# THE MATHEMATICS DEPARTMENT AT THE UNIVERSITY OF ARIZONA

by Richard S. Pierce

1984

In the beginning there was mathematics; it has been with us ever since. Formal university training in mathematics came to Arizona about five years after the Territorial assembly pronounced its blessing on higher education in 1885.

In those frontier days, mathematics was embedded in the College of Mines. The School of Mathematics in 1892 consisted of departments of Pure Mathematics and Applied Mathematics. The courses that were given under the banner of pure mathematics in those days seem to be roughly equivalent to the current offerings of most Tucson high schools: elementary algebra, plane geometry, trigonometry, and calculus. The applied Mathematics offerings were more sophisticated, though they would not be classified as mathematics today. These courses included mechanics, strength of materials, and astronomy.

To establish some perspective, it is useful to survey the status of mathematics in this country at the end of the nineteenth century. During its early years, these United States had a surprising number of scientists who clearly rank among the leading geniuses of the times. However, before about 1900, there were only a few native born Americans who could be called world class mathematicians; J. W. Gibbs, G. A. Miller, and C. S. Peirce were recognized names in 1890 or earlier. In any case, the mathematical center of gravity in this country at the turn of the century was well east of the Mississippi river. To be sure, 1900 was a time of mathematical excitement in the eastern centers such as Harvard, Yale, Princeton, and Johns Hopkins, but the ripples from this ferment did not reach the sands of Arizona. As we noted, the mathematical curriculum at the youthful University of Arizona was just a few steps above standard high school offerings. However, the same statement could probably be made about the most distinguished universities in their early years of existence.

During its first twenty years, the Mathematics Department at the University of Arizona grew at a leisurely pace. Calculus was expanded from one to three quarters; some remedial courses such as beginning algebra, geometry, and trigonometry were dropped (but of course rose from the ashes in more recent times); and the catalog picked up a few advanced specialties such as "computation," "higher plane curves," and a "synoptic course in advanced mathematics." Appropriately, mathematics education took an honored place in the curriculum. The evidence indicates that many of the mathematics majors from the University during the period between 1900 and 1920 graduated to the teaching profession.

The condensation of mathematics into an important part of the University's program started at about the time of World War One. The leader of this development was Dr. Heman Leonard. He joined the faculty in 1915, became the first officially recognized department head in 1920, and served in that role until 1938. During this time, the mathematics major was formalized with the available degrees of B.A., B.S., and M.S. The records of the Department show that a B.S. in mathematics was awarded to Josephine Walters in 1918. For lack of contrary evidence, it can be assumed that she was the first of Arizona's mathematics degree graduates. The first graduate degree in mathematics was awarded to Samuel Ridgely in 1921. His M.S. thesis dealt with the rotation of an ellipsoid about a non-principal axis, a topic in the English Tripos tradition. Clearly, mathematics in the southwest had not caught up with the exciting developments that were taking place in the main centers of learning around the country and the world. It was the period between 1910 and 1930 that saw the birth of functional analysis, algebraic topology, abstract algebra, mathematical logic, general relativity, and quantum mechanics. A non-mathematical event of importance for Arizona ocurred during that time period: Arizona became the forty-eighth member of the United States of America in 1912.

The second head of the Mathematics Department at the University of Arizona was Dr. Roy Graesser. He served in that capacity from 1938 until 1959. These were the years of World War Two and the Korean War. The times were difficult, and it is probably not surprising that the record of progress during that period is slender.

In 1959 the headship of the Mathematics Department passed from Graesser to Dr. Harvey Cohn, a distinguished number theorist. Under his direction the department grew from adolescence to young adulthood in the intellectual community of mathematics. A Ph.D. program was initiated around 1960. Its first product was Bernard Maarcus who graduated in 1962. The start of a doctoral program is generally recognized as the first step that a department takes toward academic respectability; the step was taken rather late here in Arizona. By comparison, Berkeley's mathematics department produced 34 Ph.D. graduates in 1962 even though the University of California was founded only thirty years before the University of Arizona. Nevertheless, the doctoral program here has prospered. Approximately eighty mathematical scholars have followed Bernard Marcus across an Arizona graduation platform to accept the rights and privileges that are conferred with a doctorate of philosophy. Two of those graduates are among the present faculty at Arizona.

At the time when Harvey Cohn took the assignment to build a mathematics department of high quality, there were conflicting forces in the academic marketplace. On the one hand, global technological competition was becoming evident, and there was a realization that, the race would be lost by countries without excellent mathematical training centers. As a result, the money needed to build a department was in a free flow mode by current standards. However, the supply of high quality faculty lagged behind the demand. While it was easy for the prestige institutions like Harvard, Princeton, Chicago, and Berkeley to pick and choose among the best of the doctoral graduates each year, the schools with young departments "on-the-make" found themselves scrambling for the leftovers. Unfortunately, the Mathematics Department at Arizona was in the tail-end group. In spite of the tight market, Arizona's Mathematics Department did improve during the 1960's, both in its faculty and in its programs.

One of the creative ways in which the general manpower shortage was circumvented during the 1960's was by hiring senior mathematicians who had retired from other universities. The Mathematics Department at Arizona was outstandingly effective in the use of this strategy. Two of the world's leading number theorists came to Arizona–one for a short time, the other one for several years. In 1963, Louis J. Mordell joined the Arizona faculty as a visiting Professor. An American by birth, Mordell gained his fame in the English University system, where he pursued his research on Diophantine equations and helped to build a tradition of number theory that is one of the strongest in the world. Mordell's name has surfaced in the news this year

(1984) because of an amazing conjecture of his that has recently been proved by a young German mathematician. Mordell's stay in Arizona was short, but another distinguished number theorist, Henry Mann, spent seven years at the University of Arizona from 1968 until his retirement in 1975. He was influential in the development of a strong group of number theorists within the department. Another distinguished English mathematician, Louis Milne-Thompson spent several years at the University of Arizona. He directed numerous Ph.D. theses in his specialty area, hydrodynamics.

After Cohn resigned in 1967, the departmental leadership passed through the hands of four acting heads, while a search was made for a permanent director. It was during this interim period (in 1969) that the mathematics department moved from the Physics Building to its present quarters, the X shaped Mathematics Building. Construction of the Mathematics Building was supported in part by funds from the National Science Foundation. An addition to the building was made in 1970. It is now the home of the Mathematics Department, the linguistics department, and several other smaller units of the university.

A new permanent head for the Mathematics Department was found in 1970. Dr. Hanno Rund came from Waterloo University to the University of Arizona to take the leadership of the Department during the period between 1970 and 1978. He energetically pursued the development program that had been initiated by Cohn, adding more than a dozen new faculty members.

One of the important occurrences in the history of mathematics at the University of Arizona was the introduction around 1977 of an interdisciplinary group in Applied Mathematics. Mathematics is used and taught in many of the university programs. The faculty members who view themselves as mathematicians are by no means confined to the Mathematics Department. The formal recognition of a common bond of interest that links a large group of people on campus came about with the formation of the Committee on Applied Mathematics. This committee consists of several dozen faculty members from various areas, including the Mathematics Department of course. Some of the most interesting and exciting mathematical developments in Arizona during the last eight years have occurred under the stimulus of the Applied Mathematics Committee.

The most recent department head came from among the existing faculty. Dr. Theodore Laetsch served with distinction in this position from 1978 to 1984. He has vigorously continued the work of his predecessors to raise the quality of research and teaching of mathematics in Arizona.

In general, bare statistics are deadly dull and uninformative. However, a listing of the numbers of faculty at various ranks over the last thirty years does convey some sense of the development of mathematics at the University of Arizona. Here is such a table.

| | Professor | Assoc. Prof. | ¿Asst. Prof. | Inst. | Lecturer | Total |
|---|---|---|---|---|---|---|
| 1955 | 2 | 1 | 6 | 1 | 0 | 10 |
| 1965 | 7 | 5 | 16 | 1 | 3 | 32 |
| 1975 | 18 | 16 | 3 | 3 | 5 | 45 |
| 1984 | 23 | 15 | 3 | 0 | 3 | 44 |

The moral of the table seems to be: we are growing older, not much fatter, hopefully wiser.

We conclude this brief history with comments on the flavor of current research activity in the Department of Mathematics at the University of Arizona. The goal of this exercise is to point out some of the features that distinguish the Department from its counterparts around the country, to describe the special problems and special promises of the Department, and to provide a background for the Department's plans for future development. Perhaps our greatest asset is the orientation of a large fraction of the research faculty toward science and engineering. In this respect, the Department is ahead of many other schools in the country. Many mathematics departments have neglected the applications of their subject to other sciences. At Arizona this has not been the case. In fact, the Department's orientation toward applications was a choice that was made by some of its top researchers, not out of necessity, but because they thought that the applications of mathematics provide the greatest scientific rewards. This viewpoint has had a positive effect on teaching. Some of the best and most enthusiastic students in the mathematics courses come from science and engineering. They expect to learn robust and exciting material on the frontiers of mathematics, and their eagerness stimulates the faculty to respond with high quality teaching and research.

The mathematical area that provides most of the interaction with science and engineering fields is analysis. The Department is active in several parts of analysis, including differential and integral equations, functional analysis, operator theory, and global analysis. There is also considerable effort in areas related to mainstream analysis, such as probability and stochastic processes, statistics, and numerical analysis.

Algebra and number theory are well represented in the research of the Department. Group theory, ring theory, and field theory are actively pursued by various members of the faculty. There is also some research in peripheral parts of algebra: logic, combinatorics, and lattice theory. These are aspects of algebra that have important applications to computer science and linguistics.

The Department's greatest weaknesses are in areas of geometry, such as topology, Lie groups, and algebraic geometry. Strengthening research in geometry would provide a more balanced program, and it would support the present research efforts of the Department's two most active groups: applied analysis, and algebra/number theory.

It is difficult to give a detailed portrayal of the Department's research activity that fairly represents the diverse contributions made by individual faculty members. However, some themes stand out. Describing one of them, nonlinear analysis, may give an idea of why people get excited about mathematics. Many advances in analysis over the last three centuries have come about through a better understanding of linear problems. Roughly speaking, these are problems where response is proportional to input. Some mathematicians regard nonlinear problems as belonging to a category that consists of a zoo of special examples. That perception is now changing. There is a feeling that we are coming to a global understanding of non-linear phenomena. A lot of work is still done on special examples, but there is now some feeling for the existence of orderly patterns in nonlinear systems. Nonlinear effects, rather than being a nuisance or hopeless or an invitation to go blindly to the computer, are now viewed as sources of stability and order. Examples of stability in nonlinear phenomena are the travelling waves of combustion and nerve impulses, and the solitons of fluid dynamics and nonlinear optics. In other circumstances, however, nonlinear effects are responsible for disorder and instability. The separation between stability and instability is among the most exciting problems in nonlinear science. Researchers in probability and mathematical physics have also found that nonlinearity can generate order. In very large strongly interacting systems there is the phenomenon called phase transition. Small changes in a parameter such as temperature can suddenly produce a new ordered phase (condensation or crystalization). The mathematics of this physical effect is poorly understood, but the central questions are now clear, and the mathematical tools that come naturally to hand in the study of phase transitions have remarkably fruitful analogs in probability theory.

What will the future bring? Limited resources make it impossible to develop every area of mathematics. We will have to decide what parts of mathematical research are most important–to the university, the state and the nation. The decision will have to take account of the ultimate value of mathematical research, its utility, and its prestige value to the Department.

Faculty and students need to be aware of the main current developments in mathematics. They need access to specialist who can tell them about the latest results on Thurston's classification program for three-dimensional manifolds, or the structure of finite simple groups. The mathematics faculty should also be ready to explore the new ideas that live at the frontier between mathematics and other sciences. There will be a demand for experts on such new developments as the transition to turbulence or the algebraic aspects of programming languages. Can the Department keep up with current mathematical developments and contribute to them in significant ways? It will certainly try to do so, and the evidence of the Department's past development suggests that the effort will be successful.