Q4)

From the previous questions; we got $a_0 = \frac{3}{4}$

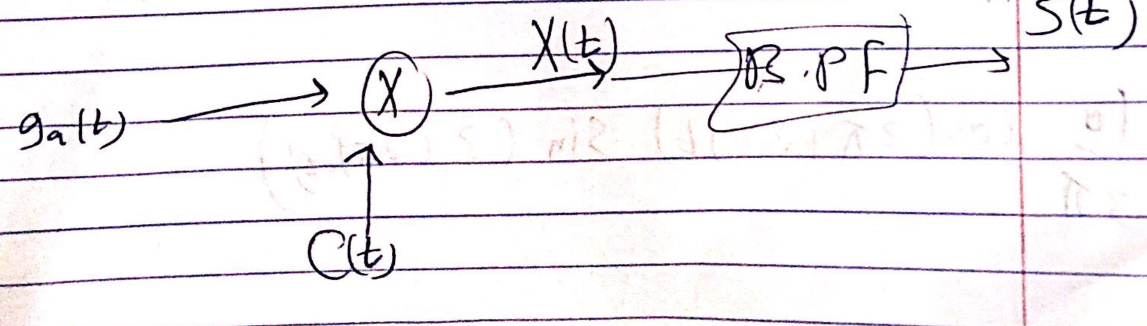
$$\{a_1, a_2, a_3\} = \left\{\frac{-2}{\pi^2}, 0, \frac{-2}{9\pi^2}\right\}$$

$$\{b_1, b_2, b_3\} = \left\{\frac{1}{\pi}, \frac{1}{2\pi}, \frac{1}{3\pi}\right\}$$

$$g_a(t) = a_0 + \sum_{n=1}^{3} (a_n \cdot \cos(n w_0 t)) + (b_n \sin(n w_0 t))$$

$$g_a(t) = \left(\frac{3}{4}\right) + \left(\frac{-2}{\pi^2} \cos(w_0 t) - \frac{2}{9\pi^2} \cos(3 w_0 t)\right)$$

$$+ \left(\frac{1}{\pi} \sin(w_0 t) + \frac{1}{2\pi} \sin(2 w_0 t) + \frac{1}{3\pi} \sin(3 w_0 t)\right)$$



$c(t) = 10\cos(2\pi(200)t)$ given

$$X(t) = 10\cos(2\pi(200)t)\left[\frac{3}{4} - \frac{2}{\pi^2}\cos(2\pi f_0 t)\right.$$

$$- \frac{2}{9\pi^2}\cos(3\cdot 2\pi f_0 t) + \frac{1}{\pi}\sin(2\pi f_0 t) +$$

$$\left.\frac{1}{2\pi}\sin(2\cdot 2\pi f_0 t) + \frac{1}{3\pi}\sin(3\cdot 2\pi f_0 t)\right]$$

$$X(t) = \frac{30}{4}\cos(2\pi(200)t) - \frac{20}{\pi^2}\cos(2\pi(200)t)\cdot\cos(2\pi f_0 t)$$

$$-\frac{20}{9\pi^2}\cos(2\pi(200)t)\cdot\cos(3\cdot(2\pi f_0 t)) + \frac{10}{\pi}\cdot\cos(2\pi(200)t)\cdot\sin(2\pi f_0 t)$$

$$+\frac{\cancel{10}^{\,5}}{\cancel{2}\pi}\cos(2\pi(200)t)\cdot\sin(2(2\pi f_0 t)) +$$

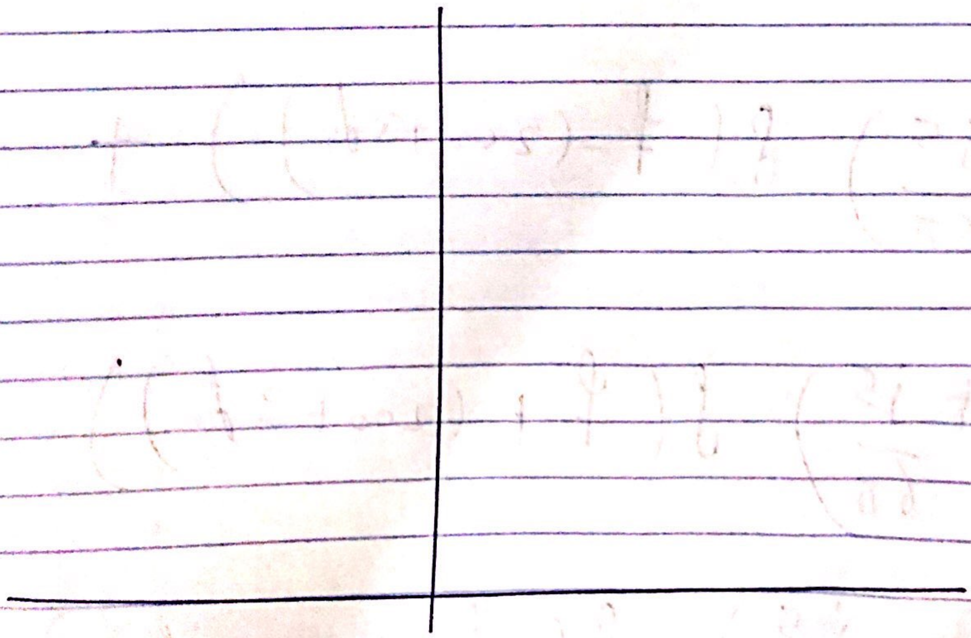
$$\frac{10}{3\pi}\cos(2\pi(200)t)\cdot\sin(3(2\pi f_0 t))$$

$$+\frac{5}{4\pi j}\left[\delta(f-(200-2f_c)) - \delta(f+(200-2f_c))\right]$$

$$+\frac{5}{6\pi j}\left[\delta(f-(200+3f_0)) - \delta(f+(200+3f_0))\right]$$

$$\frac{5}{6\pi j}\left[\delta(f-(200-3f_0)) - \delta(f+(200-3f_c))\right]$$

$$X(f) = \frac{30}{8}\left[\delta(f-200) + \delta(f+200)\right]$$



$X(f) =$

$$\frac{30}{8}\left[\delta(f-200)+\delta(f+200)\right]$$

$$+\left(\frac{5}{2\pi j}-\frac{5}{\pi^2}\right)\delta(f-(200+f_c))$$

$$+\left(\frac{-5}{\pi^2}+\frac{j5}{2\pi}\right)\delta(f+(200+f_c))$$

$$+\left(\frac{-5}{\pi^2}+\frac{j5}{2\pi}\right)\delta(f-(200-f_c))+$$

$$\left(\frac{-5}{\pi^2}-\frac{j5}{2\pi}\right)\delta(f+(200-f_c))+$$

$$\left(\frac{-5}{9\pi^2}-\frac{j5}{6\pi}\right)\delta(f-(200+3f_c))+$$

$$\left(\frac{-5}{9\pi^2}+\frac{j5}{6\pi}\right)\delta(f+(200+3f_c))$$

$$+\left(\frac{-5}{9\pi^2}+\frac{j5}{6\pi}\right)\delta(f-(200-3f_c))$$

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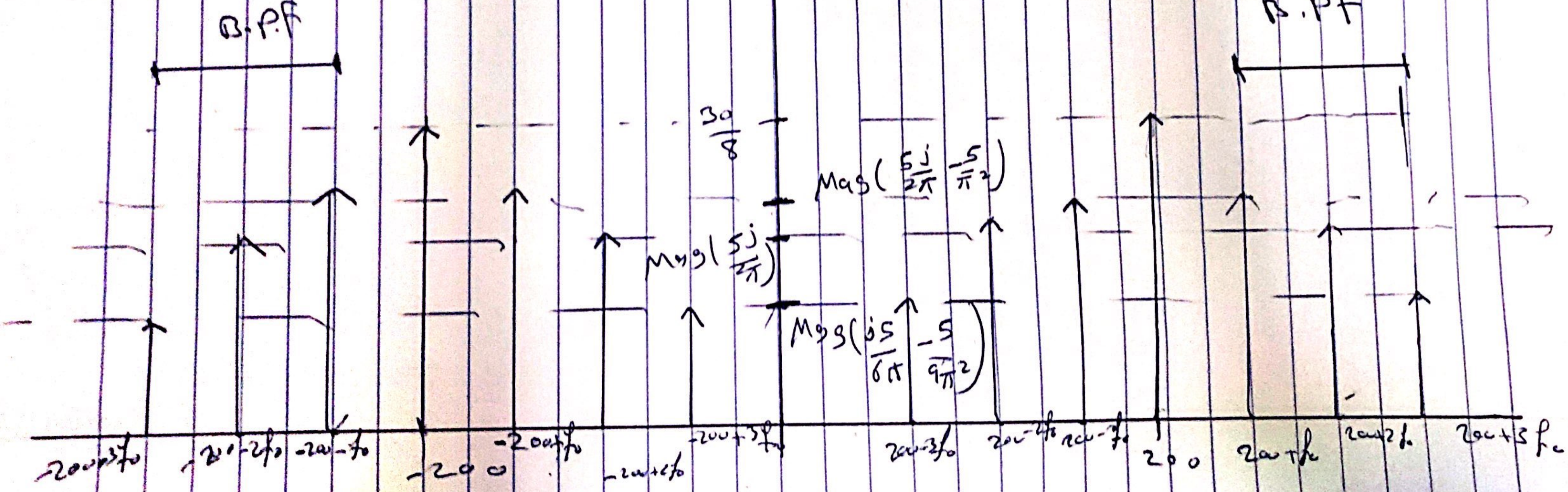
$$+\left(\frac{-5}{9\pi^2} - \frac{j5}{6\pi}\right)\delta(f+(200-3f_c))$$

$$-\frac{j5}{4\pi}\Big[\delta(f-(200+2f_c)) - \delta(f+(200+2f_c))$$

$$-\delta(f-(200-2f_0)) + \delta(f+(200-2f_0))\Big]$$

After Applying ~~$f_c = 200$ Hz~~, And the filter :

$\frac{1}{T_0} = f_0 = 10$ Hz



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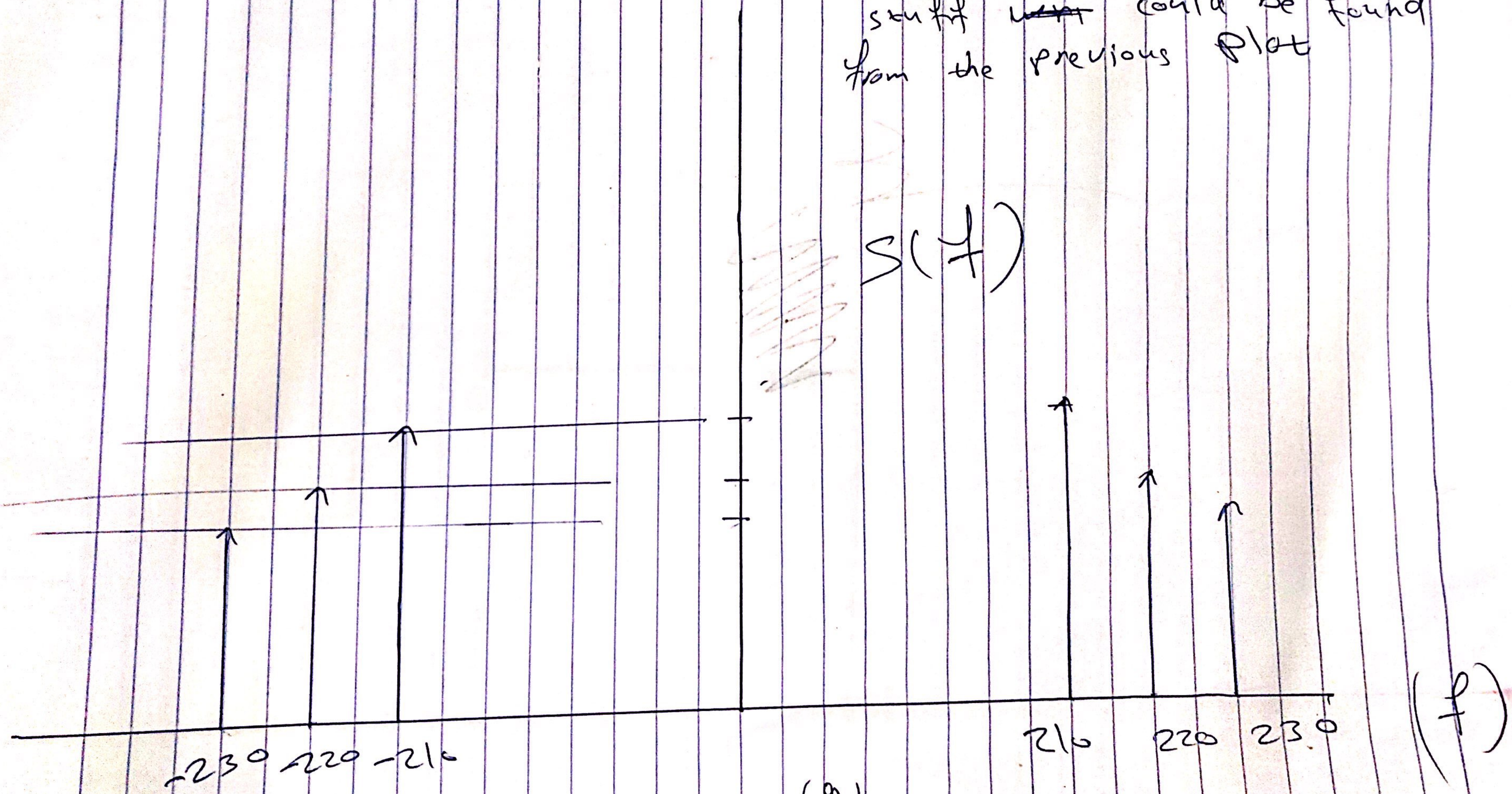
Values of Magnitudes and other stuff ~~with~~ could be found from the previous plot



(a)

Using trig formual of $\cos\alpha \cdot \cos\beta$ and other ones :-

$$X(t) = \frac{30}{4} \cos(2\pi 200 t) - \frac{10}{\pi^2} \cos(2\pi(200+f_o)t)$$

$$-\frac{10}{\pi^2} \cos(2\pi(200-f_o)t) - \frac{10}{9\pi^2}\left[\cos(2\pi(200+3f_o)t)\right.$$

$$\left. - \cos(2\pi(200-3f_o)t)\right] + \frac{5}{\pi}\left[\sin(2\pi(200+f_o)t)\right.$$

$$\left. - \sin(2\pi(200-f_o)t)\right] + \frac{5}{2\pi}\left[\sin(2\pi(200+2f_o)t)\right.$$

$$\left. - \sin(2\pi(200-2f_o)t)\right] + \frac{5}{3\pi}\left[\sin(2\pi(200+3f_o)t)\right.$$

$$\left. - \sin(2\pi(200-3f_o)t)\right]$$

$$X(f) = \frac{30}{8}\left[\delta(f-200) + \delta(f+200)\right]$$

$$-\frac{5}{\pi^2}\left[\delta(f-[200+f_c]) + \delta(f+[200+f_c])\right]$$

$$-\frac{5}{\pi^2}\left[\delta(f-[200-f_c]) + \delta(f+[200-f_c])\right]$$

$$-\frac{5}{9\pi^2}\left[\delta(f-(200+3f_c)) + \delta(f+(200+3f_c))\right]$$

$$-\frac{5}{9\pi^2}\left[\delta(f-(200-3f_c)) + \delta(f+(200-3f_c))\right]$$

$$+\frac{5}{2\pi j}\left[\delta(f-(200+f_c)) + \delta(f+(200+f_c))\right]$$

$$-\frac{5}{2\pi j}\left[\delta(f-(200-f_c)) - \delta(f+(200-f_c))\right]$$

$$+\frac{5}{4\pi j}\left[\delta(f-(200+2f_c)) - \delta(f+(200+2f_c))\right]$$

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