Prediction of compressive strength of concrete from early age test result

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Prediction of compressive strength of concrete from early age test result

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ABSTRACT: In the construction process, it is always important to know the concrete compressive strength. The recommended procedure to ensure the concrete strength is to perform cylinder test. Test result of concrete cylinder at 28th day, represents the characteristic strength of the concrete that has been prepared and cast to form the concrete work. Usually two concrete cylinders (specimen) are cast for each day's representative strength test. The 3-days or 7-days tests are done to assess the early gain of concrete strength. However, 28-days tests are mandatory as per design/construction code requirements. Waiting 28 days is quite time consuming while it is important to ensure the quality control process. This paper is an attempt to develop a simple mathematical model based on concrete's nature of strength gain to predict the compressive strength of concrete at 28th day from early age results. The model is a simple equation known as a rational polynomial. The proposed model has a good potential to predict concrete strength at different age with high accuracy.

1. INTRODUCTION

Concrete has a versatile use in the construction practice for its availability, cheap rate, flexibility of handling and giving shape to any desired form. Designing a concrete structure requires the concrete compressive strength to be used. The design strength of the concrete normally represents its 28th day strength (Hamid-zadeh et al., 2006). In case of construction work 28 days is considerable time to wait for the test results of concrete strength, while it also represents the quality control process of concrete mixing, placing, proper curing etc. Concrete mix design is a process done by using code recommendation and sometimes by experience. If due to some experimental error in mix design the test results fail to achieve the designed strength, then repetition of the entire process becomes mandatory,which can be costly and time consuming. For every failure, it is necessary to wait at least 28 days,thus the need for an easy and suitable method for estimating the strength atan early age of concrete is being felt all the time. Hence, a rapid and reliable concrete strength prediction would be of great significance (Kheder et al., 2003).

Researchers are very keen to explore the concrete behavior and for this reason prediction of the concrete strength is being marked as an active area of research. Many studies are being carried out in this area (Zainet al., 2010). Different approaches using regression functions have been proposed for predicting the concrete strength (Snell et al., 1989; Chengju, 1989; Oluokun et al., 1990, Popovics, 1998). Traditional modeling approaches are established based on empirical relation and experimental data which are improving day by day. Some smart modeling system utilizing artificial neural network (Kasperkiewicz et al., 1995; Vahid et al., 2010) and support vector mechanics (Gupta, 2007) are developed for predicting concrete compressive strength.

Objective of all studies that have been carried out was to make the concrete strength predictable and increase the efficiency of the prediction. In this paper, an attempt is made to develop a relation between concrete strength and its age and finally express this relationship with a simple mathematical equation. Once, the relationship equation of concrete strength with its age be established, it is possible to predict its strength at any time (age).

2. CONCRETE AND COMPRESSIVE STRENGTH

Concrete is inert mass which grows from a cementing medium. Concrete is a product of two major components, one is the cement paste and another is the inert mass. In order to form the cementing medium, cement would mix with water. Coarse aggregates and fine aggregates are the part of inert mass. In properly mixed concrete, these materials are completely surrounded and coated by cement paste filling all the void space between the particles (Raju, 1979). With time, the setting process of the concrete starts and it starts to gain its strength.

Strength is the design property of the concrete. Characteristics like, durability, impermeability, volume stability may be important in some case of designing concrete structure but strength is the most important one. An overall picture of concrete quality is being reflected by the concrete strength. The process of strength growth is called 'hardening'. This is often confused with 'setting' while setting and hardening are not the same. Setting is the stiffening of the concrete from its fluid state after it has been placed. On the other hand hardening is the process of strength growth and may continue for weeks or months after the concrete has been mixed and placed. The rate at which concrete sets is independent of the rate at which it hardens. There are many factors which control concrete compressive strength. Concrete mix proportioning, aggregate quality, aggregate gradation, type of cement, mixing and placing method, concrete curing and curing temperature and the most important one is the water cement ratio. Water cement (W/C) ratio has a critical impact on concrete strength characteristic. A minimum amount of water is necessary for proper chemical reaction in the concrete and extra amount of water increases the workability and reduces strength.

3. EXPERIMENTAL RESULTS

Data used for this study was taken from previous study (Garg, 2003). Total 56 sets of data were used to analyze the behavior of the concrete with time (age). Ordinary Portland cement (brand: Ambuja Cement) was used for preparing the concrete. ACI mix design method (ACI 211.1-91) was used for the mix design process and for testing the cylinders ASTM (ASTM C39) recommendation was used. Out of 56 sample data sets, randomly selected 20 sample data sets are shown in Table 1.

Table 1: Concrete mix proportion sample data sets.

Number	Concrete strength (MPa)			FM of sand	W/C ratio	Mix proportion of concrete (kg/m^3)				CA size ratio (10mm:20mm)
	7day	14day	28day			Water	Cement	FA	CA	
1	13.91	17.93	21.82	2.4	0.5	185	370	781	1055	1:1
2	14.08	17.91	19.53	2.4	0.48	185	385	767	1056	2:1
3	14.08	17.91	19.53	2.4	0.5	185	370	781	1055	2:1
4	14.44	19.06	23.06	2.4	0.48	185	385	767	1056	1:1
5	15.08	20.26	24.84	2.4	0.52	185	356	797	1057	1:1
6	15.71	18.57	25.57	2.4	0.48	190	396	744	1057	2:1
7	15.71	18.57	25.57	2.4	0.5	190	380	760	1056	2:1
8	16.11	21.77	26.84	2.4	0.48	190	396	744	1057	1:1
9	16.11	21.77	26.84	2.4	0.5	190	380	760	1056	1:1
10	16.91	20.13	25.97	2.4	0.52	190	365	775	1056	2:1
11	15.66	20.4	29.3	2.6	0.52	185	356	825	1021	2:1
12	17.13	23	28	2.6	0.52	185	356	825	1021	1:1
13	17.66	23.88	28.57	2.6	0.48	190	396	776	1021	2:1
14	17.66	23.88	28.57	2.6	0.5	190	380	790	1022	2:1
15	19.22	25.55	28.55	2.6	0.46	185	402	780	985	1:1
16	19.57	29.46	29.77	2.6	0.52	190	365	808	1023.	2:1
17	19.64	26.42	28.97	2.6	0.46	185	402	780	985	2:1
18	19.86	28.91	27.68	2.6	0.42	185	440	740	1021	1:1
19	20.4	22.73	32.55	2.6	0.48	190	396	776	1021	1:1
20	20.4	22.73	32.55	2.6	0.5	190	380	790	1022	1:1

Ranges of material properties and concrete strengths achieved for all the 56 data sets are shown in Table 2. No admixtures or additives were used in this study; only the general constituents of concrete [Cement(C), Coarse-Aggregate (CA), Fine-Aggregate (FA) and Water (W)] were used to evaluate the concrete compressive strength. Different mix proportions of the ingredients and different w/c ratio were used to study the variations. All the specimens were immersed in water until the day of testing and variation of temperature was negligible so, the temperature variation was neglected.

Table 2: Ranges of the sample data used for analysis.

Name	Unit	Range
Coarse aggregate (CA)	(kg/m^3)	985-1078
Fine aggregate (FA)	(kg/m^3)	665-826
Cement (C)	(kg/m^3)	356-475
Water (W)	(kg/m^3)	185,190
Fineness modulus (FM)of sand		2.4, 2.6
W/C ratio		0.4-0.52
CA size ratio (10mm:20mm)		1:1, 2:1
7th day test strength	MPa	13.84-27.82
14th day test strength	MPa	17.8-37.6
28th day test strength	MPa	19.53-39.37

4. PROPOSED MODEL AND PERFORMANCE

Very first step of the study was to understand the strength gaining pattern of the concrete with age. For this reason strength verses day curve was plotted for every single set. It was observed that every curve followsa typical pattern. Figure 1 is a representative figure showing the strength gaining pattern with age of concrete for three sets. MATLAB curve fitting tool (MATLAB 2010a) was used to plot these data and also for the analysis purposes. From the plotted data the best fitcurve for each set was drawn. The plotted best fit curves show a good correlation and the average value of the square of the correlation coefficient was estimated at 0.997. The value of correlation coefficients of plotted representative sets of Figure 1 is given in Table 3.

Second step of the study was to determine a general equation of these curves being plotted. Investigation shows that all the curves maintain a good correlation with the following simple equation:

$$St_n = \frac{p.(D_n)}{(D_n) + q} \qquad (1)$$

where, St_n = Strength of the concrete at n^{th} day.(n = 1,2,3,…..); D_n = Number of days; p and q are constants for each curve but different for different data sets (curves). Though this equation (Eq. 1) is formed independently, it (Eq. 1) is similar to the equation (Eq. 2) proposed by ACI committee (ACI 209-71) for predicting compressive strength at any time.

$$[(f_c]')_t = \frac{t}{a + b.t}.(f_c')_{28d} \qquad (2)$$

Here a and b are constants, $(f_c')_{28d}$ = 28-day strength and t is time and this equation (Eq. 2) can be recast to similar form of Equation 1.

To utilize the above equation (Eq. 1), just value of two constants (p and q) are to be determined.

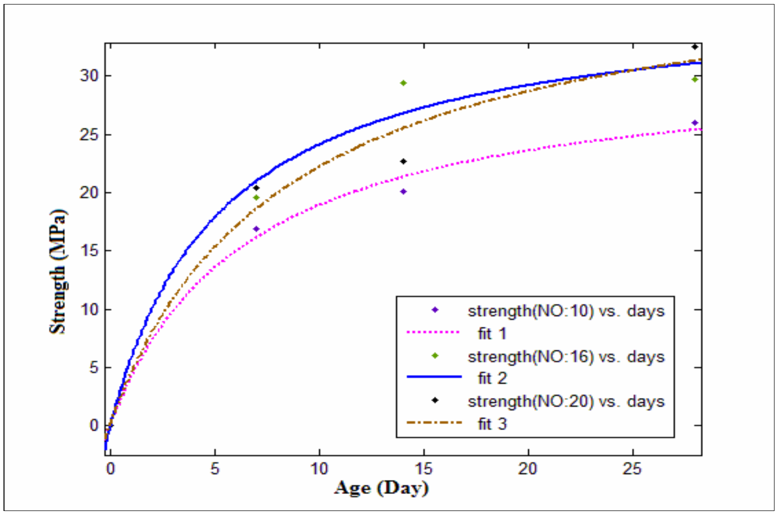

Figure 1: Strength gaining curve for representative sets.

Table 3: Representative sample sets correlation.

Number	Compressive strength test results (MPa)			Square of coefficient of correlation	Value of p	Value of q
	7 days	14 days	28 days			
10	16.91	20.13	25.97	0.99	31.33	5.56
16	19.57	29.46	29.77	0.98	36.99	5.30
20	20.4	22.73	32.55	0.98	40.56	8.31

Third step of the study was to evaluate the value of constant p and q. Table 3 shows the values of p and q for representative data sets which are obtained for the best fit curves after analyzing those. The values of p and q can be determined by putting strength test results in Equation 1 for any two days and solving it, but for this at least two test results for two different days are required. Our aim was to determine these values from only one day test result. An empirical relation was built up for this particular case (particular type of ingredients of concrete) to solve the problem. It was observed that, all values of p, q and strength of a particular day (St_n) for each set maintain a correlation of polynomial surface. In other words, values of p can be expressed as the function of q and (St_n) [which is a polynomial surface equation]. This correlation is being explored by MATLAB surface fitting tool. The equation of the correlation is given below:

$$p = a + b.q + c.St_n + d.q.St_n + e.St_n^2 \qquad (3)$$

Where, St_n= Strength of the concrete at n^{th} day. (n = 1, 2, 3 …) and a, b, c, d and e are the coefficients. This relation of p, q and St_n is valid for different day test result of concrete strength [for different n values] just the coefficients [a, b, c, d, e] of Equation 3 will be different. As we build up the correlation for 7th day test result of concrete [n=7], the values of the coefficients becomes, a = 10.23; b = -0.9075; c = 0.3412; d = 0.1721; e = 0.0112.

Putting these values in Equation 3 the following equation was obtained:

$$p = 10.23 - 0.9075q + 0.3412St_7 + 0.1721q.St_7 + 0.0112St_7^2 \qquad (4)$$

For 14^{th} day strength results [n=14] the coefficients becomes, a = -4.527; b = -1.041; c =1.373; d = 0.1406; e = -0.0125. Putting these values in to Equation 3 the following equation was obtained:

$$p = -4.527- 1.041q + 1.373St_{14} + 0.1406q.St_{14} +-0.0125St_{14}^2 \qquad (5)$$

Figure 2 shows the polynomial surface corresponding to Equation 4.

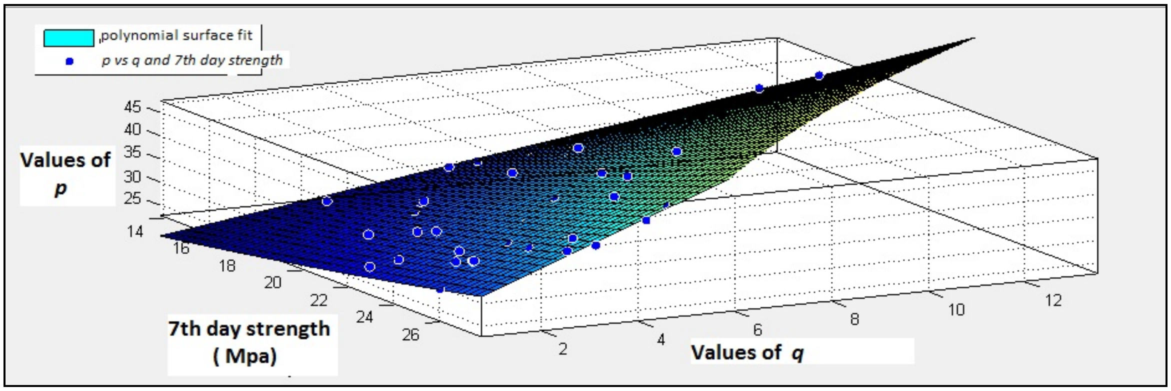

Figure 2: Polynomial surface representing Equation 3.

Table 4: Prediction of 28th day strength of concrete.

Number	Concrete strength results (MPa), A_i			Concrete Mix Ratio (C:CA:FA)	W/C Ratio	Water (kg/m^3)	CA size ratio (10mm: 20mm)	Solving Equation 1 & 4 [n=7]		Solving Equation 1 & 5 [n=14]	
								Predicted Concrete strength (MPa), P_i	Pi/Ai	Predicted Concrete strength (MPa), Pi	P_i/A_i
	7day	14day	28day					28day		28day	
1	13.84	17.8	25.6	1:2.24:2.97	0.52	185	2:1	20.77	0.81	22.43	0.88
2	13.91	17.93	21.82	1:2.11:2.85	0.5	185	1:1	20.95	0.96	22.45	1.03
3	14.08	17.91	19.53	1:1.99:2.74	0.48	185	2:1	20.73	1.06	22.45	1.15
4	14.08	17.91	19.53	1:2.11:2.85	0.5	185	2:1	20.73	1.06	22.45	1.15
5	14.44	19.06	23.06	1:1.99:2.74	0.48	185	1:1	22.69	0.98	22.94	1.00
6	15.08	20.26	24.84	1:2.24:2.97	0.52	185	1:1	24.46	0.98	23.79	0.96
7	17.55	21.95	24.77	1:1.74:2.51	0.44	185	1:1	25.10	1.01	25.29	1.02
8	18.04	23.17	24.64	1:1.62:2.4	0.42	185	1:1	27.78	1.13	23.10	1.01
9	18.62	24.24	25.33	1:1.87:2.65	0.46	185	2:1	28.55	1.13	27.61	1.09
10	18.97	22.02	23.84	1:1.87:2.65	0.46	185	1:1	23.94	1.00	25.35	1.06
11	19.9	27.22	27.82	1:1.62:2.4	0.42	185	2:1	33.35	1.20	30.90	1.11
12	20	21.53	25.97	1:1.74:2.51	0.44	185	2:1	22.39	0.86	24.89	0.96
13	22.22	28.08	32.17	1:1.49:2.28	0.4	185	1:1	32.35	1.01	31.89	0.99
14	24.75	27.66	29.77	1:1.49:2.23	0.4	185	2:1	29.39	1.00	31.41	1.06
15	15.71	18.57	25.57	1:1.88:2.67	0.48	190	2:1	20.43	0.80	22.68	0.87
16	15.71	18.57	25.57	1:2.0:2.78	0.5	190	2:1	20.43	0.80	22.68	0.87
17	16.11	21.77	26.84	1:1.88:2.67	0.48	190	1:1	26.41	0.80	25.11	0.94
18	16.11	21.77	26.84	1:2.0:2.78	0.5	190	1:1	26.41	0.98	25.11	0.94
19	16.91	20.13	25.97	1:2.12:2.89	0.52	190	2:1	22.25	0.86	23.69	0.92
20	17.82	23.2	25	1:2.12:2.89	0.52	190	1:1	27.32	1.09	26.52	1.06
21	19.98	26.75	29.32	1:1.76:2.56	0.46	190	2:1	32.21	1.10	30.37	1.04
22	22.33	25.82	25.97	1:1.76:2.56	0.46	190	1:1	28.01	1.08	29.32	1.13
23	23.15	29.51	31.48	1:1.4:2.22	0.4	190	2:1	34.21	1.09	33.57	1.07
24	23.77	27.11	34.68	1:1.63:2.45	0.44	190	1:1	29.16	0.84	30.78	0.89
25	24.33	30.6	30.66	1:1.51:2.33	0.42	190	1:1	35.13	1.16	34.87	1.14
26	24.6	29.88	31.35	1:1.63:2.45	0.44	190	2:1	33.47	1.07	34.01	1.08
27	24.66	26.68	29.88	1:1.4:2.22	0.4	190	1:1	27.82	0.93	30.29	1.01
28	26.02	35.35	37.4	1:1.51:2.33	0.42	190	2:1	43.07	1.15	40.72	1.09

The effectiveness of the proposed model is summarized below in Table 5 considering all the 56 test data.

Now if we just put the strength values in Equation 4, it becomes linear. Solving two linear Equations 1 and 3, values of p and q are obtained for each case. Finally,after finding the values of p and q the complete equation for the particular case can be formed. Using this above mentioned procedure, all 56 different concrete sets are analyzed and the model predicted values of compressive strength for 28th day was compared with experimental values. Same procedure was repeated when Equation 5 was used to solve with Equation 1 to evaluate the values of p, q. For all the cases, acceptable efficiency is obtained. The performance of the proposed equations was evaluated by three statistical parameters, mean absolute error (MAE), root mean square error (RMSE) and normal efficiency (EF); their expressions are given below.

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|P_i - A_i| \tag{6}$$

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(P_i - A_i)^2} \tag{7}$$

$$\text{EF} = \left(1 - \frac{1}{n}\sum_{i=1}^{n}\frac{|Pi - A_i|}{A_i}\right) \times 100\ \% \tag{8}$$

Here, A_i = Actual value; P_i = Predicted value; n = number of data (1, 2, 3 …).

Prediction of 28th day strength of concrete for some data sets is listed in Table 4. The fineness modulus (FM) of fine aggregate for all the sample data sets was 2.4.

Table 5: Prediction effectiveness of the proposed model for predicting 28th day compressive strength.

	Using 7th day strength result	Using 14th day strength result
Root Mean Square Error [RMSE]	3.23	3.02
Mean Absolute Error [MAE]	2.68	2.51
Efficiency [EF (%)]	90.7	91.4
Average Pi/Ai (max-min)	1.03 (0.80-1.20)	1.02 (0.87-1.19)

5. CONCLUSION

This paper represents a simple mathematical model to predict the concrete compressive strength from the early age test results [just any single day test result]. In this study, the concrete strength gain characteristic with age is modeled by a simple mathematical equation (rational polynomial) and a polynomial surface equation. Early age test data are being used in this case to get reliable values of the two constants which are required for the prediction. The proposed equations have the potential to predict strength data for every age. There are scopes for further study to evaluate the values of these constants without the help of test results of early age if the two constants can be estimated from previous test results. Herein, a simple and practical approach has been described for prediction of 28-day compressive strength of concrete and the proposed technique can be used as a reliable tool for assessing the design strength of concrete from quite early test results. This will help in making quick decision for accidental poor concreting at site and reduce delay in the execution time of large civil construction projects.

REFERENCES

ACI Committee 211, 1991 “Standard Practice for selecting properties for normal, heavy weight concrete”, (ACI211.1-91) American Concrete Institute, Detroit.Chengju G., 1989. “Maturity of concrete: Method for predicting early stage strength”. ACI Materials Journal, Vol.86 (4), pp. 341–353.

ACI Committee 209, 1971. “Creep Shrinkage Temperature in Concrete Structures” (ACI 209-71), American concrete Institute, Detroit, Michigan, 1971, sp 27-13, pp. 258-269.

ASTM C39/C39M-03. “Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens”, 2003 Annual Book of ASTM Standards, Vol.04.02, American Society for Testing Materials, Philadelphia, USA.

Garg R., 2003 “Concrete mix design using artificial neural network”. Master of Engineering thesis, Thapar Institute of Engineering and Technology, Department of Civil Engineering, Patiala, India.

Gupta, S.M., 2007. “Support Vector Machines based Modeling of Concrete Strength”. World Academy of Science, Engineering and Technology, Vol. 36.

Hamid-Zadeh N., Jamli A., Nariman-Zadeh N. and Akbarzadeh H., 2006. “A polynomial model for concrete compressive strength prediction using GMDH-type neural networks and genetic algorithm”. Proceedings of the 5th WSEAS International Conference on System Science and Simulation in Engineering, Dec. 16-18, Canary Islands, Spain, pp. 13-18.

Kasperkiewicz J., Rach J., and Dubrawski A., 1995. “HPC strength prediction using Artificial neural network”, Journal of Computing in Civil Engineering, Vol. 9(4), pp. 279-284.

Kheder, G.F., Al-Gabban, A.M. and Suhad, M.A., 2003. “Mathematical model for the prediction of cement compressive strength at the ages of 7 and 28 days within 24 hours”.Materials and Structure. Vol. 36, pp. 693-701.

MATLAB, 2010.MATLAB, the Language of Technical Computing, Version 2010a.MathWorks Inc., Natick, MA, USA.

Oluokun F.A., Burdette E.G., Harold Deatherage J., 1990. “Early-age concrete strength prediction by maturity — another look”. ACI Materials Journal, Vol. 87(6), pp. 565–572.

Popovics S., 1998. “History of a mathematical model for strength development of Portland cement concrete”. ACI Materials Journal, Vol. 95(5), pp. 593–600.

Raju N.K., Basavarajaiah B.S., Ramakrishna N, 1979. “A comparative study of concrete mix design procedures”, Indian Concrete Journal, pp. 13-16.

Snell L.M., Roekel J.V., Wallace N.D., “Predicting early concrete strength”. Concrete International 1989, Vol. 11(12), pp. 43-47.

Vahid. K. Alilou& Mohammad. Teshnehlab, “Prediction of 28-day compressive strength of concrete on the third day using artificial neural networks”, International journal of Engineering, 2010, Vol. 3,pp. 565-575.

Zain, M.F.M., Suhad, M., Abd-Hamid, R. and Jamil, M., “Potential for Utilizing Concrete Mix Properties to Predict Strength at Different Ages”. Journal of Applied Sciences, 2010, pp. 2831-2838.