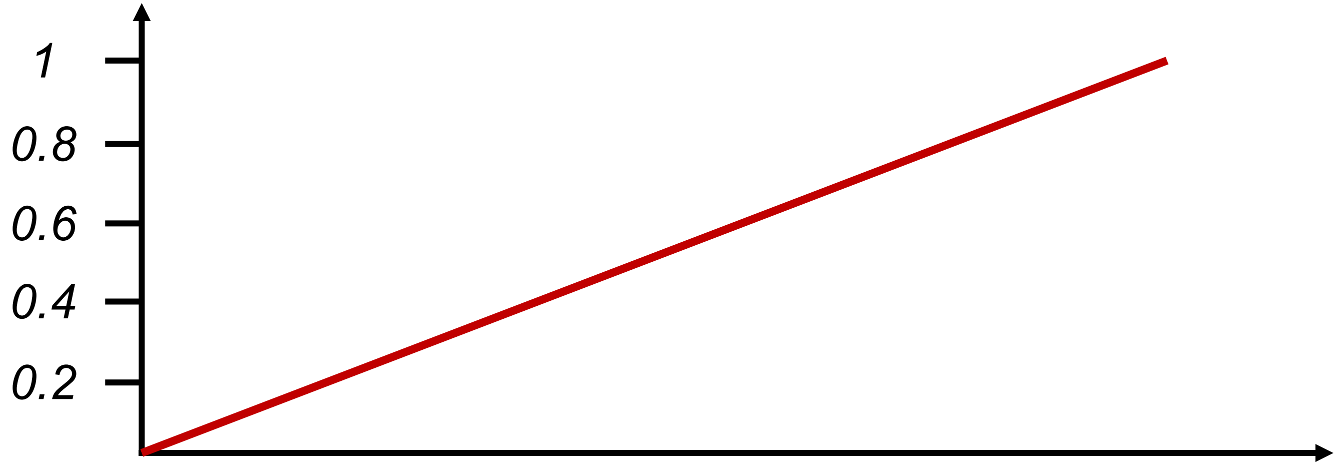
This is called Monte Carlo!

Stochasticity, Random Number and Monte Carlo

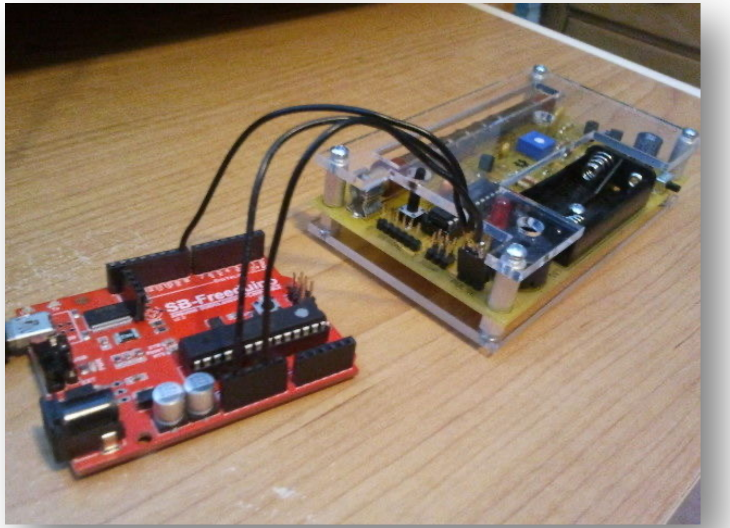
- Theoretically, what does the CDF of a deterministic parameter look like?



- Theoretically, what does the CDF of a uniform distribution look like?



Stochasticity, Random Number and Monte Carlo



As you saw, the backbone of stochastic modeling is the generation of random numbers.

But, can we generate purely random numbers?

There are two methods of generating random numbers:

(1) Based on sensing physical phenomenon e.g., measuring atmospheric noise, thermal noise;

(2) Pseudo-random number generated mathematically (computationally). In most cases we use this type of generators in simulations

Stochasticity, Random Number and Monte Carlo

An example of random number generator is Linear Congruential Scheme

$$\mathbf{Z_n = a \times Z_{n-1}\ MOD\ m}$$

Where:

a: Multiplier (usually 7^5)
Z_0: the starting integer = seed number
m: modulus (a very large number e.g., 2^{31}-1)

$$\mathbf{R_n = \frac{Z_n}{m}}$$

Where:

R_n : Random Number

Stochasticity, Random Number and Monte Carlo

Example:

Generate a random number using LCS model. Assume a=5, m= 7 , and Z_0=9

$$Z_n = 5 \times Z_{n-1} \text{ MOD } 7$$

$$Z_1 = 5 \times Z_0 \text{ MOD } 7$$

$$= 5 \times 9 \text{ MOD } 7$$

$$= 45 \text{ MOD } 7$$

$$= 3$$

$$R_1 = \frac{Z_n}{7}$$

$$R_1 = \frac{3}{7} = 0{,}4285$$

n	Z_n	R_n
0	9	
1	3	0,4285
2	1	0,1428
3	5	0,7142
4	4	0,5714
5	6	0,8751
6	2	0,2857
7	3	0,4285

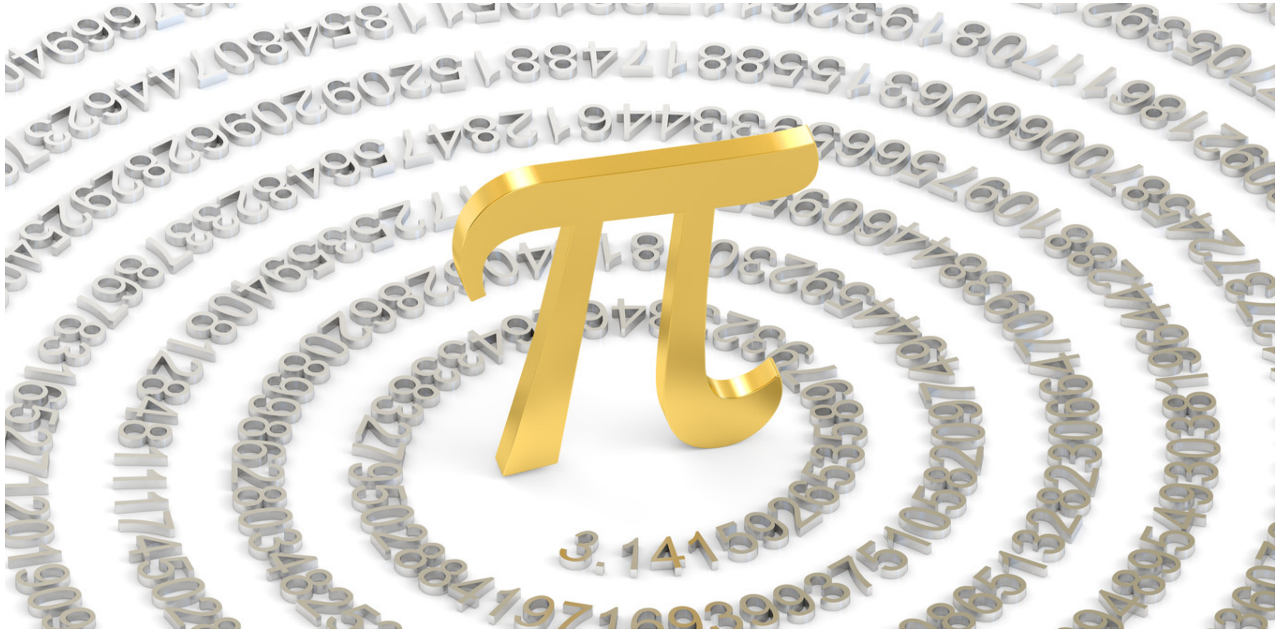
Repeated

That is why *m* must be a large number

Stochasticity, Random Number and Monte Carlo

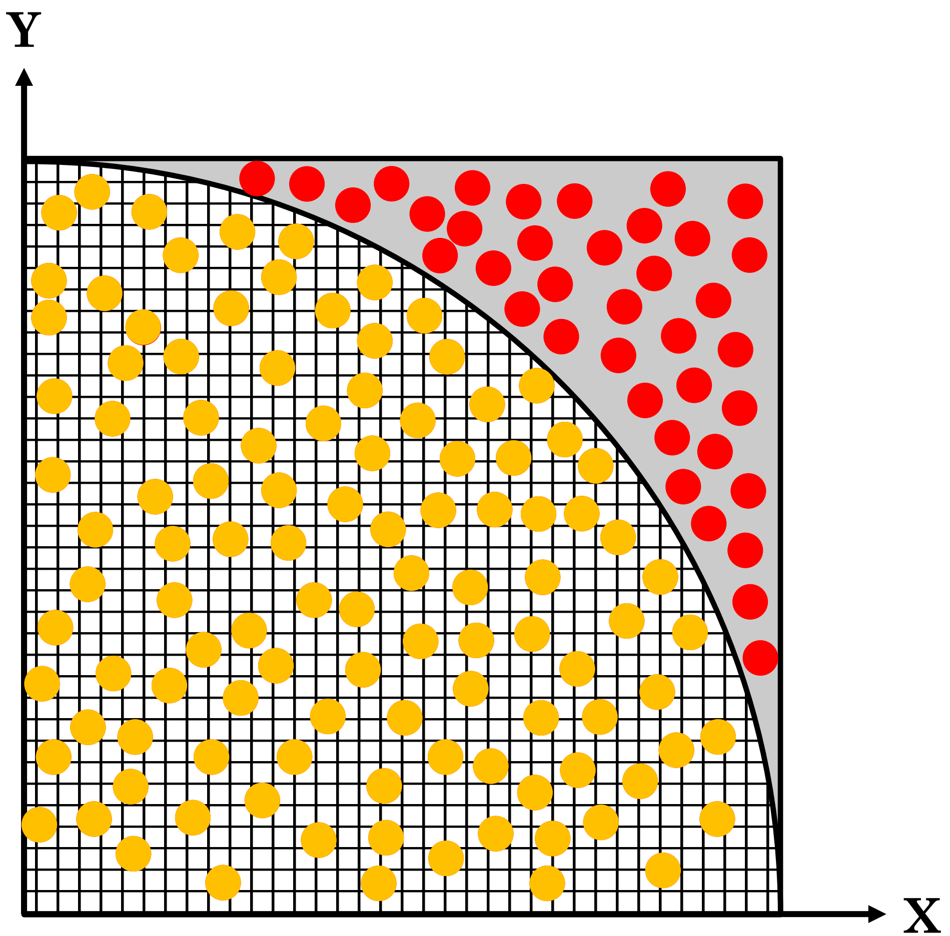
Example:

How can we calculate the number Pi (π) using stochastic simulation?

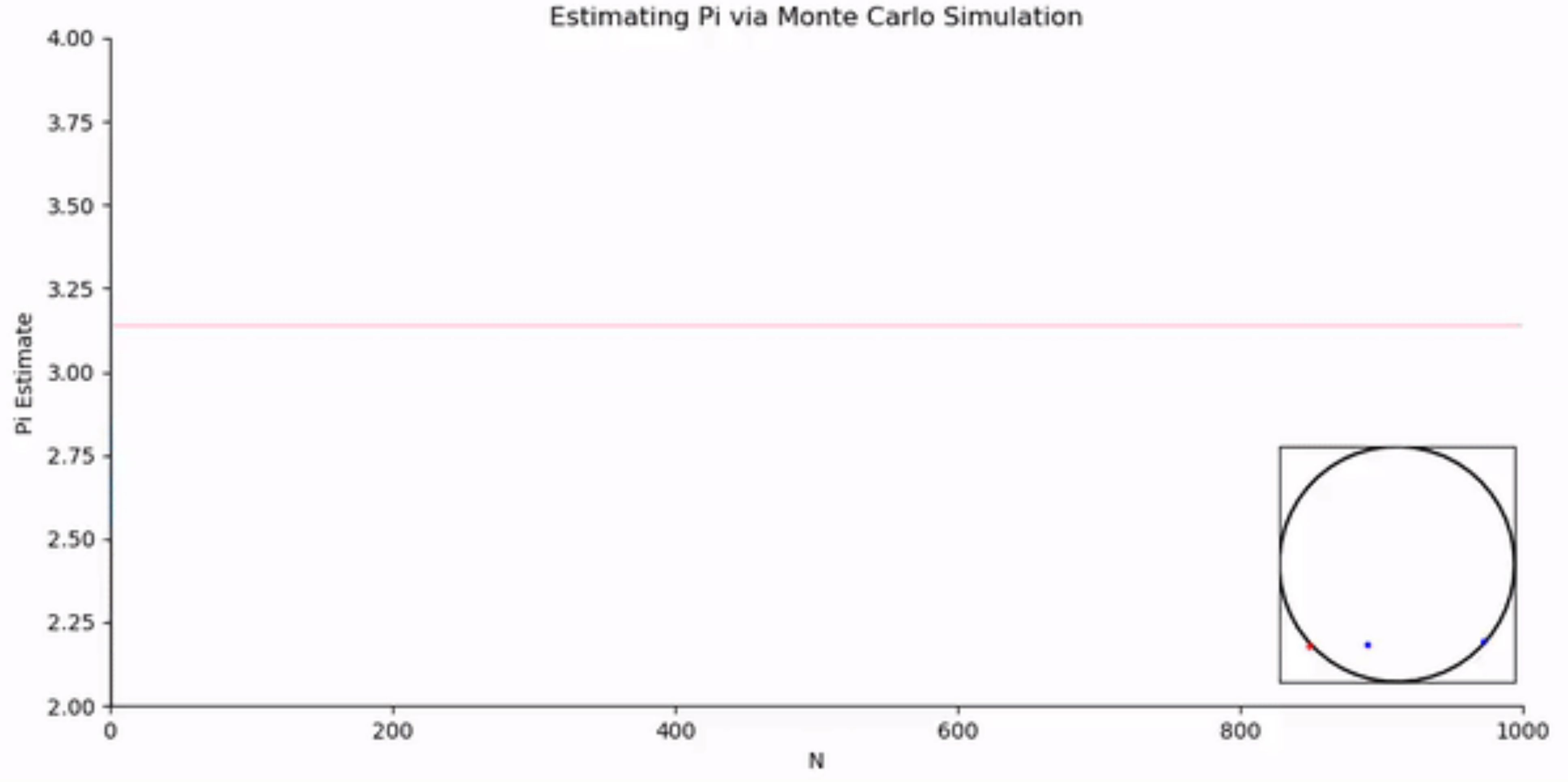


Stochasticity, Random Number and Monte Carlo

- Let's consider a 1×1 square and a circle with R=1.
- Now, lets generate a random point in this square.
- And we can generate a lot more!
- Now, if we calculate the number of points inside the circle by the total number of points, this would give an approximated value for π !!



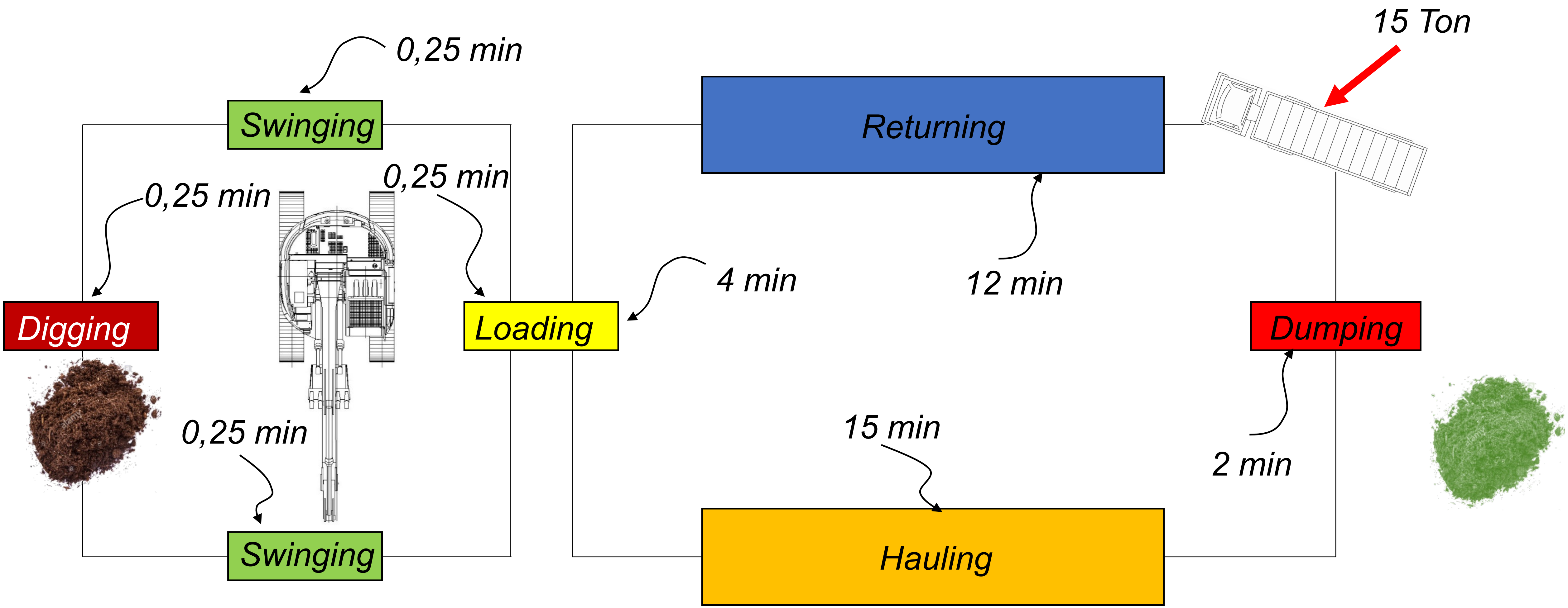
Stochasticism, Random Number and Monte Carlo



What does this have to do with Civil Engineering!!!!

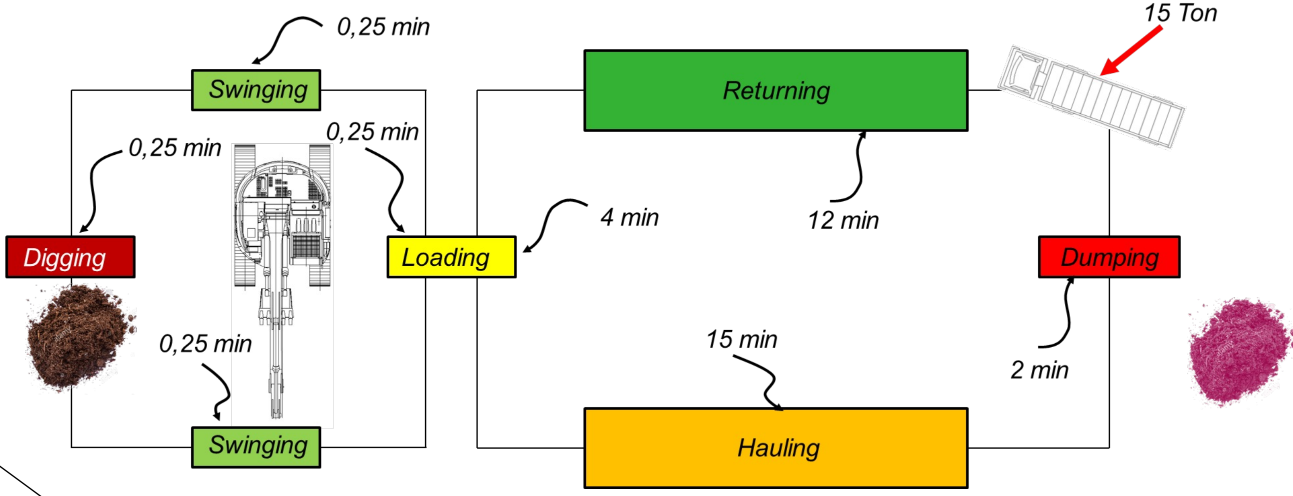
Let’s Imagine a Simple Construction Process

Deterministic Method



What is the productivity of this system?

Deterministic Method



Truck Cycle time =

Duration=4+15+2+12 = **33 min/load**

Capacity = 15 ton

Productivity = 15 ton / 33 min = **27,3 ton/hr**

Excavator Cycle time =

Duration=0,25+0,25+0,25+0,25 = **1 min/load**

Capacity = 15 ton $\times \dfrac{1\ min}{4\ min}$ = 3,75 ton

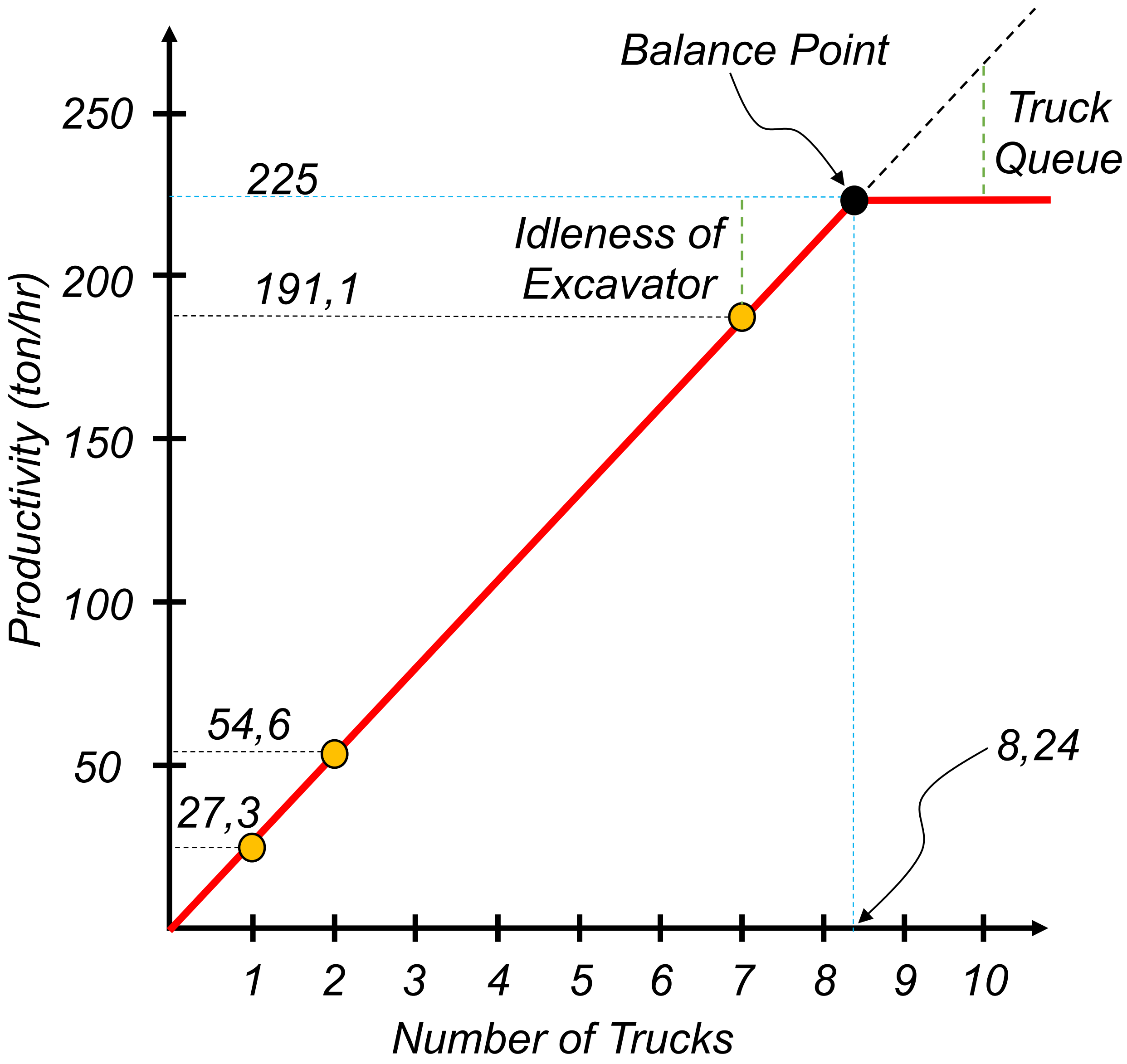
Productivity = 3,75 ton/min = **225 ton/hr**

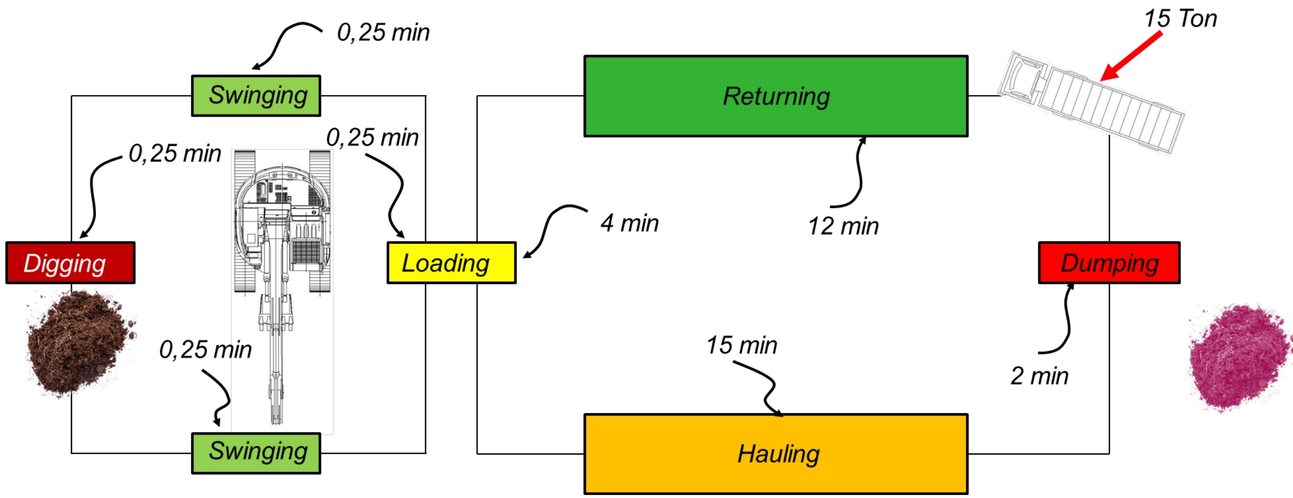
Which one is the productivity of the system?

27,3 ton/hr

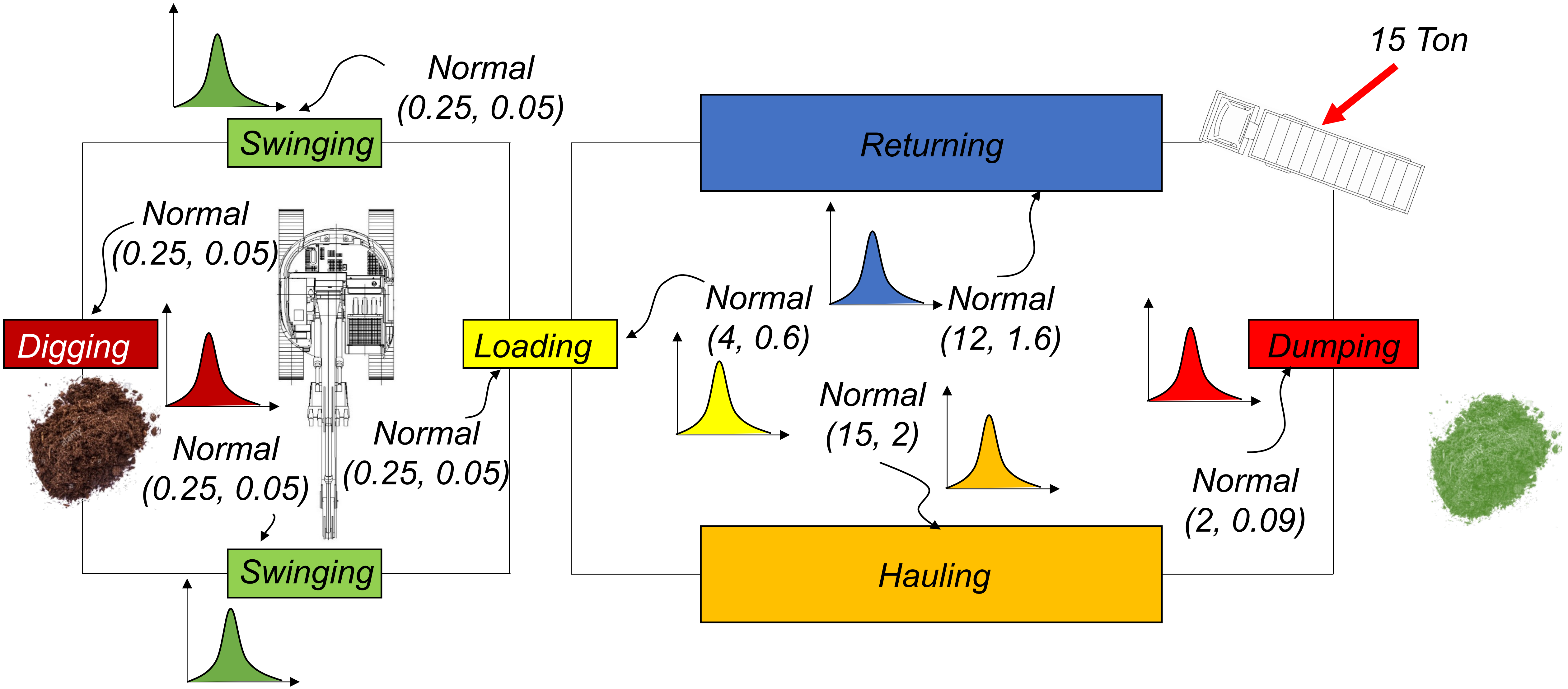
Now, what if we have two trucks?

Deterministic Method



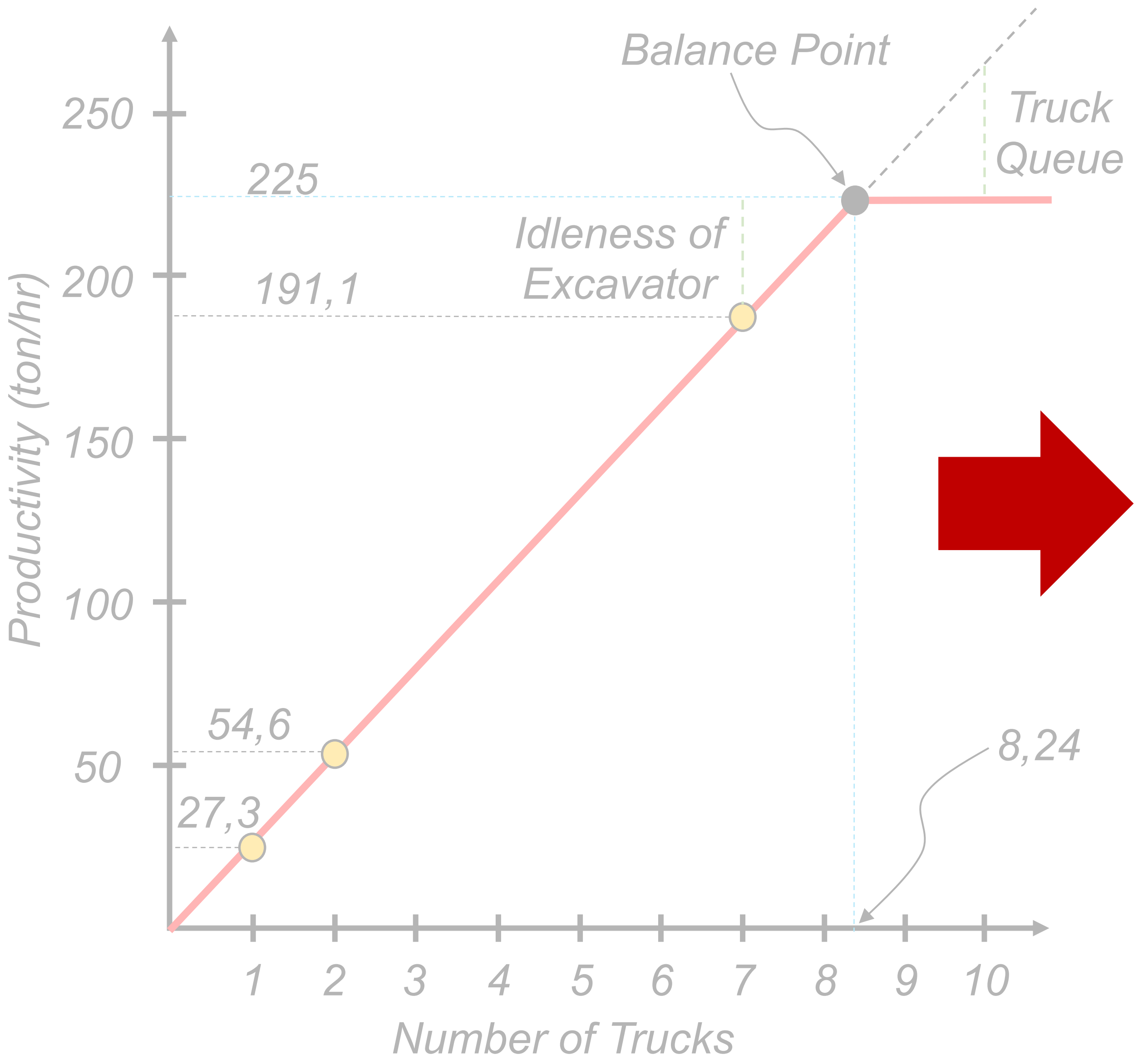


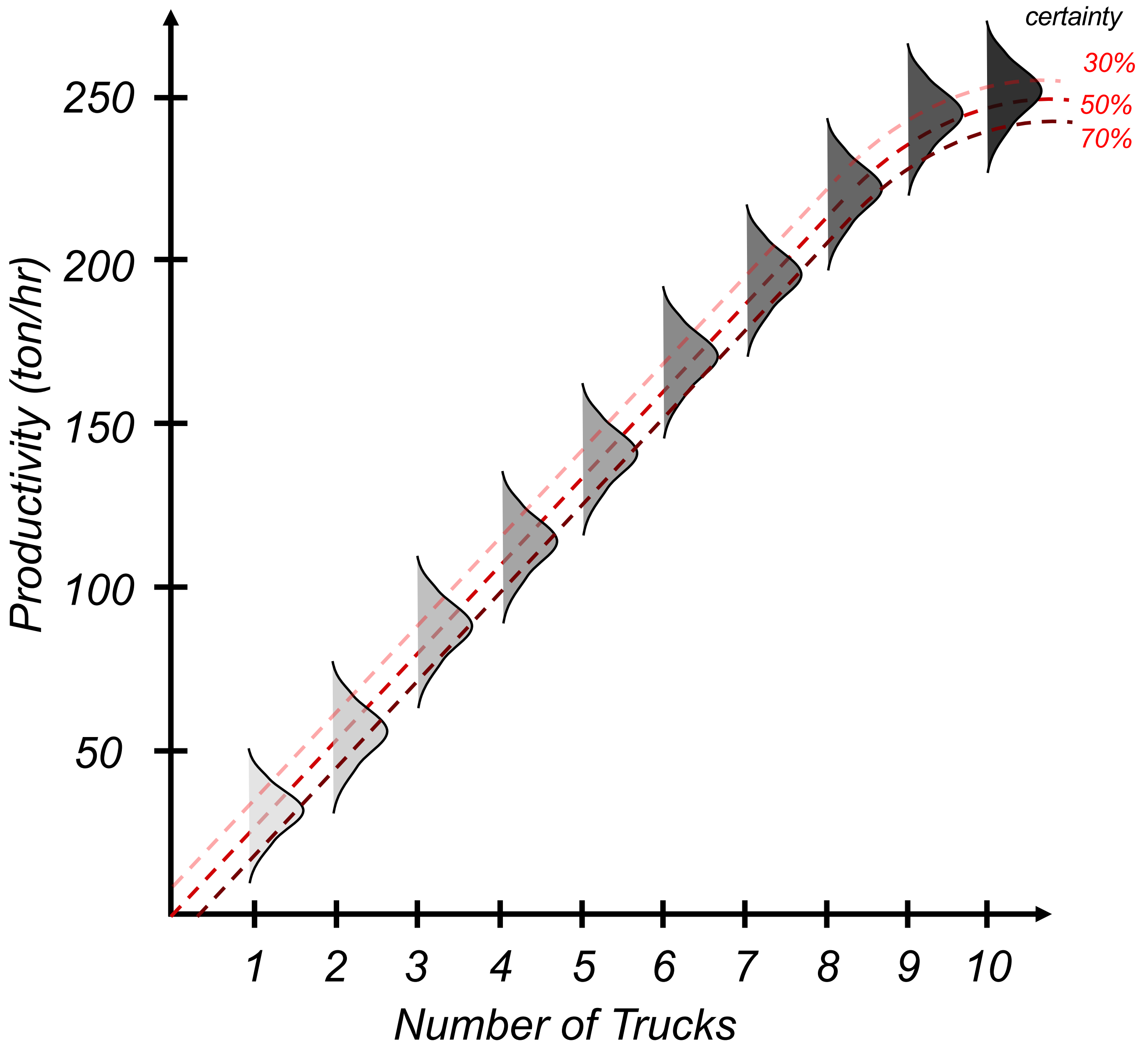
Stochastic Method



How would the representation of the system change?

Stochastic Method





Stochastic Modeling

- It is a modeling approach where you take into account **uncertainties associated with the decision-making parameters** (time, cost, quality, etc.)

- It provides decision-makers with **a measure of certainty** they can expect with different estimates

- It relies on the replication of the evaluation/simulation

- The generation of a random number is central to stochastic modeling

Burning Questions

- If the majority of the physical phenomena are stochastic, **why do people still use deterministic models**?
- Can/should we apply **Monte Carlo simulation** to deterministic models? Why?
- Can/should we apply **Monte Carlo on a stochastic single-activity process**? Why?

What did we learn in this lecture?

- Deterministic modeling
- Balance point in the system
- Stochastic Modeling
- Monte Carlo Simulation
- Random Number generator