Arbitrage Opportunities for Liquid Restaking Tokens on Pendle

Faris Chaudhry
faris.chaudhry@nethermind.io

Jian-Ren Chen
jian.chen@nethermind.io

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Abstract

Pendle is a protocol that enables the separation of yield-bearing tokens into their principal and yield components, offering users the ability to leverage and trade yield. This paper analyzes liquid restaking tokens (LRTs) on Pendle, exploring the factors influencing fixed yields, correlations among different LRTs, relationships across different maturities, and the impact of liquidity on yields. We examine contracts with varying levels of liquidity, tokens, and maturities. By applying PCA, correlation analysis, and random forest regression, we identify the key drivers of yield and interactions between YT/PT prices. Through cointegration testing, we investigate long-term equilibria between token pairs. Our findings reveal structural weaknesses in the correlation matrices of LRTs, an illiquidity factor contributing to higher APYs for certain LRTs, and evidence of cointegration between tokens and yields. Additionally, we backtest statistical arbitrage strategies based on our results.

Contents

1 Introduction

1.1 Overview of Yield Tokenization

In traditional finance, interest rate derivatives form one of the largest markets, with over $\$700$ trillion of outstanding notional value in 2023, due to their extensive use in hedging, financing, and speculation. [1] Analogously, Pendle is a decentralized finance (DeFi) protocol designed to facilitate yield tokenization. It enables users to split their yield-bearing tokens into two separate tokens: Principal Tokens (PTs) and Yield Tokens (YTs). PTs represent the ownership of the underlying asset, while YTs represent the rights to the yield generated by the asset. The PT can be redeemed for the underlying token at maturity, while the YT entitles holders to the yield that would be generated over the contract's lifetime. In traditional finance terms, PTs resemble zero-coupon bonds, and YTs are akin to floating rate notes with daily payouts.

At any point during the contract, users can mint new PTs and YTs from the underlying asset or burn PTs and YTs to retrieve the underlying asset through the Pendle Automated Market Maker (AMM). Additionally, holding YTs entitles the holder to the rewards generated by the underlying, providing an additional incentive for investors. This feature allows users to leverage yields and rewards through the liquidity and flexibility offered by Pendle without having to own the underlying asset. Pendle charges a fee on the yield generated by YTs and matured, unredeemed PTs.[1] [2] Users can buy and sell PTs and YTs on Pendle, paying a small swap and gas fee. These swap fees are adjusted such that contracts with lower time to maturity have lower swap fees to create fairer pools.

A more detailed specification of standardized yield stripping, including the standardised yield (SY) token which is omitted here, can be found on the Pendle whitepapers, [3] and [4].

1.2 Objectives and Scope of the Study

We examine LRTs on the Pendle protocol. Specifically we wish to answer the following questions:

- What are the drivers of fixed yields?
- Does liquidity influence performance, i.e., do less liquid pools tend to offer higher yields?
- How are yields correlated across different tokens of the same tenor?
- How are yields correlated across different tenors of the same token?
- Can viable arbitrage strategies be identified based on our findings?

To address these questions, we analyse a variety of contracts with different underlying tokens, tenors, protocol design, and liquidity levels. Specifically, we focus on Etherfi eETH (Jun/Sep/Dec), Zircuit eETH (Jun), Renzo ezETH (Sep/Dec), Puffer pufETH (Jun/Sep), Bedrock uniETH (Jun/Sep), and Kelp rsETH (Jun/Sep). June contracts, which have matured, provide our only complete dataset, while other contracts remain ongoing at time of writing.

Data for the YTs and PTs is sourced from the Pendle API. Underlying token price data is obtained from Yahoo Finance. Total Value Locked (TVL) data is retrieved from DefiLlama. We use daily open data for prices.

[1] At the time of writing, this fee is 3%.

2 Drivers of Fixed Yields

2.1 Pricing Schema

We first introduce some useful formulae and concepts for pricing YT and PT. Due to the mechanism of minting and burning PT and YT for the underlying, we always have the PT/YT invariant:

$$\text{PT}(t) + \text{YT}(t) = \text{Underlying}(t)$$

which the Pendle AMM maintains via arbitrage. [5] If we consider the same formula with prices in terms of the underlying, then we instead get:

$$\text{PT}_{ETH}(t) + \text{YT}_{ETH}(t) = 1 \tag{1}$$

which is more convenient to use as it removes the USD exposure of the underlying.[2] We will refer to this as the underlying being hedged, which can be done by, for example, being short a futures contract dated after maturity. A useful byproduct of the PT/YT invariant is that any position can be expressed in terms of other assets. For example, instead of going long a YT, one can long the underlying and short the PT.

We know that as we approach the expiration date, T, that the PT must converge in value to the underlying, since it can be redeemed, and the YT must converge to 0 since it has emitted all its yield. We therefore assume that

$$\lim_{t\to T} \text{PT}_{ETH}(t) = 1 \quad \text{and} \quad \lim_{t\to T} \text{YT}_{ETH}(t) = 0. \tag{2}$$

Now, since the price of the PT must be given by discounting the price of the underlying from the time to maturity, which is 1 of the underlying, we know the present value of the PT under continuous compounding is:

$$\text{PT}_{ETH}(t) = e^{-r((T-t)/365)} \tag{3}$$

where r is the annualised fixed yield when buying at t. Consequently, we know the fixed yield is given by:

$$r = -\frac{365}{T-t} \ln\left(\text{PT}_{ETH}(t)\right). \tag{4}$$

Here, r is a function of time since the fixed yield is dependent on the price of the PT. Since we have the PT/YT invariant in (1), the variability in the PT's price is related to the fluctuations in the YT's price. That is to say, an increase in the value of the YT will decrease the price of the PT.

The canonical method of gauging the sentiment of the YT is implied APY, defined in [6] as:

$$\text{Implied APY} = \left[\left(1 + \frac{\text{YT}}{\text{PT}}\right)^{\frac{365}{T-t}}\right] - 1. \tag{5}$$

Implied APY is a forward-looking measure that reflects market expectations of the underlying asset's yield and any rewards (e.g., EigenLayer points) over the term of the contract. It

[2]Note that the use of 'ETH' (almost everywhere) refers to the underlying LRT, not ETH itself.

captures consumer sentiment and predictions about future interest rates, risk premiums, and potential policy changes. The order book and bid-ask spread on Pendle are given in terms of implied APY, which facilitates a common reference for trading YTs. It enables investors to compare the expected returns of different tokens and make informed decisions based on their risk tolerance and yield expectations.

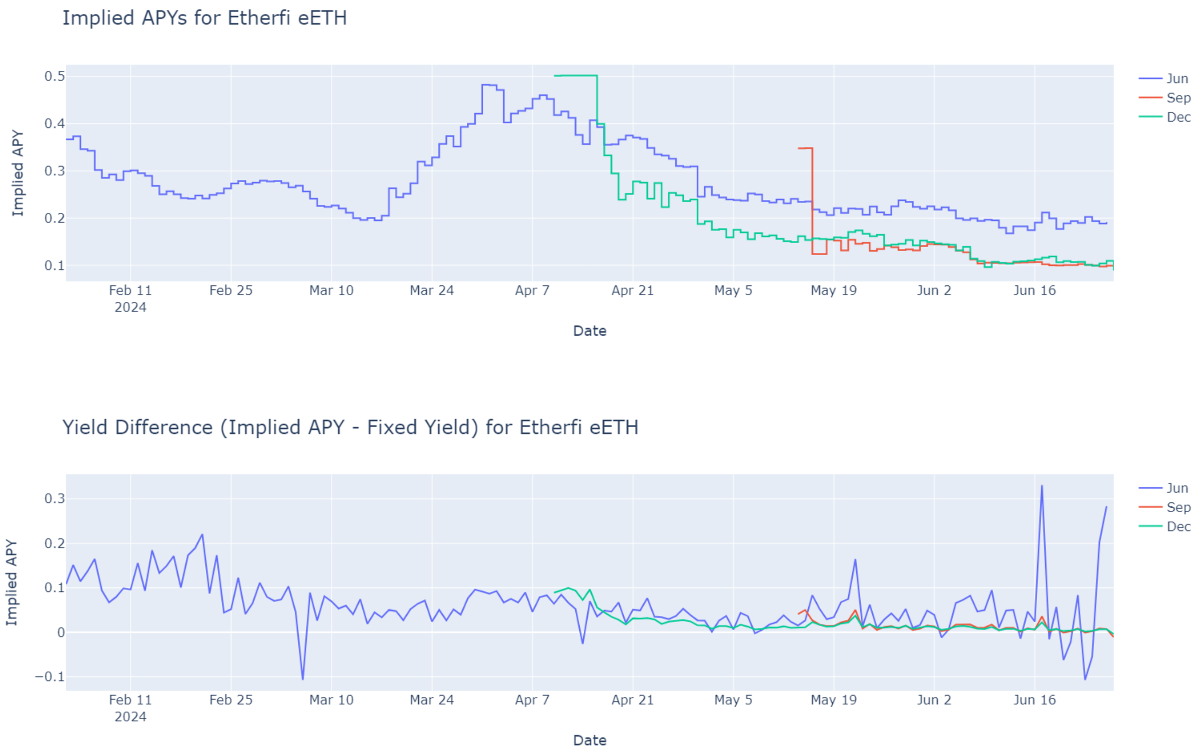

Figure 1: (Top) An example of the implied APY over time for Etherfi eETH contracts. By using the formula given in (5), we can replicate the graphs shown on Pendle. (Bottom) The difference between the implied APY and the fixed yield for Etherfi eETH contracts. The fixed yield formula used here assumes daily compounding to match with the formula used by Pendle for implied APY.

While equations (4) and (3) are useful to calculate fixed yield, they cannot be used to theoretically price YTs since the PT price and fixed yield, r, are dependent on each other. It is instead the YT that drives the pricing dynamics. If we instead consider the future yields over the duration of the contract, the price of a YT would be of the form:[3]

$$\mathrm{YT}_{ETH}(t) = \int_t^T r(u)du + \sum_{i=i}^{N} \int_t^T R_i(u)du \tag{6}$$

where $r(t)$ is the APY of the underlying token and $R_i(t)$ can be arbitrary reward functions for other incentives. These rewards are token-dependent and could be from EigenLayer, Veda, Karak, and so on. Sometimes they are combined with a boost factor that increases points for various tokens. For example, the Etherfi Sep contract has a 2x Etherfi point multiplier, while the Dec contract has a 4x Etherfi point multiplier. Some examples of how these reward functions

[3]The equation presented here is a template. Exact pricing formulae will depend on the nature of the reward functions used, for example one might use summation instead of integrals for rewards added daily. If one wishes to get the price in USD instead, then replace the operand of the first integral, $r(u)$, with $r(u) \cdot \mathrm{Underlying}(u)$.

might operate are in [7] and [8]. Generally, a reward function for points emitted daily might take the form:

$$R_i(t) = V_i(t) \cdot BoostFactor \cdot RateOfEmission \tag{7}$$

where V_i is a valuation function for 1 unit of the point. Not all rewards must take this form though. The vote-escrowed Pendle (vePENDLE) token is the staked version of the PENDLE token. [9] The fees that Pendle collects from yield generated by YTs are distributed to vePENDLE holders and protocol revenue from swap fees is given to the voters of the respective pool. [2] Therefore, the value of incentives given by vePENDLE would depend on the amount of fees being generated by YT, the amount of voters in the same pool, the amount of vePENDLE in total. Importantly, the valuation of all these rewards combined is what is used to price YTs and thus the implied APY can be far higher than the underlying APY to the extent that some contracts will say "Get a return of -99% if the underlying APY remains constant."

2.2 Correlation Analysis

We use correlation matrices as a qualitative tool to identify expected relationships and examine how well our empirical data conforms to expectations. Our analysis is separated into two phases. First, we examine correlations among the absolute price levels of YTs and PTs (in terms of the underlying) relative to fixed yields, contract durations, and implied APYs, to capture fundamental dynamics. Second, we analyze the correlations between daily returns in token prices and the percentage change in TVL, utilizing percentage values to mitigate concerns of non-stationarity and normalize the data for more accurate correlation assessments. Figures with the complete range of correlation matrices can be found in Appendix A.

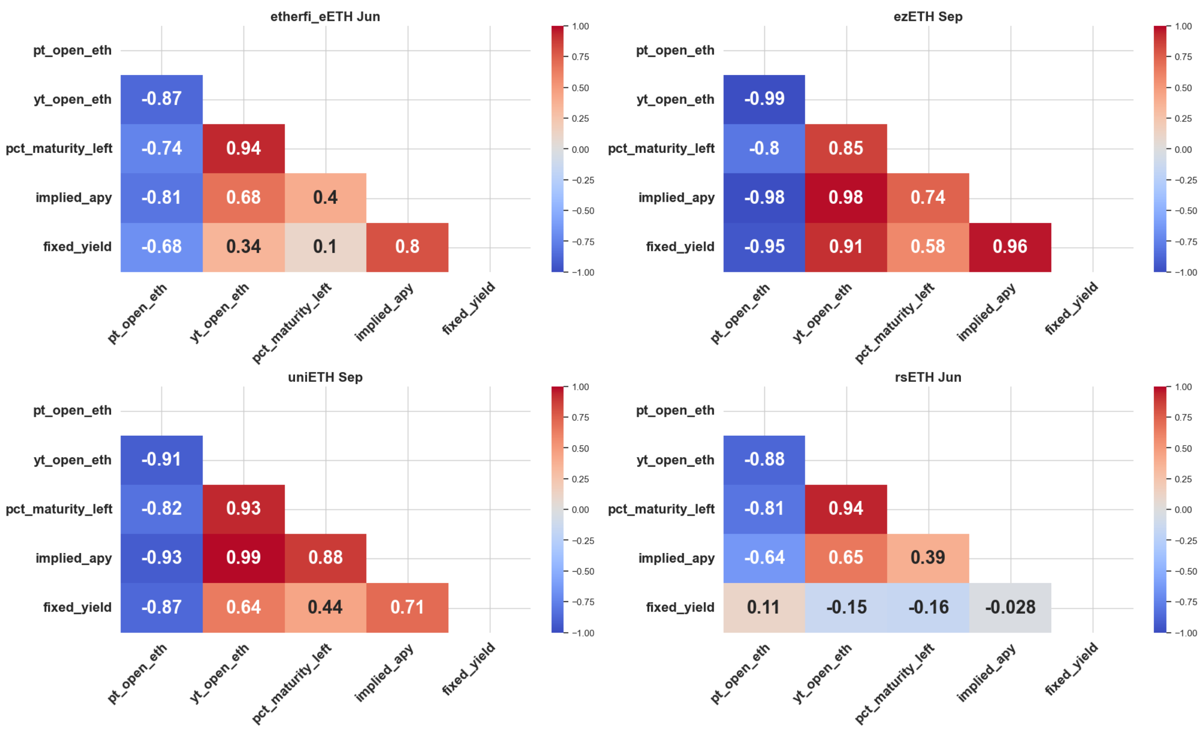

Figure 2: Correlation matrices of Etherfi eETH (Jun), ezETH (Sep), uniETH (Sep), rsETH (Jun) of price levels against fundamental factors. The diagonal and upper triangle have been hidden to preserve visual clarity.

There is a consistently strong positive correlation between implied APY and YT prices, confirming the effectiveness of implied APY as a gauge of sentiment on future yields and consequently YT prices. Maturity of the contract shows strong positive and negative correlations with YT and PT respectively; this aligns with the concept of YT value reducing as yield is emitted. Fixed yield and implied APY generally show strong positive correlations. This is expected since both increase as PT prices increase or YT prices decrease, and we know PT and YT prices counterbalance one another. Out of the examples presented, rsETH Jun shows the most structural weakness, especially in the low correlation between fixed yield and PT opening prices. We will investigate this shortly.

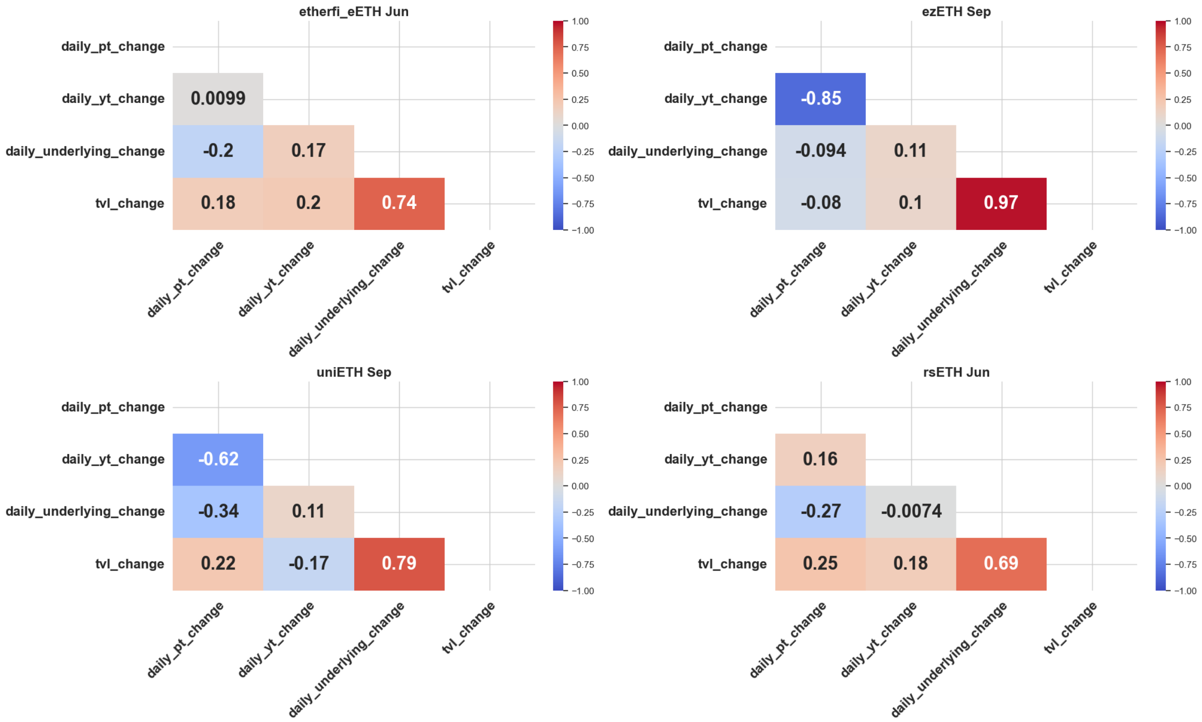

Figure 3: Correlation matrices of Etherfi eETH (Jun), ezETH (Sep), uniETH (Sep), rsETH (Jun) of daily returns and TVL change against each other.

We now examine the correlations between daily returns and TVL changes. Positive correlations are observed between changes in the underlying asset prices and YT prices. This correlation is expected as increases in the underlying asset price typically enhance the value of the base yield generated by YTs. However, these correlations are moderately weak, possibly indicating that movements in underlying prices have a limited impact due to additional yield sources such as points and incentives comprising total yield.

Similarly, correlations between changes in the underlying asset and PT prices are anticipated to be modestly positive, reflecting the PT's pricing mechanism, which is inherently tied to the underlying asset but adjusts less dramatically. This adjustment gap narrows as the maturity date approaches and prices converge. Contrary to expectations, several instances show negative correlations, suggesting that YT returns may respond more sensitively and potentially over-adjust, resulting in reduced PT returns.

There are robust correlations between increases in TVL and increases in underlying asset prices, which are consistently mirrored in both YT and PT price movements to a lesser extent. This indicates that TVL growth positively influences the pricing dynamics of both components.

2.2.1 Analysis of Anomalies

There are some tokens which deviate from theoretical expectations. For example, rsETH Jun and (near the beginning of the contract) Etherfi eETH Jun. Below is a comparison of PT and underlying prices of Etherfi eETH Jun and ezETH Jun. Etherfi eETH conforms well to the theoretical ideas. The gap between the underlying and PT prices converges over time and vanishes at maturity. When standardised in terms of the underlying, the price after the beginning of April follows a slow arc ending at 1. Before this point, there is volatility in the price that weakens some of the correlations one would expect.

On the other hand, the gap between the underlying and PT prices of rsETH Jun converges slower near maturity, with a significant gap still remaining when the contract is due to expire. This leads to large fixed yield values at the time since one can buy a PT at approximately 0.97 of the underlying and redeem it the next day to generate large time-adjusted yields, assuming anyone is willing to sell at market price. Furthermore, there is a period around May 26th where the underlying crashes and the price of the PT is temporarily higher than the price of the underlying. Consequently, the price of the yield token would also have to be negative to maintain the PT/YT invariant, which is nonsensical. Anomalies like these are likely causing unexpected correlations.

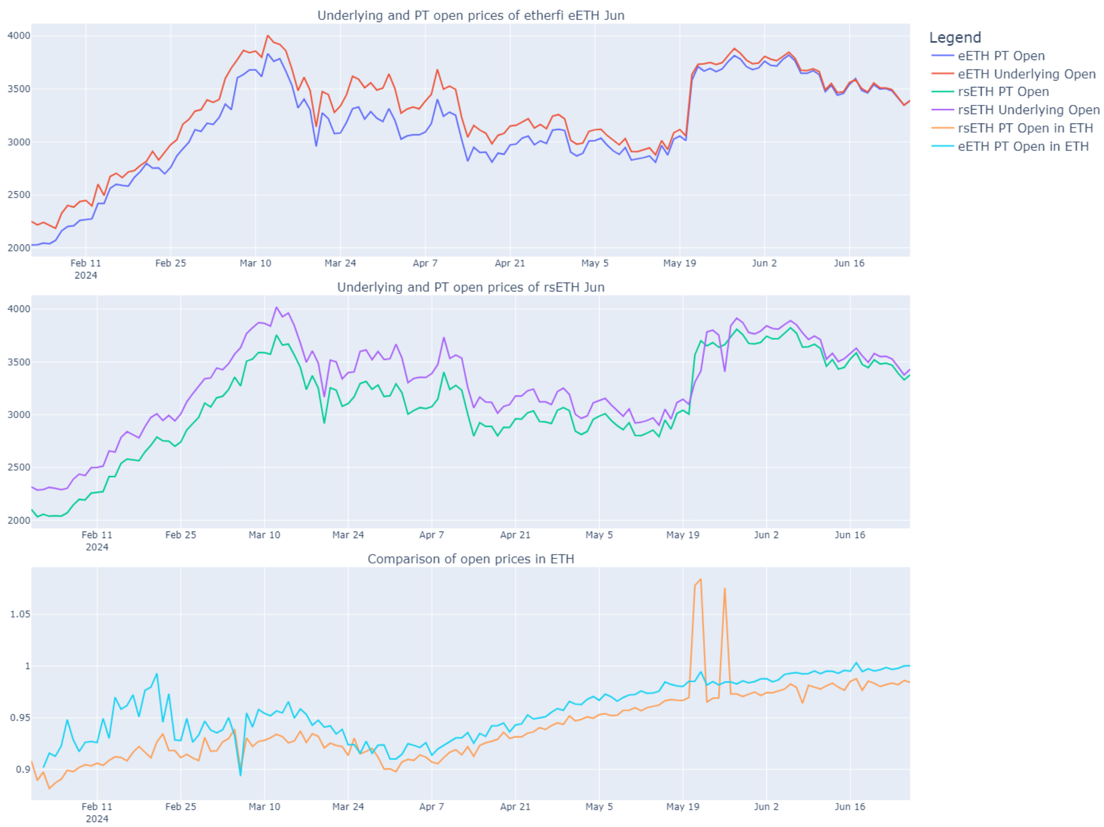

Figure 4: (Top) Underlying and PT open prices of Etherfi eETH Jun. (Middle) Underlying and PT open prices of rsETH Jun. (Bottom) A comparison of the PT open prices in nominal ETH between the two.

Below are the correlation matrices of Zircuit eETH Jun, uniETH Jun, and rsETH Jun respectively, along with Etherfi eETH Jun with and without the volatile period. Zircuit eETH Jun is the prototypical matrix one would expect in line with the dynamics of pricing PT and YT. If the matrix deviates from this realm of correlations, then there is very likely to be market inefficiencies that can be exploited by arbitrage strategies. For example, if the price of PT is higher than the underlying, as seen in the previous example, then one could buy and underlying, swap it into a PT and YT, and sell the PT while keeping or selling the YT. This would net a profit as long as the gain is greater than the swap fees.

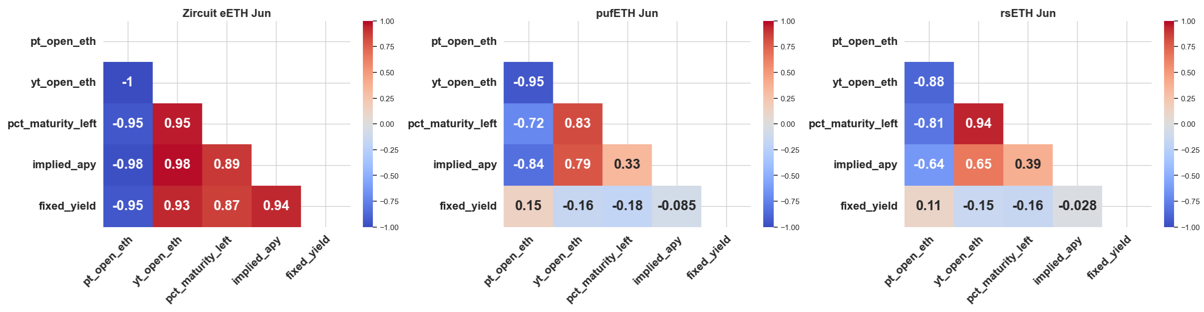

Figure 5: Correlation matrices of (Left) Zircuit eETH Jun, (Middle) uniETH Jun, (Right) rsETH Jun in nominal ETH.

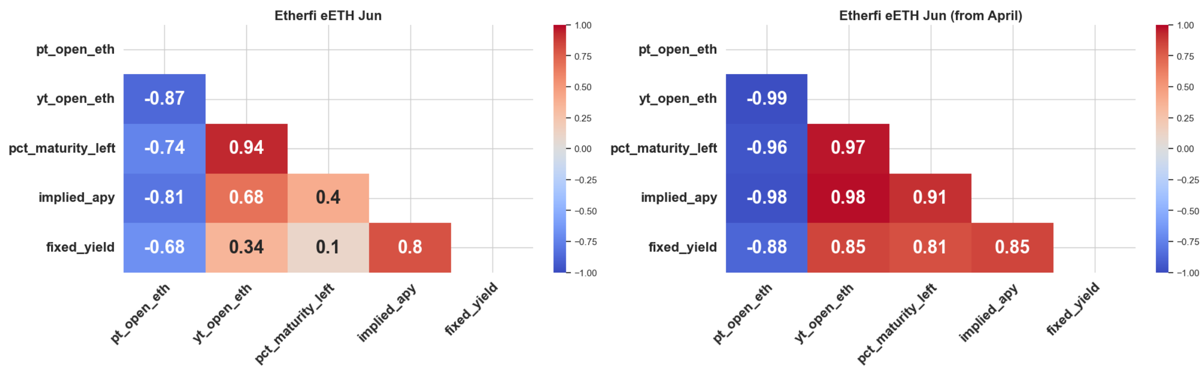

Figure 6: Correlation matrices of (Left) entire Etherfi eETH Jun contract and (Right) Etherfi eETH Jun starting from 1st April. The correlations fall in line with expectations after the early period of volatility, suggesting there were some unexpected pricing dynamics during this time. This can be observed in Figure 4 where the steep increase in price from February led to the gap between PT and underlying closing quickly and thus the PT value almost being the same as the underlying.

2.3 Principal Component Analysis

To understand the drivers of fixed yield and PT prices comprehensively, we employ Principal Component Analysis (PCA). By reducing the dimensionality of our dataset, PCA helps to uncover the principal components that account for the most variance in the data. Through PCA, we aim to isolate the key features that influence PT prices and fixed yields, providing clearer insights into how these elements interact and their overall impact on market behavior.

2.3.1 Fundamental Characteristics

The first PCA looks into the fundamental characteristics discussed previously. Namely, opening prices, implied APY, fixed yield, and the contract's remaining duration. We further solidify the interaction between these.

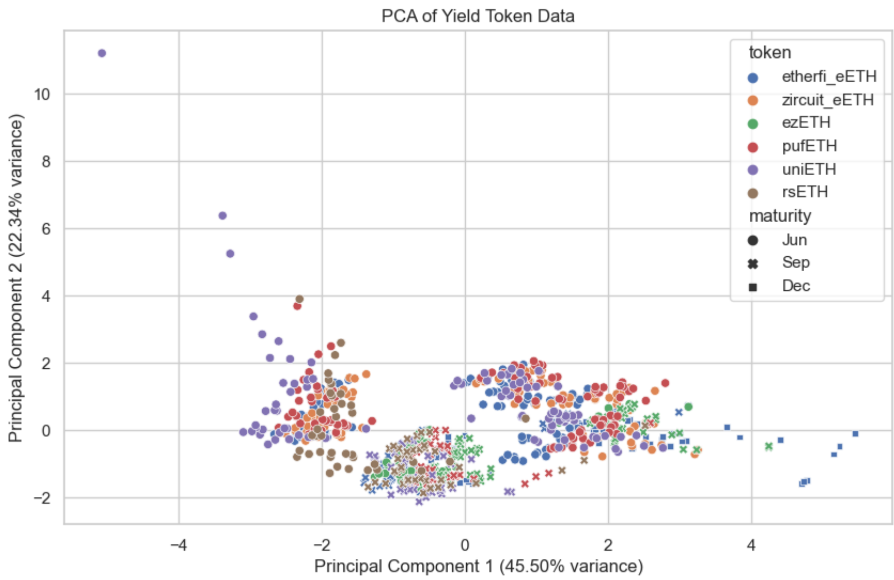

Figure 7: A PCA using PT open, YT open, underlying open, maturity of the contract, implied APY, and fixed yield.

Table 1: Loadings of the Principal Components for Fundamental Components

	PT Open	YT Open	Underlying Open	Maturity Left	Implied APY	Fixed Yield
PC1	-0.5458	0.4721	-0.4600	0.3732	0.3287	-0.1423
PC2	-0.3175	-0.3735	-0.4375	-0.5722	0.2334	0.4317

The PCA reveals that the first principal component explains approximately 45.49% of the variance. The positive loadings on YT open, percentage maturity left, and implied APY suggest that longer maturities and higher anticipated yields drive YT prices higher. This aligns with theoretical expectations where longer duration encompass more yield potential, reflected in higher YT prices. The negative coefficients for PT open and underlying open in the first principal component indicate that increases in YT values, due to higher yields and longer maturities, inversely affect PT prices. This conforms with the concept of the PT/YT invariant. Overall, the loadings of this component support the theoretical pricing model discussed.

The second principal component accounts for an additional 22.34% of the variance, with maturity left showing a strong negative loading. This suggests that PC2 may represent market conditions as contracts near expiry. PT, YT, and underlying prices have negative coefficients in this component, indicating that their prices tend to decline as PC2 increases. This pattern suggests that as contracts approach maturity, market dynamics such as increased volatility or heightened risk assessments may lead to lower values for principal tokens and their underlying assets. The proximity to expiry may amplify uncertainty or speculative trading, resulting in more significant price fluctuations in PT and YT. In contrast, the positive, though smaller, coefficient for implied APY indicates that higher expected yields may be associated with market conditions that drive PC2 upward, such as nearing maturity or declining underlying prices. This highlights how market sensitivity to yield expectations plays a crucial role, especially as contracts near their expiration. The most significant positive loading is observed in fixed yield, supporting the large fixed yields near maturity we have seen.

Recall the yield difference graph shown in Figure 1. Intuitively, it appears that the volatility in the difference is higher towards the end of the contract, which would be in accordance with the relationships in PC2. We confirm this by splitting the data into an early period and late period (with 10% of the contract to go) and using Levene's test on the June contracts. The following results are obtained:

Table 2: Levene's Test Results for Volatility in the Last 10% of the Contract Duration (Jun).

Token	Statistic	p-value
Etherfi eETH	21.14	9.27×10^{-6}
Zircuit eETH	29.64	4.38×10^{-7}
pufETH	45.27	6.19×10^{-10}
uniETH	39.43	5.68×10^{-8}
rsETH	35.49	1.83×10^{-8}

These statistically significant results suggest that the volatility of the yield difference increases as the contract approaches maturity. This heightened volatility near the end of the contract could potentially create arbitrage opportunities. However, these findings might also be influenced by noise and recent market conditions. Therefore, additional data is required to confirm these observations and assess the consistency of this pattern across different market environments.

2.3.2 Trading Volumes

Liquidity data for PT/YT token pairs on Pendle is not publicly available. We use the logarithm of the trading volume of the PT, YT, and underlying token as a proxy for liquidity. Since the trading volume on some days is 0, presumably due to data not being available, we take a 5-day moving average.

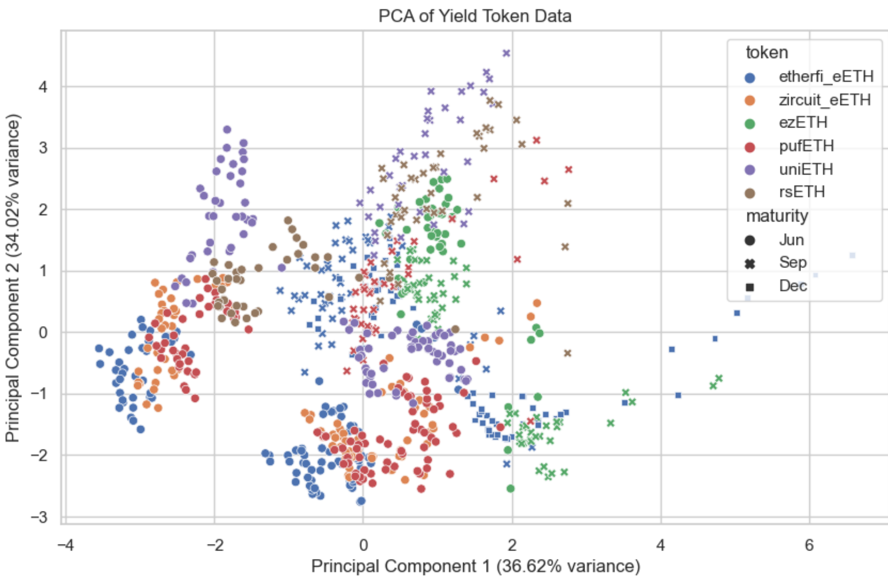

Figure 8: A PCA using PT open, YT open, underlying open, maturity of the contract, and 5-day moving averages of the log volume.

Table 3: Loadings of the Principal Components for Volumes

	PT Open	YT Open	Underlying Open	PT Vol	YT Vol	Underlying Vol	Maturity Left
PC1	-0.3927	0.5109	-0.2216	-0.3260	-0.3003	0.2064	0.5446
PC2	0.4538	-0.0833	0.5370	-0.4951	-0.4995	-0.0211	0.0601

The first principal component, accounting for approximately 36.62% of the total variance, highlights trends in trading behavior and price movements. The negative correlation between PT open prices and this component suggests increased market activity during price dips, likely as investors seek to take advantage of lower prices or sell off their assets. Conversely, higher YT open prices positively impact this component, indicating increased investor interest or speculative activity as these prices rise. A similar negative association is observed with the underlying open prices, where lower prices might signal broader market corrections or bearish sentiments. The trading volumes for PT and YT exhibit negative loadings, implying that higher volumes often coincide with lower token prices, indicative of selling pressure or phases of liquidation. The percentage of maturity left shows a strong positive loading, suggesting longer contract duration correlates with higher component values, reflecting higher risk or yield expectations from longer contracts.

The second principal component, explaining an additional 34.02% of the variance, offers insights into more nuanced behaviors. It highlights that PT and underlying open prices have large positive loadings, suggesting bullish market conditions or strong buying interest when these prices increase. In contrast, YT open prices have a minimal negative loading, indicating a slight inverse relationship and suggesting that increases in PT and underlying prices may coincide with less YT trading. This component focuses on a case where contract maturity and underlying volume impacts are relatively negligible, focusing more on immediate trading dynamics rather than long-term time evolution. PT and YT volumes have significant negative loadings, reinforcing the idea that periods of large trading volumes may depress prices due to market saturation or high liquidity or crashes in the underlying token's price.

The issue with using volumes as a proxy for liquidity is that it is not known whether high volume indicates large sell-offs and price fluctuations or stability in price due to arbitrage bots and liquidity providers. We demonstrate this by looking at the correlation between trading volume and price volatility. We use 30-day annualised exponentially weighted moving averages (EWMA) on the volatility of the PTs, YTs, and underlying tokens. Volume is plotted with respect to the corresponding volatility (e.g., PT volatility against PT volume).

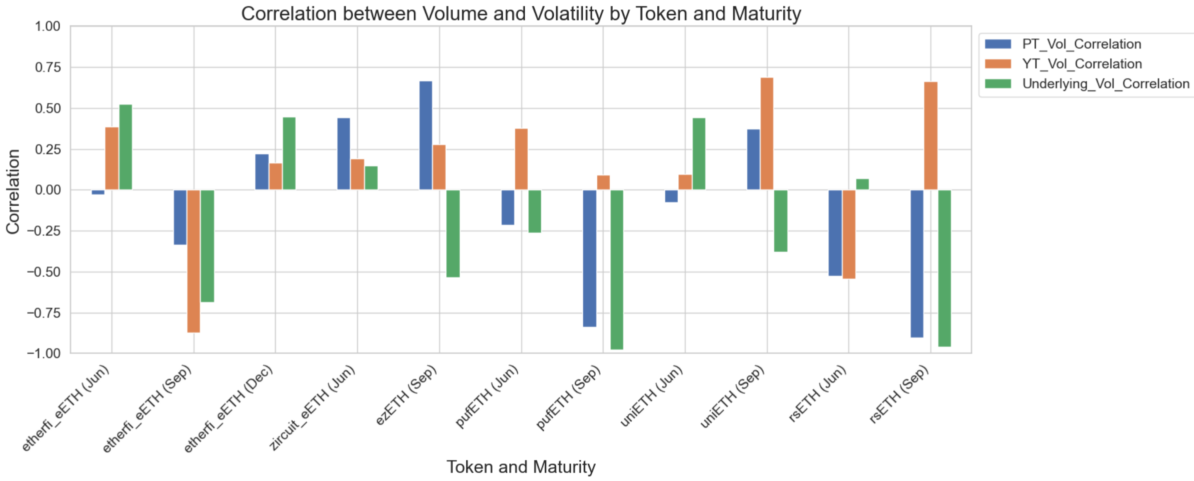

Figure 10: A bar plot of correlations between 5-day moving average of log volume against 30-day annualised EWMA of the respective asset's volatility in price. Note that contracts with the same underlying token will have different maturities since the periods of the contracts are different.

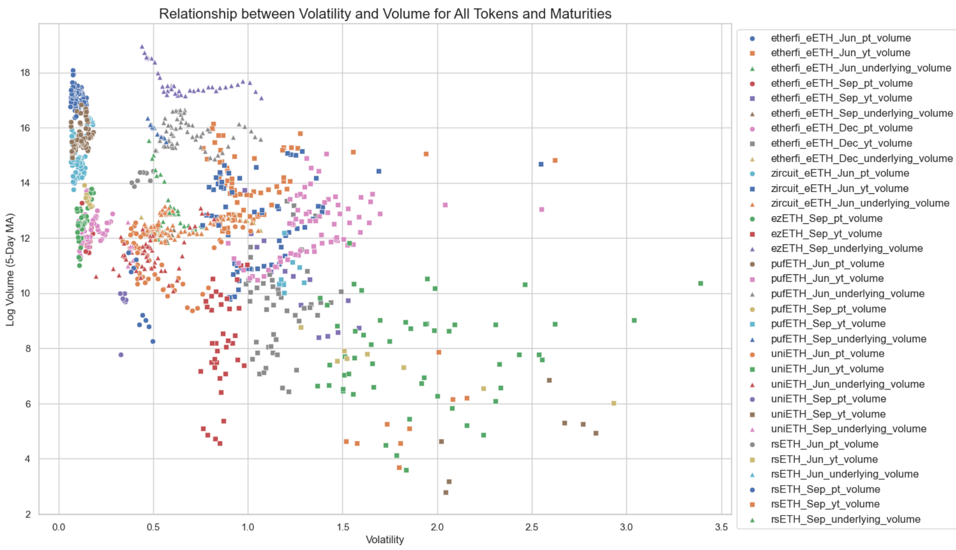

Figure 9: A scatter plot of 5-day moving average of log volume against 30-day annualised EWMA of the respective asset's volatility in price. Some well-formed clusters can be seen; tokens with very low volatility have especially tight clusters. The plot is overall quite noisy, although it could be said that the plot shows a downward trend, with less volatility in tokens with high volume. In addition, the tokens with high volatility and low volume are candidates for arbitrage as they indicate markets with greater price impact from trades.

Correlations vary significantly, as shown in Figure 10, indicating that analysis must be done on a token-by-token basis. Where correlations are positive, token prices tend to fluctuate more when trading volume is high. Where correlations are negative, token prices remain stable during periods of high trading activity. A more detailed analysis must be done, which is out of the scope of this paper: firstly, using more data for other contracts, e.g., Sep and Dec; secondly, considering non-stationarity in the volatility time series leading to fallacious correlations; thirdly, cross-referencing findings with those in liquidity data, should it become available.

2.3.3 TVL

While we will look at the effect of the underlying token's TVL on yields later, we here introduce the role of TVL as a stabilizing factor in price relationships and its utility in clustering tokens.

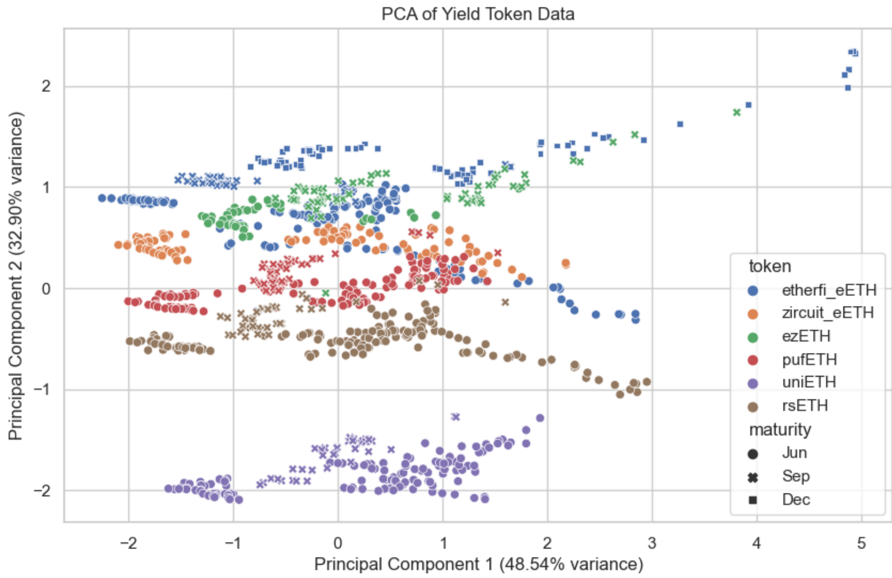

Figure 11: A PCA using PT open, YT open, and log TVL. The addition of log TVL creates strata of tokens which may be useful in clustering algorithms.

Table 4: Loadings of the Principal Components for TVL

	PT Open	YT Open	Log TVL
PC1	-0.700914	0.682236	-0.208022
PC2	-0.055200	0.238893	0.969476

The first principle component, explaining 48.54% of the variance, shows a strong inverse relationship between PT and YT open. This indicates that when PT prices increase, YT prices tend to decrease and vice versa, supporting the PT/YT invariant concept where shifts in one are counterbalanced by the other. The smaller negative loading of log TVL suggests that tokens with higher TVL show a less pronounced inverse relationship between PT and YT prices, possibly indicating more market stability or confidence in tokens with higher liquidity.

The second principle component, explaining 32.90% of the variance, is dominated by the positive loading of log TVL, indicating that variations in TVL are a significant factor distinguishing tokens at this component level. The positive loading implies that tokens with lower TVL might be subject to more volatility or less price stability in YT prices. The minimal loadings on PT on YT open suggest that, at this level, the stability impact of TVL is relatively small.

2.4 Random Forest Regression and Fourier Analysis

So far we have performed multiple PCAs and speculated how each component reflects some mechanism of market dynamics. Now, in order to get a holistic view on the importance of these features, we perform a random forest regression for feature importance on predicting the price of the PT.

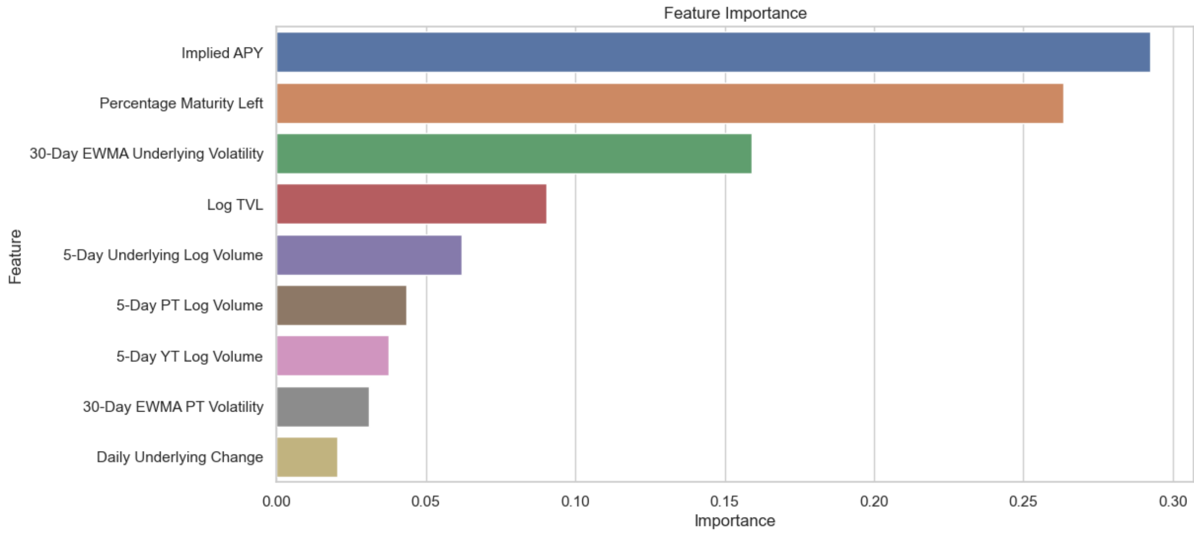

Figure 12: Bar chart of feature importance derived from random forest regression. The number of trees estimators used is 100.

The random forest regression analysis indicates that implied APY is the most influential feature, suggesting that investor sentiment on the expected return on these assets significantly drives market behavior and price movements. This is followed closely by the duration left on the contract, which underscores the critical nature of time decay and appreciation in pricing YT and PT respectively. Interestingly, log TVL also shows significant importance, reflecting how liquidity influences market stability and investor confidence. We explore this in the next section.

While PT volatility does play a role, its relative importance is overshadowed by factors directly related to returns and market activity, affirming that traders prioritize return potentials over market volatility when making investment decisions. Again, this is expected for investors and liquidity providers of fixed yield because they always have the option to hold until maturity to prevent divergence loss. However, since the daily change in the underlying token has an impact, as PTs and YTs are ultimately pegged to it, the volatility in the underlying is much more significant as it leads to potential depegs and price loss in the case when USD/ETH risk is not hedged.

The analysis also reveals that trading volumes for both PTs and YTs, though significant, have less impact than the Implied APY and maturity features. PT volume is most significant, although YT volume is similarly important. This highlights how intertwined sentiments about PT and YT are. The underlying volume is slightly ahead than the others, perhaps being a signal of the underlying market being healthy.

Our findings are further supported by a Fourier analysis, which we conducted to examine the frequency components of PT prices. The Fourier analysis, particularly through the examination of power spectral density (PSD), reveals dominant low-frequency components which corroborate the significance of long-term trends inferred from the random forest results. The relatively flat spectrum at higher frequencies implies that short-term price movements are more random and less predictable. This is consistent with the market being efficient.

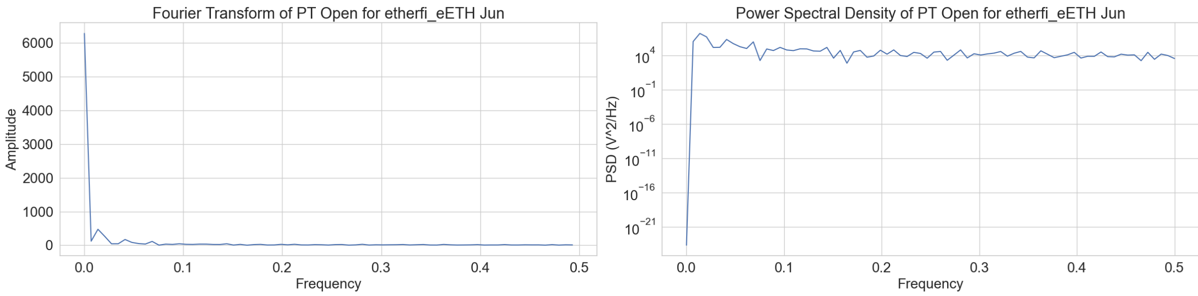

Figure 13: An example of a Fourier analysis for Etherfi eETH Jun. All plots look similar with high activity at low frequencies and a negligible contribution from higher frequencies.

3 Liquidity Impact of the Underlying Token

In many financial settings, illiquid instruments offer higher yields for the risks they come with. For example, corporate bonds have higher yields than treasury bonds due to a company's probability of defaulting being non-negligible. We attempt to quantify the existence of a similar effect by considering whether the TVL for the underlying token impacts the respective YT and PT. In particular, we use the quantiles of log TVL to partition groups of tokens and verify whether there is an implicit difference in the fixed yield and implied APY. Here, quantile 0.0 represents the lowest TVL (least liquid pools) and quantile 3.0 represents the highest TVL (most liquid pools).

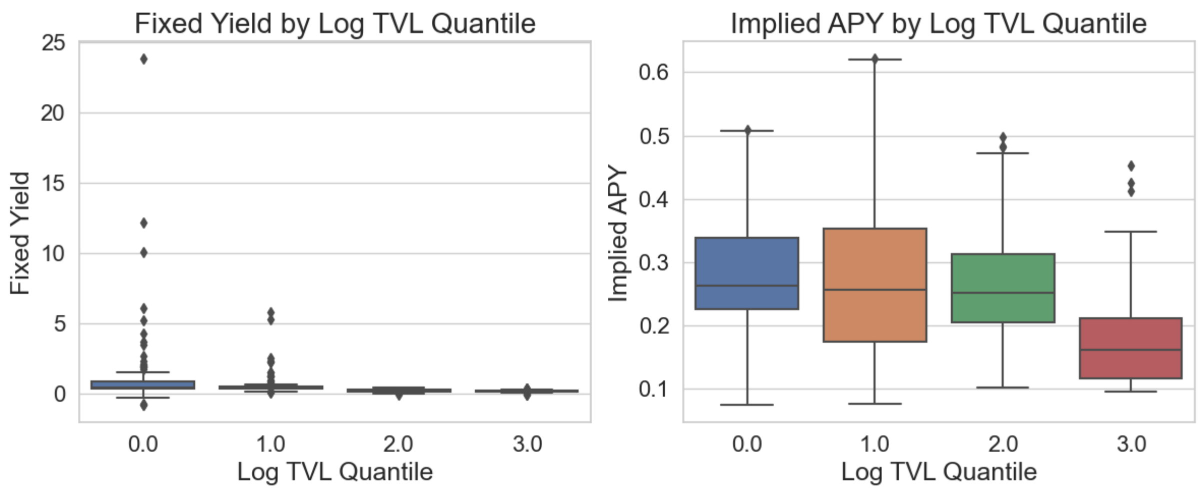

Figure 14: (Right) Box plot of fixed yields by log TVL quantiles. (Left) Box plot of implied APY by log TVL quantiles.

By using a box plot, we can see the median and spread of implied APY and fixed yield as well as well as any outliers. Notice that fixed yield plot is heavily skewed by outliers. This is related to the phenomenon we saw previously were prices were not converging as quickly as expected near maturity in some tokens, which caused exponentially-increasing fixed yields. The existence of these outliers is indicative of tokens which often break the PT/YT invariant and are inefficient, perhaps due to lower liquidity and trading volume. The differences are more apparent in the implied APY plot. Quantiles 0.0, 1.0, and 2.0 have similar medians, with the inter-quartile spread of 2.0 being the greatest. Quartile 3.0 has a significantly lower median and tighter inter-quartile spread but does have some outliers.

To confirm quantitatively, we use the Kruskal-Wallis test to measure if there are differences in medians between the group. The tested observed statistics of 316.439 (p-value: 3e-68) for implied APY and 122.422 (p-value: 2e-26), indicating statistically significant differences across log TVL quantiles. A permutation test using sample means as a test statistic demonstrates the means of the fixed yield for all groups are not the same (p-value: 2e-4). For more granularity, we use Conover's test for pairwise comparisons between quantiles. The values in the cells represent the p-values of the pairwise comparisons, adjusted using the Bonferroni correction.

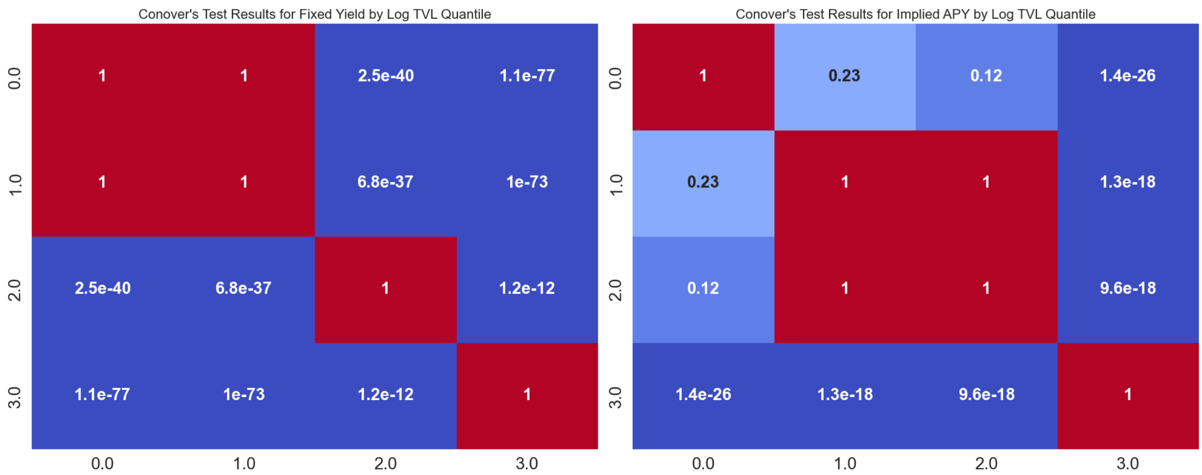

Figure 15: (Left) Conovers's test for fixed yield between log TVL quantiles. (Right) Conover's test for implied APY between log TVL quantiles.

From the above heatmap for fixed yield, we observe that quantiles 0.0 and 1.0 have the same median. We therefore get three gradations of median fixed yield by log TVL. From the heatmap for implied APY, we see that quartiles 0.0, 1.0, 2.0 have a similar median and quartile 3.0 is different, as discussed previously. The p-values are small enough to warrant further investigation, however, since volatility during this contract cycle could skew the figures. The overall distributions can be seen visually via the bootstrapping method.

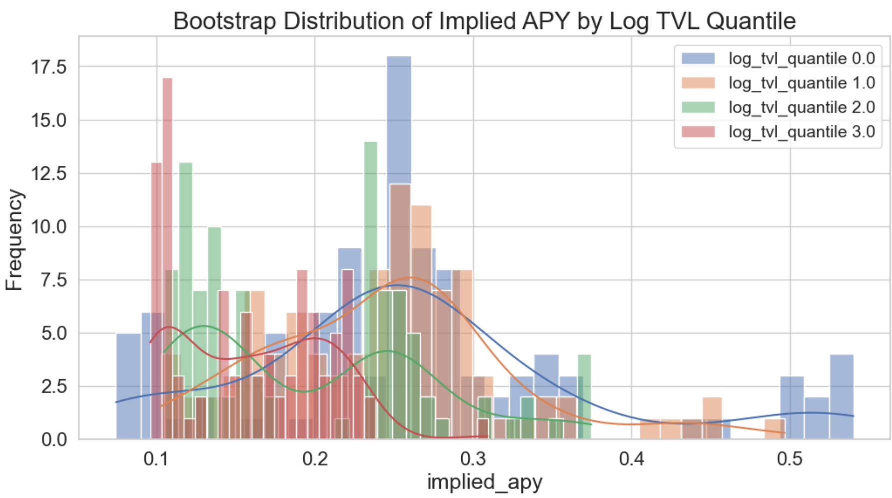

Figure 16: An example of bootstrapped distributions generated by sampling from different log TVL quantiles.

In conclusion, the analysis suggests that illiquid pools tend to offer higher fixed yields and implied APYs, supporting the hypothesis that illiquid pools outperform in terms of returns. This is likely a premium afforded to investors for the higher risk and lower liquidity associated with these pools. Here one can argue that, since all underlying assets are pegged to ETH, any price discrepancies will naturally be neutralized and therefore the higher implied APY might also be influenced by the underlying token's liquid restaking protocol to incentivise investors to choose one protocol over another.

4 Movement of LRTs With One Another

There are three features of contracts to be varied when comparing them: the expiry date (e.g., Etherfi eETH Jun vs. Etherfi eETH Sep), the underlying token (e.g., Etherfi eETH Jun vs. uniETH Jun), and the liquid restaking protocol (e.g., Etherfi eETH Jun vs. Zircuit eETH Jun). To study the movements of LRTs we use ratios between the two sets such that:

$$\mathrm{PTR} = \frac{\mathrm{PT}_1}{\mathrm{PT}_2}, \qquad \mathrm{YTR} = \frac{\mathrm{YT}_1}{\mathrm{YT}_2}, \qquad \mathrm{UR} = \frac{\mathrm{Underlying}_1}{\mathrm{Underlying}_2},$$

where these ratios are functions of time. The idea is to find how they develop through the duration of the contract and if it is possible to tell when one token pair is overvalued compared to another. Figure 17 shows these ratios for the June contracts.

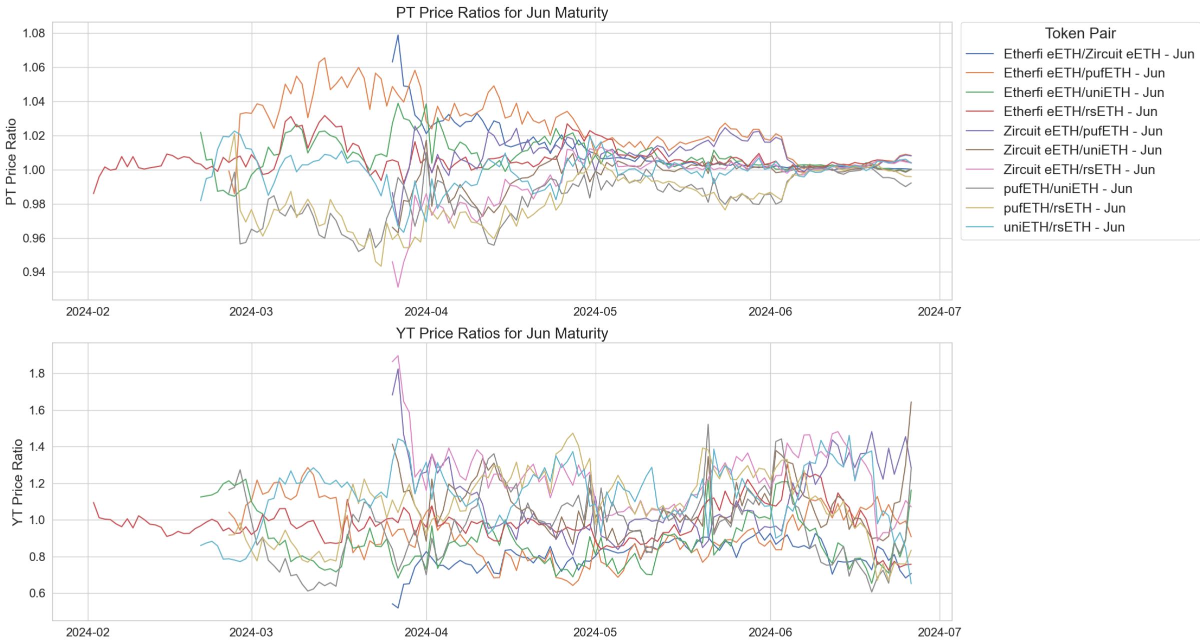

Figure 17: Ratios of the open prices of PT and YT tokens for June contracts.

Now we look at a selection of token pairs with ratios for PT, YT and the underlying plotted together. A greater range can be found in Appendix B and C.

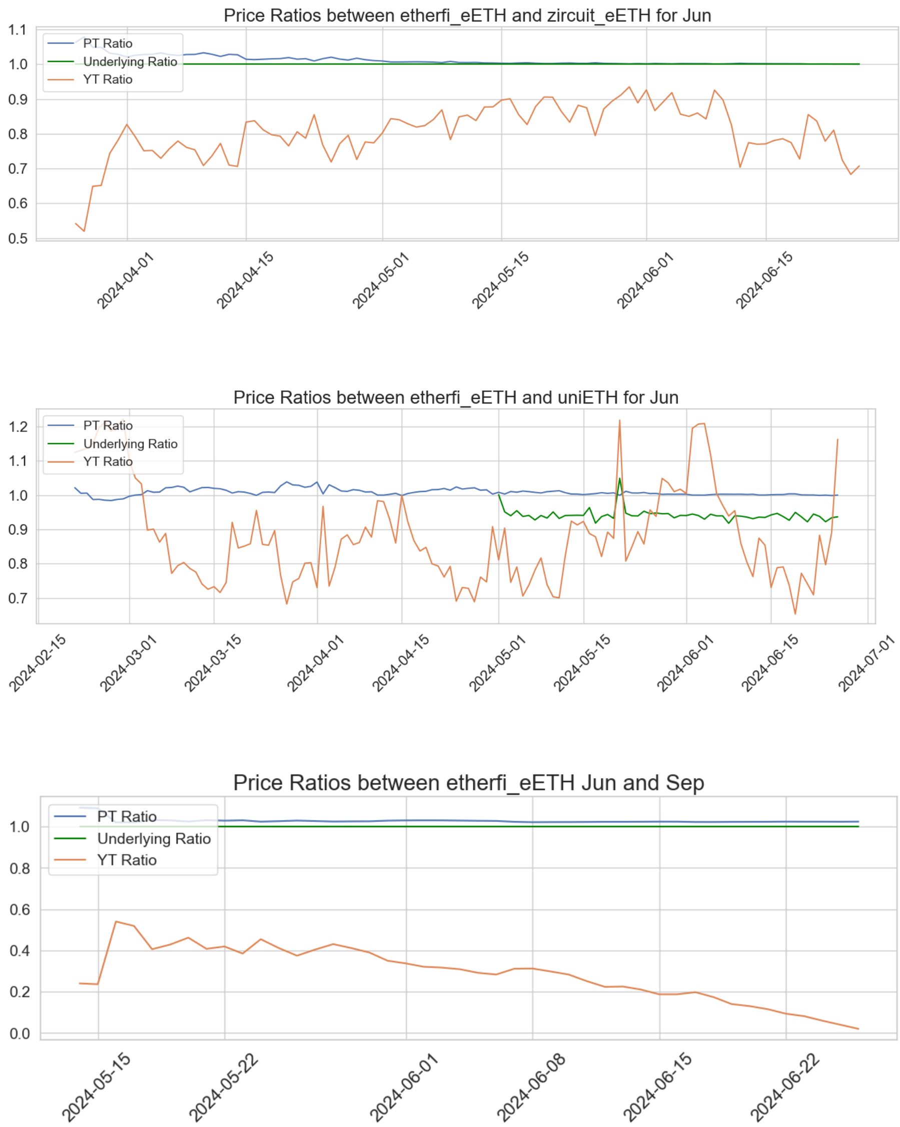

Figure 18: (Top) Price ratios between Etherfi eETH and Zircuit eETH. (Middle) Price ratios between Etherfi eETH and uniETH Jun. (Bottom) Price ratios between Etherfi Jun and Sep.

The top graph shows how contracts on the same underlying with different protocols might interact. Given they are both based on eETH, the only way there can be a price discrepancy between the two is if one offers more rewards. Here, we can see that Zircuit eETH YT is more expensive that the Etherfi one, indicating the Zircuit is offering more rewards. As the contract matures, the PT ratio will always tend towards one because both PTs can be redeemed for the same underlying.

The middle graph presents an interesting opportunity to build trading strategies. Near maturity, we know that the price of the PTs of Etherfi eETH and uniETH Jun are approximately equal whereas uniETH itself is more valuable then Etherfi eETH. Although the YT for Etherfi eETH is more expensive than for uniETH, since both YTs will be very close to worthless, it is negligible. Therefore, long uniETH PT and short Etherfi PT is likely worth it. Indeed, there are many potential statistical arbitrage ideas that appear from these ratios. For example, it appears the the volatility of the YT ratio for inter-tokens pairs is higher than that of the PT and underlying ratio. This is confirmed by Levene's test which gives statistically significant results for all pairs. One can argue that if both the YT ratio and the PT ratio are higher then the underlying ratio, and if the YT ratio is sufficiently high, then the YT ratio will follow a reverting process leading to large swings if it gets too high or low without much perturbing the PT ratio.

Finally, we look at the bottom graph of Etherfi eETH Jun against Sep. When looking at inter-tenor ratios where one has reached expiry, it will always be the case that the YT of the shorter contract will decay in price faster than the other and eventually become worthless as the contract expires. Consequently, the YT ratio will always drift towards either zero or infinity depending on the orientation of the ratio (we always ensure it tends to zero for clarity).

4.1 Johansen Cointegration Test Between Pairs

While correlation provides insights into the synchronous movements of assets over time, cointegration explores whether there exists a stable, long-term equilibrium relationship despite short-term deviations. This section focuses on employing cointegration testing to assess the relationships between different LRTs. We use the Johansen cointegration test since it is more robust for multiple time series. We use a lag of 1 day under the assumption that market prices adjust to news or events between market open times.

For inter-token ratios we assume there is no linear trend (drift) since the ratios are a stochastic process. However, for inter-tenor ratios, we assume that there is a drift, as was explained in the previous section. These assumptions are integral for this test and, although we believe they are well-justified, violating them leads to very different conclusions.

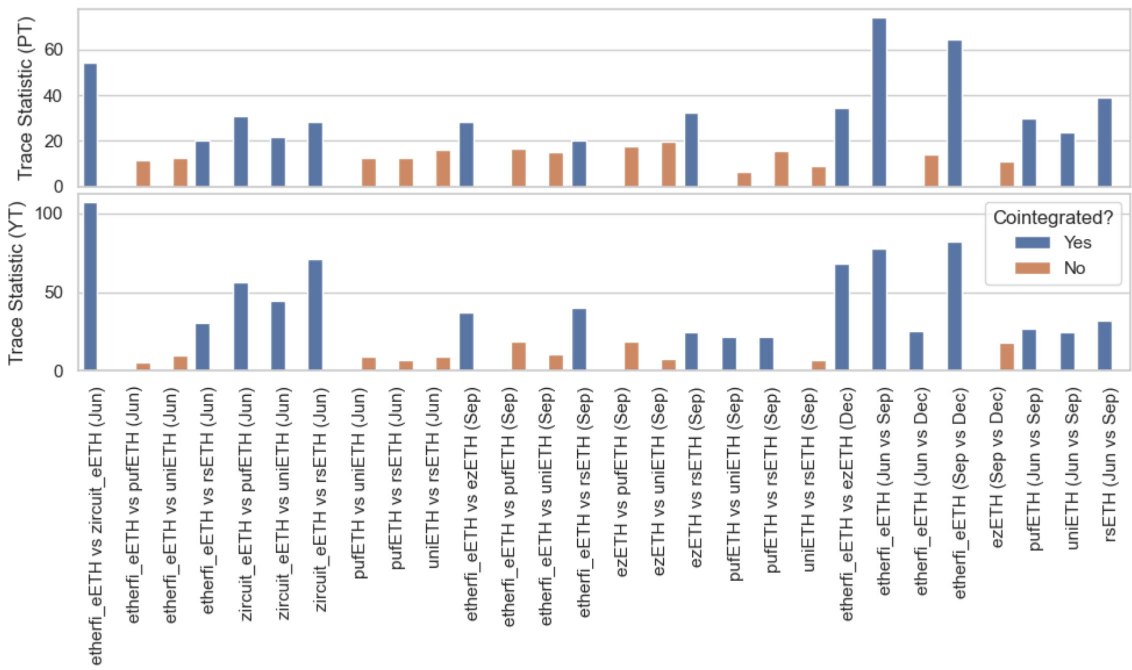

Figure 19: Johansen cointegration test for pairs of PT and YT tokens under different contracts.

The Johansen cointegration indicates a wide variety of the token pairs are cointegrated in both PT and YT. This is especially prevalent in inter-tenor pairs. Furthermore, we note that every pair with cointegrated PT has cointegrated YT, although it is not immediately obvious why. In order to understand how cointegrated dynamics play out, we use vector error correction models (VECMs) to check how strong the cointegration factors are. The implication for trading is the potential for pairs trading strategies where one would go long on one token and short on the other when the spread between them widens, betting on the spread to revert to its historical mean when error correction is sufficiently high given the contract's timescale.

Table 5: VECM results for Etherfi eETH Jun (PT).

Parameter	Coef.	Std. Err.	z	P> $\|z\|$	[0.025	0.975]
Lagged Endog. Parameters						
L1.etherfi_eETH_Jun	20.6687	3.666	5.637	0.000	13.483	27.855
L1.etherfi_eETH_Sep	0.1242	1.959	0.063	0.949	-3.715	3.963
L1.etherfi_eETH_Dec	1.2007	0.383	3.135	0.002	0.450	1.951
L1.zircuit_eETH_Jun	-16.3023	3.089	-5.277	0.000	-22.357	-10.247
L1.ezETH_Sep	-0.3465	0.460	-0.753	0.451	-1.248	0.555
L1.ezETH_Dec	-0.4061	0.496	-0.818	0.413	-1.379	0.567
L1.pufETH_Jun	-1.4881	0.667	-2.232	0.026	-2.795	-0.181
L1.pufETH_Sep	-1.0077	0.378	-2.663	0.008	-1.750	-0.266
L1.uniETH_Jun	-4.0770	1.277	-3.194	0.001	-6.579	-1.575
L1.uniETH_Sep	0.7879	0.191	4.123	0.000	0.413	1.162
L1.rsETH_Jun	1.7619	0.744	2.369	0.018	0.304	3.219
L1.rsETH_Sep	0.1001	0.442	0.227	0.821	-0.766	0.966
Loading Coefficients						
ec1	-15.9458	5.205	-3.063	0.002	-26.148	-5.743
ec2	-6.7475	1.906	-3.540	0.000	-10.483	-3.012
ec3	0.0146	0.728	0.020	0.984	-1.413	1.442
ec4	26.4411	4.514	5.857	0.000	17.594	35.288
ec5	-3.3710	0.497	-6.786	0.000	-4.345	-2.397
ec6	1.4060	0.401	3.504	0.000	0.620	2.192
ec7	1.0162	1.024	0.992	0.321	-0.991	3.024
ec8	1.1856	0.453	2.616	0.009	0.297	2.074
ec9	-7.7776	2.611	-2.978	0.003	-12.896	-2.659
ec10	-0.4062	0.196	-2.076	0.038	-0.790	-0.023

We first look at the cointegrating relationships for PT but, given the volume of data, discuss only Etherfi Jun PT as an example. We chose to employ a rank 10 VECM to avoid overfitting while capturing the most significant relationships. A rank 10 model effectively incorporates the major dynamics necessary for understanding the adjustments toward long-term equilibrium, without being overly sensitive to minor fluctuations. The VECM analysis provides insights into the speed and direction of adjustments required to return to long-term equilibrium after short-term deviations. The lagged endogenous parameters show the effect of the previous day's prices on future price.

Each component vector (ec1 to ec10) serves as a corrective mechanism, indicating how strongly each cointegrated pair contributes to the equilibrium adjustment. For example, a significant and negative loading coefficient (e.g., ec1 = -10) indicates a strong correction mechanism pulling the series back towards equilibrium when there is a deviation. This implies that the identified pairs are likely to revert to their historical mean, making them suitable candidates for pairs trading strategies. Table 11 in Appendix E gives the loadings for each error correction component.

Significant adjustments are highlighted by coefficients such as ec1 and ec4. In particular, ec1 with a coefficient of -15.9458 suggests a substantial downward correction, indicating potential overvaluation scenarios where short positions might be profitable. Conversely, ec4 with a coefficient of 26.4411 implies significant upward adjustments, highlighting scenarios where prices are undervalued and likely to increase, thus presenting opportunities for long positions.

Other coefficients like ec3 (0.0146) and ec10 (-0.4062) suggest negligible or slight adjustments, pointing to stable relationships or minor adjustments that might not offer substantial arbitrage opportunities. On the other hand, ec2, ec5, and ec9, which present notable negative adjustments, could be essential for high-frequency strategies focusing on quick corrections after positive deviations, suitable for short-selling strategies.

Potential pairs trading strategies derived from the lagged endogenous parameters can be put into three categories. For each we give examples of Etherfi Jun potential pairs trading strategies:

- **Same underlying, different protocol.**

  A positive significant coefficient for L1.etherfi_eETH_Jun (20.6687) and a negative one for L1.zircuit_eETH_Jun (-16.3023) suggests a divergence in price movements. Traders can go long on etherfi_eETH_Jun and short on zircuit_eETH_Jun when their price movements diverge. This follows from the result found in the ratio analysis that the PT price will converge since they are based on the same underlying.

- **Same underlying, different durations.**

  A positive coefficient for L1.etherfi_eETH_Dec (1.2007) indicates a direct relationship. One can go long on both tokens when their prices diverge from the norm. Furthermore, tokens of different durations can be used to hedge based a historical beta value.

- **Same duration, different underlying.**

  The negative coefficient for L1.uniETH_Jun (-4.0770) indicates an inverse relationship with etherfi_eETH_Jun. The negative coefficient for L1.pufETH_Jun (-1.4881) also suggests an inverse relationship. One idea is to go long on etherfi_eETH_Jun and short on uniETH_Jun upon divergence.

  On the other hand, a positive coefficient for L1.rsETH_Jun (1.7619) suggests a direct relationship. One can go long on both etherfi_eETH_Jun and rsETH_Jun when they diverge from their typical correlation.

  In general, this last category is the least reliable since many protocol-based factors can influence the spread. These relationships are not necessarily reproducible in live trading. In theory, shorter-term historical betas can be used to more accurately model spread.

The main takeaway is the prevalence of cointegrating relationships in the first two categories. Any token pair that fit this criteria is a candidate for pair trading. It is crucial to note that these findings are specific to PT prices, which tend to drift towards equilibrium and converge. The PT market typically shows less volatility compared to YT markets. The YT, given its higher volatility, might present a more rewarding landscape for pair trading.

4.2 Johansen Cointegration Test For Yields

So far, we discovered potential trading pairs of PTs, with a specific focus on same underlying pairs. We perform a similar process between YTs and expected yields. The reason is twofold: firstly, seeing similar relationships in YTs reinforces their existence and robustness in another setting; secondly, we introduce the possibility of trading YTs based on the historical spread between expected yields and YT prices. Expected yields are defined as implied APY $*$ underlying token price.

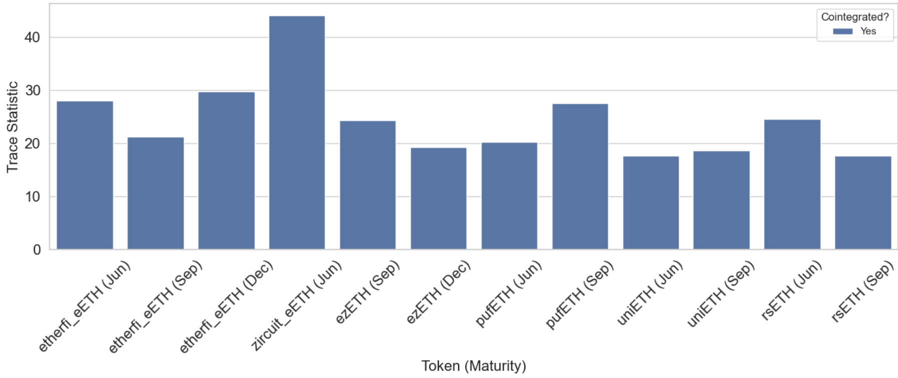

Figure 20: Johansen Cointegration test between YT prices and expected yields of the same token.

The VECM tables are omitted for conciseness. We once again saw potential in same underlying pairs with etherfi_eETH_Jun (20.6687) and etherfi_eETH_Sep (0.1242), as well as etherfi_eETH_Jun (20.6687) and etherfi_eETH_Dec (1.2007), where positive coefficients suggest a strategy of going long on both tokens during price divergences. Similarly, uniETH_Jun (-4.0770) and uniETH_Sep (0.7879), and pufETH_Jun (-1.4881) and pufETH_Sep (-1.0077) showed potential for pairs trading, albeit with mixed signals. In the same underlying, different protocol category, etherfi_eETH_Jun (20.6687) and zircuit_eETH_Jun (-16.3023) presented a clear divergence.

Finally, the inverse relationships observed between etherfi_eETH_Jun (20.6687) and uniETH_Jun (-4.0770), and pufETH_Jun (-1.4881) and rsETH_Jun (1.7619) suggest opportunities across different underlying tokens.

These findings reinforce the PT analysis, confirming consistent cointegration patterns and pairs trading potential within the same underlying pairs across durations and protocols.

5 Simulation of Trading Strategies

This section covers some miscellaneous information about pricing PTs and YTs before looking at possible arbitrage strategies. We first look at how well the PT/YT invariant is kept. We then backtest some of the trading ideas generated by our analysis so far.

5.1 Effectiveness of the PT/YT Invariant

We test how well the PT/YT invariant is kept, that is, $\text{PT}_{ETH}(t) + \text{YT}_{ETH}(t) = 1$. This is done by fixing the price of PT and plotting the theoretical $1 - \text{PT}$ price against the actual YT price. A full range of graphs can be found in Appendix F.

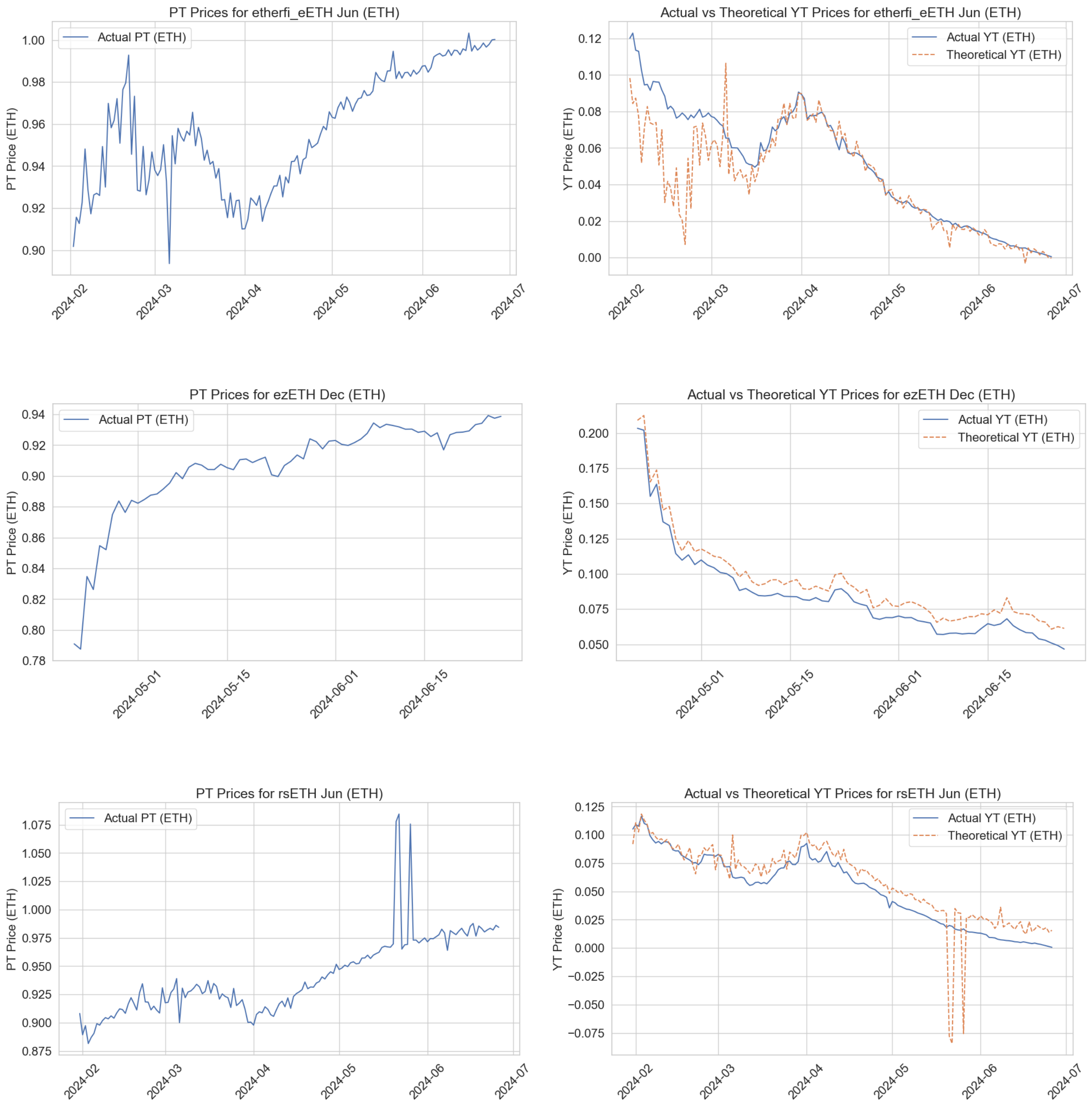

Figure 21: Actual vs. theoretical YT prices for (top) Etherfi eETH Jun, (middle) ezETH Dec, and (bottom) rsETH Jun.

The graphs in Figure 21 show some variability in the PT/YT invariant. The prices for Etherfi eETH Jun (and any contract with eETH as the underlying) are most consistent, showing small fluctuations that quickly correct. Other contracts have some discrepancy between actual and theoretical YT prices. For example, ezETH Dec has a small deviation but is likely not profitable with gas fees. In addition, rsETH had some huge swings in the price of the underlying that lead to the PT prices failing out of equilibrium temporarily. The caveat here is that we assume the gain from the trades are larger than swap and gas fees.

5.2 Backtesting

There are a few caveats to our simulations: firstly, there is an in-sample bias from the small range of contracts and trading days we backtest; secondly, we assume that transactions can be made at market prices and without price impact, allowing positions for each token to be changed by 1 for each day; thirdly, we don't account for swap and gas fees. In reality, the possible alpha from such arbitrage trades is more sparse due to fees and price impact. We assume an annual risk-free rate of 5% to calculate Sharpe and Sortino ratios. Both ratios are calculated with logarithmic returns from data starting from when the first trading signal was made, not necessarily when the contracts(s) started. This is because no capital is locked up until that point.

High Fixed Yields and Implied APYs

We have seen that PTs can occasionally have extremely fixed high yields. This strategy involves shorting the underlying and going long on the PT when fixed yields exceed a threshold, selling when they fall below another. This means that if the PT prices fall lower than expected, we can hold to obtain the fixed yield at which we originally bought by redeeming at maturity, and if PT prices are higher than expected, we can sell immediately.

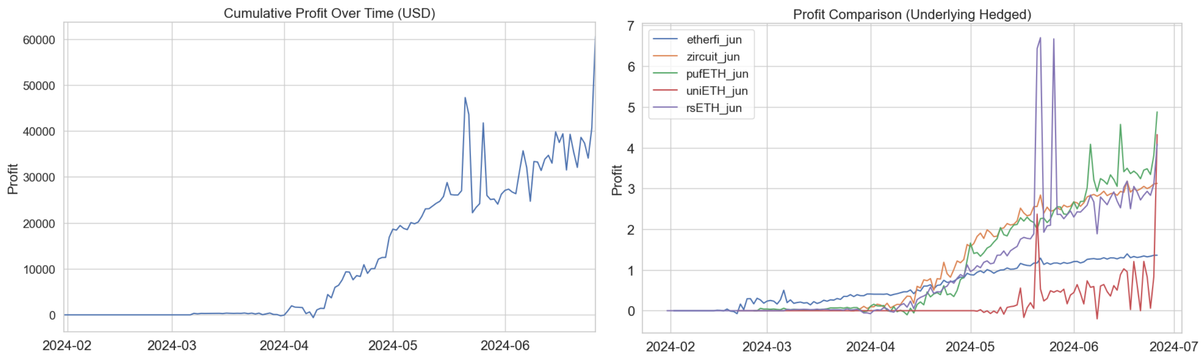

Figure 22: (Left) Overall profit in USD. (Right) Profit for each contract in underlying ETH.

Figure 22 shows this strategy for the June contracts. In this example, the buy threshold to 0.4, and the sell threshold to 0.1. The Sharpe and Sortino ratios for the strategy in USD are 0.88 and 0.85 respectively and the metrics are more promising with the underlying hedged at a 3.67 Sharpe ratio and 4.75 Sortino ratio. Ratios for individual tokens strategies are shown in Table 6:

Token	Sharpe Ratio	Sortino Ratio
Etherfi eETH	3.46	4.59
Zircuit eETH	4.44	6.77
pufETH	3.63	6.54
uniETH	1.43	2.24
rsETH	1.32	1.28

Table 6: Sharpe and Sortino ratios for high fixed yield strategy.

Despite the Sharpe ratios for uniETH and rsETH being somewhat low, this is more to do with the fact that the PT was consistently undervalued and then had a huge spike at maturity rather than a bad performing strategy. This shows how downside risk is implicitly managed by the strategy.

There are two related strategies worth mentioning. The first is to buy high fixed yields especially close to maturity to gain large returns with little risk. However, due to price impact, the capacity for this method is low. The other is to short the YT when implied APY is very high (e.g., 0.4). These expectations are unrealistic over the course of the contract even when accounting for all additional rewards.

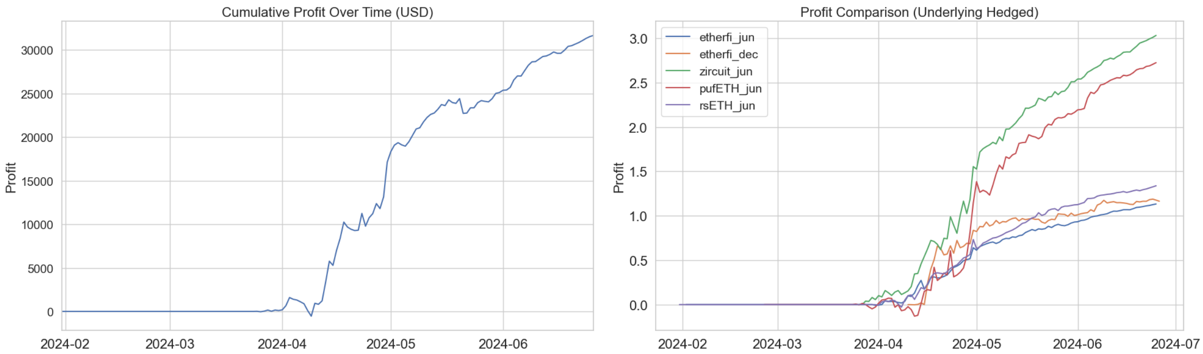

Figure 23: High implied APY strategy. (Left) Overall profit in USD. The strategy in USD has a Sharpe Ratios of 2.81 and 2.68. (Right) Profit for each contract in underlying ETH. The benefit of this strategy is that all contracts can be used, although some never create trading signals; these are removed from the graph for clarity.

Once again, the strategy performance when the underlying is hedged is better. These results must be taken cautiously since we cannot replicate the exact pricing of yield incentives. To compensate, we have been generous in assuming annual APY for all YTs, including rewards and the base rate, is 30%.

Token	Sharpe Ratio	Sortino Ratio
Etherfi eETH Jun	8.26	9.17
Etherfi eETH Dec	5.12	13.95
Zircuit eETH Jun	8.03	15.61
pufETH Jun	4.90	5.80
rsETH Jun	8.24	9.91

Table 7: Sharpe and Sortino Ratios for high implied APY strategy.

Underlying Protocol Based

We have seen that in the case of contracts based on the same underlying, such as Etherfi eETH and Zircuit eETH, that there is a long-term equilibrium in the price of their PTs, which both converge to the same value approaching maturity. This mean that we can long the token with the higher yield and short the token with the lower yield. Equivalently, we can short the more expensive PT, long the less expensive PT, and wait for convergence. One benefit of this long/short strategy is low capital requirement.

In our collected data, the only test case we have is Etherfi eETH and Zircuit eETH. In general, one can compute which token offers better rewards (likely to be the protocol with lower TVL as they must offer better incentives to compete).

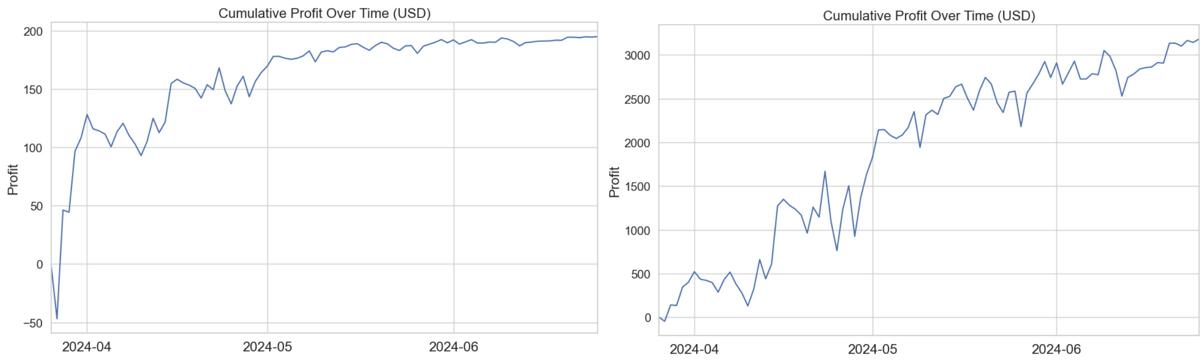

Figure 24: (Left) Profit from buying the spread at contract inception and holding. Sharpe ratio of 3.01 and Sortino ratio of 7.94. (Right) Profit from buying the spread daily since the percentage changes become smaller over time. Sharpe ratio of 2.91 and Sortino ratio of 4.22.

TVL Factor

Given the liquidity effect we discovered, we can generalise the previous strategy to a factor strategy on multiple tokens. We rank contracts in order of their TVL and attach a sell signal to the top n YTs and buy signal to the bottom n YTs to capitalise on the difference. An additional adjustment to the rank can be based on volatility; in this case, we use 30-day volatility of the underlying. The signals are only created if there exist at least $2n$ contracts to maintain long/short balance.

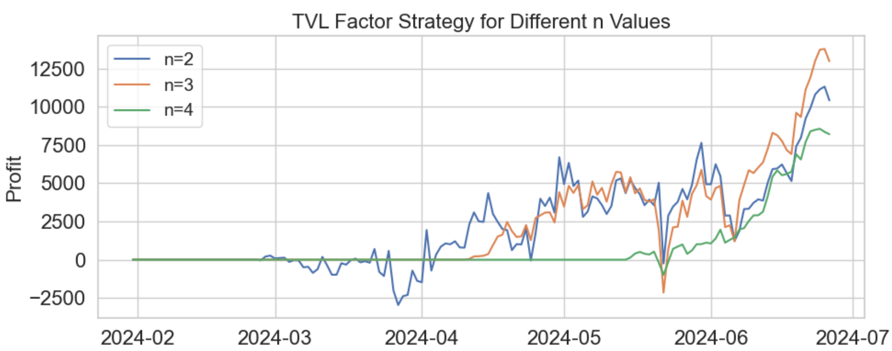

Figure 25: TVL Factor strategy for different values of n.

n	Sharpe Ratio	Sortino Ratio
2	2.25	2.80
3	4.44	7.75
4	6.20	5.88

Table 8: Sharpe and Sortino Ratios for different values of n in the TVL factor strategy.

Cointegration Beta Trading

Finally, we consider the cointegrated pairs identified in previous analyses, specifically focusing on pairs with the same underlying assets. The strategy operates by continuously monitoring the spread between two cointegrated tokens, adjusting positions of the YTs based on deviations from the expected spread.

OLS regression over a rolling window is used to estimate the beta between the two tokens over a specified period, allowing for dynamic adjustments in response to changing market conditions. The strategy involves trading the YTs according to the recent Z-score of the spread: when the Z-score exceeds a predetermined threshold, the strategy shorts the YT of the overperforming token and takes a long position in the YT of the underperforming token. The tested strategy outputs a signal of either -1, 0, 1 at each step representing short, close, and long the spread respectively. The main disadvantage is that multiple contracts of the same underlying have to be simultaneously active and sufficient data for estimating beta and the Z-score of the spread must be collected, hence a period when no trades can be made.

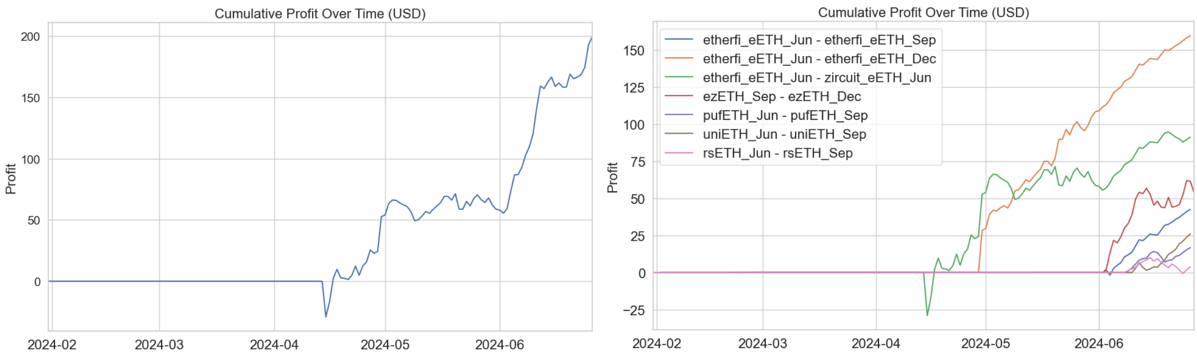

Figure 26: Cointegrated pairs trading strategy. The window is is set to 10 days. The Z-score threshold is set to 1. The overall strategy has a Sharpe ratio of 3.70 and Sortino ratio of 4.73.

A natural extension is to use beta as a dynamic hedge ratio where the underlying position is adjusted based on the estimated beta to maintain market neutrality. This approach allows for capitalizing on short-term price discrepancies while maintaining a balanced risk profile. An example of such a strategy is below. Similar to the above, the strategy monitor the spread between expected yield and the underlying token price. If the yield is increasing faster than the underlying token price, then it goes long PT and hedges with the underlying. Conversely, if the yield is falling faster than the underlying token price, it goes long YT and hedges with the underlying. A beta is fitted for the PT and YT separately to determine hedge ratios.

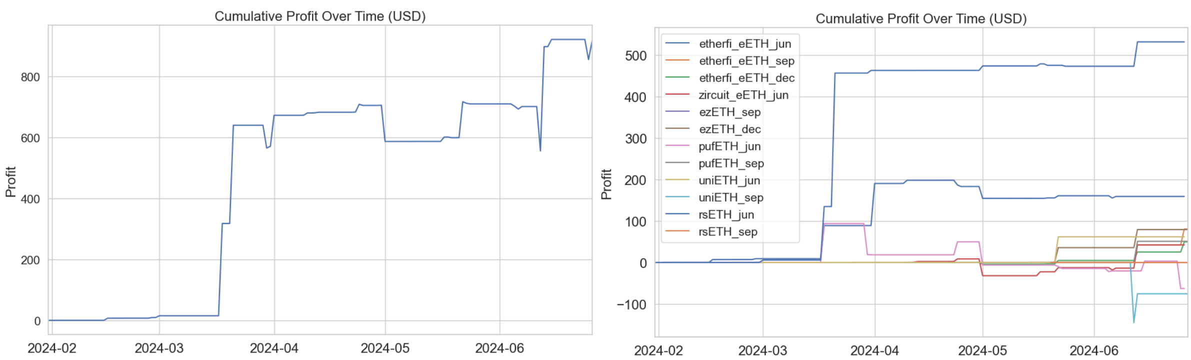

Figure 27: Cointegration trading on the expected yield spread. The window is set to 10 days. The Z-score threshold is set to 2. The overall strategy has a Sharpe ratio of 3.01 and Sortino ratio of 11.29.

6 Concluding Remarks

6.1 Summary of Key Investment Strategies and Observations

- **Long-Term Trends and Fundamental Characteristics:** The study revealed that long-term trends and fundamental market characteristics like time appreciation in PTs and yield decay in YTs are the most significant drivers in token pricing.
- **Arbitrage from PT/YT Invariant:** We observed instances where the PT/YT invariant did not hold. These situations offer arbitrage opportunities by minting or redeeming tokens at non-equilibrium prices to realise a profit.
- **Fixed Yield vs. Implied APY:** We have seen high correlation between fixed yields and implied APYs. Furthermore, we found that fixed yields may become extremely high is convergence of the PT price is not fast enough at maturity. Finally, we noticed that high implied APYs are rarely sustainable and tend to correct quickly.
- **Liquidity Effect:** We found that tokens with higher TVL tend to have lower median yields with a tighter range, whereas tokens with less TVL may offer greater yields to remain attractive despite their relative illiquidity.
- **Pairs Trading from Cointegration:** Our analysis found cointegrated trading pairs. While some of these have in-sample biases that can't be generalised on unseen data robustly, key ideas include tokens with the same underlying and different durations or protocols. Strategies include mean reversion between pairs, basket strategies with asset allocation based on cointegration, and dynamic hedging using cointegration beta.
- **Statistical Arbitrage from Ratios and Factors:** The ratios between PTs, YTs, and their underlying assets offer statistical arbitrage opportunities. Deviations from expected ratios indicate inefficiencies that can be exploited, especially as contracts approach maturity and prices are expected to converge. We explored this idea based on TVL; the framework generalises to other factors.
- **Hedging Strategies:** Hedging the USD/underlying risk in PT and YT investments can free up strategies, allowing traders to focus less on currency exposure. This can be done by rolling futures.
- **Lower Risk in PT Investments:** Since PTs can always be held to maturity to collect the fixed yield, they inherently carry lower risk compared to YTs. In the worst case, the payoff will be the fixed yield the PT was bought for. We note that every YT strategy can be expressed in terms of PT and the underlying, which in many cases can reduce volatility of a portfolio.

6.2 Further Areas of Study

Given the temporal limitations of our dataset, primarily working with the June contracts, it's crucial to extend this analysis across a broader range of contracts and time frames. This expansion would help mitigate the influence of specific market conditions inherent to our current dataset and potentially unveil more robust patterns. Pendle is continuously adding new contracts providing opportunity further exploration.

Studying the specifics of each protocol and relating events to movements in the respective PT and YT. We know that the YT is priced based on the expectation of all future yields, including the liquid restaking protocol and the liquid staking protocol. Event-based trading strategies are likely to be very effective, just as in the rest of the cryptocurrency sphere.

The availability of liquidity data from Pendle could significantly enhance our understanding of liquidity providers' impact on market stability and efficiency. Furthermore, a comprehensive volatility analysis to expound on the relationship between volatility and volume in addition to the effect of liquidity providers will give a rich understanding of price movements on a token-by-token basis.

Future research could also benefit from backtesting the arbitrage models on unseen data to prevent any in-sample bias. Use of a more sophisticated engine to incorporate fees, gas, and price impact is advisable. The order book must be taken into account since some arbitrage opportunities might be restricted from a reluctance to trade at market prices, which also affects the possibility of trading strategies requiring frequent rebalancing of multiple tokens. Some strategies may benefit from higher frequency data, at least at the hourly level, to measure the rate of intraday arbitrage opportunities.

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[8] EigenLayer. Restaked Points. docs.eigenlayer.xyz/eigenlayer/restaking-guides/restaking-user-guide/restaked-points, 2024.

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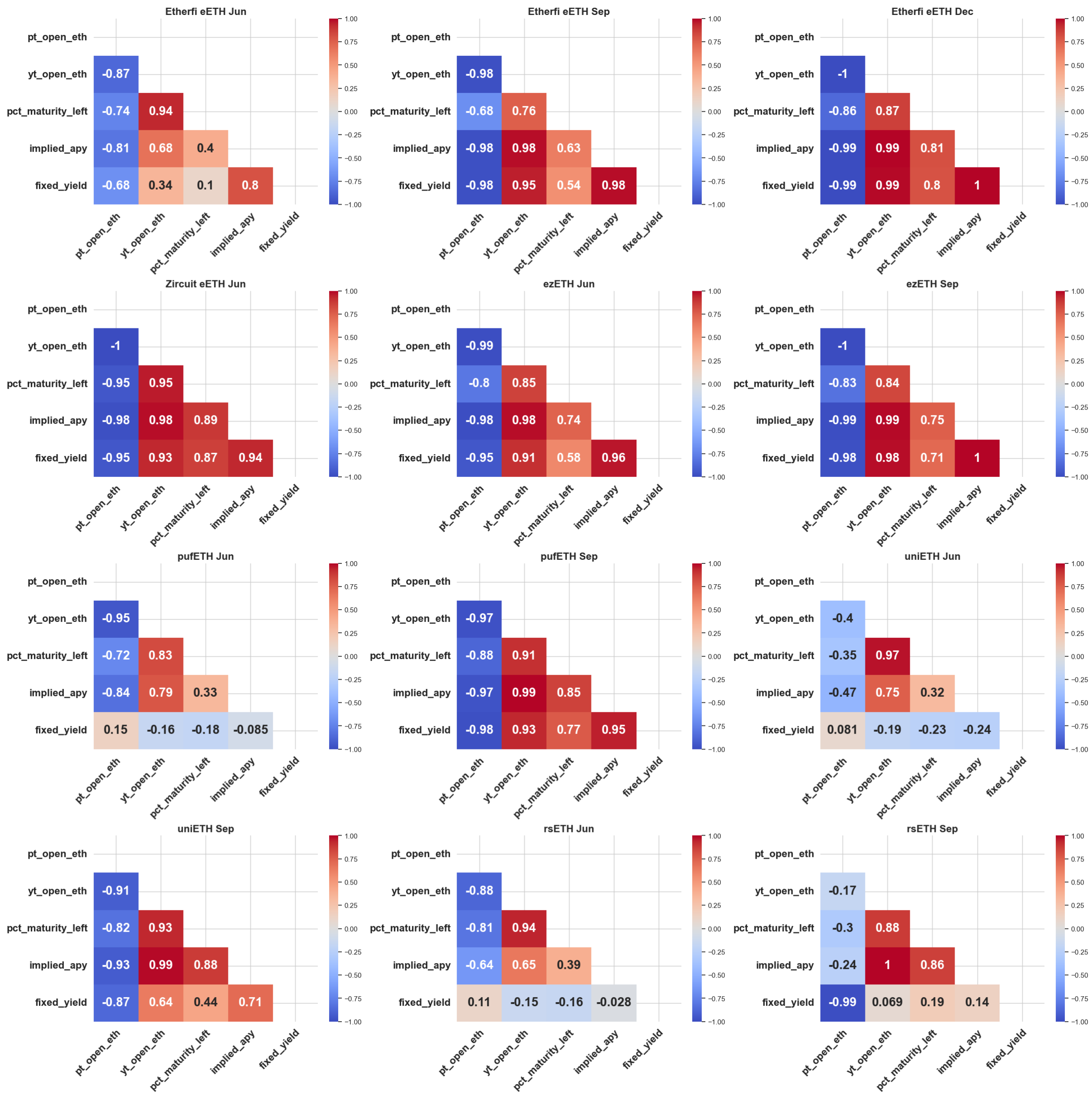
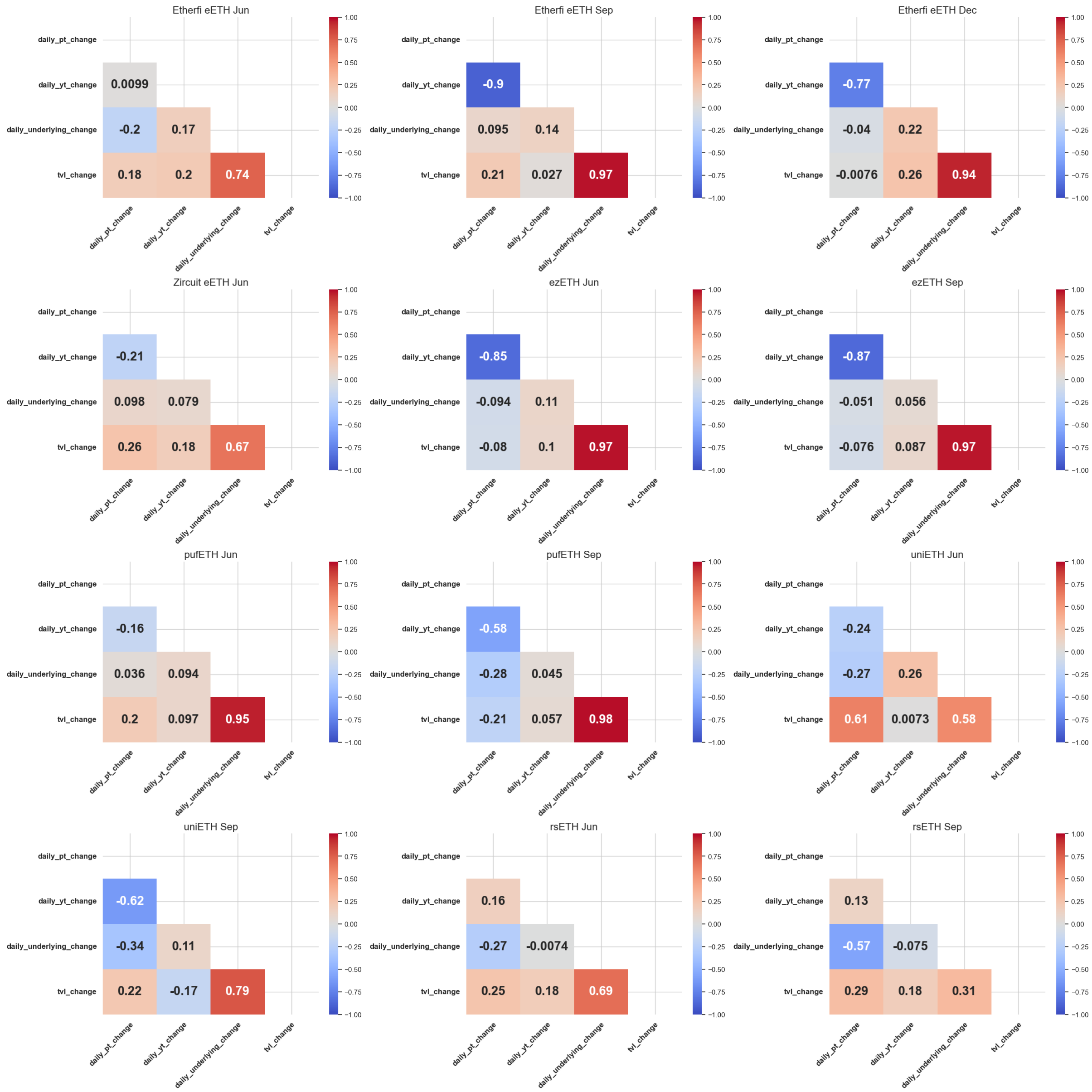
A Complete Correlation Matrices




Etherfi eETH Jun
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-0.2
0.17
0.18
0.2
0.74
Etherfi eETH Sep
-0.9
0.095
0.14
0.21
0.027
0.97
Etherfi eETH Dec
-0.77
-0.04
0.22
-0.0076
0.26
0.94
Zircuit eETH Jun
-0.21
0.098
0.079
0.26
0.18
0.67
ezETH Jun
-0.85
-0.094
0.11
-0.08
0.1
0.97
ezETH Sep
-0.87
-0.051
0.056
-0.076
0.087
0.97
pufETH Jun
-0.16
0.036
0.094
0.2
0.097
0.95
pufETH Sep
-0.58
-0.28
0.045
-0.21
0.057
0.98
uniETH Jun
-0.24
-0.27
0.26
0.61
0.0073
0.58
uniETH Sep
-0.62
-0.34
0.11
0.22
-0.17
0.79
rsETH Jun
0.16
-0.27
-0.0074
0.25
0.18
0.69
rsETH Sep
0.13
-0.57
-0.075
0.29
0.18
0.31
daily_pt_change
daily_yt_change
daily_underlying_change
tvl_change
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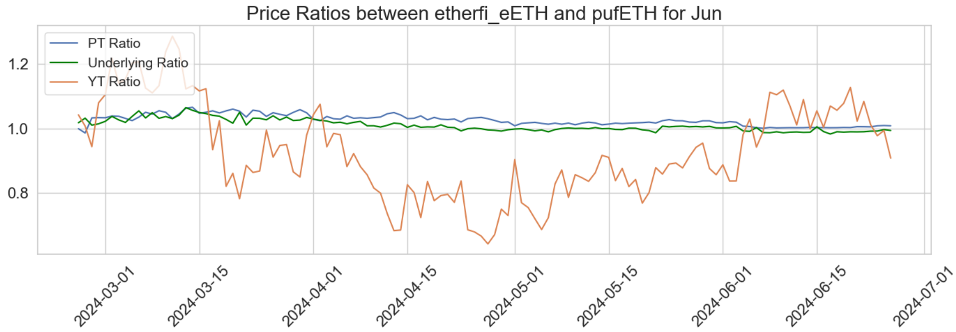
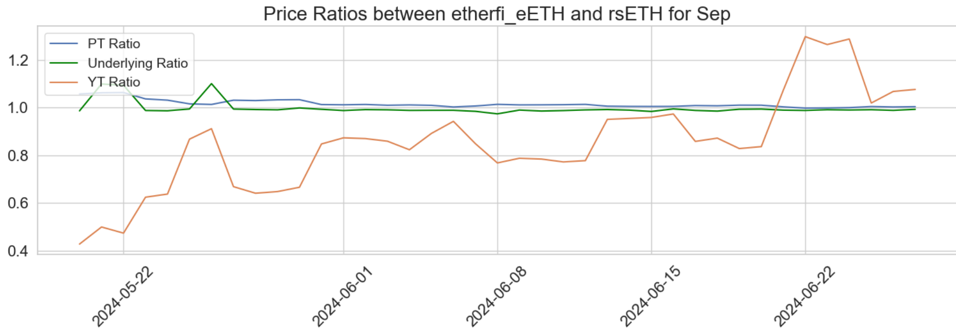
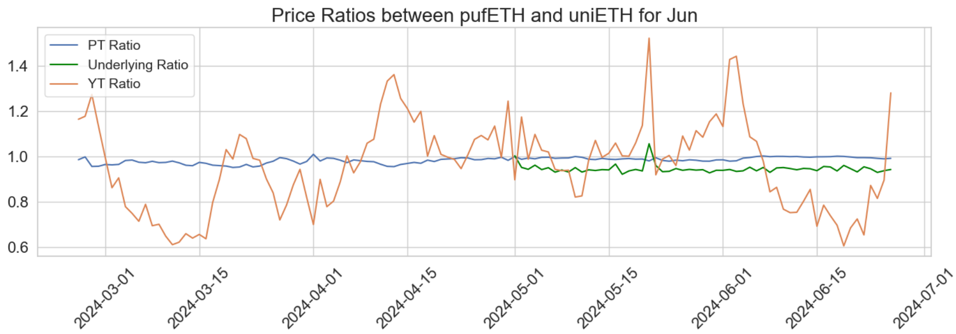
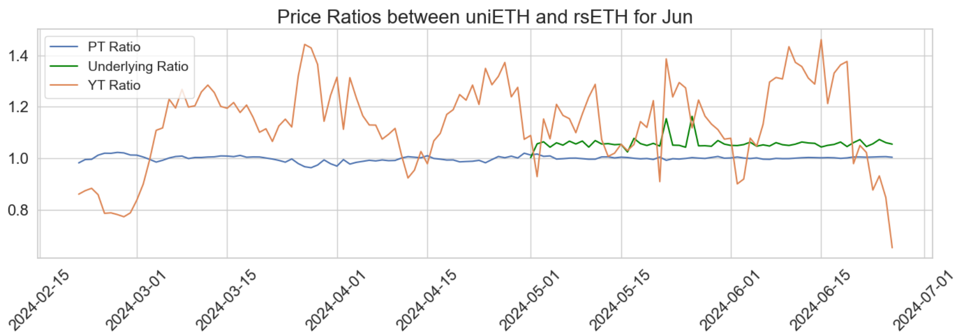
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00

B More Inter-token Ratio Examples

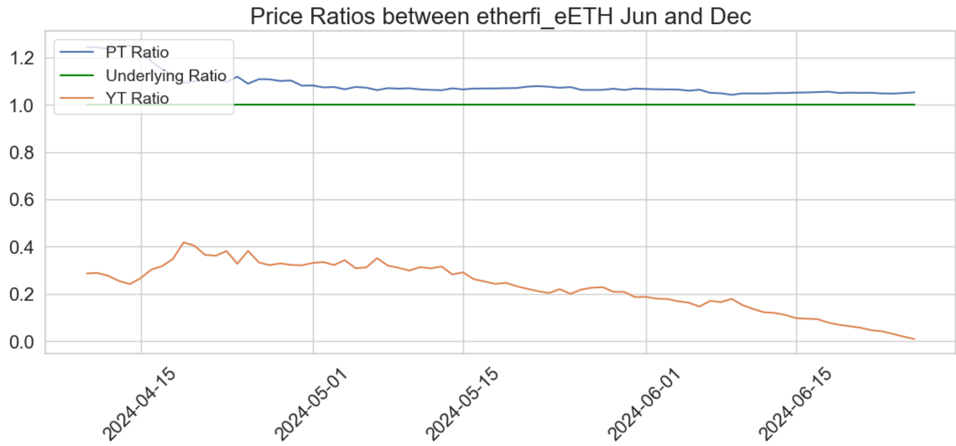


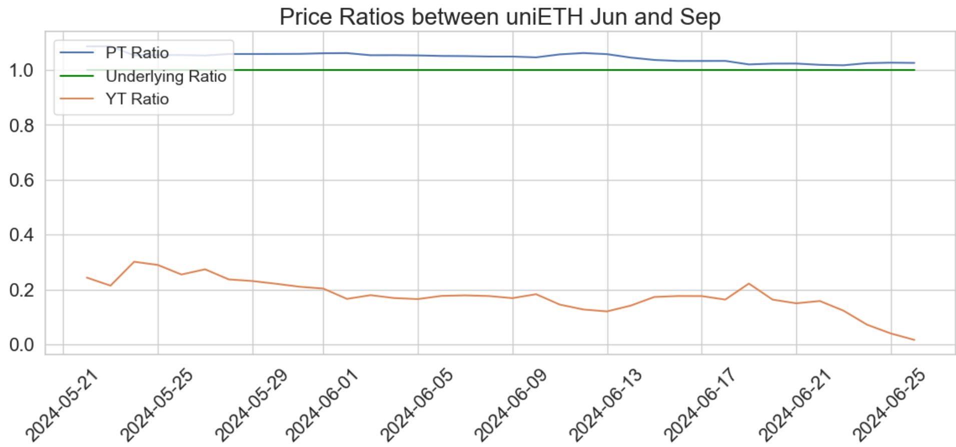


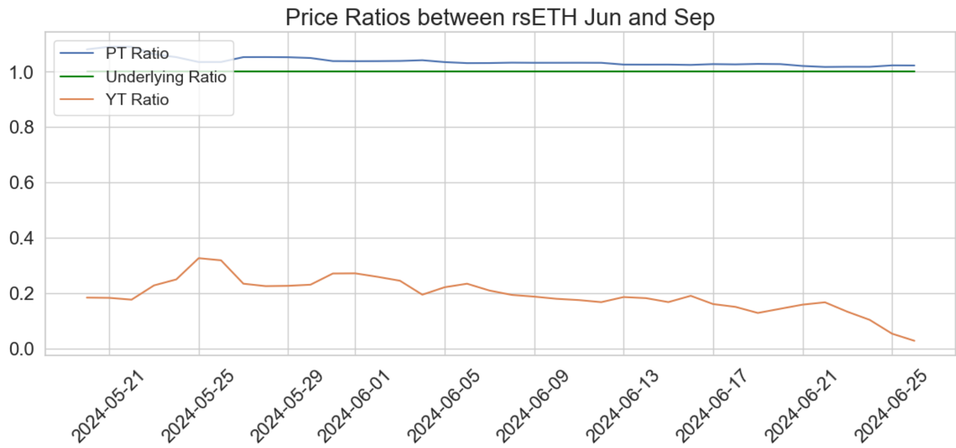




C More Inter-tenor Ratio Examples







D Johansen Cointegration Test Tables

Critical values at the 1% significance level are 19.9349 for inter-token ratios and 23.1485 for inter-tenor ratios. Critical value at the 1% significance level is 16.364 for YT and expected yield.

Table 9: Johansen Cointegration Test Results for PT and YT Open Prices.

Token Pair	**PT Statistic**	**Cointegrated?**	**YT Statistic**	**Cointegrated?**
Etherfi eETH vs Zircuit eETH (Jun)	54.53	Yes	107.36	Yes
Etherfi eETH vs pufETH (Jun)	11.68	No	5.58	No
Etherfi eETH vs uniETH (Jun)	12.61	No	10.05	No
Etherfi eETH vs rsETH (Jun)	20.19	Yes	30.45	Yes
Zircuit eETH vs pufETH (Jun)	30.98	Yes	56.31	Yes
Zircuit eETH vs uniETH (Jun)	21.76	Yes	44.28	Yes
Zircuit eETH vs rsETH (Jun)	28.19	Yes	71.44	Yes
pufETH vs uniETH (Jun)	12.63	No	9.05	No
pufETH vs rsETH (Jun)	12.30	No	6.51	No
uniETH vs rsETH (Jun)	16.31	No	9.07	No
Etherfi eETH vs ezETH (Sep)	28.23	Yes	37.45	Yes
Etherfi eETH vs pufETH (Sep)	16.44	No	18.58	No
Etherfi eETH vs uniETH (Sep)	15.09	No	10.92	No
Etherfi eETH vs rsETH (Sep)	20.12	Yes	39.83	Yes
ezETH vs pufETH (Sep)	17.83	No	18.35	No
ezETH vs uniETH (Sep)	19.66	No	7.86	No
ezETH vs rsETH (Sep)	32.23	Yes	24.57	Yes
pufETH vs uniETH (Sep)	6.34	No	21.69	Yes
pufETH vs rsETH (Sep)	15.62	No	21.69	Yes
uniETH vs rsETH (Sep)	9.02	No	7.00	No
Etherfi eETH vs ezETH (Dec)	34.38	Yes	67.86	Yes
Etherfi eETH (Jun vs Sep)	74.10	Yes	77.73	Yes
Etherfi eETH (Jun vs Dec)	14.02	No	25.22	Yes
Etherfi eETH (Sep vs Dec)	64.53	Yes	82.36	Yes
ezETH (Sep vs Dec)	11.24	No	17.60	No
pufETH (Jun vs Sep)	29.82	Yes	26.46	Yes
uniETH (Jun vs Sep)	24.00	Yes	24.35	Yes
rsETH (Jun vs Sep)	38.92	Yes	31.67	Yes

Table 10: Johansen Cointegration Results for YT and expected yield for same token.

Token	Maturity	Trace Statistic	Cointegrated?
etherfi_eETH	Jun	28.094660	Yes
etherfi_eETH	Sep	21.214811	Yes
etherfi_eETH	Dec	29.794729	Yes
zircuit_eETH	Jun	44.114495	Yes
ezETH	Sep	24.304318	Yes
ezETH	Dec	19.307841	Yes
pufETH	Jun	20.267669	Yes
pufETH	Sep	27.534837	Yes
uniETH	Jun	17.692669	Yes
uniETH	Sep	18.642395	Yes
rsETH	Jun	24.636377	Yes
rsETH	Sep	17.622516	Yes

E Cointegration Relation Vectors from VECM

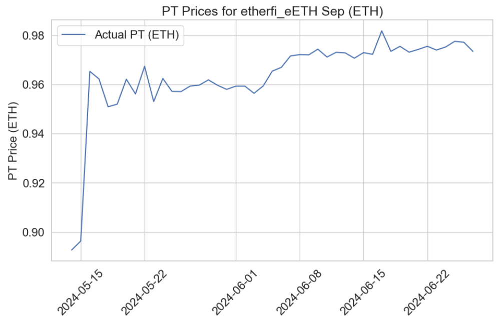
The following are for the VECM modelling token pairs. Below are the loadings for each error correction component with the following mapping:

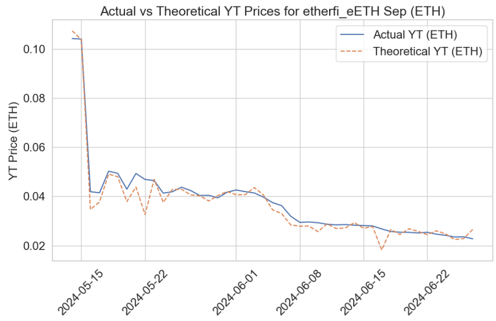
- **etherfi_eETH_Jun**: beta.1
- **etherfi_eETH_Sep**: beta.2
- **etherfi_eETH_Dec**: beta.3
- **zircuit_eETH_Jun**: beta.4
- **ezETH_Sep**: beta.5
- **ezETH_Dec**: beta.6
- **pufETH_Jun**: beta.7
- **pufETH_Sep**: beta.8
- **uniETH_Jun**: beta.9
- **uniETH_Sep**: beta.10
- **rsETH_Jun**: beta.11
- **rsETH_Sep**: beta.12

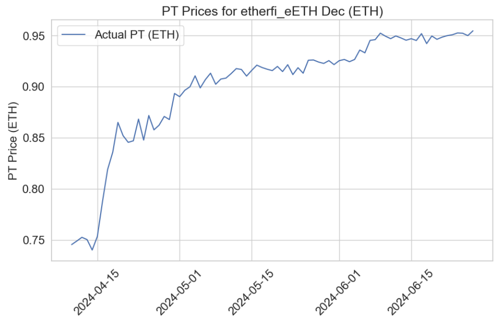
Table 11: Cointegration Relations for Loading-Coefficients Across Columns

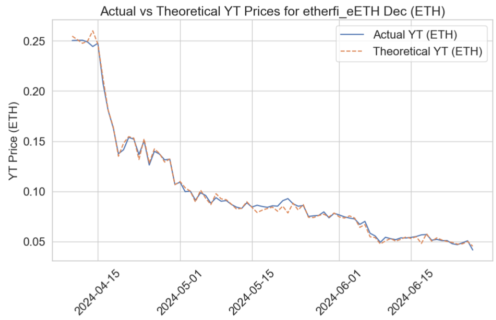
Beta	**ec1**	**ec2**	**ec3**	**ec4**	**ec5**	**ec6**	**ec7**	**ec8**	**ec9**	**ec10**
beta.1	1.0000	7.889e-18	8.198e-16	-2.355e-15	4.973e-15	-4.773e-16	-7.353e-15	1.76e-14	4.332e-15	5.056e-15
beta.2	1.772e-15	1.0000	3.733e-15	1.024e-15	2.661e-15	1.206e-16	1.534e-15	-1.479e-15	-1.502e-15	1.845e-15
beta.3	-1.219e-15	2.582e-15	1.0000	-1.058e-15	3.383e-15	-4.518e-16	1.491e-15	7.07e-15	-1.562e-15	1.803e-16
beta.4	2.807e-15	-5.294e-15	-1.702e-15	1.0000	-6.171e-15	-2.201e-16	1.104e-14	-3.47e-15	-1.644e-15	-1.222e-14
beta.5	4.309e-18	-1.334e-16	-2.659e-15	1.131e-16	1.0000	2.78e-16	-4.604e-15	-3.297e-15	1.532e-16	4.813e-15
beta.6	6.324e-17	-1.124e-15	3.281e-15	1.748e-17	-1.229e-15	1.0000	-3.666e-15	2.836e-15	1.432e-15	-3.03e-15
beta.7	8.657e-16	9.732e-16	9.848e-15	-2.393e-17	3.479e-15	-4.726e-16	1.0000	2.439e-15	-1.739e-15	-8.247e-15
beta.8	-1.006e-15	-2.46e-16	-8.905e-16	-4.862e-16	-3.029e-16	-5.309e-16	-9.126e-15	1.0000	1.348e-15	9.464e-16
beta.9	2.238e-15	6.646e-16	-2.352e-15	1.207e-15	3.984e-15	-4.714e-16	6.688e-15	-2.195e-14	1.0000	1.272e-14
beta.10	2.14e-16	-3.708e-16	-9.824e-16	-9.788e-17	-1.463e-16	2.161e-16	-1.153e-15	-2.648e-16	6.636e-16	1.0000
beta.11	-2.0384	1.3092	5.4542	-2.1717	2.1942	-0.9906	7.4150	7.5686	-3.8538	-5.7232
beta.12	1.4460	-3.4884	-9.3493	1.6818	-4.4300	0.2104	-12.0761	-12.0811	4.1891	7.7978
const	-0.0737	0.1062	0.3504	-0.0830	0.1222	-0.0491	0.4531	0.3885	-0.1775	-0.3415

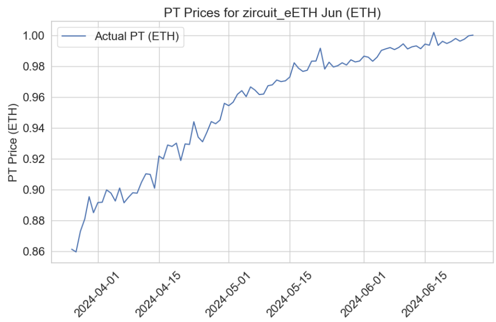
F Complete Graphs of the PT/YT Invariant

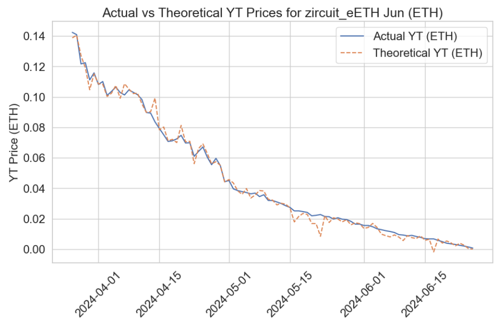


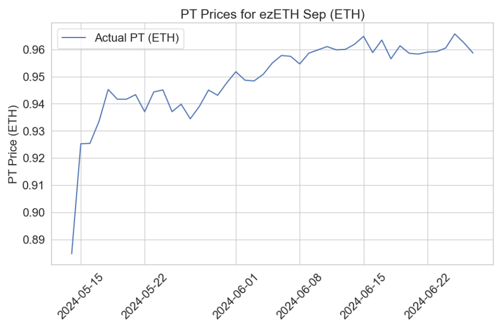


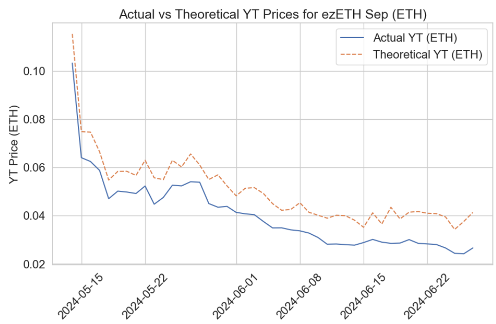


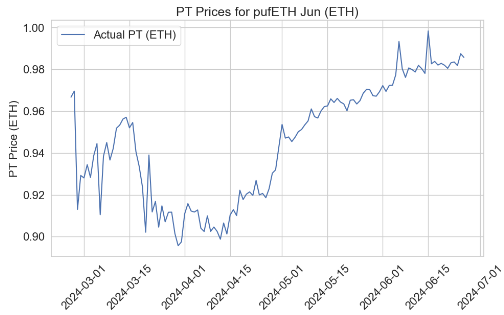


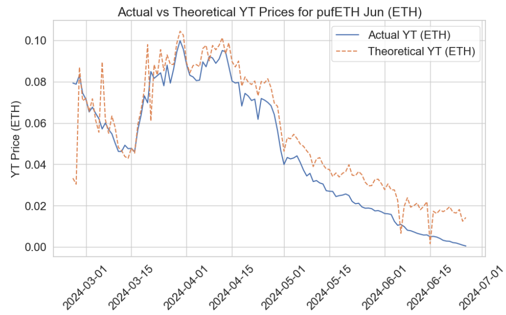









PT Prices for pufETH Jun (ETH)
Actual PT (ETH)
PT Price (ETH)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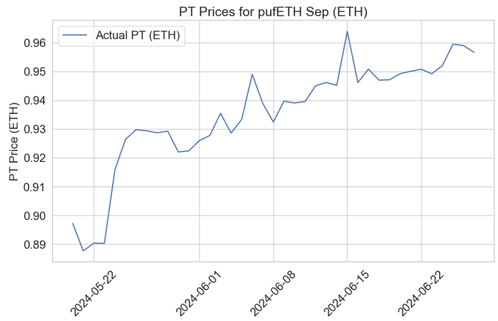
Actual YT (ETH)
Theoretical YT (ETH)
YT Price (ETH)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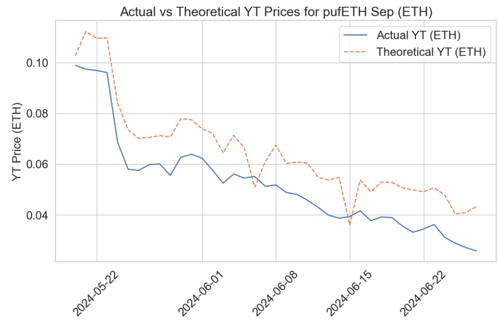
Actual PT (ETH)
PT Price (ETH)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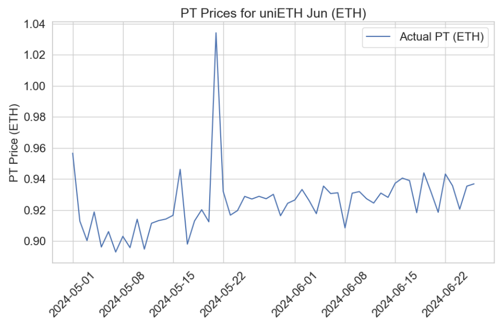
Actual YT (ETH)
Theoretical YT (ETH)
YT Price (ETH)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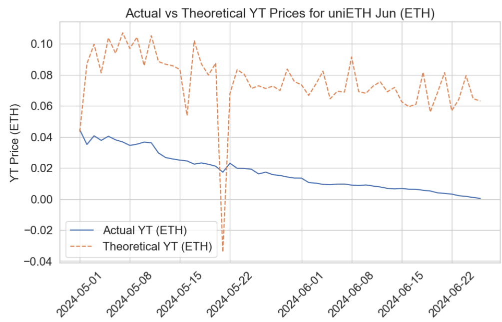
Actual PT (ETH)
PT Price (ETH)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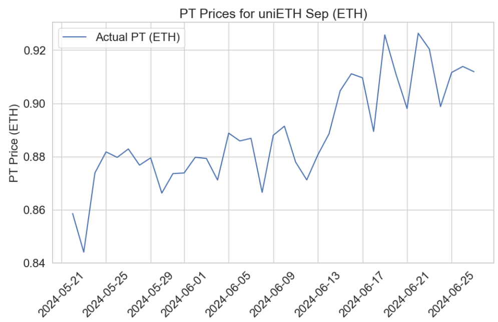
YT Price (ETH)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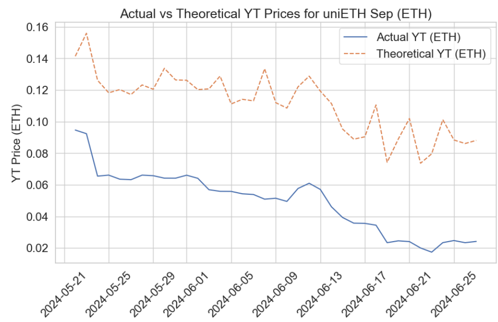
PT Price (ETH)


Actual vs Theoretical YT Prices for uniETH Sep (ETH)
Actual YT (ETH)
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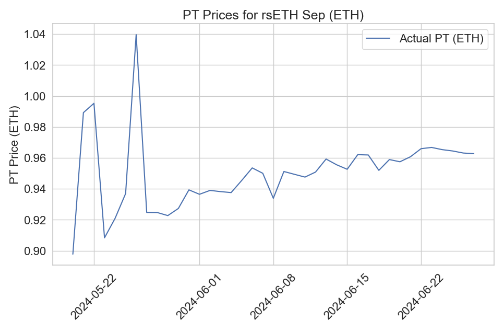
YT Price (ETH)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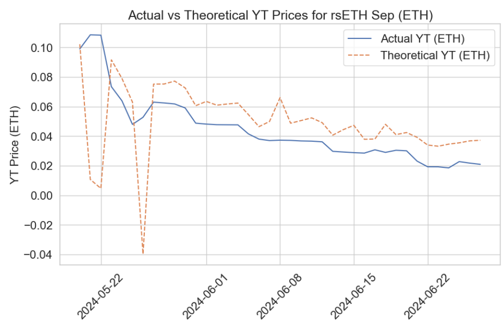
Actual PT (ETH)
PT Price (ETH)


Actual vs Theoretical YT Prices for rsETH Sep (ETH)
Actual YT (ETH)
Theoretical YT (ETH)
YT Price (ETH)