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Scientific paper

Internal Viscoelastic Modulus Associated with Autogenous Shrinkage in Cementitious Materials

Will Hansen¹, Zhichao Liu^{2*} and Eduard A. B. Koenders³

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Abstract

The principal objective in this paper is to determine the internal viscoelastic modulus associated with autogenous shrinkage of cementitious materials. This is accomplished by analyzing autogenous shrinkage of plain and reinforced paste and mortar specimens. The internal restraint from steel reinforcement creates uniaxial tensile stress condition increasing with shrinkage. Data analysis demonstrates a unique internal shrinkage modulus, which eliminates the need for stress relaxation modulus calculations. The internal modulus is in the range of 8000-9000 MPa, while the external composite modulus known as Young's modulus is 32 000-36 000 MPa for the low water-binder ratio (0.35) concrete.

1. Introduction

Self-induced tensile stresses develop in concrete if the movements caused by cement hydration reactions are restrained (Bosnjak 2000). During early age hydration two active mechanisms are involved in producing these movements, starting with thermal effects which dominate during the first 24-48 hours. Self-desiccation is another consequence of cement hydration as this process consumes water into solid hydration products (Copeland and Bragg 1955). As hydration proceeds internal pore drying develops with associated internal stress development from capillary tension in the pores (Koenders 1997). These stresses are transferred to the hydration products as compression and subsequent shrinkage. This type of shrinkage is known as autogenous shrinkage. It is characterized by a uniform volume reduction and at any time is a material property (that is, no moisture gradient), whereas drying shrinkage development is size-dependent and non-uniform. Thermal stresses are relatively short-term acting throughout the concrete composite, while the time-dependent shrinkage stresses are internal acting primarily on the porous hydration products.

Autogenous shrinkage is intensified in high performance concrete (HPC) of low water-binder (W/B) ratio (relative to conventional concrete) due to its generally higher cement content, reduced W/B, and pozzolanic mineral admixtures. Prior results indicate that cementitious systems containing slag cement produces greater

autogenous shrinkage at later ages (Hanehara *et al.* 1999; Lura 2003; Lee *et al.* 2006). The early age cracking problem in HPC has become important due to the increased use of these materials (Bentz *et al.* 2001; Kovler *et al.* 1993; Toma *et al.* 1999; RILEM Report 2003). The reasons were generally attributed to the higher chemical shrinkage, the finer pore structure, the removal of calcium hydroxide as a shrinkage restraint, and a reduction in pore humidity associated with pozzolanic reactions.

The conventional method to calculate tensile stress development in concrete due to the thermal and hygral effects is to apply theory of viscoelasticity, developed for polymers, to concrete based on the assumed analogy between creep compliance function and stress relaxation modulus (Bažant 1977, 1988; Bažant and Huet 1999). The outcome in viscoelastic stress calculations is a reduction in concrete modulus by 30%-50% due to stress relaxation under full external deformation restraint (Atrushi 2003; Grasley 2006; Pane and Hansen 2008).

Recent numerical simulations, using the lattice model for predicting stress development from a mini temperature stress testing machine (TSTM) on small paste specimens, where thermal effects from heat of hydration are minimal, concluded that a poor agreement was obtained between predicted stresses by using three different models for obtaining stress relaxation modulus (Schlangen *et al.* 2006). The first model was based on an assumption of instant relaxation that internal stresses will cause instant deformation of the micro-porous hydration products. In this model there is no relaxation over time. The second model assumes relaxation based on hydration and it uses an exponential relaxation factor approach (van Breugel 1980). In the last method, stress relaxation modulus is a reduced Young's modulus, since the relaxation is the amount of stiffness lost over time. It was concluded that a better relaxation model is needed.

An intriguing and novel test procedure was presented (Bjøntegaard and Sellevold 1998; Bjøntegaard 1999) for simultaneous measurements of stress development in a

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fully restrained test (TSTM) and free autogenous shrinkage measurements (i.e. without external restraint). Although the focus of his study was the early age period (0-7 days) typical results show that once thermal contraction has ceased (typically within 24-48 hours), the two curves of stress and autogenous shrinkage development are parallel with a constant net internal viscoelastic modulus of about 11 000 MPa, considerably smaller than the Young's modulus. Furthermore, these results suggest that tensile stress relaxation is either instant or not a major factor.

A methodology for analyzing shrinkage stresses in cementitious materials has been reported (JCI 1998; Hansen *et al.* 2014). Simultaneous measurements of free and partially restrained autogenous shrinkage using steel rebar reinforcement are employed in this study for shrinkage stress and internal modulus analysis.

2. Experimental Program

A series of cementitious mixes of a 0.35 water-cement (W/C) ratio and different aggregate contents by volume were prepared in the laboratory according to ASTM C192 (2000) and the mix design is listed in **Table 1**. The first part of the mix denotation is the W/C ratio, which is followed by the aggregate content. Commercially available Type I portland cement was used. Fine aggregate was silica sand with a fineness modulus of 2.43, the absorption capacity of 1.6% and the specific gravity of 2.64. Coarse aggregate was lime stone with a 25 mm nominal maximum size, the absorption capacity of 1.2% and the specific gravity of 2.66. Cylindrical samples were cast and cured for one day before demoulding. Then they were sealed cured for different ages before

the following test procedures were carried out.

- (1) Compressive strength and split tensile strength were tested on 100 mm × 200 mm cylinders according to ASTM C39 (2000) and C496 (2000), respectively. Three specimens were tested for each age and both the average and individual results were reported.
- (2) Static modulus of elasticity was obtained from the stress-strain curve of 300 mm × 600 mm cylinders based on the simultaneous measurement of uniaxial compressive load by a static hydraulic system and linear deformation by a motion capture system (**Fig. 1**). Three sets of markers were mounted on the specimen, one on the front and the other two on the lateral sides (**Fig. 1(a)**). A duo-camera system facing the specimen front continuously monitored the change in X-Y-Z coordinates of each marker while the specimen was loaded in compression. The averaged vertical deformation from the three sets of markers was used to calculate the compressive strain history of the specimen. The stress history was obtained from the recorded force of the static hydraulic system. A stress-strain curve was constructed by synchronizing the time from stress and strain histories, as shown in **Fig. 1(b)**.
- (3) Autogenous shrinkage and temperature was measured simultaneously on duplicate mortar or paste specimens of 60 mm × 100 mm × 1000 mm where double polystyrene films were used to seal the specimen. While there was a short period of temperature fluctuation in the early age due to cement hydration, an isothermal condition at 20±1 °C was achieved in the long run by circulating water through channels embedded into the sides and bottom of the rigs. In addition to the restraint imparted only by aggregate particles, extra restraint was introduced by the embedment of four symmetrically located rebars in the specimen. The shrinkage strain was plotted against the maturity time to account for the temperature effect.

Table 1 Mix design (kg/m³).

	cement	gravel	sand	water
035-paste	1497	0	0	524
035-20%agg.	1198	0	528	419
035-40%agg.	899	0	1055	314
035-60%agg.	594	936	646	208
035-70%agg.	450	1093	753	157

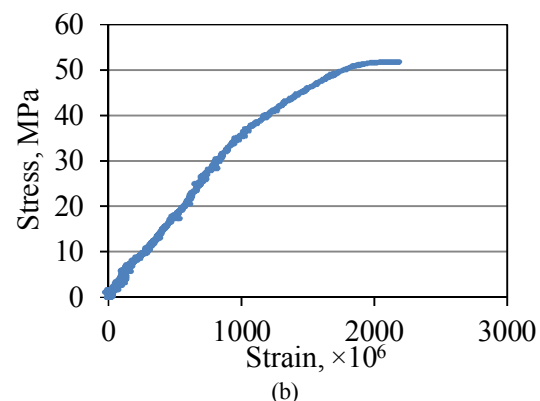
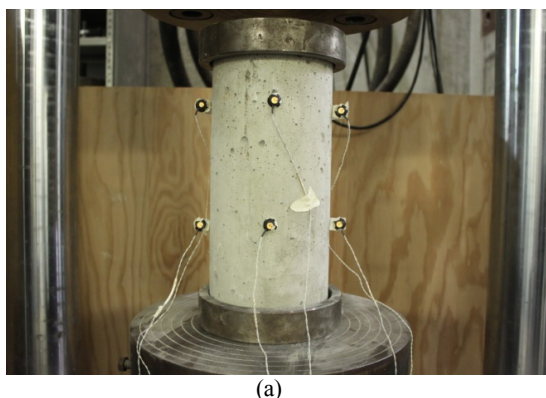


Fig. 1 (a) Static modulus measurement by a combination of a hydraulic test system and a motion capture system and (b) measured 7-day stress-strain curve of a concrete mix (035-70%agg.).

3. Results and Discussion

Measurements and modeling of autogenous shrinkage of the low W/C systems (0.35) for different internal restraint conditions (aggregate particle and steel reinforcement) form the basis for the proposed activity. Autogenous shrinkage is a form of drying shrinkage, but without shrinkage gradients (Fig. 2). A uniform internal stress develops due to the hydration process that proceeds without exchange of water (i.e. sealed curing). Internal stresses increase with increasing hydration resulting in more specimen shrinkage.

In this study, the averaged internal stresses acting directly on the hydration products can be computed by the measurement of the deformation of the reinforced cementitious materials with four symmetrically embedded rebars (Fig. 3).

3.1 Bond stress analysis

One question that arises regarding this method is that if there is any debonding between the four embedded steel rebars and the cementitious material during the test. In order to examine the bonding condition, a simple calcu-

lation is made considering the compressive force in the rebars is equilibrated by the shear force transferred from the cementitious material (Fig. 3).

The compressive force in the rebars is

$$F_{steel}(t) = \varepsilon_s(t) E_s A_s \quad (1)$$

where $\varepsilon_s(t)$ = elastic strain of steel rebars in reinforced mix, E_s = elastic modulus of steel, A_s = area of the four steel rebars and $A_s = 4 \times \frac{\pi D^2}{4} = \pi D^2$, D is the diameter of the rebars.

The total shear force transferred to the rebars is

$$F_{shear}(t) = \tau(t) A_{shear} \quad (2)$$

where $\tau(t)$ is the average shear stress, A_{shear} is the bonding area and $A_{shear} = 4\pi DL$, L is the embedded length of the rebars.

Force equilibrium yields

$$\varepsilon_s(t) E_s A_s = \tau(t) A_{shear} \quad (3)$$

Thus, the shear stress can be calculated as

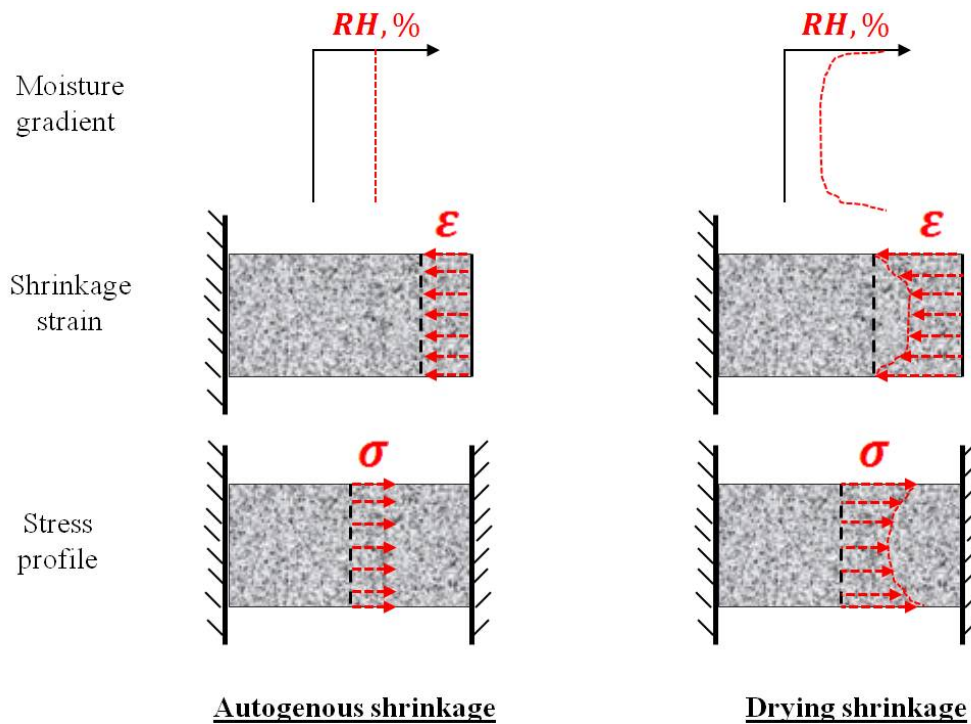


Fig. 2 Schematic comparison between autogenous shrinkage and drying shrinkage.

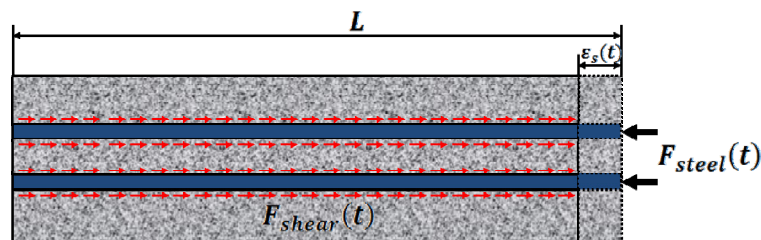


Fig. 3 Illustration of bonding condition analysis.

$$\tau(t) = \frac{\varepsilon_s(t) E_s A_s}{A_{\text{shear}}} = \frac{\varepsilon_s(t) E_s \pi D^2}{4 \pi D L} = \frac{E_s D}{4 L} \varepsilon_s(t) \quad (4)$$

The shear stress is calculated for the 035-40% agg. mortar mix from its rebar restrained shrinkage strain. The tensile strength of the mortar is estimated to be ~65% of the split tensile strength. As shown in **Fig. 4**, the shear stress is almost negligible compared with the

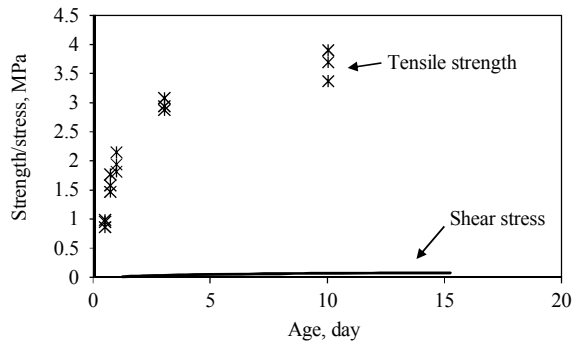


Fig. 4 Comparison between calculated tensile strength of 035-40% agg. mortar and its shear stress with the re-bars.

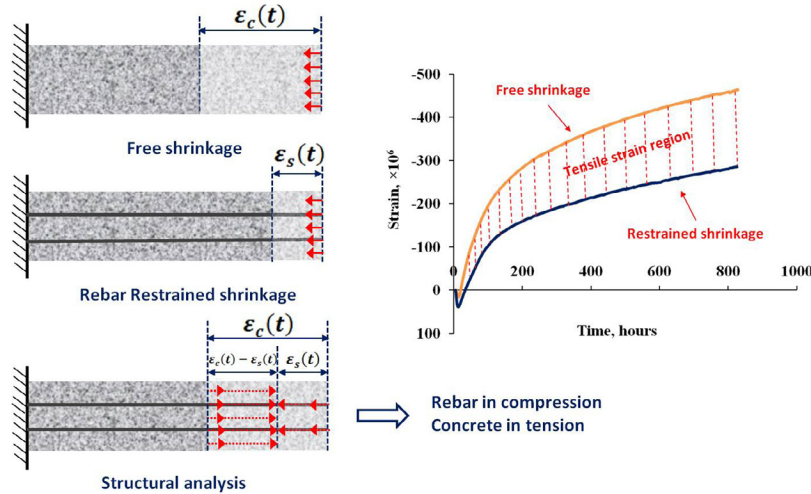


Fig. 5 Illustration of free shrinkage and rebar restrained shrinkage by concentrically placed rebars.

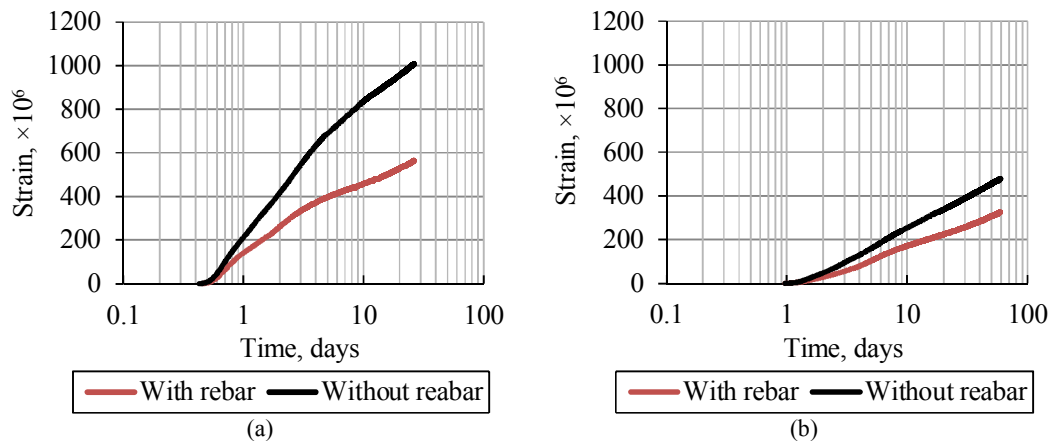


Fig. 6 Autogenous shrinkage with and without rebar restraint in (a) paste and (b) mortar containing 40% aggregate by volume (0.35 W/C).

tensile strength of the cementitious material, thus no bond slippage should occur during the test.

3.2 Internal modulus

The uniform stress condition associated with autogenous shrinkage allows for a straightforward tensile stress analysis when symmetrically placed reinforcement bars are used and 100% bonding condition is achieved (**Fig. 5**). Autogenous shrinkage results are shown in **Fig. 6** versus time. Force equilibrium analysis yields a linear correlation between free shrinkage and rebar restrained shrinkage (**Eqs.5-6**), from which a constant internal modulus associated with the hydration products is obtained by minimizing the sum of errors between the predicted line (dashed) and the measurements (**Eq.7** and **Fig. 7**).

$$\varepsilon_s(t) E_s A_s = [\varepsilon_c(t) - \varepsilon_s(t)] E_v A_c \quad (5)$$

Thus,

$$\varepsilon_c(t) = 1 + \frac{A_s E_s}{A_p E_v} \varepsilon_s(t) \cong (1 + n_v \rho_s) \varepsilon_s(t) \quad (6)$$

or

$$E_v = E_s \rho_s / (\frac{\varepsilon_c(t)}{\varepsilon_s(t)} - 1) \quad (7)$$

Where $\varepsilon_c(t)$ = free shrinkage of plain mix, A_c = area of plain mix, $\rho_s = A_s / A_c$ = steel ratio, E_v = internal modulus, $n_v = E_s / E_v$.

3.3 Pickett model for autogenous shrinkage prediction

The Pickett shrinkage model is perfect for modeling autogenous shrinkage of concrete as it is developed for a uniform paste stress distribution within a cross section and accounts for the aggregate volume fraction and restraining effect (Pickett 1956).

$$\varepsilon_c = \varepsilon_p (1 - V_a)^n \quad (8)$$

where V_a is relative aggregate volume fraction and n is the shrinkage exponent, a measure of aggregate particle restraining effect.

Free shrinkage results for different paste volume fractions can be fitted using the Pickett model with an exponent $n \sim 1.5$ in this case (**Fig. 8**). This model is a powerful tool for evaluating effect of paste content ($1 - V_a$) on concrete shrinkage.

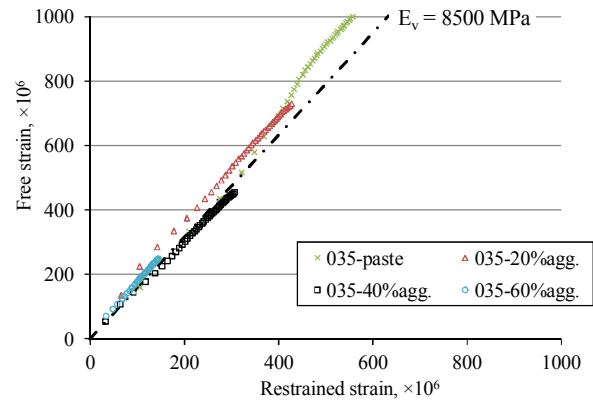


Fig. 7 Effect of steel reinforcement on internal modulus (E_v).

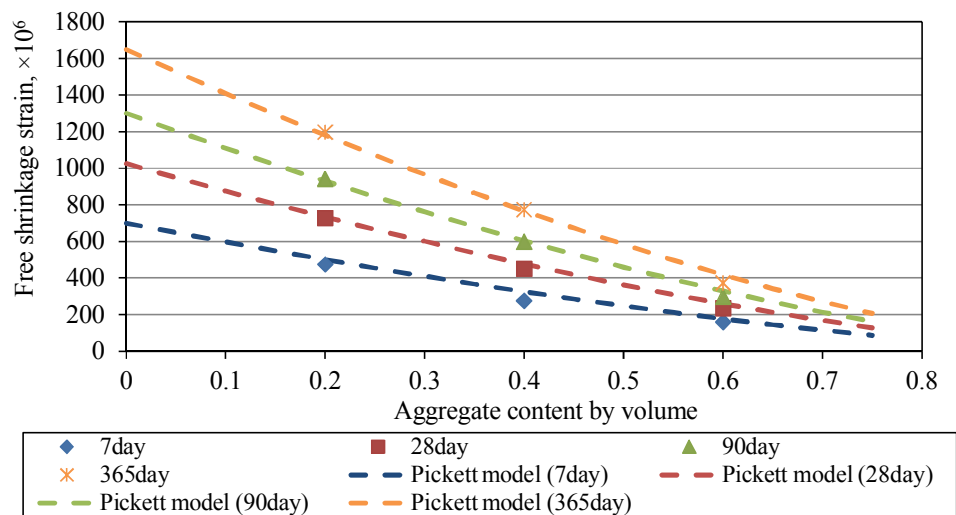


Fig. 8 Prediction of shrinkage strain by Pickett model.

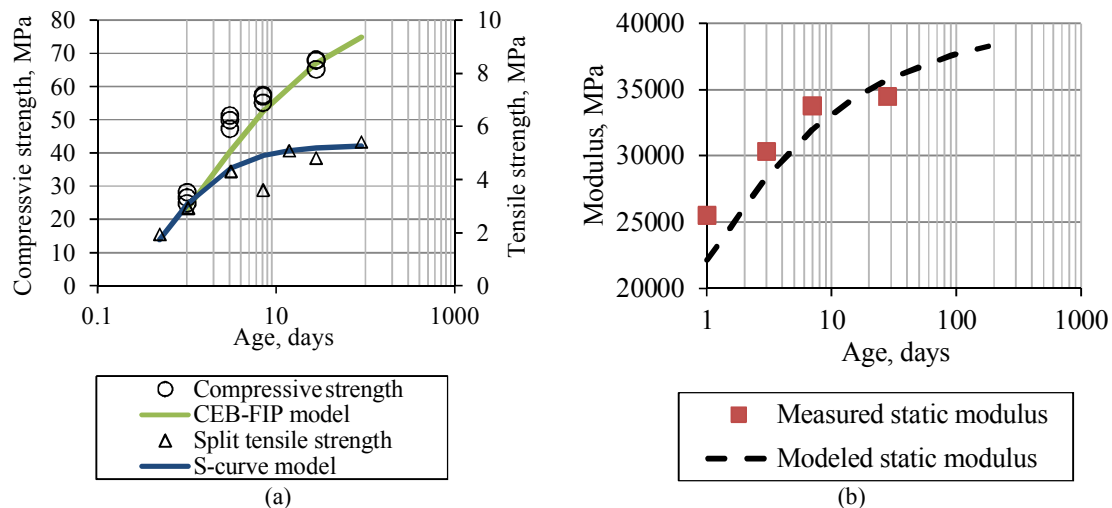


Fig. 9 Measurement and prediction on (a) compressive and split tensile strength and (b) static modulus.

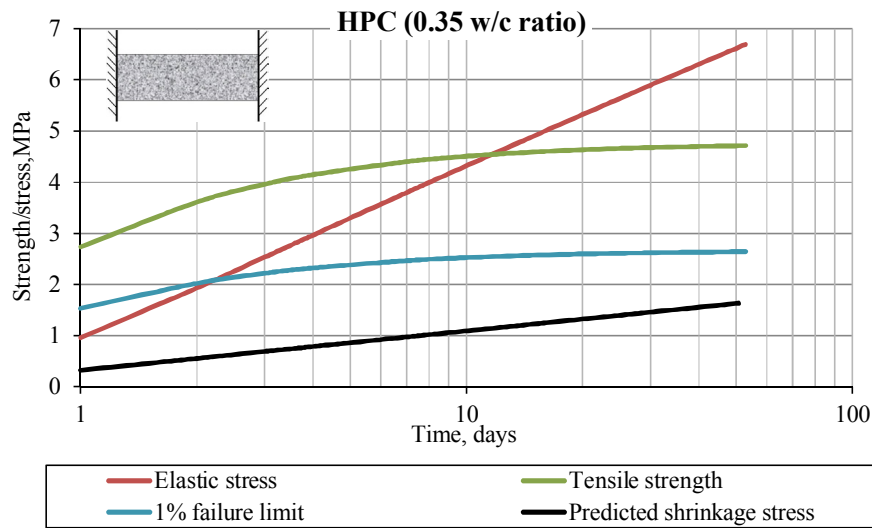


Fig. 10 Self-desiccation stress development in high performance concrete ($W/C = 0.35$) subjected to full shrinkage restraint based on different stress prediction methods.

3.4 Total stress in HPC due to full shrinkage restraint

Based on the measurement and modelling of the compressive strength, split tensile strength (Fig. 9(a)) and static modulus (Fig. 9(b)) of concrete mix, the predicted elastic and viscoelastic shrinkage stress development in sealed cured specimens under a full deformation restraint is shown in Fig. 10. The shrinkage stresses calculated with a constant internal modulus are significant and increasing over time, thus reducing the crack resistance of HPC. Typically a failure limit of 1% is used. This corresponds to an allowable stress/strength ratio of 0.56 (Lokhorst 1999).

Furthermore, the total time-dependent tensile stress $\sigma_t(t)$ in plain concrete subjected to external deformation restraint can be calculated by incorporating the thermal effect:

$$\sigma_t(t) = \sigma_{\text{thermal}}(t) + \sigma_{\text{shrinkage}}(t) \quad (9)$$

and

$$\sigma_{\text{thermal}}(t) = \Delta T \times \text{CTE} \times E_c(t) \times R_f \quad (10)$$

$$\sigma_{\text{shrinkage}}(t) = \varepsilon_{\text{shrinkage}} \times E_v \times R_f \quad (11)$$

where ΔT is the temperature difference and CTE is the coefficient of thermal expansion, $E_c(t)$ is the elastic modulus of concrete, $\varepsilon_{\text{shrinkage}}$ is the autogenous shrinkage strain, R_f is the degree of restraint (0-1) and it is defined as $1 - \varepsilon_r / \varepsilon_0$, where ε_r is the restrained strain and ε_0 is the strain without any restraint.

4. Conclusions

Shrinkage strain analysis for plain and reinforced cementitious binder was used to determine the internal viscoelastic modulus associated with autogenous shrinkage. The internal modulus is much lower (8000-

9000 MPa) than the external macro-modulus (32 000-36 000 MPa). This modulus is a material property which is needed to quantify shrinkage stresses.

Total stress analysis therefore has two moduli properties, an elastic Young's modulus for instant loading (mechanical and thermal) and an internal modulus for calculating shrinkage stresses directly.

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