**Faculty of Engineering & Technology**

**Department of Electrical and Computer Engineering**

**COMMUNICATION SYSTEMS ENEE3309**

**Course Assignment**


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**Section:** 3

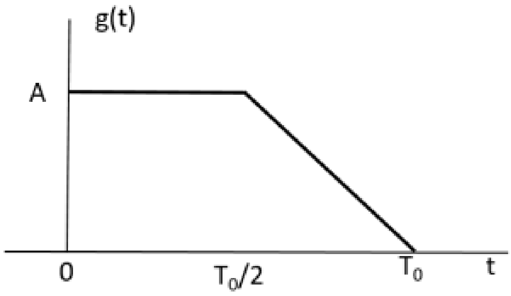
**Date:** 22/1/2023

# Abstract

The main objective of this assignment is to get our hands on the technical part of the course (coding) in such a language like MATLAB; to find the numeric values, and plotting certain signals. For a better view of the Assignment you can check it on my Github

# Main Question

**Consider the periodic signal g(t), for which one period is shown in the figure below**



One Period of the periodic signal g(t)

**where A=1 and $T_0 = 0.1\ sec$. This signal can be expanded in a trigonometric Fourier series as:**

$$g(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

**Now, consider the approximate signal:**

$$g_a(t) = a_0 + \sum_{n=1}^{K}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

## Explanation

The question gives us a signal g(t), and its needed values like **A** in one period **T0** as given in the above figure, and told us that the exact signal could be found using trigonometric Fourier series using the formula:

$$g(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

Since we can't do summation to infinity in programming languages, an approximate function ga(t) is given as:

$$g_a(t) = a_0 + \sum_{n=1}^{K}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

# Q1

1. Find $a0$, $a1$, $a2$, $a3$, $b1$, $b2$, $and$ $b3$ (you can use matlab or any other code to find numerical values of the coefficients)

## Code

```
syms t
T0 = 0.1;   %   period
W0 = 2.*pi/T0;
gt = heaviside(t) + (1/(-T0/2)).*(t-T0/2).*heaviside(t-T0/2) +
(1/(T0/2)).*(t-T0).*heaviside(t-T0); %original function
% Define the original function g(t) as a piecewise function with three parts:
% 1. The Heaviside function
% 2. A linear function with a slope of 1/(-T0/2) and starts at t = T0/2
% 3. A linear function with a slope of 1/(T0/2) and starts at t = T0
a0 = (1/T0).* int(gt,t,0,T0) % Calculate the DC component of the Fourier series
expansion
an = 0; bn = 0;
xan = []; xbn = [];
for n = 1:1:3
   harmonicA = 2/T0 * int(gt.*cos(n*W0*t),t,0,T0); %  % calculate the nth
harmonic cosine term of the Fourier series expansion
   harmonicB = 2/T0 * int(gt.*sin(n*W0*t),t,0,T0); % calculate the nth harmonic
sine term of the Fourier series expansion
   an = an  + harmonicA *cos(n*W0*t);
   bn = bn  + harmonicB * sin(n*W0*t);
    xan = [xan,harmonicA]; % list of all an from 1 to 3, and appends values
   xbn = [xbn, harmonicB]; % list of all bn from 1 to 3
end
display(xan) % prints values of an, a1, a2,a3
display(xbn) % prints values of an, b1, b2,b3

%Q2
gat = a0+an + bn;
figure;
fplot(gat,'g','LineWidth',1.5)
hold on % to make 2 signals on one plot
grid on
axis([-0.005,0.11,-1,2])
fplot(gt,'r','LineWidth',1.5)
xlabel('t')
ylabel('g(t), ga(t)')
title('g(t) VS ga(t)')
legend({'y=ga(t)', 'x = g(t)'})
```

## Results

```
>> q1

a0 =

3/4


xan =

[-2/pi^2, 0, -2/(9*pi^2)]


xbn =

[1/pi, 1/(2*pi), 1/(3*pi)]
```

## Explanation

The code  used to calculate the first three harmonics (an and bn) of the Fourier series expansion of g(t), Xan is list of the three values of an, a1, a2, a3 from left to right, same for Xbn

# Q2

**Use matlab to plot $g(t)$ and $ga\ (t)$ for K = 3, on the same figure for one cycle of $g(t)$.**

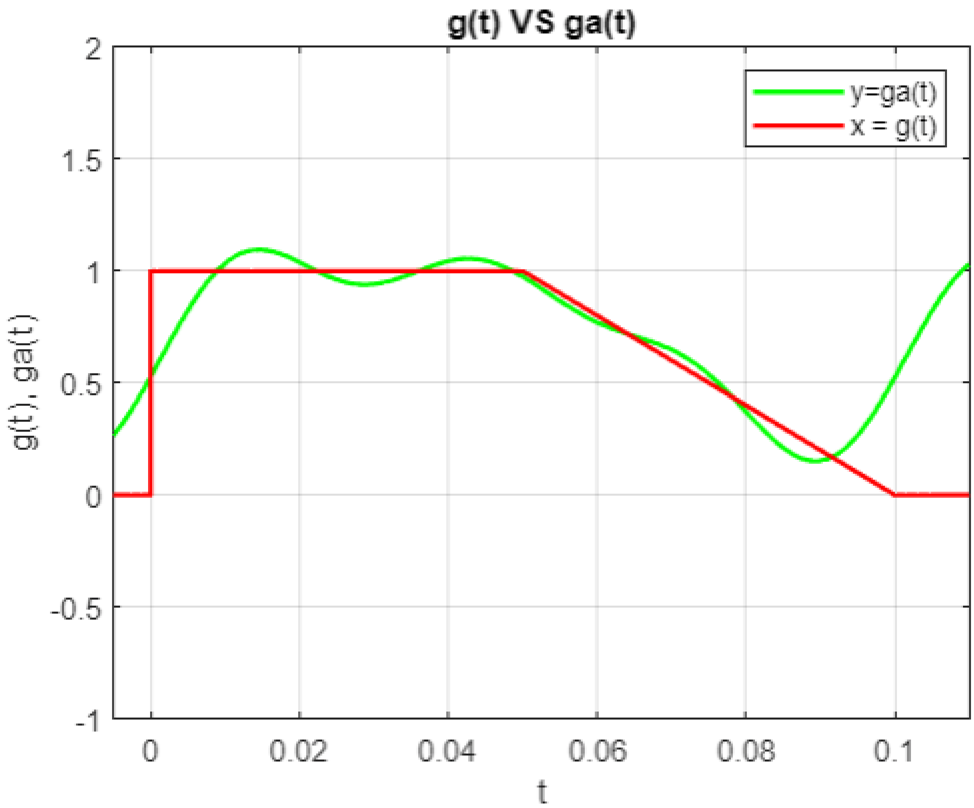
## Code

Last section of the **Q1** code:

```
%Q2
gat = a0+an + bn;
figure;
fplot(gat,'g','LineWidth',1.5)
hold on % to make 2 signals on one plot
grid on
axis([-0.005,0.11,-1,2])
```

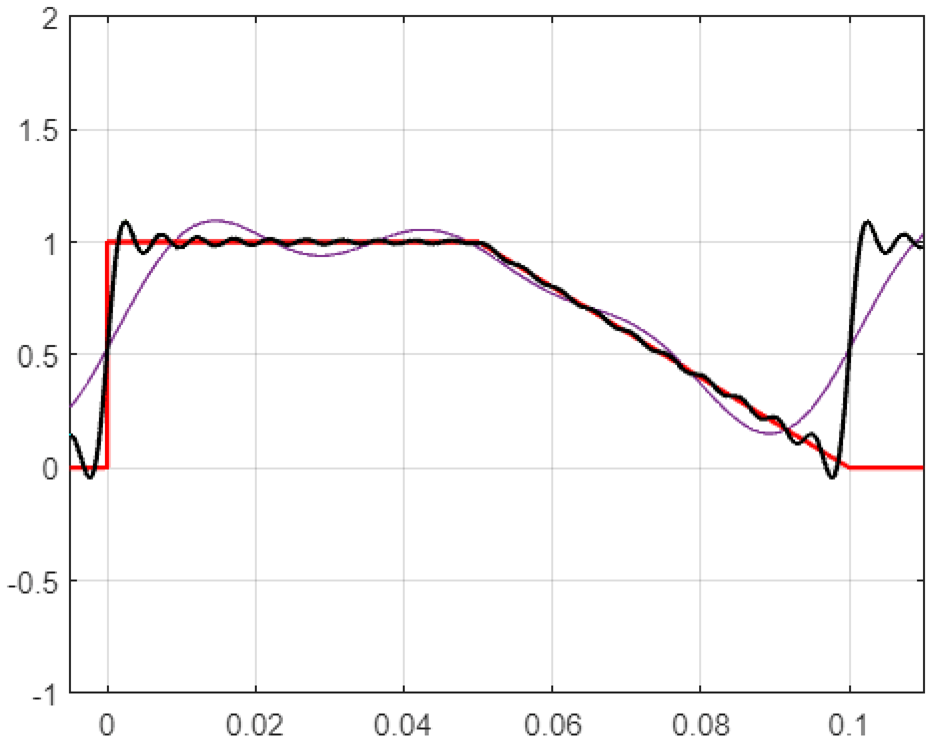
```
fplot(gt,'r','LineWidth',1.5)
xlabel('t')
ylabel('g(t), ga(t)')
title('g(t) VS ga(t)')
legend({'y=ga(t)', 'x = g(t)'})
```

## Plot




## Explanation

From the Plot we can see that ga(t) looks a bit similar to g(t), since **K=3**, it wouldn't look as much as the original function, but as much as we increase the value of **K,** the more that ga(t) will look like g(t)




The black signal shows the ga(t) when K = 20

# Q3

**3. The mean square error between $g(t)$ and $g_a(t)$ is defined as**

$$MSE = \frac{1}{T_0}\left(\int_0^{T_0} \left(g(t) - g_a(t)\right)^2 dt\right)$$

## Code

```
syms t
T0 = 0.1    %   period
W0 = 2.*pi/T0
gt = heaviside(t) + (1/(-T0/2)).*(t-T0/2).*heaviside(t-T0/2) +
(1/(T0/2)).*(t-T0).*heaviside(t-T0); %original function

a0 = (1/T0).* int(gt,t,0,T0);
an = 0; bn = 0;
xan = []; xbn = [];
```

```
MSE = [];
for n = 1:1:3

   harmonicA = 2/T0 * int(gt.*cos(n*W0*t),t,0,T0); % for each iteration
   harmonicB = 2/T0 * int(gt.*sin(n*W0*t),t,0,T0);
   an = an  + harmonicA *cos(n*W0*t);
   bn = bn  + harmonicB * sin(n*W0*t) ;
   absoluteDiffSquare = ((an + bn +a0) - gt )^2;
   MSN_each_iter = (1/T0) * int(absoluteDiffSquare,t,0,T0);
   MSE = [MSE,MSN_each_iter];

   xan = [xan,harmonicA]; % list of all an from 1 to 3, and appends values
   xbn = [xbn, harmonicB] ;% list of all bn from 1 to 3

end
MSE_THREE_VALUES = sprintf('MSE:\n');
MSE_THREE_VALUES=  MSE_THREE_VALUES + sprintf("%f
\n%f\n%f\n",MSE(1),MSE(2),MSE(3))
```

## Results

```
MSE_THREE_VALUES =

    "MSE:
     0.032974
     0.020309
     0.014427
     "

>>
```

## Explanation

The code is almost the same as the previous ones, but the difference is we added the list **MSE,** and inside the loop we find **MSE** each time (when n=1,n=2,n=3) respectively, using the formula given in the question. We can notice that the more we increase k, the less the **MSE** becomes.

# Q4

**Question 4 is attached individually as a PDF file, since it's 9 pages of handwriting solution. Please check it out**

# Appendix

The whole MATLAB  script:

```
syms t
T0 = 0.1;   %   period
W0 = 2.*pi/T0;
gt = heaviside(t) + (1/(-T0/2)).*(t-T0/2).*heaviside(t-T0/2) +
(1/(T0/2)).*(t-T0).*heaviside(t-T0); %original function
% Define the original function g(t) as a piecewise function with three parts:
% 1. The Heaviside function
% 2. A linear function with a slope of 1/(-T0/2) and starts at t = T0/2
% 3. A linear function with a slope of 1/(T0/2) and starts at t = T0
a0 = (1/T0).* int(gt,t,0,T0) % Calculate the DC component of the Fourier series
expansion
an = 0; bn = 0;
xan = []; xbn = [];
MSE = [];
for n = 1:1:3
  harmonicA = 2/T0 * int(gt.*cos(n*W0*t),t,0,T0);   % calculate the nth harmonic
cosine term of the Fourier series expansion
  harmonicB = 2/T0 * int(gt.*sin(n*W0*t),t,0,T0); % calculate the nth harmonic
sine term of the Fourier series expansion
  an = an  + harmonicA *cos(n*W0*t);
  bn = bn  + harmonicB * sin(n*W0*t);
  absoluteDiffSquare = ((an + bn +a0) - gt )^2;
  MSN_each_iter = (1/T0) * int(absoluteDiffSquare,t,0,T0);
  MSE = [MSE,MSN_each_iter];

  xan = [xan,harmonicA]; % list of all an from 1 to 3, and appends values
  xbn = [xbn, harmonicB]; % list of all bn from 1 to 3
end
display(xan) % prints values of an, a1, a2,a3
display(xbn) % prints values of an, b1, b2,b3
% q3
MSE_THREE_VALUES = sprintf('MSE:\n');
MSE_THREE_VALUES =  MSE_THREE_VALUES + sprintf("%f
\n%f\n%f\n",MSE(1),MSE(2),MSE(3))
%Q2
gat = a0+an + bn;
figure;
fplot(gat,'g','LineWidth',1.5)
hold on % to make 2 signals on one plot
grid on
axis([-0.005,0.11,-1,2])
fplot(gt,'r','LineWidth',1.5)
xlabel('t')
```

```
ylabel('g(t), ga(t)')
title('g(t) VS ga(t)')
legend({'y=ga(t)', 'x = g(t)'})
```