Department of Physics
School of Natural Sciences, NUST

PHY-411 Group Theory

Mid Term Exam

Instructor: M.Ali Paracha

Time Allowed: 2 Hrs. Marks: 30.

02-11-2024

Instructions: Please attempt all questions. No books and helping material are allowed. Please do not write anything on the question paper

Q.1(a) Prove that the identities (i) $e^{-1} = e$, (ii) $a^{-1}a = e$, (iii) $ea = a$ for all $a \epsilon G$ follow from the basic axioms of the group. (4)

Q.1(b) Consider the following two elements of symmetry group S_5

$$g_1 = (135)(24)$$
$$g_2 = (12)(345)$$

Find a third element g of this group such that (6)

$$g^{-1} g_1 g = g_2$$

Q.2 Consider the dihedral group D_4 which is a symmetry group of square consisting of rotations around the center and reflections about the vertical, horizontal and diagonal axes.

(a) Enumerate the irreducible representations. (3)

(b) Construct the character table for these representations. (4)

(c) Decompose them into classes. (3)

Q.3(a) Consider a 3-dimensional coordinate transformation of the form (4)

$$x_\mu{}' = \lambda_{\mu\nu} x_\nu$$

where $\lambda_{\mu\nu}$ is a 3 dimensional rotation matrix. By making an infinistesimal transformation of rotation matrix $\lambda_{\mu\nu}$, dtermine the rotation parmater and find its nature.

Q.3(b) Consider a group of 2-dimensional coordinate transformation of the form

$$x' = ax + by + c$$
$$y' = dx + ey + f$$

How many parameters does this group has? Construct an infinitesimal operators X_a, X_b and X_c in differentia form. (6