Problem 4

a) $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$, $x \equiv 3 \pmod 7$

Product $N = 3 \cdot 5 \cdot 7 = 105$

Compute Contributions:

$N_1 = 105/3 = 35$, $35 \equiv 2 \pmod 3$, inverse of 2 mod 3 is 2. Term: $1 \cdot 35 \cdot 2 = 70$

$N_2 = 105/5 = 21$, $21 \equiv 1 \pmod 5$, inverse 1. Term $2 \cdot 21 \cdot 1 = 42$

$N_3 = 105/7 = 15$, $15 \equiv 1 \pmod 7$, inverse 1.

Term $3 \cdot 15 \cdot 1 = 45$

Sum $= 70 + 42 + 45 = 157$ Reduce mod 105:

$157 \equiv 52 \pmod{105}$.

$\therefore x = 52 \pmod{105}$

Q1.

$x \equiv 5 \pmod{11}$, $x = 19 \pmod{29}$, $x \equiv 15 \pmod{31}$

Product $N = 11 \cdot 29 \cdot 31 = 9889$

Compute contributions:

$N_1 = 9889/11 = 899$, $899 \equiv 8 \pmod{11}$; inverse of 8 mod 11 is 7. Term: $5 \cdot 899 \cdot 7 = 31465$

$N_2 = 9889/29 = 341$, $341 \equiv 22 \pmod{29}$, inverse of 22 mod 29 is 4. Term: $19 \cdot 341 \cdot 4 = 19096$

$N_3 = 9889/31 = 319$, $319 \equiv 9 \pmod{31}$, inverse of 9 mod 31 is 7. Term: $15 \cdot 319 \cdot 7 = 33495$

Sum $= 31465 + 19096 + 33495 = 84056$.

Reduce mod 9889: $84056 \equiv 4944 \pmod{9889}$

$x \equiv 4944 \pmod{9889}$

c) $x \equiv 5 \pmod 6$, $x = 4 \pmod{11}$, $x \equiv 3 \pmod{17}$

Product $N = 6 \cdot 11 \cdot 17 = 1122$

Compute contributions:

$N_1 = 1122/6 = 187$, $187 \equiv 1 \pmod 6$, inverse 1.

Term: $5 \cdot 187 \cdot 1 = 935$

$N_2 = 1122/11 = 102$, $102 \equiv 3 \pmod{11}$, inverse of 3 mod 11 is 4, Term: $4 \cdot 102 \cdot 4 = 1632$

$N_3 = 1122/17 = 66$, $66 \equiv 15 \pmod{17}$, inverse of 15 mod 17 is 8. Term: $3 \cdot 66 \cdot 8 = 1584$

Sum $= 935 + 1632 + 1584 = 4151$. Reduce mod 1122: $4151 \equiv 785 \pmod{1122}$

$x \equiv 785 \pmod{1122}$.